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A MESHLESS NUMERICAL APPROACH FOR THE ANALYSIS OF EARTHING SYSTEMS IN ELECTRICAL INSTALLATIONS

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Abstract

In the last three decades some numerical formulations have been developed for solving potential problems in electrical engineering applications. In the particular case of the grounding analysis area, in recent years we have developed a general numerical approach based on the Boundary Element Method for homogeneous and isotropic soil models, which has been succesfully applied to the analysis of large grounding systems. This numerical approach has been recently extended for the study of earthing grids embedded in stratified soils, which enables to solve some frequent practical cases, such as the two-layered soil models. Nevertheless, boundary element approaches imply a considerable computational effort when applied to the grounding analysis buried in more stratified soils or completely heterogeneous. The difficulty of the extremely high cost also arises with the use of standard numerical techniques (Finite Differences or Finite Elements) which require the discretization of the whole domain: the ground.

Since early nineties, several numerical methods where meshes are unnecessary ("meshless methods") have been proposed in several engineering applications. In this paper, we briefly review some of these meshless techniques, and propose the use of a Moving Least Square methodology with a point collocation scheme for solving problems in electrical engineering. Furthermore, the use of enrichment procedure in these meshless formulations is explored to improve results and decrease the computational cost required.

1. INTRODUCTION

In last decades, the development of high efficient numerical methods, such as Finite Elements (FE), Boundary Elements (BE), Finite Volumes (FV) or Finite Differences (FD), and advances in computer sciences have allowed to spread the numerical simulation in most of the fields in engineering applications. As a general rule, these numerical methods are based on dividing the solution domain of the problem (or/and its boundary) into a number of subdomains, where the integration process required to solve it is performed. However, this mesh generation process (specially in 3D problems) frequently becomes the bottle neck. Thus, in some practical applications such as problems with moving boundaries, with discontinuities in the domain, or with a very complicated geometry, the discretization of the domain can involve more computing effort (in memory storage and CPU time) than the integration and solving processes themselves. For this reason, some numerical methods "free-of-meshes" have been proposed in last years.

The first meshless methods were derived from finite difference approaches in early seventies[1]; and the first class of these "particle methods" (the Smooth Particle Hydrodynamics method, SPH) were developed in the computational physics field[2,3]. However, the application of SPH method in engineering problems have been recently performed in order to solve problems in solid and fluid mechanics[4,5].

A different type of meshless methods (the Diffuse Element Method, DEM), was proposed by Nayroles[6] in 1992. In this method, a basis function and a weighting function are used to form a local approximation based on a set of arbitrary nodes. In 1994, Belystchko *et al.* modify and refine this method, proposing the Element Free Galerkin method (EFGM)[7,8], in which a moving-least squares (MLS) interpolation is used to define the local approximation. On the other hand, Liu *et al.* have proposed a meshless technique (the Reproducing Kernel Particle Method, RKPM) based on a convolution integral, which it is similar to SPH method, although several correction functions and refinements are introduced in order to assure consistency near boundaries and for nonuniform spacing[9].

A class of meshless methods that can provide an efficient way to perform h-p adaptivity are those based on partitions of unity (hp-Clouds method[10] and the Partition of Unity Finite Element Method [11]). They introduced a partition of unity with a moving-least square interpolation and the enhancement of the polynomial order of the approximation through an extrinsic basis, which can be added locally to nodes[12].

On the other hand, Oñate *et al.* [13,14] have proposed a method which combines the moving least square approximation with a point-collocation approach to compute the integral terms, in convective transport and fluid flow problems. This method completely avoids the necessity of mesh generation, because no auxiliary grid is required. Furthermore, different techniques can be derived if the weighting function is "fixed" (Diffuse Least Square method, DLS) or it depends on the point where the approximated value is computed (Moving Least Square method, MLS).

On the basis of this MLS method, we have recently proposed a numerical approach to solve potential problems in electrical engineering applications[15,16], such as grounding analysis, where the use of standard numerical methods (such as finite elements) are precluded due to the complexity of the domain[17]. In this paper, we review this MLS interpolation with a point collocation approach for solving boundary value problems; furthermore, the use of enrichment functions is analyzed for the solution of this kind of problems.

2. MOVING LEAST SQUARES APPROXIMATION

The Moving Least Squares methodology is an effective numerical technique for the approximation of a function by using a set of disordered data. It consists of a local weighted least square fitting, valid on a small neighbourhood of a point and based on the information provided by its n closest points (subdomain Ω_k). The local character of the approximation comes from a "moving" weighting function which takes its maximum value at this point and vanishes outside a surrounding region[12,14].

In order to define properly the approximation at every point, it is necessary that all subdomains Ω_k cover all the interpolation domain. Hereby, these subdomains must overlap, and the common areas have to include enough nodal points in order to ensure the convergence of the method[10]. The selection of the nodal points included in the subdomain of a given nodal point can be performed by using a effective technique based on the "four-quadrants" criteria[14].

If Ω_k is the interpolation domain of a function $u(\boldsymbol{x})$, it can be approximated by

$$u(\boldsymbol{x}) \cong \hat{u}(\boldsymbol{x}) = \sum_{i=1}^{m} p_i(\boldsymbol{x})\alpha_i = \boldsymbol{p}^t(\boldsymbol{x})\boldsymbol{\alpha} \tag{1}$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_m]^t$ is a set of unknown coefficients, $\boldsymbol{p}(\boldsymbol{x})$ contains a base of interpolating functions (monomial terms, generally) which order is m. These base interpolating functions can be normalized within each subdomain Ω_k by dividing for the maximum distance d between each point i of the domain and the surrounding points; thus, it is possible to define normalized coordinates ($\boldsymbol{\xi} \equiv [\xi, \eta, \zeta]$) within a subdomain Ω_k as,

$$\boldsymbol{\xi}(\boldsymbol{x}) = \left[\frac{x - x_i}{d}, \frac{y - y_i}{d}, \frac{z - z_i}{d}\right] \tag{2}$$

On the other hand, function $u(\boldsymbol{x})$ can be sampled in the n points belonging to Ω_k as,

$$\boldsymbol{u}^h = \begin{pmatrix} u_1^h \\ u_2^h \\ \vdots \\ u_n^h \end{pmatrix} \cong \begin{pmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \vdots \\ \hat{u}_n \end{pmatrix} = \begin{pmatrix} \boldsymbol{p}_1^t \\ \boldsymbol{p}_2^t \\ \vdots \\ \boldsymbol{p}_n^t \end{pmatrix} \boldsymbol{\alpha} = \boldsymbol{S}\boldsymbol{\alpha} \tag{3}$$

where u_j^h are the values of unknown function evaluated in nodal points of Ω_k ($u_j^h = u(\boldsymbol{x}_j)$, $j = 1, \ldots, n$), $\hat{u}_j = \hat{u}(\boldsymbol{x}_j)$ are their approximated values, and $\boldsymbol{p}_j$ contains the normalized base interpolating functions evaluated in $\boldsymbol{\xi}_j$ (where $\boldsymbol{\xi}_j = \boldsymbol{\xi}(\boldsymbol{x}_j)$).

The approximation defined in (3) can also be understood as a generalization of the finite element interpolation, since this approach is obtained if the number of subdomain points n is chosen equal to order m of the polinomials base[18]. In general, if $n > m$, $\boldsymbol{S}$ is a rectangular matrix and the approximation cannot fit all the u_j^h values. However, approximated values $\hat{u}(\boldsymbol{x})$ can be determined by minimizing the weighted sum of the square differences between the exact value u_j^h and the approximation $\hat{u}(\boldsymbol{x}_j)$ at each nodal point $\boldsymbol{x}_j$ belonging to the domain of node $\boldsymbol{x}_k$. The weighting function is usually built in such a way that it equals unity in point $\boldsymbol{x}_k$ and vanishes outside domain Ω_k.

In the Moving Least Square approach, this functional can be written as

$$J(\boldsymbol{x}) = \sum_{j=1}^{n} \omega_k(\boldsymbol{x}_j, \boldsymbol{x}_k)(u_j^h - \hat{u}(\boldsymbol{x}_j))^2 \tag{4}$$

where $\omega_k(\boldsymbol{x}_j, \boldsymbol{x}_k)$ is the weighting function computed in $\boldsymbol{x}_j$, which shape and span depend on $\boldsymbol{x}_k$. It must be pointed out that $\boldsymbol{x}_k$ represents an arbitrary position and can be replaced for a generic coordinate $\boldsymbol{x}$.

Now the minimization of functional (4) allows to obtain[12,18]:

$$\boldsymbol{\alpha} = \boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x})\boldsymbol{u}^h \qquad \boldsymbol{A}(\boldsymbol{x}) = \boldsymbol{P}\boldsymbol{W}(\boldsymbol{x})\boldsymbol{P}^t \qquad \boldsymbol{B}(\boldsymbol{x}) = \boldsymbol{P}\boldsymbol{W}(\boldsymbol{x}) \tag{5}$$

being auxiliar matrices $\boldsymbol{P}$ and $\boldsymbol{W}(\boldsymbol{x})$:

$$\boldsymbol{P} = [\boldsymbol{p}(\boldsymbol{\xi}_1) \quad \ldots \quad \boldsymbol{p}(\boldsymbol{\xi}_n)] \qquad \boldsymbol{W}(\boldsymbol{x}) = diag\left[\omega_k(\boldsymbol{x}_j, \boldsymbol{x})\right], \quad j = 1, ..., n \tag{6}$$

Now, the substitution of (5) in (1) allows to obtain an approximation to function $u(\boldsymbol{x})$ in Ω_k in the form,

$$\hat{u}(\boldsymbol{x}) = \boldsymbol{p}^t(\boldsymbol{\xi}(\boldsymbol{x}))\boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x})\,\boldsymbol{u}^h. \tag{7}$$

From this expression, we can define "shape functions" ($\boldsymbol{N}^t(\boldsymbol{x})$ in Ω_k) in a similar way as in finite elements:

$$\boldsymbol{N}^t(\boldsymbol{x}) = \boldsymbol{p}^t(\boldsymbol{\xi}(\boldsymbol{x}))\boldsymbol{C}(\boldsymbol{x}); \qquad \boldsymbol{C}(\boldsymbol{x}) = \boldsymbol{A}^{-1}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x}). \tag{8}$$

It is important to remark that the local values of the approximating function do not fit the nodal unknown values ($\hat{u}(\boldsymbol{x}_j) \neq u_j^h$) due to the least-squares character of the approximation. It must be pointed out that if $n = m$ the FEM type approximation is recovered and no effect of weighting is presented[14]. Besides, if the weighting function is constant and equals the unity, the standard least square method is reproduced.

3. ENRICHMENT OF MLS APPROXIMATIONS

The enrichment of a Finite Element numerical formulation is an effectiveness technique to avoid a high refinement of the mesh in some problems in computational mechanics. In general, the enrichment process consists of introducing some information about the solution of the problem in the trial functions (e.g. its behaviour near singularities or discontinuities in the domain). In finite element approaches, these enriched techniques were developed in mid-seventies and succesfully applied to different problems[12].

Recently, it was found out that it is possible to incorporate information about the solution in applications of meshless methods is simpler and easier than in finite elements formulations[12,19]. The enrichment of meshless methods may be carried out extrinsically —i.e. adding a set of enrichment functions to the trial functions— or intrinsically, that is the enhancement functions are included in the MLS interpolation basis.

In extrinsic enrichment of MLS meshless approximations, a function —or a set of functions— closely related to the solution of the problem is included in the polynomial interpolation, as for example:

$$\hat{u}(\boldsymbol{x}) = \boldsymbol{p}^t(\boldsymbol{x})\boldsymbol{\alpha} + \sum_{j=1}^{n_f} \Big(k_j F_j(\boldsymbol{x})\Big) \tag{9}$$

where $\hat{u}(\boldsymbol{x})$ is the approximation to function $u(\boldsymbol{x})$, $\boldsymbol{p}(\boldsymbol{x})$ is a complete polynomial basis in the spatial coordinates, $\boldsymbol{\alpha}$ is the vector of unknowns associated to the basis, n_f is the number of enrichment functions added $F_j(\boldsymbol{x})$, and k_j $(j = 1, n_f)$ are global unknowns associated with functions F_j. Now coefficients $\boldsymbol{\alpha}$ can be obtained in a similar way to those in (1) by using a MLS methodology. However, additional terms arise from the inclusion of the enrichment functions. Therefore, the MLS functional J in (4) must be rewritten in terms of the new approximation (9). If the minimization process of this new functional is performed[12], the final expression for $\hat{u}$ results in

$$\hat{u}(\boldsymbol{x}) = \sum_{i=1}^{n} N_i(\boldsymbol{x}) \left[u_i^h - \sum_{j=1}^{n_f} \Big(k_j F_j(\boldsymbol{x}_i)\Big)\right] + \sum_{j=1}^{n_f} \Big(k_j F_j(\boldsymbol{x})\Big) \tag{10}$$

where shape functions $N_i(\boldsymbol{x})$ $(i = 1, n)$ are identical to previously defined in (8).

Another type of extrinsic enrichment in meshless methods, simpler and computationally faster than (10), can be obtained by using partition of unity methods[12]. In this case, the approximation is modified by adding a basis of enrichment functions extrinsically to the existing MLS approximation. These new functions can be polynomials of higher order than the MLS interpolants basis, or functions contained in the exact solution of the problem, which are smoothly added to the MLS approximation by multiplying it by a partition of unity[12,18]. Since shape functions in MLS approximations are partitions of unity, this extrinsic enrichment procedure frequently takes the form

$$\hat{u}(\boldsymbol{x}) = \sum_{i=1}^{n} N_i(\boldsymbol{x}) \left(u_i^h + \sum_{j=1}^{n_f(i)} k_{ij} F_j(\boldsymbol{x})\right) \tag{11}$$

where $nf(i)$ is the number of enrichment functions of nodal point i (in general, nf can be different for each nodal point), and k_{ij} are the unknowns associated to the basis of enrichment functions.

The partition of unity method provides a good tool for local enrichment. Thus, since the consistency is assured by the partition of unity $\boldsymbol{N}(\boldsymbol{x})$ formed by using the basis of MLS interpolants, the enrichment of the approximation may be performed locally by adding extrinsically functions of a new basis. Obviously, it should be remarked that this enrichment must be added to each node belonging to the domain of influence into the region to be enhanced[12].

Intrinsic enrichment of meshless methods consists of including special functions in the complete polynomial basis of the MLS interpolation. In contrast to the extrinsic procedures, this method involves no additional unknowns. However, since the size of the basis increases, additional computational effort is required to obtain shape functions $\boldsymbol{N}(\boldsymbol{x})$ in (8) —due to the inversion of matrix $\boldsymbol{A}(\boldsymbol{x})$—, and some problems of ill-conditioning can arise. Furthermore, when this enrichment is used at any node of the domain it must be used at all nodal points. Since it is not possible to delete functions from the basis because it produces discontinuities in the approximation, a special technique must be used to mix nodal points with different basis functions[12].

In this paper, we will consider the extrinsic enrichment technique based on partition of unity method for MLS meshless methods. Although the number of degrees of freedom increases with its use (if a nodal point i is enriched, then the total number of unknowns to obtain for it is $nf(i)+1$, instead of one), this enhancement procedure can be applied locally in different parts of the approximation, and it is also quite easy to implement in a meshless code.

4. STATEMENT OF THE DISCRETIZED EQUATIONS OF A BVP

In previous sections we have presented the MLS interpolation and the different kind of enrichments that can be performed. In this section we review how obtain the discretized equations of a boundary value problem. Thus, if $\mathcal{A}$ and $\mathcal{B}$ are two differential operators, Ω the domain of our problem and Γ its boundary ($\Gamma = \Gamma_t \cup \Gamma_u$), a scalar BVP can be written as,

$$\mathcal{A}(u) = b \qquad \text{in} \quad \Omega \tag{12}$$

with boundary conditions,

$$\begin{aligned} \mathcal{B}(u) &= t \qquad \text{in} \quad \Gamma_{\mathrm{t}} \\ u - u_p &= 0 \qquad \text{in} \quad \Gamma_{\mathrm{u}} \end{aligned} \tag{13}$$

where u is the solution, b and t represent the actions over Ω and along the boundary Γ_t, and u_p is the prescribed value of u along Γ_u.

Application of the weighted-residuals method allows to obtain a variational form of the above problem, in terms of the trial approximation function $\hat{u}$ of the unknown u, as

$$\int_{\Omega} W_j\,[\mathcal{A}(\hat{u})-b]d\Omega+\int_{\Gamma_t}\widehat{W}_j\,[\mathcal{B}(\hat{u})-t]d\Gamma+\int_{\Gamma_u}\widehat{\widehat{W}}_j\,[\hat{u}-u_p]d\Gamma \;=\; 0, \qquad j=1,...,n_p \quad (14)$$

which must hold for all members of the set of n_p functions W_j, $\widehat{W}_j$ and $\widehat{\widehat{W}}_j$ of a suitable class of test functions defined on Ω, Γ_t and Γ_u[14].

Now, the different selection of test functions in the general variational form (14) allows to derive different numerical formulations. In order to take advantage of the meshless character of the approximation, we can use a point-collocation approach ($W_j = \widehat{W}_j = \widehat{\widehat{W}}_j = \delta_j$, where δ_j is Dirac delta)[14,18]. Other authors[6,7,8,9,10] have proposed other integral methods, but require some kind of auxiliar grid to evaluate the resulting integrals. With a point-collocation scheme, the following set of equations is obtained:

$$\begin{aligned} [\mathcal{A}(\hat{u})]_j - b_j = 0, &\quad j=1,...,n_p \quad \text{in} \quad \Omega \\ [\mathcal{B}(\hat{u})]_j - t_j = 0, &\quad j=1,...,n_p \quad \text{in} \quad \Gamma_t \\ \hat{u}_j - u_p = 0, &\quad j=1,...,n_p \quad \text{in} \quad \Gamma_u \end{aligned} \qquad (15)$$

Now, if we do not perform any enrichment, given a set of n_p shape functions defined on Ω, approximation $\hat{u}$ to the solution u can be discretized as,

$$\hat{u} = \sum_{i=1}^{n_p} N_i u_i^h = \boldsymbol{N}^t \boldsymbol{u}^h, \qquad (16)$$

being n_p the total scattered points of the solution domain, and where $\boldsymbol{N}^t$ can be built by using the previous MLS methodology. Finally, we obtain the following system of linear equations:

$$\boldsymbol{K}\boldsymbol{u}^h = \boldsymbol{f} \qquad (17)$$

where coefficient matrix $\boldsymbol{K}$ is sparse but not necessary symmetric ($K_{ji} = [\mathcal{A}(N_i)]_j + [\mathcal{B}(N_i)]_j$), $\boldsymbol{f}$ is also known (contains the contributions from terms b and t and prescribed values u_p), and $\boldsymbol{u}^h$ contains the unknown values of the function evaluated in nodal points.

Obviously, as it has been explained in the previous section, if enrichment functions are used to define the MLS interpolation, then it will be necessary to use more collocation-points in (15) in order to obtain all the unknowns.

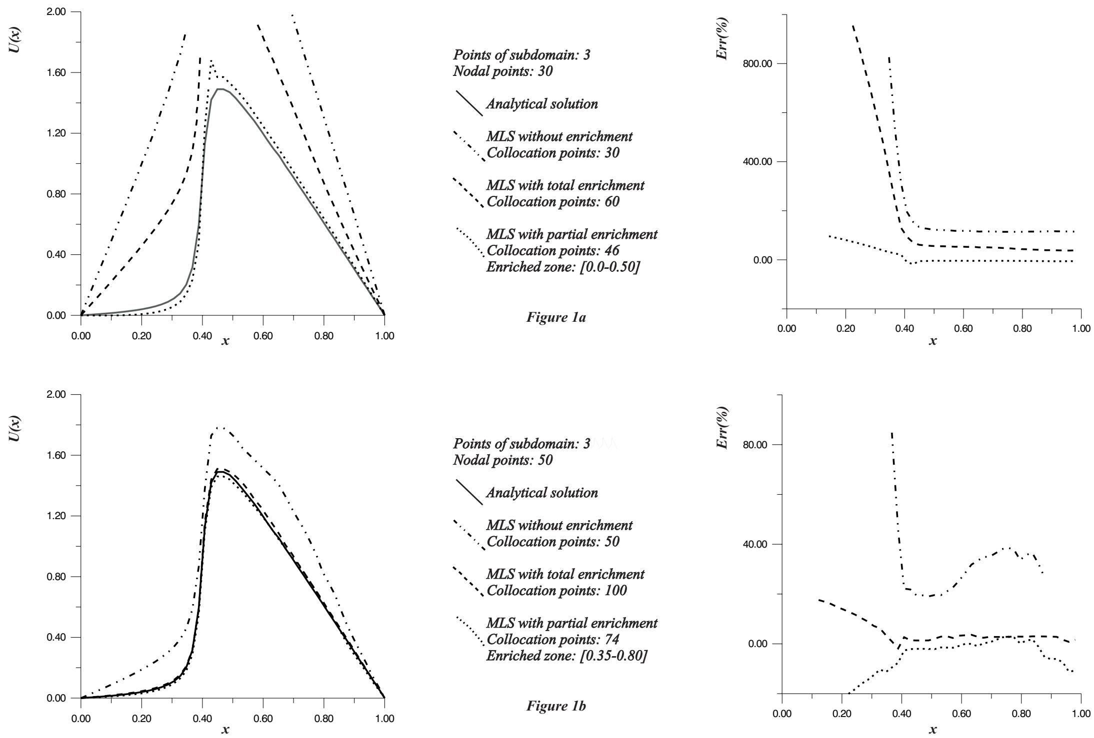


Fig. 1.1D numerical example: Comparison of results obtained by using different MLS formulations.

5. NUMERICAL EXAMPLES

In this section we present a very significant numerical test of the MLS approach applied to the solution of a 1D boundary value problem. This test (also used in finite element analysis[20]) allows to obtain sharp or smooth functions, depending on the choice of different parameters. Thus, it will be compared results obtained by using a standard MLS and an enriched MLS approach (applied in the whole domain or in a local zone of the domain), when quadratic interpolating is used.

The differential equation considered is:

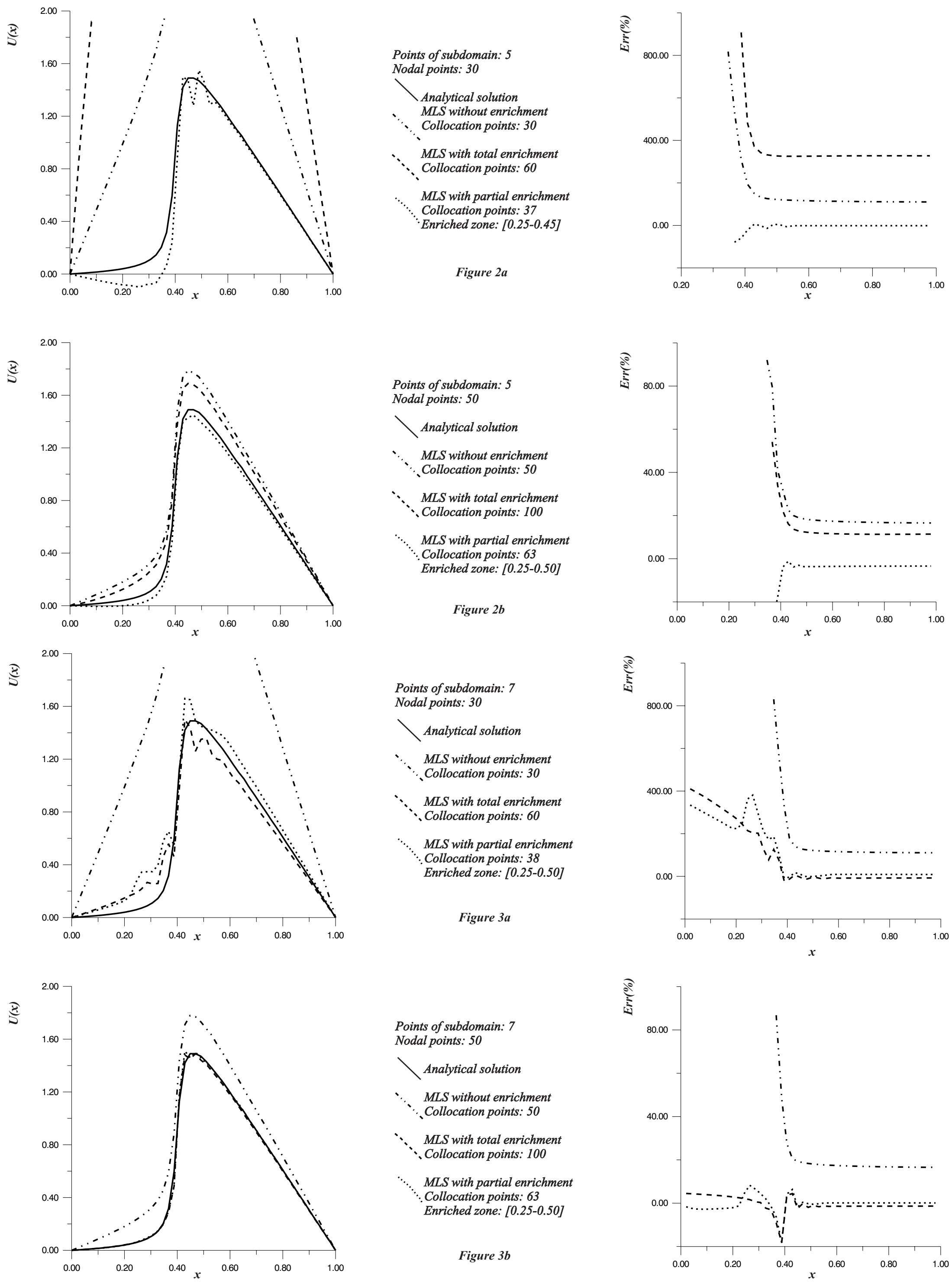
$$-\frac{d^2u}{dx^2} + u = f(x)\ , \qquad 0 \leq x \leq 1\ , \quad u(0) = 0 \quad u(1) = 1 \tag{18}$$

being

$$f(x) = \frac{2\rho[1+\rho^2(1-\beta)(x-\beta)]}{[1+\rho^2(x-\beta)^2]^2} + (1-x)[\text{atan}(\rho(\text{x}-\beta)) + \text{atan}(\rho\beta)] \tag{19}$$

The analytical solution is given by:

$$u(x) = (1-x)[\text{atan}(\rho(\text{x}-\beta)) + \text{atan}(\rho\beta)] \tag{20}$$



Figs. 2. and 3.- 1D numerical example: Comparison of results obtained by using different MLS meshless formulations.

For a given set of n_p trial functions N_i defined on the domain, and for the enrichment function atan(x), the approximation $\hat{u}$ to the solution u can be written in the form:

Without enrichment functions: $\hat{u} = \sum_{i=1}^{n_p} N_i u_i^h$

With enrichment functions: $\hat{u} = \sum_{i=1}^{n_p} N_i (u_i^h + a_{1,i} F_1(x)).$

Total enrichment: $F_1(x) = \text{atan(x)}, \quad 0 \leq \text{x} \leq 1.$

Local enrichment: $\begin{cases} F_1(x) = 0, & 0 \leq x < x_0; \\ F_1(x) = \text{atan(x)}, & x_0 \leq x \leq x_1; \\ F_1(x) = 0, & x_1 < x \leq 1. \end{cases}$

being n_p the total number of nodal points of the solution domain. The weighting function used is the truncated gaussian[14] with $\alpha = 0.25$ and $k = 1.1$. Parameters ρ and β are given by $\rho = 50.0$ and $\beta = 0.4$; these values produce a sharp function very difficult to approximate.

In figure 1 we compare the approximations obtained by using or not enrichment functions for different number of scattered nodal points (30 and 50), when subdomains of 3 points are considered (i.e., a finite element approach). As it is shown, in both cases, the use of an appropriate local enrichment allows to obtain a great improvement in the results with only a small increase in the number of collocation points. In this example, the computational cost is very similar when standard MLS and local enrichment MLS are used; while is much higher if the enrichment is performed in the whole domain.

In the same way as in figure 1, the advantages of the use of local enrichment functions in MLS interpolations are shown in figure 2. In these tests, the subdomain of each point are formed by 5 nodes, and the total number of scattered nodal points is 30 and 50.

The comparison of results obtained by using MLS interpolants with subdomains of 7 nodal points are shown in figure 3. In this case, as the size of subdomains increases, the enrichment solution improves considerably. It can also be noticed that the application of enrichment functions in local parts of the domain improves the approximation, in some cases, more than if it is performed in the whole domain.

In the light of these 1D test examples it can be concluded that approximations obtained by using meshless methods can be substantialy improved if certain zones of the domain are enhanced with suitable enrichment functions. In the next section, we will try to apply these ideas to potential problems in electrical engineering applications.

6. APPLICATION TO GROUNDING ANALYSIS

6.1. Mathematical model of the problem

A grounding system is an essential installation in electrical substations. A safe earthing electrode must guarantee the integrity of equipments and the continuity of the service under fault conditions –providing means to carry and dissipate electrical currents into the ground–, and safeguard that persons in the surroundings of the installation are not exposed to dangerous electrical shocks. In order to achieve these goals, the equivalent electrical resistance of the grounding must be low enough to assure that fault currents dissipate mainly through the electrode into the earth, and maximum potencial differences between close points on the earth surface are kept under certain tolerances.

The physical phenomena that underlyes the dissipation of fault currents into the earth can be modelled by means of Maxwell's Electromagnetic Theory[17]. If the analysis is constrained to the electrokinetic steady-state response, and the inner resistivity of the grounding electrode is neglected, the 3D problem associated to an electrical current derivation to earth can be written as

$$\begin{array}{c} \mathbf{div}\boldsymbol{\sigma} = 0, \qquad \boldsymbol{\sigma} = -\boldsymbol{\gamma}\,\mathbf{grad}V \;\text{ in }\; E; \\ \boldsymbol{\sigma}^t \boldsymbol{n}_E = 0 \;\text{ in }\; \Gamma_E; \quad V = V_\Gamma \;\text{ in }\; \Gamma; \quad V \longrightarrow 0, \;\text{ if }\; |\boldsymbol{x}| \to \infty; \end{array} \tag{21}$$

where E is the earth, $\boldsymbol{\gamma}$ its conductivity tensor, Γ_E the earth surface, $\boldsymbol{n}_E$ its normal exterior unit field and Γ the electrode surface. Thus, when the electrode attains a voltage V_Γ (Ground Potential Rise or GPR) relative to a distant grounding point, the solution to (21) gives potential V and current density $\boldsymbol{\sigma}$ at an arbitrary point $\boldsymbol{x}$. On the other hand, since V and $\boldsymbol{\sigma}$ are proportional to the GPR, it can be assumed that $V_\Gamma = 1$ without any restriction.

In last years, authors have developed a high efficient numerical approach based on the Boundary Element Method to analyze grounding systems of electrical substations embedded in uniform and stratified soils[17,21]. However, the application of these techniques based on boundary elements in the cases of heterogeneous or multi-layer soils implies a considerable computational effort. On the other hand, the specific geometry of earthing systems in practice (a grid of interconnected buried conductors) precludes the use of standard numerical techniques, since the obtention of sufficiently accurate results would imply unacceptable computing efforts because the discretization of the domain (the earth) is required[17]. For these reasons, we have turned our attention to investigate the applicability of numerical formulations based on meshless methods[15] for the solution of this kind of problems. In accordance with this, we present two examples. The first one is a 1D potential problem. In this test, we will study the solution by using MLS interpolants with enrichment functions, as a first stage of a project that is being developed in order to apply these techniques to 3D potential problems. In the second example, we analyze the fault current dissipation into the earth through a toroidal grounding electrode by using a MLS meshless method with no enhancement.

6.2. A 1D Numerical Test

As a first test to study the performance of the standard and the enriched MLS interpolations applied to a potential problem, we will consider the following boundary value problem:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = 0 \ , \quad 1 \le r \le L \ ; \qquad V(1) = 1, \quad V(L) = 0, \tag{22}$$

which analytical solution is given by $V(r) = \frac{1}{L-1}\left(\frac{L}{r} - 1\right)$.

Thus, for the enrichment function $1/r$ and for a given set of n_p trial functions N_i defined on the domain, the approximation $\hat{V}$ to the solution V can be written in the form:

Without enrichment functions: $\hat{V} = \sum_{i=1}^{n_p} N_i u_i^h.$

With enrichment functions: $\hat{V} = \sum_{i=1}^{n_p} N_i (u_i^h + a_{1,i} F_1(r)).$

Total enrichment: $F_1(r) = 1/r, \quad 1 \le r \le L.$

Local enrichment: $\begin{cases} F_1(r) = 0, & 1 \le r < r_0; \\ F_1(r) = 1/r, & r_0 \le r \le r_1; \\ F_1(r) = 0, & r_1 < r \le L. \end{cases}$

being n_p the total number of nodal points of the solution domain. The weighting function used[18] is the truncated gaussian with $\alpha = 0.25$ and $k = 1.1$, and the total length of the domain L is 100.

In figure 4, it can be shown the comparison of the approximations obtained by using MLS interpolants with or without enrichment functions, and with a different number of scattered nodal points (10 and 30) and the number of points of the subdomains (5 and 7 nodes). In this case, the use of enhanced approaches allows to obtain very good results, specilly in those cases in which a local enrichment is performed. As it is shown, in comparison with standard MLS, an appropriate local enhancement successfully improves the approximation only adding a few collocation points and, therefore, with a minimal increase in the computational cost.

6.3. Analysis of a toroidal grounding electrode

The example presented in this paper consists of a toroidal electrode horizontally buried to a depth of 7 m. The interior diameter of the ring is 20 m and the electrode diameter is 3 m. Characteristics of the soil model used in this example are shown in figure 5, and the relationship between the scalar conductivities of each part of the soil are: $\gamma_2 = 4\,\gamma_1$ and $\gamma_3 = 2\,\gamma_1$. Due to the axial symmetry of the problem, solution

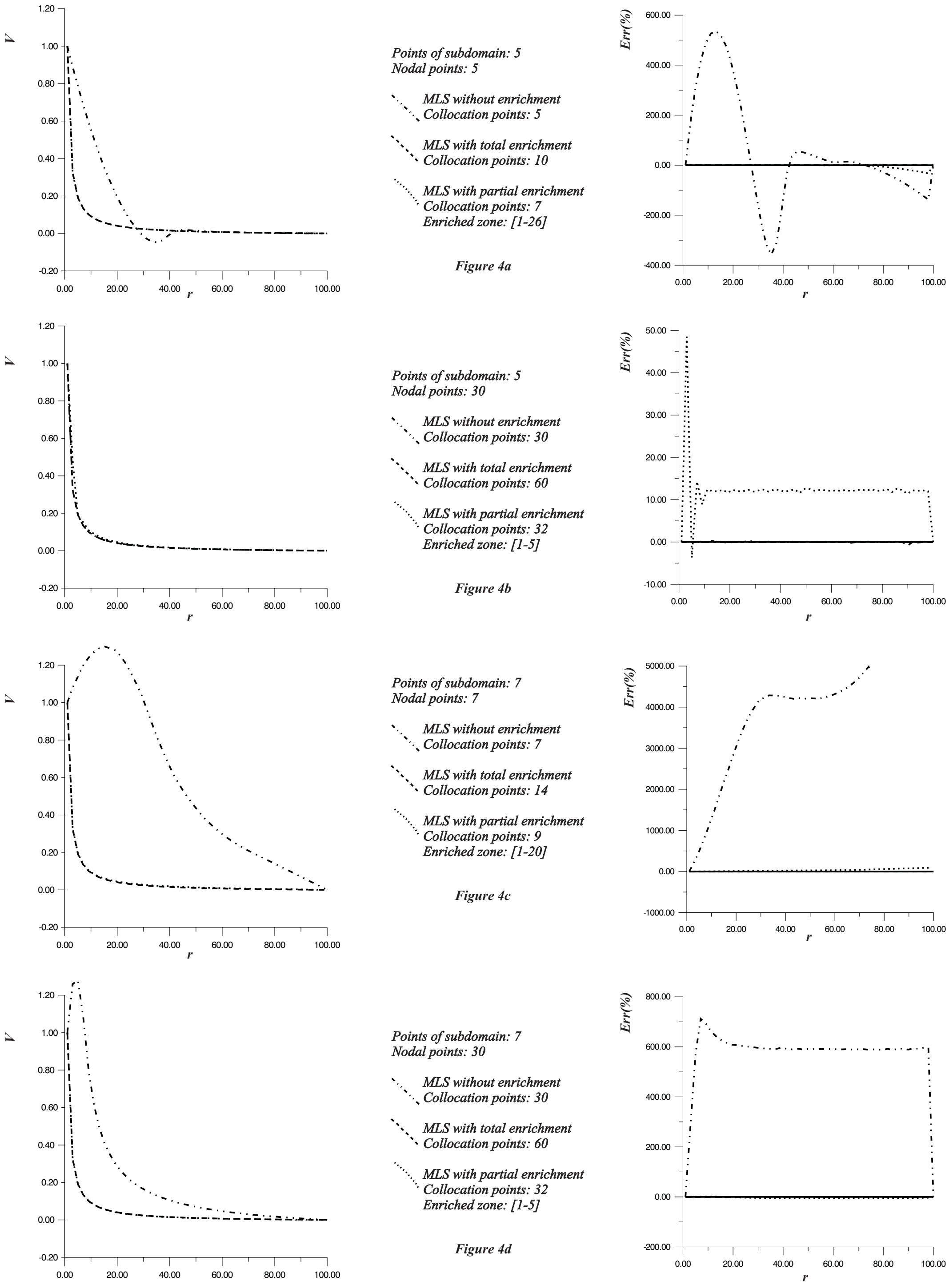


Fig. 4.- Comparison of results obtained by using different MLS approaches for the 1D numerical test.

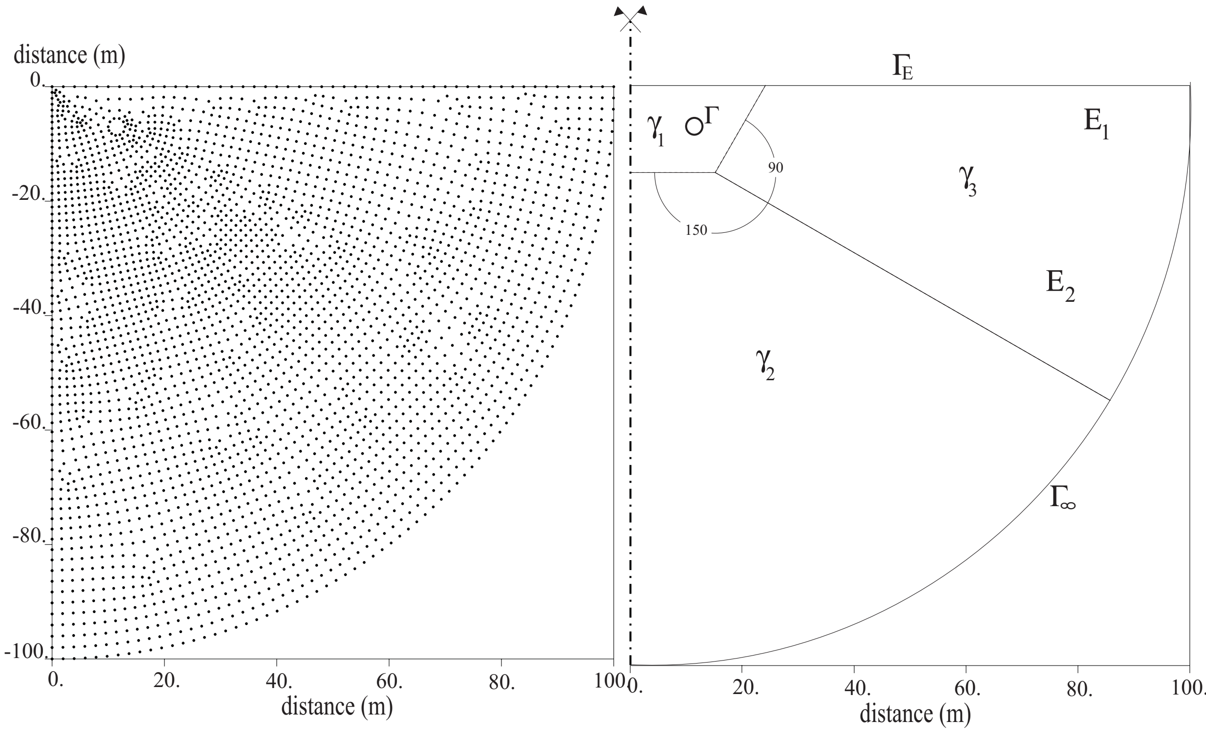


Fig. 5.- Distribution of nodal points used in the numerical resolution and scheme of the geometry of the problem.

can be obtained by using a 2D model. A MLS meshless method with a point collocation scheme has been used (in this example, no enrichment procedure has been carried out), and the distribution of nodal points (3019) has been obtained by means of the program GEN4U (figure 5)[22]. The base interpolating functions are linear and all subdomains contain at least five points. Figure 6 shows the contour lines and the potential distribution around the torodial electrode considering a non-homogeneous soil model. These numerical results agree significantly with those obtained by using a very dense point distribution. Nowadays, in the actual stage of the development of the project, we are working to include extrinsic enrichment techniques to this kind of problems in order to improve the results, and decrease the actual computational cost. Furthermore, with these enhancement methods we hope to solve problems with a different distribution and number of electrodes of the grounding system.

7. CONCLUSIONS

In this paper, a Moving Least Squares interpolation method with a point collocation approach has been presented for the analysis of potential problems in electrical engineering applications. The approximations to the solution can be improved by using a total or partial enrichment in the basis functions of the MLS interpolants.

In this kind of problems, in which the use of standard numerical methods is precluded

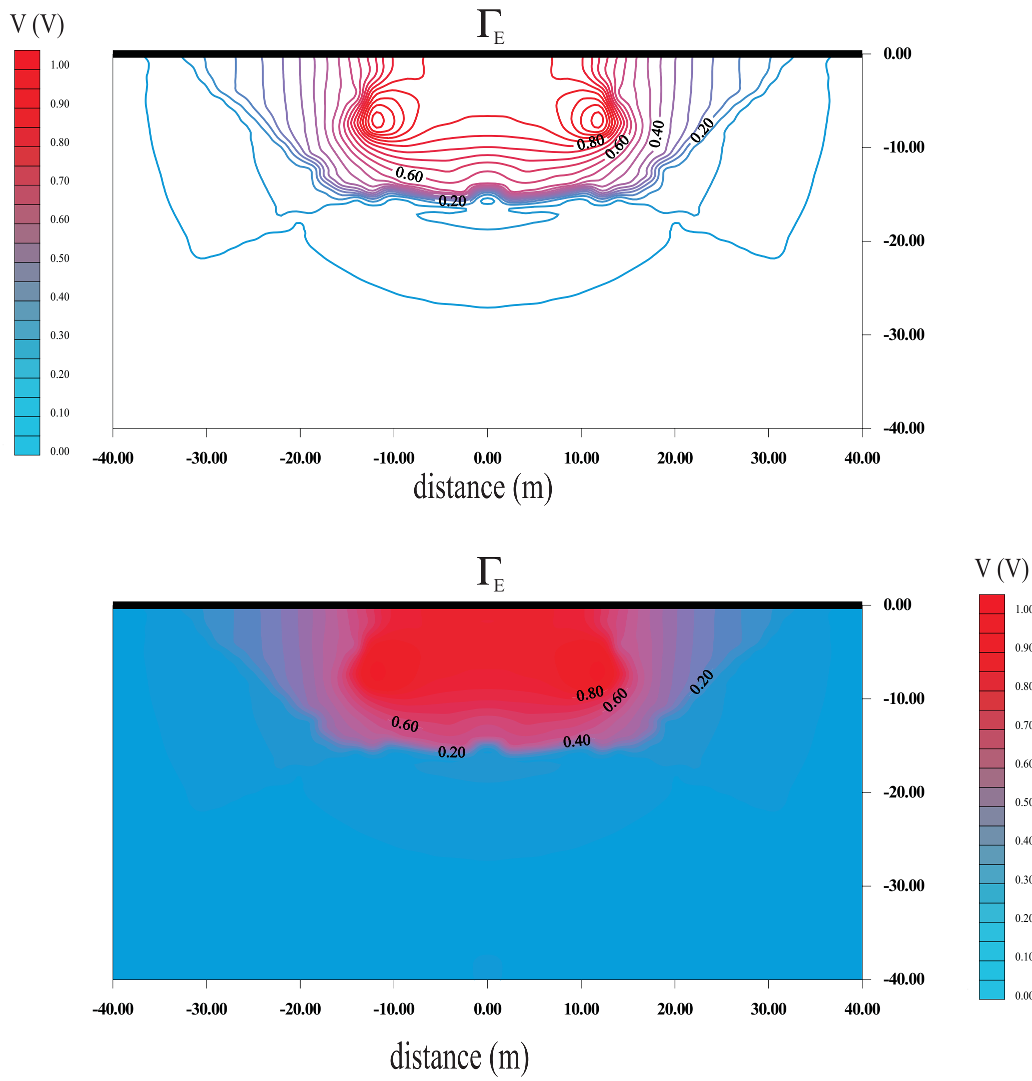


Fig. 6.- Contour lines and potential distribution intro the ground around a toroidal electrode during a fault current derivation.

due to large computing efforts required in the discretization process, the meshless character of these MLS approximations may represent an important improvement in their computational analysis. In the grounding analysis, since we know some information related to the type of the function of the solution, the use of enrichment functions in the MLS interpolation can also be very profitable. In this paper, we have presented some examples in 1D cases. The good performance can be noticed in the tests carried out. Furthermore, it has been verified that very interesting advantages can be obtained, specially if local enrichment is used.

The next step will be the comparison between standard and enriched MLS meshless methods in 2D problems. The standard MLS approach has already been developed, and it has been applied to the solution of a toroidal grounding system. Results obtained for different point distributions, even with a stratified soil model, are very promising and require a reasonable computational cost. Moreover, further analysis related with mathematical and numerical aspects must be done to assess the practical feasibility of this approach.

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