# RUSSIAN ALPHAMAGIC SQUARES

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It has been twenty-two years since Dutch Mathematician Lee C. F. Sallows defined certain "magic" squares of numbers to be "alphamagic" squares. He then wrote: "*Alphamagic* is the word I use to describe any magic array…that remains magic when all of its entries are replaced by numbers representing the word length, in letters, of their conventional written names (thus one (1) becomes 3)." **Such an alphamagic array represents an extraordinary confluence of "magic" in the world of numbers with "magic" in the world of words.** Sallows conceived of a method to discover alphamagic squares involving what he termed "Logorithms" (from the Greek root "Logos" for "word," and "Arithmos" for "number and as distinct from the more conventional mathematical "Logarithm"). The "logorithm" of a number is defined to be the number of letters required in a given language's alphabet to spell out the first number's name. In order to find third-order (3 X 3) alphamagic squares, Sallows and colleague Victor Eijkhout devised a Pascal-based computer program that processed the numbers of diverse Roman-alphabet languages and their logorithms through famed number theorist Eduoard Lucas' (1842-1891) general formula for finding third-order magic squares:

| | | |
|---|---|---|
| A + B | A – B - C | A + C |
| A – B + C | A | A + B – C |
| A - C | A + B + C | A – B |

Using this computer program, Sallows found numerous alphamagic squares in English and in other Roman-alphabet languages. Moreover, he found that there are even "translation" relationships among them: either two primary magic squares could share the same secondary logorithmic magic square, or one primary magic square could be alphamagical in more than one language. As for the non-Roman-alphabet languages, he wrote: "Besides those like our own employing Roman letters, *there remain others using the Greek, Hebrew, and Cyrillic alphabets* (italics here ours)." But he did not seek to discover any of these and, to our knowledge, no other non-Roman-alphabet alphamagic square has been discovered and published since. So, inspired by Sallows' work published twenty-two years ago, we set about to find Russian alphamagic squares.

The first task is to find the Sallows' logorithms of each of the Russian number names as written in the Cyrillic alphabet, as **ноль** (= 0, logo=4), **один** (=1, logo= 4), **два** (=2, logo=3), **три** (=3, logo=3), **четыре** (=4, logo=6), **пять**

(=5, logo=4), **шесть** (=6, logo=5), **семь** (=7, logo=4)…and so on. Just this process turns up to the careful observer some alphamagic possibilities. For instance, there are only three numbers having a constant difference between them (as there must be in any series of numbers through the center of a third-order magic square) in the constituency of logorithm 6, and these are **восемь** (=8, logo=6), **девять** (=9, logo=6), and **десять** (=10, logo=6). So this enables the construction of a "trivial" magic square (not all cells of the array being different) that yields a secondary square of logorithms all of the values of which are six (6).

| | | |
|---|---|---|
| 9<br>**девять**<br>logo 6 | 8<br>**восемь**<br>logo 6 | 10<br>**десять**<br>logo 6 |
| 10<br>**десять**<br>logo 6 | 9<br>**девять**<br>logo 6 | 8<br>**восемь**<br>logo 6 |
| 8<br>**восемь**<br>logo 6 | 10<br>**десять**<br>logo 6 | 9<br>**девять**<br>logo 6 |

Here you can see that the number square is magic, having a constant sum of 27 in any row, column, or diagonal. And, the logorithms (the numbers of the letters to spell each of the Russian number's names in the Cyrllic alphabet) are also magic, having a constant sum in any row, column, or diagonal of 18. The finding of this "trivial" example of an alphamagic square in Russian Cyrillic gave us hope of finding better examples. We also noticed that there are precisely four Russian numbers belonging to the constituency of the logorithm 4: **ноль** (=0), **один** (=1), **пять** (=5), and **семь** (=7). Since a fourth-order magic square (unlike any odd-order square) may be constructed of *any* four numbers, this enables the formation of the first fourth-order Russian Cyrillic alphamagic square:

| | | | |
|---|---|---|---|
| 0<br>**ноль**<br>logo 4 | 1<br>**один**<br>logo 4 | 5<br>**пять**<br>logo 4 | 7<br>**семь**<br>logo 4 |
| 7<br>**семь**<br>logo 4 | 5<br>**пять**<br>logo 4 | 1<br>**один**<br>logo 4 | 0<br>**ноль**<br>logo 4 |
| 1<br>**один**<br>logo 4 | 0<br>**ноль**<br>logo 4 | 7<br>**семь**<br>logo 4 | 5<br>**пять**<br>logo 4 |
| 5<br>**пять**<br>logo 4 | 7<br>**семь**<br>logo 4 | 0<br>**ноль**<br>logo 4 | 1<br>**один**<br>logo 4 |

In this 4 X 4 square, the Russian numbers have a constant sum of 13 in any row, column, or diagonal, and the sum of the logorithms of these Russian numbers (i.e. the sum of the number of Cyrillic letters in these numbers' names) is 16 in any row, column, or diagonal. So this is a fourth-order Russian Cyrillic alphamagic square, albeit a "trivial" one. Similar fourth-order "trivial" alphamagic squares can be formed from any four Russian numbers sharing the same logorithm. These discoveries offer us hope that a better (i.e. "non-trivial") fourth-order Russian Cyrillic alphamagic square will be discovered.

Trial-and-error not availing, a Javascript computer program authored by Samuel Comi was then used to process the Russian numbers and their Cyrillic logorithms through the Lucas formula for finding third-order magic squares. This is the sole eligible array among the first one hundred Russian numbers (1-100):

| | | |
|---|---|---|
| **74<br>Семьдесят<br>Четыре<br>Logo 15** | **50<br>Пятьдесят<br>Logo 9** | **92<br>Девяносто<br>Два<br>Logo 12** |
| **90<br>Девяносто<br>Logo 9** | **72<br>Семьдесят<br>Два<br>Logo 12** | **54<br>Пятьдесят<br>Четыре<br>Logo 15** |
| **52<br>Пятьдесят<br>Два<br>Logo 12** | **94<br>Девяносто<br>Четыре<br>Logo 15** | **70<br>Семьдесят<br>Logo 9** |

The sum of any row, column, or diagonal of the Russian numbers is 216, and the sum of any row, column, or diagonal of the logorithms (i.e. the number of Cyrillic letters needed to spell each of the Russian numbers' names) is 36. **This then (i.e. 74, 50, 92/90, 72, 54/ 52, 94, 70) is the first non-trivial third-order Russian Alphamagic Square discovered. It is also the first proper non-Roman-alphabet (i.e. Cyrillic) Alphamagic Square.**

Indeed it turns out that the only other alphamagic square in Russian Cyrillic with non-repeating numbers less than two hundred (200) can be constructed from this one merely by adding the hundreds digit 1-, signifying 174, 150, 192/190, 172, 154/152, 194, 170 (i.e. adding the Russian word for 100 or "**сто**," with logorithm of 3). Lee Sallows has termed such an alphamagic square a "second harmonic" of the first. But, if we expand the search into higher Russian numbers, there are hundreds of others. The fact that the Russian word for 200 ("**двести**") has a logorithm of 6—double the logorithm of the word for 100, means that numerous triples of constant difference can be produced from a sub-100 number combined with a 100+ that number and a 200+ that number to form one of the lines through the center of a third-order square, to which the permutations of two other such triples are added as needed. An example of one of these is:

| | | |
|---|---|---|
| 154<br>**Сто**<br>**Пятьдесят**<br>**Четыре**<br>Logo 18 | 48<br>**Сорок**<br>**Восемь**<br>Logo 11 | 251<br>**Двести**<br>**Пятьдесят**<br>**Один**<br>Logo 19 |
| 248<br>**Двести**<br>**Сорок**<br>**Восемь**<br>Logo 17 | 151<br>**Сто**<br>**Пятьдесят**<br>**Один**<br>Logo 16 | 54<br>**Пятьдесят**<br>**Четыре**<br>Logo 15 |
| 51<br>**Пятьдесят**<br>**Один**<br>Logo 13 | 254<br>**Двести**<br>**Пятьдесят**<br>**Четыре**<br>Logo 21 | 148<br>**Сто**<br>**Сорок**<br>**Восемь**<br>Logo 14 |

In this third-order Russian array, the numbers sum to 453 in any row, column, or diagonal, while the Cyrillic logorithms sum in any row, column, or diagonal to 48. Notice that in a third-order magic square the constant sum is always three times the number in the center cell. But this is not all. Other even higher-numbered squares are likely to result in similar fashion as a consequence of the fact that the Russian number for 400 ("**четыреста**") has a Cyrillic logorithm of 9, thus making possible numerous triples of concentric constant difference numbers less than a hundred, with 200 + numbers (adding 6 to the logorithm) and 400+ numbers (adding 9 to the logorithm). As the number triples increase by 200, the concentric logorithms increase by 3—fertile ground for the discovery of alphamagic squares.

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## ODES FROM THE ODD TOPICS SOCIETY

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*Butler's Odd Topics Society meets every now and then to discuss Odd Topics of any nature. Professor Baetzhold is the Poet Laureate Odd Topics Society.*

On the Anniversary of the Jumping Frog Derby
of Calaveras County

Whenever I sit down to woo the muse
In order to fashion a pome for youse,
I think of Old Horace, wise ol' duck
Who said that a pome should delight and instruct.

Today it was touch; inspiration had fled.
All I could do was to scratch my head;
But then the light broke! Today marks the date
When Jim Smiley's frog first met his sad fate!

You all know the story, as told by Mark Twain—
How the champion jumper struggled in vain
To out-hop his foe. But, oh, he could not!
The stranger had filled him with five pounds of shot.

The story's delightful—with that we won't quarrel,
But what in the word should we find as a moral?
'Tis simple; in truth, can there be any doubt?
If you want to succeed, "Get the lead out!"