# SUDOKU LINES FACTORISED

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A. It is a demerit of Sudoku (vis-a-vis e.g. the crossword or chess problem) that there is nothing of memorable interest in the final diagram, once the pleasure of solving is over. One way to mitigate this is to factorise individual lines, each of which contains the nine digits unrepeated. These digits can be permuted in 9! = 362880 ways, so that there are that many different S-numbers ranging from 123456789 = $3^2.3607.3803$ (A1) to 987654321 = $3^2.17^2.379721$ (A2). We may note that all S-numbers are divisible by 9; none is divisible by both 2 and 5; to be divisible by 11, the sums of the alternate digits must be 17 and 28; and where the middle triad is the sum of the first and third triads and its digits sum to 18, the number is divisible by 7, 11 and 13.

B. Fewest prime factors The theoretical minimum of 3 is found for all S-numbers of form $3^2.p$. The lowest four are:-

123458679 = $3^2$. 13717631 (B1)
123458967 = $3^2$. 13717663 (B2)
123468957 = $3^2$. 13718773 (B3)
123469587 = $3^2$. 13718843 (B4)

C. Most prime factors Since $2^{27}.3^2$ has ten digits, the theoretical maximum is 28. The most I have found is 18 in 918245376 = $2^{12}.3^3.19^2.23$ (C1).

D. Fewest different prime factors Since no S-number is a power of 3, the theoretical minimum is 2 in the form $3^m.p^n$, as in B above or 735982641 = $3^2.9043^2$ (D1) or 185742639 = $3^6.254791$ (D2).

E. Most different prime factors Since $2.3^2.7.11.13.17.19.23.29$ has ten digits, the theoretical maximum is 8, but I have not found an example. I have found 64 examples of 7: 55 of these are divisible by 7, 11 and 13, including the lowest example 127495368 = $2^3.3^2.7.11.13.29.61$ (E1), the roundest example 283459176 = $2^3.3^4.7.11.13.19.23$ (E2) and the only two odd examples 278693415 =

$3^2.5.7.11.13.23.269$ (E3) and $746981235 = 3^2.5.7^2.11.13.23.103$ (E4); 8 are divisible by 7 and 11 (but not by 13), including the highest example $918567342 = 2.3^2.7.11.47.59.239$ (E5); and one is divisible by 7 and 13 (but not by 11), namely $537912648 = 2^3.3^2.7.13.19.29.149$ (E6). The 7 examples listed below came nearest to the theoretical maximum in that the quotient left after the 6 lowest factors had been divided out was large enough to allow for 2 more factors, but that quotient proved to be prime or, in one tantalising case, a square, as follows:-

$167549382 = 2.3^2.7.11.13.17.547$ (E7)
$382576194 = 2.3^2.7.11.13.17.1249$ (E8)
$475693218 = 2.3^2.7.11.13.17.1553$ (E9)
$541927386 = 2.3^2.7.11.13.19.1583$ (E10)
$657981324 = 2^2.3^2.7.11.13.19.31^2$ (E11)
$745963218 = 2.3^2.7.11.13.19.2179$ (E12)
$782936154 = 2.3^2.7.11.13.19.2287$ (E13)

F. Highest powers The highest powers I have found for the first four primes are $2^{12}$ in C1 above, $3^9$ in $784269135 = 3^9.5.13.613$ (F1), $5^7$ in $314296875 = 3^3.5^7.149$ (F2) and $7^5$ in $129783654 = 2.3^3.7^5.11.13$ (F3). For the next four primes the highest power is $17^4$ in $589324176 = 2^4.3^2.7^2.17^4$ (F4).

G. Roundest numbers The seven roundest S-numbers I have found are:-

(i) $423579618 = 2.3^6.7^4.11^2$ (G1) and $847159236 = 2^2.3^6.7^4.11^2$ (G2)
(ii) F3 above and $418693275 = 3^2.5^2.7.11^2.13^3$ (G3)
(iii) F4 above
(iv) $243918675 = 3^3.5^2.7.11.13.19^2$ (G4) and $249567318 = 2.3^8.7.11.13.19$ (G5).

It will be seen that G2 is twice G1, and there are other pairs of S-numbers which are similarly closely related.

H. Squares Table 64 in Albert Beiler's "Recreations in the Theory of Numbers" (Dover, New York, 1966) lists 30 S-numbers which are perfect squares, including D1, F4and G2 above. The lowest of them is 139854276 = $2^2.3^8.73^2$ (H1), and the highest is 923187456 = $2^8.3^4.211^2$ (H2). The most different prime factors is 4, shown in F4 and G2 and also in 714653289 = $3^2.7^2.19^2.67^2$ (H3). The largest prime whose square divides a S-number is 9043 in D1 above.

There are no doubt other interesting aspects of the factorisation of S-numbers, and I hope that the 34 examples I have given may stimulate further investigation.

## A POEM

MARTIN GARDNER
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This is an excerpt from Gardner's 1969 book *Never Make Fun Of A Turtle, My Son* (Simon and Schuster, illustrated by John Alcorn).

### Scribble Scamp

A Scribble Scamp's a horrid girl
Who scribbles everywhere.
She scribbles on the tablecloth,
She scribbles on the chair.

She writes her name upon the walls,
She draws upon the floor.
She colors up the kitchen sink,
She decorates the door.

She *never* scribbles on a sheet
Of paper as she should.
She'd rather use the lampshade,
Or the ceiling — if she could!

She thinks she is an artist
But she's really a disgrace.
And it takes her poor dear mother
Several weeks to clean the place.