LITERARY CRYPTARITHMETIC BY COMPUTER

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Editor's Note: In the November 1989 **Word Ways**, *Peter Newby requested readers to design literary cryptarithms: "apt or ironic" messages capable of encipherment in arithmetic terms in a unique manner. To stimulate interest, he offered a copy of his new book,* Pears Word Games, *for the best example devised. Several readers sent in entries, including a very fine one by Eric LeVasseur:*

$$PI \times R^2 = AREA \quad 96 \times 7^2 = 4704$$

However, the palm must be awarded to Leonard Gordon, who harnessed the PC to this task. This article describes the results.

In the November 1989 **Word Ways**, Peter Newby presented a set of literate cryptarithms. This name was coined by Newby, but it is a well-known concept: these are cryptarithms in which the code letters form interrelated words. They are discussed in Mathematics on Vacation (1966) by Joseph Madachy, editor of the Journal of Recreational Mathematics. He calls them alphametics, a word coined by J.A.H. Hunter in 1955.

Newby's examples gave me the idea of developing computer aids for various forms of the problem. Some results are presented in this article. I assume they are new; I am not very familiar with the literature on this subject. I have restricted all my work to the common base 10 (problems in other bases are given in Madachy's book, but they are not appropriate for **Word Ways**).

Let us first consider the simple cryptarithm ABCDE x n = FGHIJK where n is a number between 2 and 9. There are 11 different letters; if we allow 2 (and only 2) to be the same, can we find an n that produces a cryptarithm with a unique solution? Below I give a table of cryptarithms (in numerical form) with that property:

66485x2	= 132970	A=B	87902x7	= 615314	G=J	59107x4	= 236428	F=J
35384x5	= 176920	A=C	90526x9	= 814734	H=K	35694x3	= 107082	G=I
92895x8	= 743160	A=D	55934x3	= 167802	A=B	97321x5	= 486605	H=I
83429x9	= 750861	A=I	36309x6	= 217854	A=C	91704x9	= 825336	I=J
26956x5	= 134780	B=E	62956x5	= 314780	A=E	77648x4	= 310592	A=B
47935x6	= 287610	B=H	43358x5	= 216790	B=C	64865x2	= 129730	A=D
52103x9	= 468927	B=J	57817x6	= 346902	B=E	97542x4	= 390168	A=G
95626x5	= 478130	C=E	70123x8	= 560984	B=H	35954x3	= 107862	B=D
71928x7	= 503496	C=J	86453x2	= 172906	B=K	31567x6	= 189402	B=F
83657x6	= 501942	D=F	34189x3	= 102567	C=F	71365x6	= 428190	B=I
56941x3	= 170823	E=F	29566x5	= 147830	D=E	30465x6	= 182790	B=K
70693x6	= 424158	F=H	68917x6	= 413502	D=G	72981x5	= 364905	C=I
30841x9	= 277569	G=H	78413x5	= 392065	E=F	56944x3	= 170832	D=E

30541x9 = 274869 D=H	20527x7 = 143689 A=D	30521x9 = 274689 D=F
83169x3 = 249507 E=H	62709x9 = 564381 A=G	86719x6 = 520314 D=J
74038x4 = 296152 F=K	89295x8 = 714360 B=D	64879x8 = 519032 E=H
65817x6 = 394902 G=I	50796x4 = 203184 B=G	71089x6 = 426534 F=K
91706x9 = 825354 H=J	71635x6 = 429810 B=J	73864x8 = 590912 G=I
91367x6 = 548202 I=K	80447x7 = 563129 C=D	94703x6 = 568218 H=K
66417x6 = 398502 A=B	73012x8 = 584096 C=I	

The following examples of literate cryptarithms illustrate its use. The idea is to find words which match the numbers; if you think up a pair of words, use the table to see if they constitute a cryptarithm with a unique solution. Some examples are

PIANOx3 = ARTFUL	RHYMEx6 = STANZA	DRINKx2 = CHASER
PIANOx5 = OUTCRY	COMETx3 = GALAXY	DRINKx6 = STUPOR
DAISYx4 = GARDEN	COMETx6 = GALAXY	DRINKx5 = BOTTLE

Other than making more tables like the above, there is little one can do to find cryptarithms other than by trial and error: choose words and hope for a unique solution. All too often, apt phrases lead to non-unique solutions; here are a few examples.

WATER	37941	67941	BEER	4003	SMOKE	70543	78543
SLAKES	867248	837248	MAKES	59208	OZONE	58593	50593
THIRST	905189	905189	MEN	501	CANCER	129136	129136
			DRUNK	63712			

With WRONG + WRONG = RIGHT, Newby has apparently rediscovered a famous old cryptarithm that has no fewer than 21 solutions. The first of the following is his; the second is given in 536 Puzzles and Curious Problems by H.E. Dudeney, who states that there are many others. The third is the only solution that uses the numbers 2 to 8 exclusively. The other examples are ways to modify the problem to get something with a unique solution. In the last case, I stipulate that 0 may not be used.

WRONG	24153	25938	37846
WRONG	24153	25938	37846
RIGHT	48306	51876	75692

WRONGx7 = RIGHTx2 = 105686
WRONGx4 = RIGHT = 67832

Here is a more elaborate literate cryptarithm. Two solutions with a minor difference betwen them (5 and 6 exchange) is the best I was able to find. Several "sentences" using the words **sin**, **pain**, **snake**, **evil** in combination with those below had 4 solutions.

DEVIL +	VIPER +	APPLE +	EVE =	EXILE +	ADAM
19485	48296	32259	949	97859	3130
19486	48295	32269	949	97869	3130

After writing a separate computer program for each of the above cases, I realized how to write a general program for a specific format. This made testing easier. Here are cryptarithms with unique solutions. This particular format seems to be the one most likely to produce unique solutions in base 10.

CELLO 57441	WATER 90687	MUSIC 36491	ALASKA 878308
VIOLIN 261468	DESERT 183876	CHARMS 172834	SPILL 35177
SONATA 318909	FRUITS 274563	ANIMAL 209325	MISHAP 913485
MOATS 85413	GRAND 25796	CANTOR 391807	COATS 48796
TOWERS 157693	GARDEN 275689	CHANT 34918	COLLAR 482273
CASTLE 243106	FLOWER 301485	HEBREW 426725	PRIEST 531069
TIGER 86913	ROBIN 51892	ABHOR 28741	PRIEST 694271
PARROT 423378	PIGEON 690412	UGSOME 390465	MOANS 35087
MENAGE 510291	MENAGE 742304	ORGASM 419206	SERMON 729358
OCEAN 68034	RABBI 79662	PRIEST 726185	
SCHOOL 185667	PARSON 497318	SINGS 86308	
WHALES 253701	URBANE 576980	SERMON 812493	

Here are a few near-misses, with two solutions in each case: CHIELD + LUCRE = HUBRIS (a reply to SEND MORE MONEY?), TREES + TIGERS = JUNGLE, CHANGE + HABIT = BEHAVE, PRIEST + ROARS = SERMON, SHIPS + TROOPS = ARMADA, MONKEY + WANTS = TOKENS, MONKEY + WANTS = STEAKS, OFFER + MONKEY = BANANA, and RABBLE - LEADER = RIOTS.

The editor called my attention to an example Philip Cohen found in the Recreational Mathematics Magazine that is doubly true: mathematically and cryptarithmetically.

ONE +	TWO +	FIVE =	EIGHT
621	846	9071	10538

Unfortunately, this solution is not unique; N, W, and V can be assigned 2, 4, and 7 in any permutation. Here are two more doubly-true examples. The first uses all ten digits but has 4 different solutions; the second has a unique solution but uses only nine digits.

SEVENx4 =	TWENTY +	EIGHT	SEVENx9 =	TWENTYx3 +	THREE
64940	214028	45732	62529	182913	14022

At this point I decided to expand my computer program to accommodate a fourth word, still keeping the 5 and 6-letter word format. To find groups to test, I chose 3 related words and then searched my OSPD and Chambers word lists for fourth words with allowable letters. This usually produced about 500 to 1000 words, of which maybe a dozen were related to the others. Here are the results:

BEHOLD 302671	COFFEE 651122	COFFEE 763355	COFFEE 469955
FLESH 47092	CREAM 60287	CREAM 72501	- CREAM 41503
MODEL 56107	SUGAR 94380	SUGAR 84902	- SUGAR 78201
FEMALE 405870	AROMAS 805789	GRACES 920758	MEAGER 350251
DROOL 48007	HORSE 57489	ABBOT 54476	BREAD 12673
DROOL 48007	HORSE 57489	ABBOT 54476	PEANUT 467890
DROOL 48007	HORSE 57489	WANTED 350698	BUTTER 190062
DROOL 48007	HORSE 57489	BREAD 42948	EATERS 670625
FEMALE 325672	APACHE 363259	BUTTER 416692	
MODEL 50427	RIDER 41094		
MARVEL 568127	PARADE 634309		

BUSTY 43298	LUSTY 14578	SCHEME 845070	TWAIN 47625
BLONDE 417065	BUSTY 64578	- RHYME 15670	WANTED 765438
LOSES 17252	BLONDE 612039	-REASON 102893	BREAD 91368
BOODLE 477615	BELLES 691195	MAYHEM 726507	BUTTER 904431

We add a new dimension to cryptarithms when the solver is required to find n. What is the lowest value of n which will make the equation true, and what is the lowest value which will produce a unique solution? What is the lowest and highest n that will produce a unique solution?

BRIDEx2	+	DEALER	=	BIGAMY	
60957		571270		693184	
BRIDEx4	+	LEADER	=	BIGAMY	
54716		360164		579028	
LOVERx2	+	BROKER	=	TROKIA	
50126		360726		460978	
LOVERxn	+	VIRGIN	=	GRAVID	when n=2, there are 3 solutions
38462x8		412719		720415	
SHEEPxn	+	FARMER	=	MARKET	there are no solutions with n less than 4
24776x4	+	831971	=	931075	there is one other solution for n=5
39881x5	+	452682		652087	
FAULTxn	=	ALASKA	+	QUAKE	there is no solution for n=1
91023x2	=	121871		60175	both of these solutions are unique
19025x51		828438		90837	

In conclusion, I offer the following piece de resistance, for which a special computer program was required. I guessed the words and fortunately found a unique solution for this doubly-true equation.

NINExFOUR	+	FIVE	=	FORTY-ONE
9895 3074		3865		30421 095