JOSEPHUS WORDS

DANA RICHARDS
Fairfax, Virginia

In the articles on "Magic Spells" (*Word Ways*, Feb and May 2010) it was proposed that tricks could be performed with a deck of n letter cards. The deck would be prearranged, spelling some word u. There would be a "skip sequence" of integers $k_1, k_2, \ldots, k_n$; the more natural the sequence the better. The magician would spell a new word $w = w_1w_2w_3 \cdots w_n$ as follows: skip k_1 cards and set the next card aside making it w_1, skip k_2 cards and set the next card aside making it w_2, etc. Each skipped card is returned to the bottom of the deck. Note that the skip sequence defines a permutation π of the the original deck order; $w = \pi(u)$. We say w is a fixed-point if $w = \pi(w)$. For any given permutation there exists a skip sequence, though it might be hard for a magician to incorporate.

The logological question is to find pairs of words u and w, and a well-motivated skip sequence relating them, that a magician could use with suitable patter. I am not a magician, however, so in this article I will just give pairs of common words. (Pairs using an uncommon word were found but are not reported.)

The story of Josephus Flavius is well-known in recreational mathematics. Forty men stood in a circle and every third man, still standing, was killed. (The puzzle is to find where Josephus should stand to survive to the end.) In our terminology we would say $k_i = 2$ for all i. However k_1 might be different depending on where you want to start. Let J_a^b be the skip sequence where $k_1 = a$ and $k_i = b$ for $i > 1$. Choosing $a = 0$ or $a = b$ would be natural in a trick.

We first looked at six letter words. For J_1^1 there is `teaset/estate`, and `veined/endive`. For J_2^2 there is `mimosa/Maosim`. For J_3^3 there is `neuter/tenure`, `settee/testee`, and `opuses/spouse`. For J_4^4 there is `ginned/ending`, and `parsec/escarp`. For J_5^5 `affair/raffia`. Letting $a = 0$ we have `begird/bridge`, and `preset/pester` for J_0^3. Also we have `Bosnia/bonsai`, and `stored/strode` for J_0^5. There are also some fixed points for J_0^4 (`addend`, `attest`, `eggnog`, `beetle`, `needle` among many others) and for J_0^5 (`coffee`, `yippee`, `halloo`, etc.). The word `tattoo` is a fixed point for both J_3^3 and J_0^5!

There are fewer seven letter examples. For J_4^4 there is `striate/artiest`. For J_0^3 there is `Devries/diverse` and `obtrude/outbred`. For J_0^5 there is `perusal/pleuras`. We found none for eight letter words.

We could also define other skip sequences. An obvious idea is to choose longer and longer skips, which we call "rhopalic". Let R_a be the skip sequence with $k_1 = a$ and $k_i = k_{i-1} + 1$ for $i > 1$. A six letter example, for R_0 is `starer/Sartre`. For seven letter words R_0 has `piastre/parties` and R_1 has the fixed points `eeriest` and `oospore`. The skip sequence R_0 for eight letter words yields a large number of fixed points including `addendum`, `announce`, `assassin` and `innuendo`.

There remains many more skip sequences to explore.