APPLICATION OF TIME MODULATION IN THE SYNTHESIS OF SUM AND DIFFERENCE PATTERNS BY USING LINEAR ARRAYS

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ABSTRACT

In this paper, time modulation is applied to a small number of elements of a linear array that radiates either sum or difference patterns, in order to take control over their sidelobe levels. The simulated annealing (SA) technique helps to obtain the optimum time pulses applied to such elements in terms of sideband minimization.

1. INTRODUCTION

At the end of the 1950s, a noteworthy paper was published [1] in which the possibility of the application of time as a "fourth dimension" in antenna theory was presented. Concerning a linear array antenna, time modulation consists in using electronic switches between the source and the radiating elements. The main drawback of this technique is the energy losses produced by unwanted harmonics, due to the fact that the elements are periodically open–circuited. The mathematical demonstration of the appearance of such harmonics (sideband radiation, SR) was studied by Shanks et al. [1], Kummer et al. [2] and Weeks [3]. Kummer et al. were pioneers in using time modulation by synthesising low/ultra-low side-lobe sum patterns applied to slot radiators; however, in their work the SR was not properly minimised. Such a problem was firstly faced by using optimization techniques like the differential evolution algorithm, by S. Yang et al. [4], or the Simulated Annealing technique (SA), used more recently by Fondevila et al. [5].
In this paper, time modulation is applied to an equispaced linear array in order to obtain sum and difference radiation patterns with very acceptable side-lobe levels (SLLs) whereas their corresponding SR are kept under control. This is achieved by using either uniform or generalised Villeneuve (GV) [6] excitation distributions and then optimizing the time pulses applied to some elements with the aid of SA. In this manner, sum patterns, especially those obtained from a uniform excitation distribution, can be improved by modulating only a few elements located near the edges of the array. Conversely, difference patterns are accomplished if the modulated elements are the central ones. This leads to a kind of reconfigurability of the array because, with a specified amplitude excitation distribution, the designer can obtain, on the one hand, two types of patterns and, on the other hand, ameliorated sidelobe levels (compared with the non-modulated cases). Uniform distributions are always desirable because they constitute the simplest way of feeding an array antenna. In principle, it is easier to construct and adapt the time-modulation circuitry to the antenna geometry, in order to

obtain a desired power pattern (provided the sideband radiation has been minimized previously, during the synthesis procedure), than to construct and adapt the appropriate geometry of a conventional feeding when the excitation distribution is not uniform. In case the linear array has been previously synthesised with a specified excitation distribution, such as a GV (or, for example, a Dolph-Chebyshev) one, the designer can insert the corresponding switching circuitry in order to reduce the sidelobe level of the non-modulated case. Besides, by properly changing the phase distribution of half of the array (see next section) and re-optimizing the time pulses that correspond to a few elements, difference patterns with improved sidelobe levels can also be obtained. These techniques are explained in next sections.

2. MATHEMATICAL PRELIMINARIES

Consider a linear array of $2N$ isotropic antennas located along the z axis. The corresponding expression of the array factor, with the carrier angular frequency ω explicitly indicated, is

$$F(\theta,t)=e^{j\omega t}\sum_{\substack{n=-N\\n\neq 0}}^{N} I_n e^{jkz_n cos\theta} \tag{1}$$

Where I_n is the relative excitation amplitude; $k=\frac{2\pi}{\lambda}=\frac{2\pi f}{c}=\frac{\omega}{c}$ is the wavenumber widh speed of light c, and carrier frequency f; θ is the spherical polar coordinate; and z_n is the position of each element. If each antenna is fed with temporal pulses of width $\tau_n \in (0,T_p]$, being $T_p>>T=1/f$ the maximum period among all of the elements, equation 1 becomes a sum of infinite terms (modes) [5] of the form:

$$F(\theta,t)\big|_q = e^{j\omega t}\sum_{\substack{n=-N\\n\neq 0}}^{N} I_n \xi_n \left[\frac{sin(q\pi\xi_n)}{q\pi\xi_n}\right] e^{jq\left(\frac{2\pi t}{T_p}-\pi\xi_n\right)} e^{jkz_n cos\theta}$$
$$q=0,\pm1,\pm2,\pm3...;\quad \xi_n=\frac{\tau_n}{T_p}\quad \in(0,1], \tag{2}$$

where ξ_n is the normalised time-pulse value and q represents the harmonic mode.
When q=0, equation 2 refers to the main pattern (or fundamental mode), as follows:

$$F(\theta,t)\big|_{q=0} = e^{j\omega t}\sum_{\substack{n=-N\\n\neq 0}}^{N} I_n \xi_n e^{jkz_n cos\theta}. \tag{3}$$

This expression agrees with equation (1) when setting $\xi_n I_n$ (which are called hereafter *dynamic excitations*), instead of I_n (*static excitations*) into the summation. This equation suggests a kind of reconfigurability, which constitutes an auxiliary degree of freedom in the synthesis procedure, because with a given linear array and with specified static excitations I_n it is possible to obtain several radiation patterns at the carrier frequency ω by inserting strategically some on–off switching devices in the array. Furthermore, from a specified real amplitude static distribution that radiates a sum pattern, the designer can obtain corresponding difference diagrams -as is well known- by setting an

antisymmetric phase distribution (with values equal to 0 or 180º), and this is exploited here as an additional advantage.
To simplify expressions, only one half of the summation given in (3) is used so as to establish, for sum patterns, $I_n \xi_n = I_{-n}\xi_{-n}$, whereas for difference patterns, $I_n \xi_n = -I_{-n}\xi_{-n}$. These conditions about symmetry lead to the following (taking the right hand side of the array, and simplifying a multiplicative term that does not alter the relative power pattern results):

$$F(\theta,t)\big|_q = e^{j\omega t} \sum_{n=1}^{N} I_n \xi_n \left[\frac{sin(q\pi\xi_n)}{q\pi\xi_n} \right] e^{-jq\pi\xi_n} f(kz_n cos\theta)$$
$$\text{being } f(x) = \begin{cases} \cos(x) \text{ for sum patterns} \\ \sin(x) \text{ for difference patterns} \end{cases} \tag{5}$$

As specific cases, and following the preliminary concepts given in the introduction, it will be considered here uniform and GV real amplitude distributions to synthesise sum and difference patterns with minimized SLL and SR, as seen below.

3. THE METHOD

The simple optimization process applied to the linear array –and run with the aid of the SA technique [7]– is depicted in Figure 1. Once the real–valued static excitation amplitudes I_n have been specified, the algorithm begins by setting all ξ_n (n=1,2,…N) to 1. Then, the dynamic excitations are perturbed using the SA, by changing slightly M values of ξ_m with m $\in \Omega_{s(d)}$, and where $\Omega_s=\{N-M, N-M+1, ... N\}$ for the sum pattern or $\Omega_d=\{1,2, ... M\}$for the difference pattern. The selection of M is arbitrary, and we found that setting M=2 to 4 is sufficient for the purposes established here (see next Section). The random perturbation of ξ_m is sequentially repeated in order to change the cost function

$$Cost\ Function = C_1\ H(\Delta SLL)\Delta SLL^2 + C_2\ H(\Delta Maxh)\Delta Maxh^2 \tag{6}$$

up to obtaining a minimum value, in terms of the SA parameters [7]. In this equation,

ΔSLL= (SLL_o – SLL_d), is the difference between the obtained sidelobe level (SLL_o) and the desired sidelobe level (SLL_d) of the main pattern (q=0).

For $\Delta Maxh$= ($Maxh_o$ – $Maxh_d$), if $Maxh$ is defined as the maximum value of the SR, (any of the $|q|$>0 modes), then $\Delta Maxh$ is the difference between the obtained and the desired maximums of the harmonics.

Note that –see equation (2)– the maximum amplitudes of the harmonics decrease as $|q|$ increases, and therefore q=1 mode is sufficient when calculating $\Delta Maxh$, for the purposes of reduction of the algorithm computational time.

$H(x)$ represents the Heaviside step function, defined as:

$$H(x) = \begin{cases} 1 \text{ if } x > 0 \\ 0 \text{ otherwise} \end{cases} \tag{7}$$

C_1 and C_2 are weighting coefficients that indicate to the SA the importance of each term.

4. EXAMPLES

We considered an equispaced linear array of 30 isotropic elements centred with respect to z=0 and separated 0.7λ apart, and three static excitation distributions: a uniform one (case A) and two GV ones, with $\bar{n}$ =3, SLL ≤ –20 dB and ν=0 or ν=1 (cases B or C, respectively) [6]. The parameters $\bar{n}-1$ and ν represent the number of controlled sidelobes (one side of the diagram) and the coefficient that controls the farthest side-lobe level envelope, respectively. The uniform distribution was used to optimize a sum pattern and a corresponding difference pattern. Cases B and C were used to optimize difference patterns.

By setting $\xi_n = 1$, we obtain the SLL values displayed in Table 1, and they correspond to the static case (no time modulation applied).

After the optimization process, we obtained significant improvements. The results are shown in Table 2 (compare with those of Table 1). For the sum pattern, it was obtained an SLL value 2.79 dB below that of the non–optimised case, whereas for the difference pattern, the SLL was 5.98 dB below that of the static distribution. Cases B and C achieved 5.19 and 4.79 dBs below the static distributions, with regard to the corresponding difference patterns. In all cases the maximum sideband levels were close to –30 dB. Figure 2 shows the difference pattern synthesised from the uniform static distribution, whereas figure 3 shows the corresponding sum pattern. Figure 4 shows how the harmonic (maximum) levels of this last example decrease progressively as q increases, as mentioned before (the diagram is symmetric with respect to the q=0 axis).

Tables 3–4 list the static $\left(I_n\right)$ and dynamic $\left(I_n \xi_n\right)$ excitations of each case. Note that all dynamic excitations are real and their changes are small. It should be pointed out that the authors performed several additional examples –not shown here for brevity–, being all of them generated under further restrictive conditions, concerning the number of perturbed elements, the ratio between maximum and minimum amplitudes of the dynamic excitations, and so forth. In such cases, additional improvements in SLLs (about 2 dB) can be achieved.

4. CONCLUSIONS

The novelty of this method consists in improving some characteristics of sum and difference patterns by applying time–modulation, with the aid of the SA, to a minimum number of elements of a linear array. Besides, the technique represents an advantage for reconfigurability purposes, since it can be easily applied to the array system by simply adding it the corresponding switching circuitry, once the static excitation distribution has been fixed. Although the method was applied to linear arrays, it can also be extended to planar and even conformal arrays. It is readily seen that the dynamic excitations can be used in a standard array (no time modulation performed) as static excitations, due to the straight analogy between equations (1) and (3).

ACKNOWLEDGEMENT

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LEGENDS FIGURES AND TABLES:

FIGURES

Figure 1. Flux diagram used during the optimization process.

Figure 2. Difference pattern obtained with some optimised ξ_n from the uniform static distribution.

Figure 3. Sum pattern obtained with some optimised ξ_n from the uniform static distribution.

Figure 4. Maximum value of patterns for the first thirty harmonics obtained from the example in figure 3.

TABLES

Table 1. Initial values of SLL calculated from the power patterns generated by the static excitation distributions (not optimised values).

Table 2. Summary of the results obtained for several distributions after optimisation (*n*=1,2... counted from the center of the array).

Table 3. Dynamic $\left(I_n \xi_n\right)$ and uniform static $\left(I_n\right)$ excitations of one half of the linear array for sum and difference patterns (*n*=1,2... counted from the center of the array). Bold types indicate the time-modulated elements.

Table 4. Dynamic $\left(I_n \xi_n\right)$ and Villeneuve static $\left(I_n\right)$ excitations of one half of the linear array for sum patterns (*n*=1,2... counted from the center of the array). Bold types indicate the time-modulated elements.

FIGURES

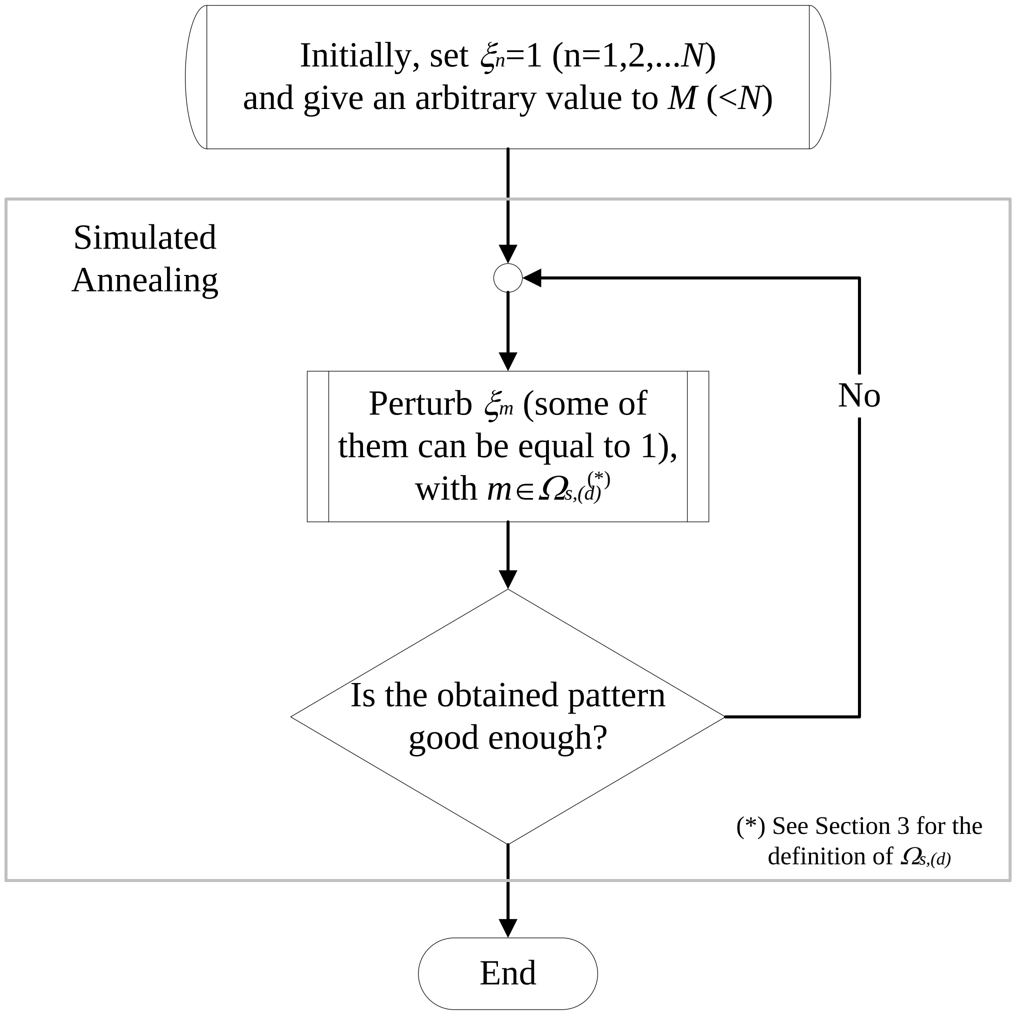
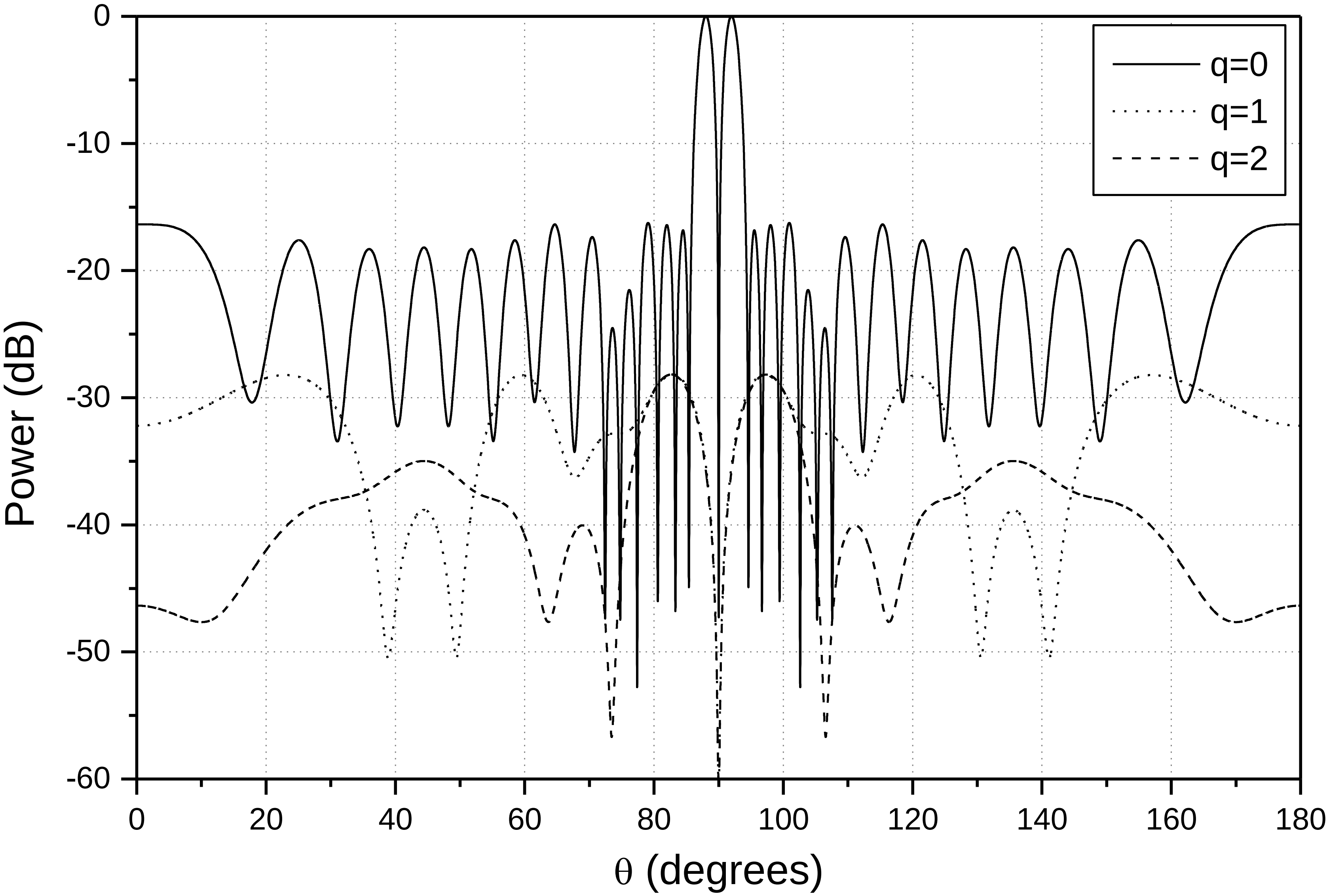


Figure 1.



0
-10
-20
-30
-40
-50
-60
Power (dB)
0
20
40
60
80
100
120
140
160
180
θ (degrees)
q=0
q=1
q=2

Figure 2.

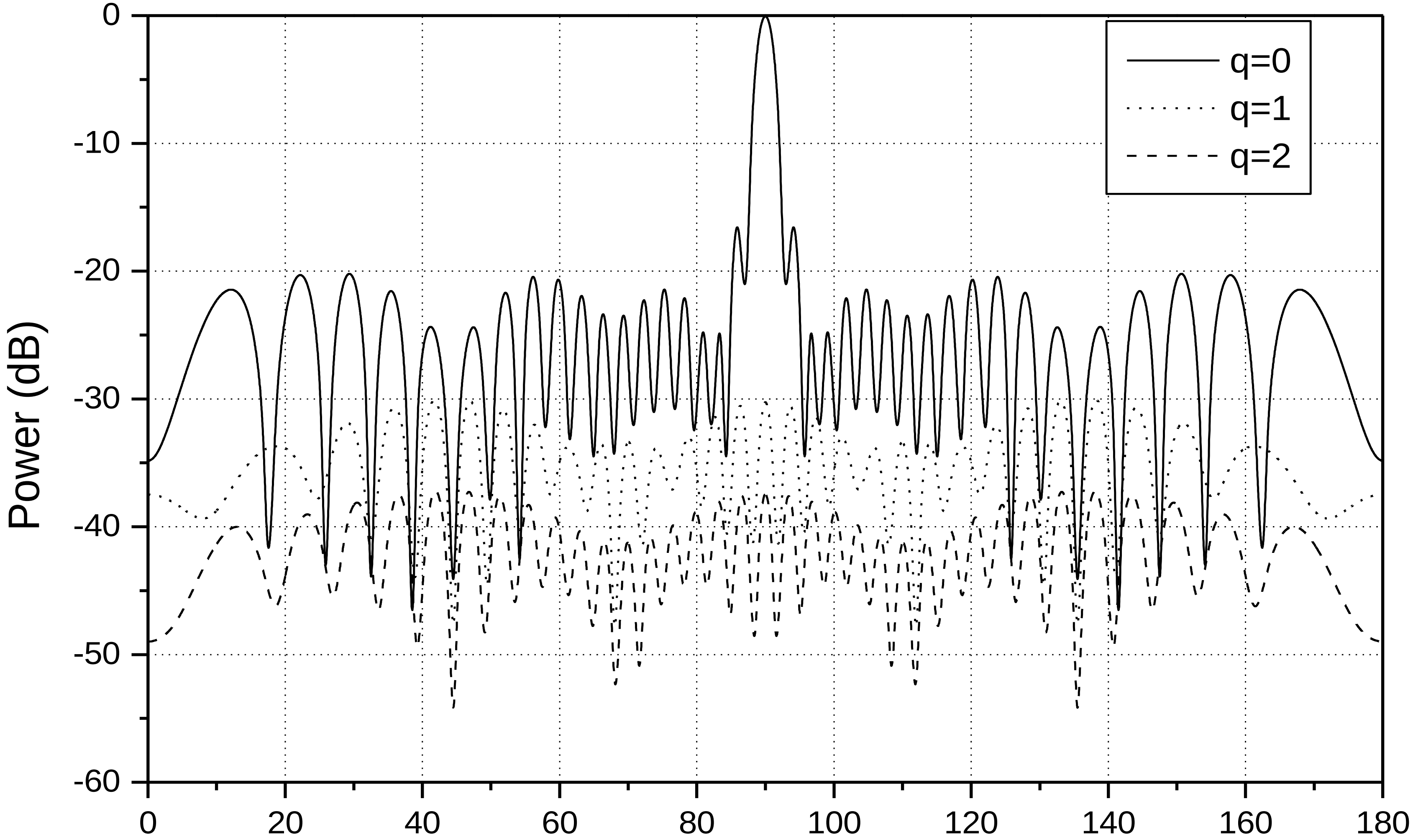
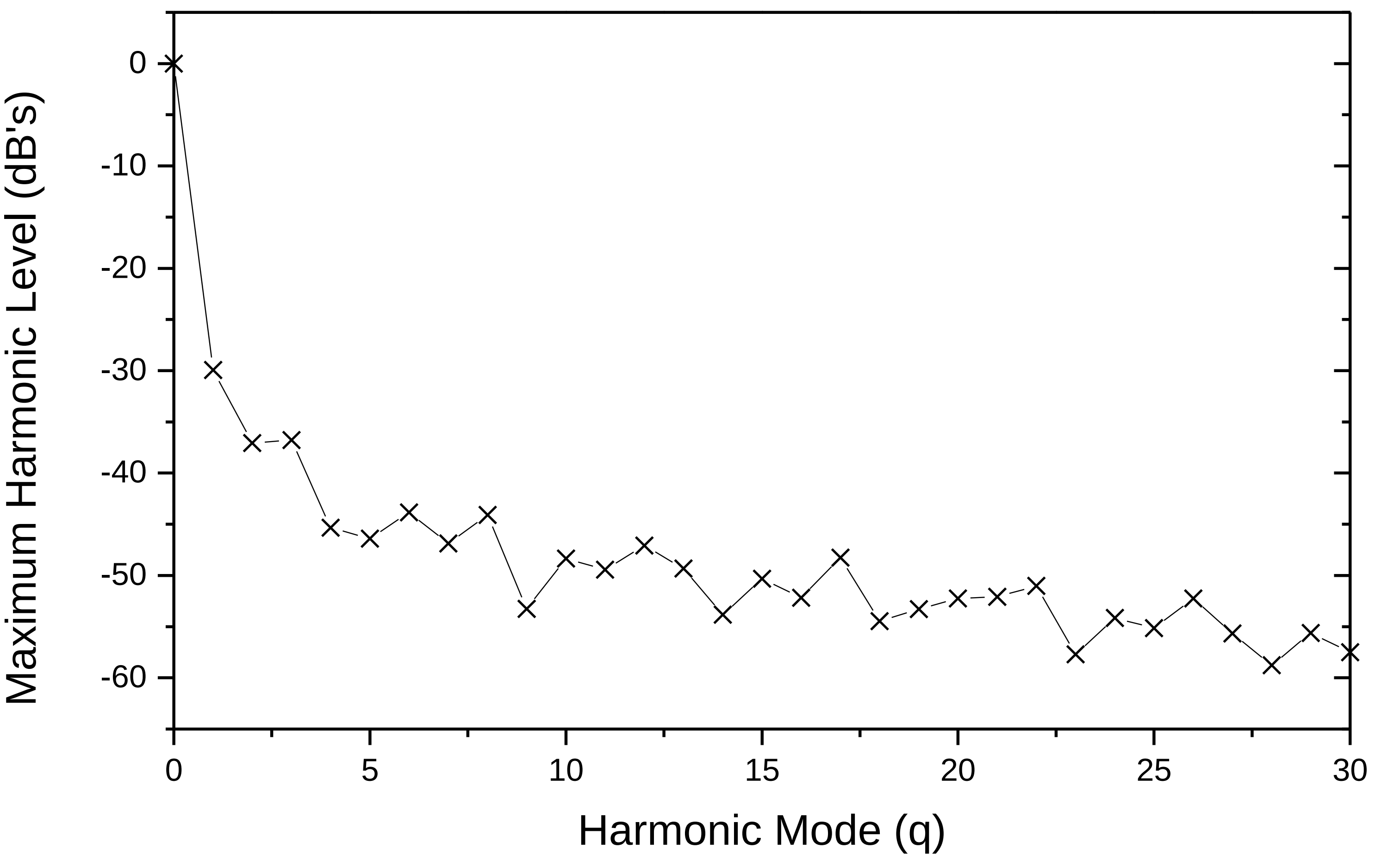

Power (dB)
0
-10
-20
-30
-40
-50
-60
0
20
40
60
80
100
120
140
160
180
q=0
q=1
q=2

Figure 3.



Maximum Harmonic Level (dB's)
0
-10
-20
-30
-40
-50
-60
0
5
10
15
20
25
30
Harmonic Mode (q)

Figure 4.

TABLES

Pattern Type	Static (non optimizad) Excitation Distribution Cases	SLL (dB)
Sum	A (Uniform)	–13.05
Difference	A (Uniform)	–10.44
	B (G.V., $\bar{n}=3; \nu=0; SLL \le -20\,\mathrm{dB}$)	–9.73
	C (G.V., $\bar{n}=3; \nu=1; SLL \le -20\,\mathrm{dB}$)	–10.42

G.V.: Generalised Villeneuve distribution (see text)

Table 1.

Pattern Type	Static Excitation Distribution Case	Number of changed ξ_n	% of changed ξ_n	SLL_o in dB (q=0)	$Maxh_o$ in dB (q=1)
Sum	A	3	20.0	–15.84	–29.93
Difference	A	4	26.7	–16.42	–28.14
	B	4	26.7	–14.92	–29.56
	C	2	13.3	–15.21	–30.43

Table 2.

	Excitations Case A		
	I_n	$I_n\xi_n$	
Element		**Sum pattern**	**Difference pattern**
1	1.0000	1.0000	**0.8868**
2	1.0000	1.0000	**0.1525**
3	1.0000	1.0000	**0.2347**
4	1.0000	1.0000	**0.7058**
5	1.0000	1.0000	1.0000
6	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000
11	1.0000	**0.5633**	1.0000
12	1.0000	1.0000	1.0000
13	1.0000	**0.2114**	1.0000
14	1.0000	1.0000	1.0000
15	1.0000	**0.9777**	1.0000

Table 3.

	Excitations			
	Case B		Case C	
Element	I_n	$I_n\xi_n$	I_n	$I_n\xi_n$
1	1.0000	**0.0756**	1.0000	**0.1883**
2	0.9891	**0.0756**	0.9865	**0.1087**
3	0.9679	**0.7854**	0.9606	0.9606
4	0.9373	**0.8993**	0.9245	0.9245
5	0.8990	0.8990	0.8809	0.8809
6	0.8547	0.8547	0.8331	0.8331
7	0.8066	0.8066	0.7840	0.7840
8	0.7569	0.7569	0.7361	0.7361
9	0.7077	0.7077	0.6903	0.6903
10	0.6613	0.6613	0.6456	0.6456
11	0.6194	0.6194	0.5986	0.5986
12	0.5838	0.5838	0.5425	0.5425
13	0.5559	0.5559	0.4673	0.4673
14	0.5367	0.5367	0.3600	0.3600
15	0.5269	0.5269	0.2067	0.2067

Table 4.