## MODULAR LADDERS

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Most Word Ways readers are familiar with word ladders in which one changes letters one at a time, as CART-CARE-CORE-BORE-BONE. Such ladders can be embedded in word networks, first discussed in the May and August 1973 issues of Word Ways. In particular, one can define the distance between two words in a network as the number of steps in the minimum-length ladder joining these words, and the span of the network as the maximum distance taken over all possible pairs of words in the network.

In a word network, one can move in either direction along a ladder, but in a directed word network one is constrained to move in a given direction. The first directed network, discussed in the August 1991 Word Ways, consisted of ladders of the form SHE-HER-ERA-RAN-ANY. Such networks have an elaborate structure; words in the core (called insiders), each of which can be reached from any other, though not by the same path; words leading into the core (called starters if no other word leads into them, and preceders if they join starters with insiders); and words leading out of the core (called enders if one can go no further, and followers if they join insiders with enders). The one-way span associated with a directed network is defined as before, but it is joined by the two-way span (restricted to insiders) which is the maximum distance between insider $A$ back to itself via insider $B$, taken over all values of $A$ and $B$.

Other directed networks are possible. This article defines a module as one which changes all the letters of an n-letter word in order, as in CAT-rat-rot-ROB, JETS-lets-lots-loss-LOST, or BOATS-coats-chats-chits-chins-CHINA. A modular ladder is created by the end words of successive modules, as JETS-LOST-MALL-FEES; a starter is a word for which there exists no module ending with it, and an ender is a word for which there exists no module starting with it.

The modular network of this article is based on a set of 1510 common four-letter words; a different vocabulary would lead to different details, but the essential structure would be preserved. 533 of these words cannot appear as end-words in a module, and therefore cannot appear in a modular network. Of the remainder, the great majority form a connected network of starters, preceders, insiders, followers and enders--plus two words (HAND and YEAR) that do not fit the basic classification and are called bypassers.

There are 179 insiders, listed below:

|  | ball cane colt |  |  |  | s |  |  |  | t | e |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | fees | le | 1 | d | ns |  | ps | gill | goat | ead | eld |
|  | hill | hind | hope | hop | hos | h | hump | lack | lane | lash | las |
| en | lens | lent | lick | lift | $\operatorname{limp}$ | line |  | lisp | list | lope | p |
| lore | lose | 10 | 10 | 1 u | le |  |  |  | mash | mass | ast |
|  | mill | mind | mine |  | 1 | ole | olt | ope | ops | most |  |
|  | us | nick | pace | pack | pans | $n$ | s | past | pens | pent | ier |
|  |  | pins |  |  |  |  |  |  |  |  | ape |
|  |  |  | ride |  | ripe | rips |  |  |  |  |  |
|  |  | sack | sags | sail | sane | sank |  | sill | ins |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  | g |
| ail | war |  | we | wild | wile | will | wind | wits |  |  |  |

Naturally, the diagram of their interconnections is too complicated to reproduce here. However, there are a number of inhomogeneities, or funnel points. For example, 15 insiders lead to FEES, which only leads to SUNK, and this in turn leads only to BASS within the core. Similarly, HEAD leads only to LORE, LIFT only to SORE, and LIMP only to PULL. Some modules are reversible: MINT-PACE, CODS-GAPE, RIPS-LOSE, CANE-LOST-MINE, WAYS-RIDE, BALL-HEAD, ROBS-CAPE. The one-way span appears to be nine: WILL-head-lore-this-past-line-tubs-cape-robs-CUTE or FEES-sunk-bass-lope-cast-line-tubs-cape-robs-CUTE. (It is much quicker to go from CUTE back to FEES, via the intermediate step MALL; CUTE returns to WILL almost as fast, via MOLD-HERE.) The two-way span is not known, but probably lies between 11 and 17.

Counting both within-core and outside-core connections, the most fecund word is LOST, with 63 inputs and 16 outputs. Other remarkable words include BASK (53-1), MALL (52-16), LENT (32-18), WILL (26-24) and TILL (9-25). The most fecund starter is KILL ( $0-26$ ), and the most fecund ender is PEAL (68-0).

Most other words in the main network cluster closely around the core, like barnacles on a ship or sperm on an egg. There are 438 starters that connect directly with the core, some via preceders. Preceders are more unusual; with starters in parentheses, they are:

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(flat)-SEND
(turf)-SELL
(live,five, hive, dive)-GAGS, -HALL, -HALT-MOSS
(whip)-SLAT-FELL
(cock,dock,hock,lock,rock,sock)-MUSE
(bum)-TOWS
(grit)-WANE
(jigs,rigs,pigs)-WADE
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(fire,hire,mire,sire,tire)-TONS
(pigs,rigs,wigs,bogs,cogs,dogs,fogs,logs,hogs)-JUTE
(gilt,hilt,jilt,lilt,kilt,tilt)-WEND
(coin,join)-LAWS
(mice,vice,rice,nice,hike,mime,time)-LANK
(pelt,welt,melt)-MOSS
(cite,kite,bite,done,come,home)-SALT-MOSS
(duel)-FELT-MOSS
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There are 200 enders that the core connects with, some via followers. Followers are also unusual.; with enders in parentheses, they are:

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PART-(died)
WARN-(book,boor,boot)
WANT-PEAR-, TEAR-, SUED-REAR-, LAND,SAND-BEAR-(soup,soul)
BAND-(leaf,leak,lean,leap,seek,seem,seen,seep,sees)
GILD-(moon, moor, moot)
MAIN-LOAD-, DOLL,ROLL-PAIN-LOAD-(reef,reek,reel)
MAIN-, DOLL,ROLL-PAIN-(loaf)
WINK-(such,sacs)
PINT-(hurl)
FEED,HEED-(shoe,shop, shot, show)
PAWN,LAWN-(seek, seem, sees, seep, seed)
SEAT-(book,boor,boon)
HEAD, PEAT-(book, boor,boon, slip, slim,spin)
HEAT,PEAT-SLOW-, LAND,SAND-BEAT-SLOW-(grey)
BASK,CASK-(much)
SUED-(real,reap)
LAND-(seek,seem,seen,seep,sees,beak,beam)
SAND-(beak,beam,leaf,leak,leap)
LAND,SAND-BEAN-(moor, mood)
LAND,SAND-BEAT-(slip,slim,spin)
RATS-(cube)
RATS-BIND-(fool,foot,folk,week)
RATS-BIND-WEEP-(stow)
DOLL,ROLL-(pair,paid)
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There are two main network words, YEAR and HAND, that join starters (or preceders) with enders (or followers) but have no connection with the core:
(coin, join)-LAWS (paws,jaws,saws)-YEAR-(soup,soul)
(cock, dock,lock, mock, rock, sock)-HAND-LEAN-(moor, moot, mood)
-BEAR-(soul,soup)
-BEAT-(slip,slim,spin)
-BEAT-SLOW-(grey)
-(leaf,leak,leap,seek, seem,seep, sees, seen, beak, beam)

These are termed bypassers, since they bypass the core.

In addition to the network of starters, preceders, insiders, followers, enders and bypasssers, there are a number of other networks that form degenerate cases: no core, but merely isolated words that join starters and enders. Three of the largest ones:
(tray,fray,bray,gray, xray)-plot-(span,spin)
-plow-(step,stem,grey)
-(plod)

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(vied)-dean-(moor,moot,mood)
    -dear-(soup,soul)
    -(deal,deaf)
(bier)-(peas,peak,peal)
    -peat-(slip,slim,spin,book,boor,boon)
        -slow-(grey)
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These networks have words overlapping the main network; they are distinct by virtue of the fact that their starters don't have connections with the main network. Fully-distinct networks (no words in common) are much smaller and hard to find: clad-golf, odds-airy, plum(sway,swap,swat,swab), (akin,thin)-(spur,spat), furs-oath, blew-stop.

The final concept worth mentioning is the fan--the full set of words that can be reached from a specified word. Starters have the largest fans; insiders all have fans of the same size; followers have very small fans. The words having the largest fan is probably LIVE (and its mates PIVE, DIVE, HIVE); this leads to preceders GAPS, HALT and MOSS as well as enders HALP and HALO which are unreachable from the core. To these one adds the 179 insiders, the 200 enders that are joined to the core, and 33 followers, for a fan size of 417. Even a starter which does nothing more than enter the core has a fan size of 412.

Similar calculations can be made for reverse fans. The champion is probably GREY, which is fed by followers SLOW, HEAT, PEAT, LAND, SAND and BEAT, bypasser HAND, starters FEAT, MEAT, NEAT, BIER, QUIT, starters XRAY, GRAY, BRAY, PRAY and TRAY which lead to PLOW, and starters BLOW and FLOW-in addition to 179 insiders, 19 preceders and 438 other starters. In all, 656 words lead to GREY.

One can play various games with modular ladders. For instance, can the three cardinals FOUR, NINE and FIVE be connected? NINE and FIVE are starters, and FOUR an ender, so the only possible connections are shown below (with minimum steps):

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FIVE-gags-lend-sail-FOUR
NINE-tubs-rode-bits-pent-wail-FOUR
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One can look for ways to join Carrollian opposites such as PIND-bale-must-line-wags-LOSE and LOSE-rips-town-dare-FIND. How about trying to connect body parts such as HEAD, FOOT, LEGS, GUTS, NECK, HAND,

HEEL and RUMP? SICK, a starter, goes to WELL: SICK-lure-cant-WELL. LOVE and HATE are both starters, and GAIN has a fan of size four, not including LOSS. WARM and WORK are insiders, but COOL and PLAY form no modules with any other word. HARD and SOFT are both starters.

Finally, modules themselves can lead to droll images: dogs-bark, sideband, fell-hurt, dock-hand, cold-girl, lust-male, kiss-mate, hugs-lady, mice-rats, bets-lose, tone-bass, fold-here, goat-bell, gold-mine, sanewild, Jets-lose (they had the worst record in 1995 professional football), logs-bark, made-wish, make-work, mend-sail, save-copy, soft-lens, testruns, face-lift, wild-goat, pipe-rack, lent-cash and port-tack.

## THE OXPORD LARGE PRINT DICTIONARY

This handsome large-size ( $81 / 2 \times 11$ ) dictionary resembles the more diminutive Merriam-Webster paperback dictionary in word coverage, but there are two important differences: its large print is far easier to read, and it places names of people and places (CLINTON, JUDAEA) in the main text. Definitions are brief and to the point (POPPYCOCK nonsense, RYEGRASS forage grass or coarse lawn grass); user-friendly pronunciation, not IPA symbols, prevails. Words at the edge include ones like JOKY, JOJOBA, JUDDER, JOEY in Oxford but not in Merriam-Webster, and JOBHOLDER, JONGLFUR, JITNEY, JUMPSUIT in Merriam-Webster but not in Oxford. This dictionary could profitably be used as a standard for logological investigations in which a relatively small vocabulary of both words and names is desired, a task for which the Merriam-Webster paperback is unsuited.

