

# OPTIMIZING UNIFORMLY EXCITED LINEAR ARRAYS THROUGH TIME MODULATION

**J. Fondevila, J. C. Brégains (Student Member, IEEE),  
F. Ares (Senior Member, IEEE), and E. Moreno (Senior Member, IEEE)**

*Radiating Systems Group, Department of Applied Physics,  
Faculty of Physics, Campus Sur, Univ. of Santiago de Compostela  
1578, Santiago de Compostela, Spain  
Emails: [fajavi](mailto:fajavi@usc.es); [fajulio](mailto:fajulio@usc.es); [faares](mailto:faares@usc.es); [famoreno](mailto:famoreno@usc.es)}@usc.es*

## ABSTRACT

This paper shows that, by the proper use of time modulation in equispaced linear arrays with uniform excitation distribution, it is possible to maintain the sidelobe zone under certain -previously stipulated- level whereas the undesired harmonics intensities are minimized. In addition to that, the further extension of the technique to non-equispaced arrays permits to obtain broadband response, by simply searching the positions of the elements that reach to the desired power pattern behaviour within the required bandwidth.

## I. INTRODUCTION

The use of time modulation in antenna arrays appeared in a recent work [1] brings back to the technical and scientific community the initial concepts introduced by Kummer et al. [2]. In Kummer's paper, the authors show how this technique is exploited to maintain under a desired value the sidelobe level of a linear array with non-uniform excitation distribution. A few years later, the concepts related to this technique are briefly re-described by some other authors [3, 4]. This simple model is based on the insertion of on-off switching devices in the feeding network of the array that allow to control, by means of periodic rectangular pulses, the power distribution of the antenna, in addition to the variables customarily used by the designers in pattern synthesis –i.e. excitations and positions of the array elements. The main problem with this kind of control is the undesired appearance of harmonics –usually called sideband radiation– whose levels are related to the Fourier coefficients used in the expansion of the element excitations so controlled. In one of the abovementioned papers, written by S. Yang et al. [1], the sideband radiation pattern is minimized through the use of the differential evolution algorithm. That suppression is applied to a Taylor excitation distribution. By contrast, in this paper it is shown that, with the appropriate use of the Simulated Annealing technique [5], it is possible to maintain the power pattern sidelobe

region of a *uniformly excited* (which constitutes the novelty of the method presented here) linear equispaced array under certain prestablished value, by perturbing the time pulses, whereas its sideband radiation is minimized. To take further advantage of this technique, the bandwidth of such a linear array is enhanced by releasing the equispacing between contiguous radiators.

## II. THEORETICAL BACKGROUND

Let us consider a linear array of  $N$  isotropic elements lied along the  $z$  axis. Its traditional array factor expression is given by:

$$F(\theta, t) = \sum_{n=0}^{N-1} I_n e^{j(kz_n \cos\theta + \omega t)} \quad (1)$$

where  $z_n$  is the position of every element,  $k = 2\pi/\lambda$  is the wavenumber,  $\theta$  is the usual spherical polar coordinate, and  $\omega = 2\pi/T$  represents the operation frequency of the RF signal. If the complex current amplitude  $I_n$  of each element is considered to be periodic function of time, with a period  $T_p \gg T$ , it can be expanded as a Fourier series, giving [3]:

$$I_n = \sum_{p=-\infty}^{\infty} a_{pn} e^{jp\omega_0 t}; \quad a_{pn} = \frac{1}{T_p} \int_0^{T_p} I_n e^{-jp\omega_0 t} dt \quad (2)$$

One of the easiest ways of implementing the time-modulation is through the use of on-off switching devices (diodes, for example). If the  $n$ -th “switch-on” time is represented by  $\tau_n$ , with  $0 \leq \tau_n \leq T_p$ , after some manipulations  $a_{pn}$  can be found to be:

$$a_{pn} = I_n \frac{\tau_n}{T_p} \left[ \frac{\sin(pu_n)}{pu_n} \right] e^{-jp u_n}; \quad u_n = \pi \frac{\tau_n}{T_p} \quad (3)$$

where  $p$  takes the values  $0, \pm 1, \pm 2, \dots$ . It can be readily seen that the expression of the operating frequency is mainly determined by the  $p=0$  component, whose pattern is given by the following expression:

$$F(\theta) \Big|_{p=0} = \sum_{n=0}^{N-1} I_n \frac{\tau_n}{T_p} e^{jkz_n \cos\theta} \quad (4)$$

It is well known that antenna designers synthesize desired radiation patterns with the usual adjustment of the excitation distribution  $I_n$  and the  $z_n$  elements positions. The appearance of the  $t_n = \tau_n/T_p$  ratio adds another dimension –time– to those available parameters. The method applied in this work takes advantage of this additional degree of freedom –in a similar way made by the previously mentioned authors [1–2]–, and this is explained in next sections.

### III. THE METHOD

The simple implementation of the on-off switchers in the feeding network leaves to the designer the option of setting  $I_n$  –which we will call hereafter static distribution– equal to 1 (i.e. uniform distribution)  $\forall n \in [0, N-1]$ , since the adjustment of the power pattern is achieved by the appropriate selection of the  $t_n$  ratios. This is a very desirable static excitation distribution, because simplifies the array feeding. In a first look –see equations (1) and (3)– and under the named condition  $I_n=1$ , the direct replacement of the numerical values of  $t_n$  by any known static distribution normalized to its maximum (a Chebyshev one, or any of the Taylor’s with a specific sidelobe level, for example) leads to a desired response of the  $p=0$  (fundamental component) power pattern, but at the price of obtaining undesirable high sideband radiation. This behaviour obligates to recalculate the  $t_n$  coefficients, searching for a minimization of the sideband patterns ( $|p| \geq 1$ ), and trying to maintain the initial characteristics of the fundamental one. The work presented here is based on these main characteristics. The Simulated Annealing (SA) –a very good global optimizer algorithm widely known due to its versatility [5]– is chosen to perform the following examples.

### IV. DESCRIPTION AND EXAMPLES

#### A. Sidelobe lowering, at a single frequency $f$ , of an equispaced linear array of isotropic elements.

As a first example, an array of 30 elements with a constant spacing of  $0.7\lambda_f$  between contiguous elements (which implies  $z_n = 0.7n\lambda_f$ , being  $\lambda_f$  the wavelength at frequency  $f$ ) is taken. The SA perturbs the  $t_n$  values in order to minimize the following cost function:

$$C_A = c_{A,1}H(\Delta_f)\Delta_f^2 + c_{A,2}H(\Omega_f)\Omega_f^2$$

$$\text{with} \begin{cases} \Delta_f = \text{SLL}_{o,f} - \text{SLL}_{d,f} \\ \Omega_f = M_{o,f} - M_{d,f} \end{cases} \quad (5)$$

where SLL represents the sidelobe level of the fundamental power pattern. The subscripts “o” and “d” mean obtained (during the optimization procedure) and desired, respectively, at certain frequency  $f$ .  $H$  is the Heaviside step function.  $M$  represents the maximum value of any ( $|p|=1$  or  $2$ ) of the sideband patterns.  $c_{A,1}$ , and  $c_{A,2}$  are weighting parameters selected to obtain a better operation of  $C_A$ . In this example,  $\text{SLL}_{d,f} = -20$  dB,  $M_{d,f} = -30$  dB, both relative to the maximum of the fundamental power pattern main beam, and  $f_p=1/T_p=f/60$ . The patterns obtained with this optimization are shown in figure 1, in which the fundamental and the two first harmonics are

represented (sidelobe level at  $-20.04$  dB, maximum of the first harmonic at  $-30.08$  dB). Figure 2 shows the  $t_n$  distribution that, when replaced in (3), generates the abovementioned patterns.

If we define the dynamic range ratio  $\delta_{rr}$  (in this special case with a uniform static excitation distribution) as  $\{t_n\}_{\max}/\{t_n\}_{\min}$ , we obtain a high value (15.75) after calculating it with the values shown in figure 2. Neglecting elements 1, 3, 4 and 5 the  $\delta_{rr}$  is reduced to 5.71, which leads to a very similar fundamental power pattern, with a sidelobe level slightly increased (0.5 dB). After those suppressions, the sideband radiations maintain their maximum values under  $-30$  dB.

### B. Sidelobe lowering, at two frequencies ( $f$ and $2f$ ), of a non-equispaced linear array of collinear dipoles.

To see the scope of the method, and in order to obtain a more realistic example, we now consider a specific linear array of 30 non-equispaced elements of collinear dipoles. This can be achieved by the sole inclusion of the element factor of a dipole –whose axis coincides with the array line– into the abovementioned array expression (1), which now becomes:

$$F(\theta, t) = \left[ \frac{\cos(kL \cos \theta) - \cos(kL)}{\sin \theta} \right] \sum_{n=0}^{N-1} I_n e^{j(kz_n \cos \theta + \omega t)} \quad (6)$$

being  $2L$  the length of every dipole, taken to be  $0.5\lambda_f$  in this example.

The conditions of the optimization, achieved by perturbing both  $t_n$  and  $z_n$  values, can be summarized as:

- 1) Minimize the fundamental power pattern sidelobes at two different frequencies  $f$  and  $2f$ , under two desired values  $SLL_{d,f}$  and  $SLL_{d,2f}$ . It is known that the duplication of the RF frequency raises the levels of the sidelobes into grating lobes if the elements are equispaced. This justifies the perturbation of  $z_n$  that optimizes the array pattern behaviour at those frequencies.
- 2) Try to suppress sideband radiations at  $f$  and  $2f$ , lowering them below  $M_{d,f} = M_{d,2f}$  (the same for both frequencies), and taking, as in the example of paragraph A,  $f_p = 1/T_p = f/60$ .

Setting  $SLL_{d,f} = -17$  dB,  $SLL_{d,2f} = -15$  dB,  $M_{d,f} = M_{d,2f} = -30$  dB, and now using a cost function analogous to  $C_A$ :

$$C_B = \sum_{q=1}^2 \left[ c_{B,1,qf} H(\Delta_{qf}) \Delta_{qf}^2 + c_{B,2,qf} H(\Omega_{qf}) \Omega_{qf}^2 \right] \quad (7)$$

$$\text{with } \begin{cases} \Delta_{qf} = \text{SLL}_{o,qf} - \text{SLL}_{d,qf} \\ \Omega_{qf} = M_{o,qf} - M_{d,qf} \end{cases}$$

the power patterns shown in figures 3 and 4 are obtained ( $\text{SLL}_f = -17.77$ ,  $\text{SLL}_{2f} = -16.58$ ,  $M_f = M_{2f} = -31.93$ , all of them expressed in dB), both generated by the array whose  $t_n$  and  $z_n$  can be observed in table 1. For a better control, the  $z_n$  were constrained to be within the range  $[0.7\lambda_f, 0.95\lambda_f]$ . If we calculate the  $\delta_{\pi}$ , a high value (236.69) is obtained if the elements with low  $t_n$  are considered. They can be further neglected (i.e. the elements 3, 22, 24, 25 and 27 are removed from the array), and in this case the patterns keep almost unaltered (fundamental and sidebands change very slightly), obtaining a very remarkable reduction of  $\delta_{\pi}$  (1.6).

### C. Analysis of some characteristics of the fundamental and sideband power patterns behaviours between $f$ and $2f$ .

To see how the array, with the configuration obtained in example B, behaves between  $f$  and  $2f$ , a succinct bandwidth analysis is made. Computations of the maximum sidelobe level of the fundamental pattern and maximum value of any of the two first sidebands versus frequency appear in figure 5. As can be seen, the obtained configuration has broadband response.

## V. CONCLUSIONS

It has been shown that the insertion of on-off switching devices in the feeding network of a uniformly excited linear array gives to the antenna designer the possibility of taking control over the sidelobe level of the power pattern at certain RF frequency, by optimizing their time-pulse values. If the positions of the elements are released to be unequally spaced, the array has a broadband response. It is readily seen that the same technique can be straightforwardly applied to planar or conformal arrays.

## ACKNOWLEDGEMENT

This work has been supported by the Spanish Ministry of Science and Technology, under project TIC2002–04084–C03–02.

## REFERENCES

- [1] Shiwen Yang, Y. Beng Gan, and A. Qing, "Sideband Suppression in Time-Modulated Linear Arrays by the Differential Evolution Algorithm", *IEEE Antennas and Wireless Propagation Letters*, Vol. 1, pp. 173-175, 2002.
- [2] W. H. Kummer, A. T. Villeneuve, T. S. Fong, and F. G. Terrio, "Ultra-Low Sidelobes from Time-Modulated Arrays", *IEEE Trans. on Antennas and Propagat.*, Vol. 11, pp. 633-639, 1963.
- [3] W. L. Weeks, "Antenna Engineering", *Mc Graw-Hill Electronic Science Series*, 1968.
- [4] R. W. Bickmore, "Time Versus Space in Antenna Theory", *Microwave Scanning Antennas*, Vol. III, Chapter 4, ed. R. C. Hansen, *Peninsula Publishing*, 1985.
- [5] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, "Numerical Recipes in C", *Cambridge University Press*: pp. 444-455, 1992.

## LEGENDS FOR FIGURES AND TABLES

**Fig. 1.** Power patterns obtained with the perturbation of  $t_n$  values, in which a SLL < -20 dB ( $p=0$ ) was desired, whereas the two first harmonics levels were minimized under -30 dB.

**Fig. 2.** Pulse distribution along the elements that generates the patterns shown in figure 1.

**Fig. 3.** Power patterns obtained with the perturbation of  $t_n$  and  $z_n$  values, in which a SLL < -17 dB ( $p=0$ ) was desired, whereas the two first harmonics levels were minimized under -30 dB at frequency  $f$ .

**Fig. 4.** Power patterns obtained with the perturbation of  $t_n$  and  $z_n$  values, in which a SLL < -15 dB ( $p=0$ ) was desired, whereas the two first harmonics levels were minimized under -30 dB at frequency  $2f$ .

**Fig. 5.** Computed values of different parameters of the power pattern generated by the array obtained in example B.

**Table 1.** “Switching-on” time pulses ( $t_n$ ), and positions ( $z_n$ ) of the elements of the array that that generates the patterns shown in figures 3 and 4. It is understood that the  $z_n$  values are calculated at  $f$ .

FIGURES

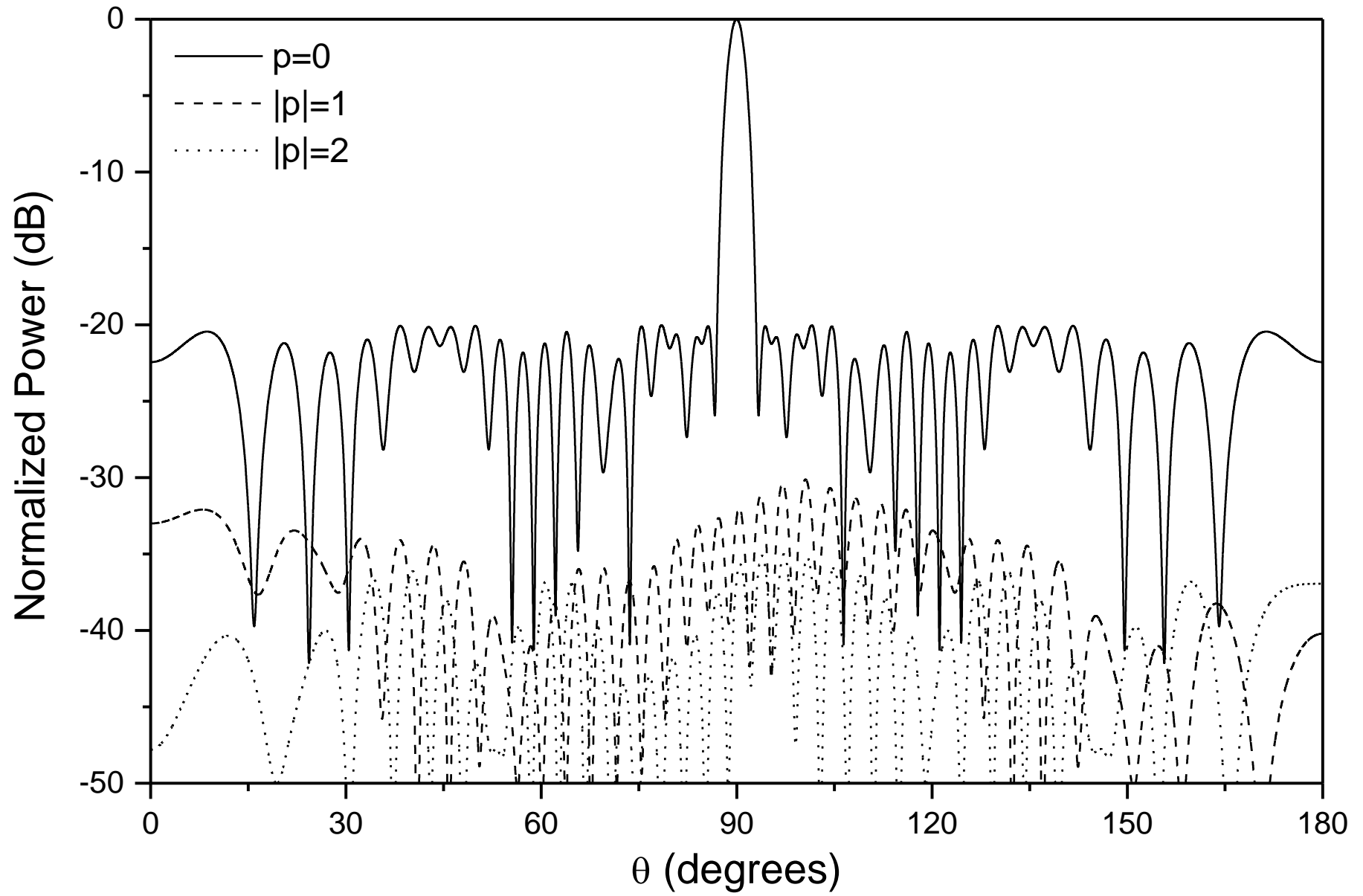


Fig. 1



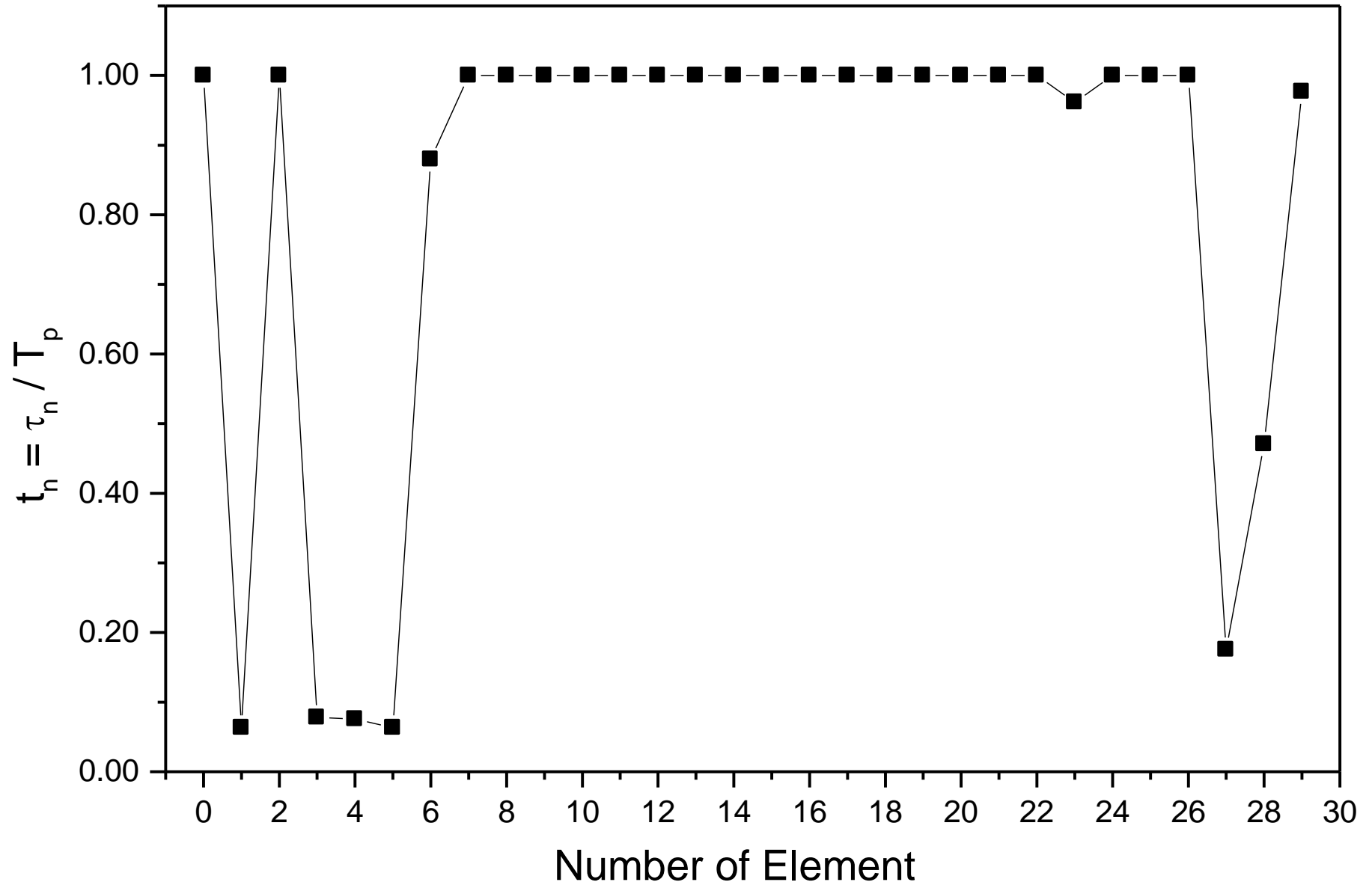


Fig.2

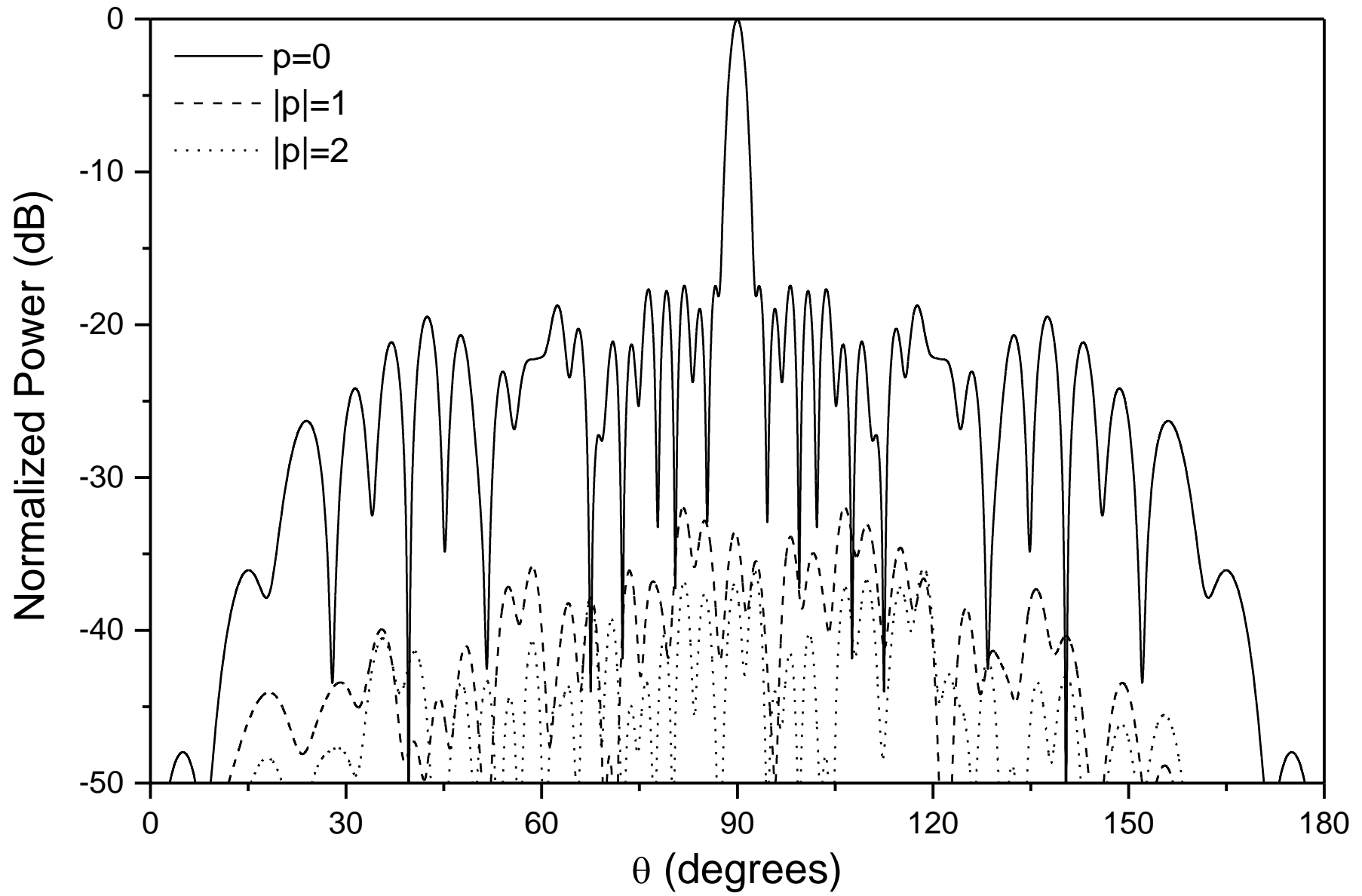


Fig. 3

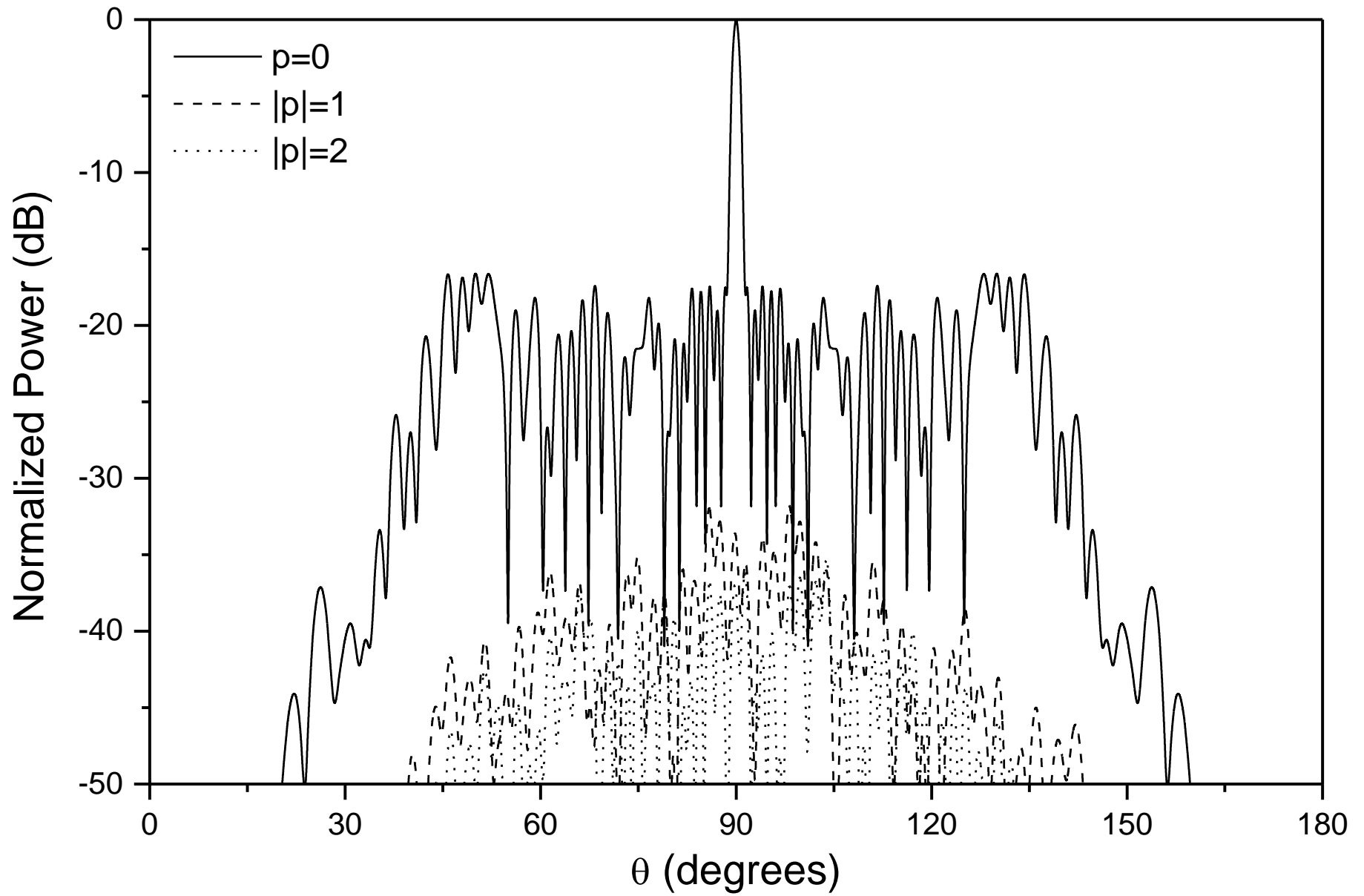


Fig. 4

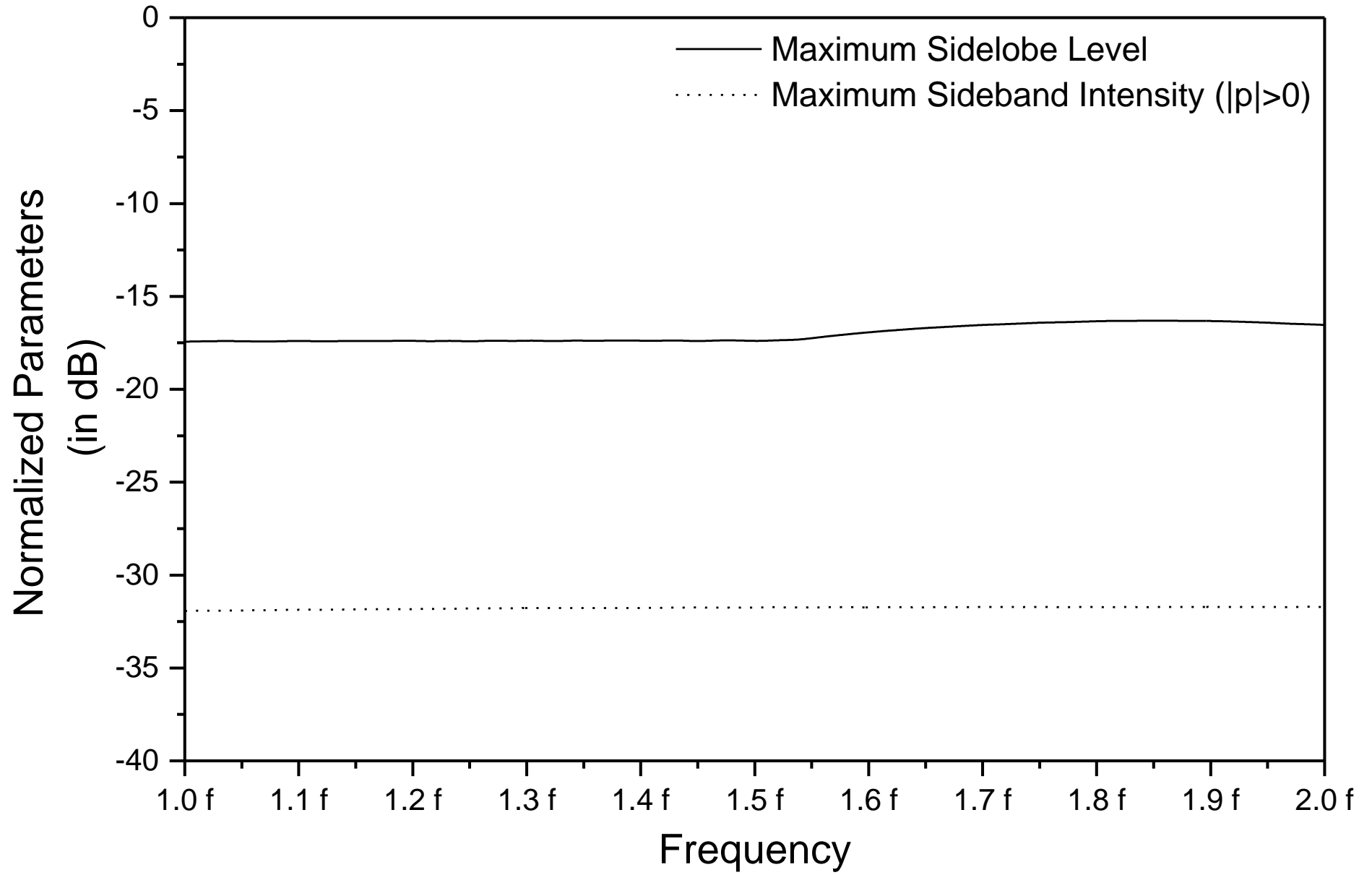


Fig. 5

Table 1

No. of Element	$z_n$ (in $\lambda_f$ )	$t_n$
0	0.000	0.982
1	0.700	0.887
2	1.400	1.000
3	2.105	0.005
4	2.820	1.000
5	3.770	1.000
6	4.497	1.000
7	5.201	1.000
8	5.905	1.000
9	6.612	1.000
10	7.316	1.000
11	8.021	1.000
12	8.779	1.000
13	9.566	1.000
14	10.516	1.000
15	11.466	1.000
16	12.166	1.000
17	12.866	1.000
18	13.567	1.000
19	14.517	0.626
20	15.224	0.916
21	16.174	1.000
22	16.883	0.120
23	17.834	1.000
24	18.535	0.009
25	19.483	0.005
26	20.184	1.000
27	20.884	0.004
28	21.584	0.899
29	22.284	1.000