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NUMERICAL COMPUTATION OF GROUNDING GRIDS

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ABSTRACT

Grounding systems are designed to preserve human safety and grant the integrity of equipments under fault conditions. To achieve these goals, the equivalent electrical resistance of the system must be low enough to ensure that fault currents dissipate (mainly) through the grounding electrode into the earth, while maximum potential gradients between close points on the earth surface must be kept under certain tolerances (step and touch voltages) [1,2].

In this paper we present a Boundary Element approach for the numerical computation of grounding systems. In this general framework, former intuitive widespread techniques (such as the Average Potential Method) can be identified as the result of specific choices for the test and trial functions, as well as suitable assumptions introduced in the BEM formulation to reduce computational cost. Linear and higher order elements can be used in order to increase accuracy avoiding excessive segmentation. On the other hand, computing time is kept under acceptable levels by means of analytical integration techniques and semi-iterative methods for solving linear equations systems. Finally, an application to a real problem is presented.

INTRODUCTION

Fault currents dissipation into the earth can be modelled by means of Maxwell's Electromagnetic Theory [4]. On a regular basis, the analysis can be constrained to the obtention of the electrokinetic steady-state response, and the inner resistivity of the earthing electrode can be neglected. In these terms, the 3D potential problem can be written as

$$\begin{gathered} \boldsymbol{\sigma} = -\underset{\sim}{\boldsymbol{\gamma}}\ \mathbf{grad}\, V, \qquad \operatorname{div}(\boldsymbol{\sigma}) = 0 \quad \text{in} \quad E, \\ \boldsymbol{\sigma}^t \boldsymbol{n}_E = 0 \quad \text{in} \quad \Gamma_E, \quad V = V_\Gamma \quad \text{in} \quad \Gamma, \quad V \longrightarrow 0 \quad \text{if} \quad |\boldsymbol{x}| \to \infty, \end{gathered} \tag{1}$$

where E is the earth, $\underset{\sim}{\boldsymbol{\gamma}}$ its conductivity tensor, Γ_E the earth surface, $\boldsymbol{n}_E$ its normal exterior unit field and Γ the earthing electrode surface [5]. The solution to this problem gives the potential V and the current density $\boldsymbol{\sigma}$ at an arbitrary point $\boldsymbol{x}$ when the earthing electrode is energized to potential V_Γ (Ground Potential Rise or GPR) with respect to remote earth. On the other hand, being $\boldsymbol{n}$ the normal exterior unit field to Γ, the leakage current density σ at an arbitrary point on the earthing electrode surface, the total surge current I_Γ leaked into the earth and the equivalent resistance R_{eq} of the earthing system can be written as

$$\sigma = \boldsymbol{\sigma}^t \boldsymbol{n}, \qquad I_\Gamma = \int\!\!\int_\Gamma \sigma \; d\Gamma, \qquad R_{eq} = \frac{V_\Gamma}{I_\Gamma}. \tag{2}$$

For most practical purposes [3], the soil conductivity tensor $\underset{\sim}{\boldsymbol{\gamma}}$ can be replaced by a meassured apparent scalar conductivity γ (hypothesis of homogeneus and isotropic soil). Since V and $\boldsymbol{\sigma}$ are proportional to the GPR value, the normalized boundary condition $V_\Gamma = 1$ is not restrictive at all. Further practical simplifications (flat earth surface) allow to rewrite problem (1) as a Dirichlet Exterior Problem [4,5].

In most of cases, the earthing electrode consist of a number of interconnected bare cylindrical conductors, horizontally buried and supplemented by a number of vertical rods, which lenght/diameter ratio uses to be relatively high ($\approx 10^3$). Because of this kind of geometries, analytical solutions to problem (1) can not be derived for practical cases. On the other hand, standard numerical techniques (such as Finite Differences or Finite Elements) require discretization of domain E, which leads to unacceptable memory storage and computing time.

Since computation of potential is only required on the earth surface and the equivalent resistance can be easily obtained in terms of the leakage current (2), we turn our attention to a Boundary Element approach, which would only require discretization of the earthing grid surface Γ.

VARIATIONAL STATEMENT OF THE PROBLEM

Further analytical work [5,6] on problem statement (1) allows to express the potential V at an arbitrary point $\boldsymbol{x}$ on the earth E in terms of the leakage current density σ, in the integral form

$$V(\boldsymbol{x}) = \frac{1}{4\pi\gamma} \int\int_{\boldsymbol{\xi}\in\Gamma} k(\boldsymbol{x}, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi})\; d\Gamma, \tag{3}$$

$$k(\boldsymbol{x}, \boldsymbol{\xi}) = \left(\frac{1}{r(\boldsymbol{x}, \boldsymbol{\xi})} + \frac{1}{r(\boldsymbol{x}, \boldsymbol{\xi}')}\right), \qquad r(\boldsymbol{x}, \boldsymbol{\xi}) = |\boldsymbol{x} - \boldsymbol{\xi}|, \tag{4}$$

where $\boldsymbol{\xi}'$ is the symmetric of $\boldsymbol{\xi}$ with respect to the earth surface.

Since this expression holds on Γ, the boundary condition $V_\Gamma = 1$ leads to a Fredholm integral equation of the first kind with quasi-singular kernel (4), which solution is the unknown leakage current density function $\sigma(\boldsymbol{\xi})$. Moreover, for all members $w(\boldsymbol{\chi})$ of a suitable class of test functions on Γ, this problem leads to the weaker variational form

$$\int\int_{\boldsymbol{\chi}\in\Gamma} w(\boldsymbol{\chi})\; (V(\boldsymbol{\chi}) - 1)\; d\Gamma \;=\; 0. \tag{5}$$

Now, for given sets of 2D boundary elements and trial functions defined on Γ, both the earthing electrode surface Γ and the leakage current density σ can be discretized. Then, for a given set of test functions defined on Γ, variational statement (5) is reduced to a system of linear equations [8], which coefficients matrix is full. In addition, the computation of each term requires double integration on a 2D domain [7], which is unacceptable for practical purposes. Therefore, it seems necessary to introduce some additional simplifications.

APPROXIMATED 1D BOUNDARY ELEMENT FORMULATION

With this scope, it seems reasonable to consider that the leakage current density is constant around the cross section of the cylindrical electrode [7,8]. This hypothesis is widely used in most of the practical methods related in the literature [1,2,3], and seems no restrictive if we take into account the real geometry of grounding grids.

Let L be the whole set of axial lines of the buried conductors, and let $\widehat{\boldsymbol{\xi}} \in L$ be the orthogonal projection of a generic point $\boldsymbol{\xi} \in \Gamma$. Let $\phi(\widehat{\boldsymbol{\xi}})$ be the conductor diameter, and let $\widehat{\sigma}(\widehat{\boldsymbol{\xi}})$ be the approximated leakage current density at this point (assumed uniform around the cross section). In this terms we can write (3) as

$$\widehat{V}(\boldsymbol{x}) = \frac{1}{4\gamma} \int_{\widehat{\boldsymbol{\xi}} \in L} \phi(\widehat{\boldsymbol{\xi}})\, \bar{k}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}}) \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\, dL, \tag{6}$$

being $\bar{k}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})$ the average of kernel (4) around cross section at $\widehat{\boldsymbol{\xi}}$ [7,8].

Because the leakage current is not really uniform around the cross section, variational equality (5) does not hold anymore. Therefore, we must restrict the class of trial functions to those with circumferential uniformity, obtaining

$$\frac{1}{4\gamma} \int_{\widehat{\boldsymbol{\chi}} \in L} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}(\widehat{\boldsymbol{\chi}}) \left[\int_{\widehat{\boldsymbol{\xi}} \in L} \phi(\widehat{\boldsymbol{\xi}})\, \bar{\bar{k}}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})\, \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\, dL \right] dL = \int_{\widehat{\boldsymbol{\chi}} \in L} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}(\widehat{\boldsymbol{\chi}})\, dL, \tag{7}$$

for all members $\widehat{w}(\widehat{\boldsymbol{\chi}})$ of a suitable class of test functions on L, being $\bar{\bar{k}}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$ the average of kernel (4) around cross sections at $\widehat{\boldsymbol{\chi}}$ and $\widehat{\boldsymbol{\xi}}$ [7,8].

Now, for given sets of 1D boundary elements and trial functions defined on L, the whole set of axial lines of the buried conductors L and the unknown leakage current density $\widehat{\sigma}$ can be discretized. Then, for a given set of test functions defined on L, variational statement (7) is reduced to a linear equations system [7,8]. The matrix of coefficients of this approximated 1D problem is still full. However, on a regular basis we can say that the computational cost has been drastically reduced, since the actual discretization (1D) for a given problem will be much simpler than before (2D). On the other hand, the computation of each term seems to require double integration on a 1D domain instead of a 2D domain. Nevertheless, in this way no real advantage has been obtained up to this point, since evaluation of the averaged kernel $\bar{\bar{k}}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$ for a given couple of points $\widehat{\boldsymbol{\xi}}$ and $\widehat{\boldsymbol{\chi}}$ requires double integration by itself [7].

SIMPLIFIED 1D BOUNDARY ELEMENT APPROACH

If we use suitable unexpensive approximations [7,8] to evaluate the averaged kernels $\bar{k}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})$ and $\bar{\bar{k}}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$ [7], we can really take advantage of the fact that double integration is performed on a 1D domain. Nevertheless, computation of the remaining integrals is not obvious, and the cost of numerical integration is still out of range due to the undesirable behaviour of the integrands. For this reason, highly efficient analytical integration techniques have been derived by the authors [10].

Within this simplified boundary element approach, for specific choices of the trial and test functions we obtain different formulations. The simplest of these can be identified with widespread previous methods based on intuitive ideas, such as superposition of punctual current sources and error averaging [1,2,3]. Thus, for constant leakage current elements, a Galerkin type choice for the trial functions lead to the Average Potential Method [1,2,3].

For Galerkin type formulations the coefficients matrix of the linear equations system is symmetric and positive definite [9]. Furthermore a conjugate gradient type method can be used, which partially overcomes the non-sparsity of the matrix [7,8,10], and higher order elements can be used in order to increase accuracy avoiding excessive segmentation.

CONCLUSIONS

A Boundary Element approach for the analysis of substation earthing systems has been presented. Some reasonable assumptions allow to reduce a general 2D BEM formulation to a simplified 1D version for practical problems. By means of new analytical integration techniques, accurate results can be obtained in real cases with acceptable computing requirements.

This approach has been applied to a real case: the *E. Balaidos II* substation grounding (close to the city of Vigo, Spain). The plan, 3D view of the grid, characteristics and numerical model are presented in Figure 1. Results (such as equivalent resistance, fault current, potential distribution and a 3D view of potential level on ground surface) obtained with this formulation implemented in a Computer Aided Design System are given in Figure 2. Each bar was discretized in one single parabolic element. The model (174 elements and 315 degrees of freedom) required only 84 seconds of cpu time on a Vax-4300 computer. At the scale of the whole grid, results are not noticeably improved by increasing discretization.

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DATA		1D BEM MODEL	
Earth Resistivity:	$6,000\ \Omega cm$	Type of Elements:	Parabolic
Ground Potential Rise:	$1\ V$	Number of Nodes:	315
Installation Depth:	$0.800\ m$	Number of Elements:	174
Number of Horizontal Bars:	107		
Horizontal Bar Diameter:	$1.285\ cm$		
Number of Vertical Bars:	67		
Vertical Bar Diameter:	$1.400\ cm$	Vertical bars marked with black points	
Vertical Bar Length:	$2.500\ m$		

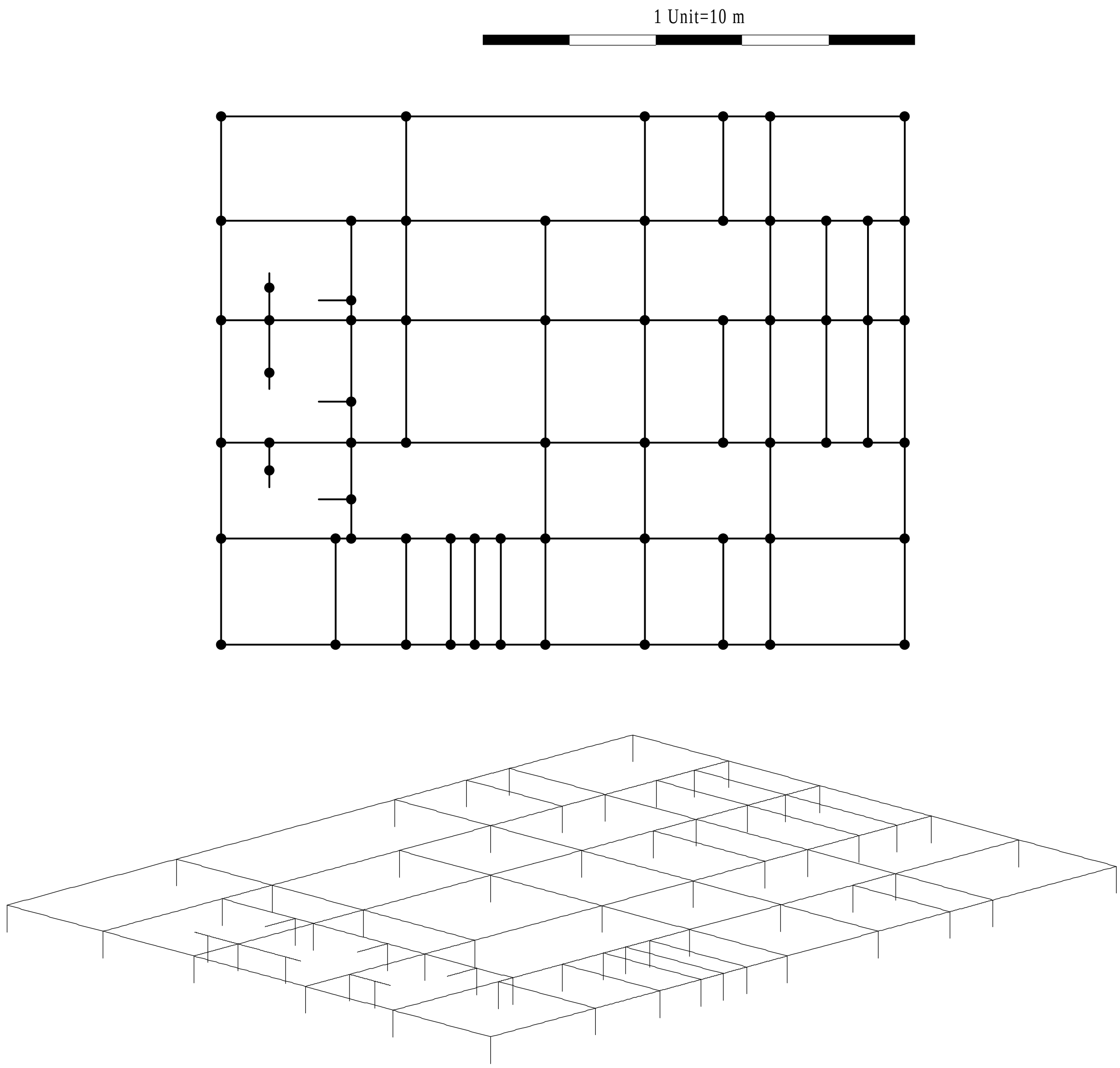

Fig 1.—E. Balaidos II Grounding System: Plan, 3D View of the Grid, Problem Characteristics and Numerical Model.

RESULTS		SURFACE POTENTIAL
Fault Current:	2.500 A	Contours plotted every 0.02 V
Equivalent Resistance:	0.400 Ω	Thick contours plotted every 0.10 V
CPU Time (in VAX–4300 Computer):	84 sec	

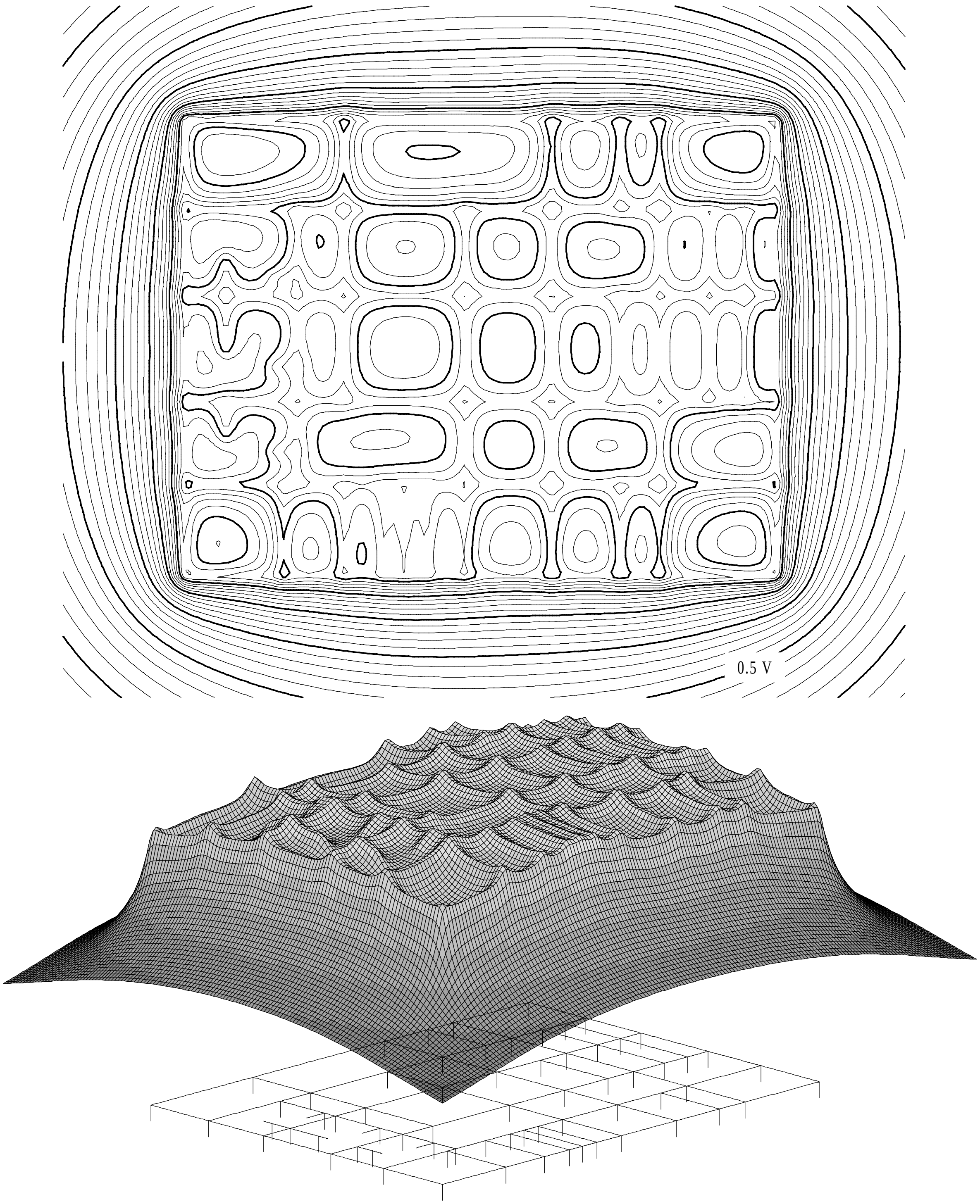

Fig 2.—E. Balaidos II Grounding System: Results by BEM, Potential Distribution and 3D View of Potential Level on Ground Surface.