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A BEM Approach for Grounding Grid Computation

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SUMMARY

Grounding systems are designed to preserve human safety and grant the integrity of equipments under fault conditions. To achieve these goals, the equivalent electrical resistance of the system must be low enough to ensure that fault currents dissipate (mainly) through the grounding electrode into the earth, while maximum potential gradients between close points on the earth surface must be kept under certain tolerances (step and touch voltages) [1,2].

In this paper, we present a Boundary Element approach for the numerical computation of grounding systems. In this general framework, former intuitive widespread techniques (such as the Average Potential Method) are identified as the result of specific choices for the test and trial functions, while the unexpected anomalous asymptotic behaviour of these kind of methods [3] is mathematically explained as the result of suitable assumptions introduced in the BEM formulation to reduce computational cost. On the other hand, the use of high order elements allow to increase accuracy, while computing time is drastically reduced by means of new analytical integration techniques. Finally, an application example to a real problem is presented.

INTRODUCTION

Fault currents dissipation into the earth can be modelled by means of Maxwell's Electromagnetic Theory [4]. On a regular basis, the analysis can be constrained to the obtention of the electrokinetic steady-state response, and the inner resistivity of the earthing electrode can be neglected. In these terms, the 3D potential problem can be written as

$$\begin{gathered} \boldsymbol{\sigma} = -\underset{\sim}{\boldsymbol{\gamma}}\ \mathbf{grad}\, V, \qquad \operatorname{div}(\boldsymbol{\sigma}) = 0 \quad \text{in} \quad E, \\ \boldsymbol{\sigma}^t \boldsymbol{n}_E = 0 \quad \text{in} \quad \Gamma_E, \quad V = V_\Gamma \quad \text{in} \quad \Gamma, \quad V \longrightarrow 0 \quad \text{if} \quad |\boldsymbol{x}| \to \infty, \end{gathered} \tag{1}$$

where E is the earth, $\underset{\sim}{\boldsymbol{\gamma}}$ its conductivity tensor, Γ_E the earth surface, $\boldsymbol{n}_E$ its normal exterior unit field and Γ the earthing electrode surface [5]. The solution to this problem gives the potential V and the current density $\boldsymbol{\sigma}$ at an arbitrary point $\boldsymbol{x}$ when the earthing electrode is energized to potential V_Γ (Ground Potential Rise or GPR) with respect to remote earth. On the other hand, being $\boldsymbol{n}$ the normal exterior unit field to Γ, the leakage current density σ at an arbitrary point on the earthing electrode surface, the total surge current I_Γ leaked into the earth and the equivalent resistance R_{eq} of the earthing system can be written as

$$\sigma = \boldsymbol{\sigma}^t \boldsymbol{n}, \qquad I_\Gamma = \int\!\!\int_\Gamma \sigma \; d\Gamma, \qquad R_{eq} = \frac{V_\Gamma}{I_\Gamma}. \tag{2}$$

For most practical purposes [3], the soil conductivity tensor $\underset{\sim}{\boldsymbol{\gamma}}$ can be replaced by a meassured apparent scalar conductivity γ (hypothesis of homogeneus and isotropic soil). Since V and $\boldsymbol{\sigma}$ are proportional to the GPR value, the normalized boundary condition $V_\Gamma = 1$ is not restrictive at all. Further practical simplifications (flat earth surface) allow to rewrite problem (1) as a Dirichlet Exterior Problem [4,5].

In most of cases, the earthing electrode consist of a number of interconnected bare cylindrical conductors, horizontally buried and supplemented by a number of vertical rods, which lenght/diameter ratio uses to be relatively high ($\approx 10^3$). Because of this kind of geometries, analytical solutions to problem (1) can not be derived for practical cases. On the other hand, standard numerical techniques (such as Finite Differences or Finite Elements) require discretization of domain E, which leads to unacceptable memory storage and computing time.

Since computation of potential is only required on the earth surface and the equivalent resistance can be easily obtained in terms of the leakage current (2), we turn our attention to a Boundary Element approach, which would only require discretization of the earthing grid surface Γ.

VARIATIONAL STATEMENT OF THE PROBLEM

Further analytical work [5,6,7] on problem statement (1) allow to express the potential V at an arbitrary point $\boldsymbol{x}$ on the earth E in terms of the leakage current density σ, in the integral form

$$V(\boldsymbol{x}) = \frac{1}{4\pi\gamma} \int\!\!\int_{\boldsymbol{\xi}\in\Gamma} k(\boldsymbol{x}, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) \; d\Gamma, \tag{3}$$

$$k(\boldsymbol{x}, \boldsymbol{\xi}) = \left(\frac{1}{r(\boldsymbol{x}, \boldsymbol{\xi})} + \frac{1}{r(\boldsymbol{x}, \boldsymbol{\xi}')}\right), \qquad r(\boldsymbol{x}, \boldsymbol{\xi}) = |\boldsymbol{x} - \boldsymbol{\xi}|, \tag{4}$$

where $\boldsymbol{\xi}'$ is the symmetric of $\boldsymbol{\xi}$ with respect to the earth surface.

Since this expression holds on Γ, the boundary condition $V_\Gamma = 1$ leads to a Fredholm integral equation of the first kind with quasi-singular kernel

(4), which solution is the unknown leakage current density function $\sigma(\boldsymbol{\xi})$. Moreover, this problem can be written in the weaker variational form:

$$\int\!\!\int_{\boldsymbol{\chi}\in\Gamma} w(\boldsymbol{\chi})\ (V(\boldsymbol{\chi})-1)\ d\Gamma\ =\ 0 \tag{5}$$

for all members $w(\boldsymbol{\chi})$ of a suitable class of test functions on Γ.

BOUNDARY ELEMENT APPROACH

For given sets of 2D boundary elements $\{\Gamma^\alpha; \alpha = 1,\dots,\mathcal{M}\}$, and trial functions $\{N_i(\boldsymbol{\xi}); i = 1,\dots,\mathcal{N}\}$ defined on Γ, the earthing electrode surface Γ and the leakage current density σ can be discretized as

$$\Gamma = \bigcup_{\alpha=1}^{\mathcal{M}} \Gamma^\alpha, \qquad \sigma(\boldsymbol{\xi}) = \sum_{i=1}^{\mathcal{N}} \sigma_i\, N_i(\boldsymbol{\xi}), \qquad \boldsymbol{\xi}\in\Gamma, \tag{6}$$

which leads to the following discretized form of (3):

$$V(\boldsymbol{x}) = \sum_{i=1}^{\mathcal{N}}\sum_{\alpha=1}^{\mathcal{M}} \sigma_i\, V_i^\alpha(\boldsymbol{x}); \qquad V_i^\alpha(\boldsymbol{x}) = \frac{1}{4\pi\gamma}\int\!\!\int_{\boldsymbol{\xi}\in\Gamma^\alpha} k(\boldsymbol{x},\boldsymbol{\xi})\, N_i(\boldsymbol{\xi})\ d\Gamma. \tag{7}$$

Finally, for a given set $\{w_j(\boldsymbol{\chi}); j = 1,\dots,\mathcal{N}\}$ of test functions defined on Γ, variational statement (5) is reduced to the linear equations system

$$\sum_{i=1}^{\mathcal{N}} R_{ji}\sigma_i = \nu_j, \qquad j = 1,\dots,\mathcal{N}; \tag{8}$$

$$R_{ji} = \sum_{\beta=1}^{\mathcal{M}}\sum_{\alpha=1}^{\mathcal{M}} \int\!\!\int_{\boldsymbol{\chi}\in\Gamma^\beta} w_j(\boldsymbol{\chi})\, V_i^\alpha(\boldsymbol{\chi})\, d\Gamma; \qquad \nu_j = \sum_{\beta=1}^{\mathcal{M}} \int\!\!\int_{\boldsymbol{\chi}\in\Gamma^\beta} w_j(\boldsymbol{\chi})\, d\Gamma. \tag{9}$$

It can be easily verified that the $\mathcal{N}\times\mathcal{N}$ matrix in (8) is not sparse. In addition, computation of coefficients R_{ji} in (9) requires integration on a 4D domain, since 2D integration must be performed twice over the electrode surface [8]. Again we must introduce some additional simplifications in order to reduce computational cost under acceptable levels.

APPROXIMATED 1D VARIATIONAL STATEMENT

With this scope, it seems reasonable to consider that the leakage current density is constant around the cross section of the cylindrical electrode [8,9]. This hypothesis is widely used in most of the practical methods related in the literature [1,2,3], and seems not restrictive whatsoever if we take into account the real geometry of grounding grids.

Let L be the whole set of axial lines of the buried conductors, and let $\widehat{\boldsymbol{\xi}} \in L$ be the orthogonal projection of a generic point $\boldsymbol{\xi} \in \Gamma$. Let $\phi(\widehat{\boldsymbol{\xi}})$ be the conductor diameter, and let $\widehat{\sigma}(\widehat{\boldsymbol{\xi}})$ be the approximated leakage current density at this point (assumed uniform around the cross section). In this terms we can write expression (3) in the form

$$\widehat{V}(\boldsymbol{x}) = \frac{1}{4\gamma} \int_{\widehat{\boldsymbol{\xi}} \in L} \phi(\widehat{\boldsymbol{\xi}})\, \bar{k}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}}) \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\, dL, \tag{10}$$

being $\bar{k}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})$ the average of kernel (4) around cross section at $\widehat{\boldsymbol{\xi}}$ [8,9].

Because the leakage current is not really uniform around the cross section, variational equality (5) does not hold anymore if we use expression (10) instead of (3). Therefore, we must restrict the class of trial functions to those with circumferential uniformity, obtaining

$$\frac{1}{4\gamma} \int_{\widehat{\boldsymbol{\chi}} \in L} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}(\widehat{\boldsymbol{\chi}}) \left[\int_{\widehat{\boldsymbol{\xi}} \in L} \phi(\widehat{\boldsymbol{\xi}})\, \bar{\bar{k}}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})\, \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\, dL \right] dL = \int_{\widehat{\boldsymbol{\chi}} \in L} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}(\widehat{\boldsymbol{\chi}})\, dL, \tag{11}$$

for all members $\widehat{w}(\widehat{\boldsymbol{\chi}})$ of a suitable class of test functions on L, being $\bar{\bar{k}}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$ the average of kernel (4) around cross sections at $\widehat{\boldsymbol{\chi}}$ and $\widehat{\boldsymbol{\xi}}$ [8,9].

APPROXIMATED 1D BOUNDARY ELEMENT APPROACH

For given sets of 1D boundary elements $\{L^\alpha; \alpha = 1, \ldots, m\}$, and trial functions $\{\widehat{N}_i(\widehat{\boldsymbol{\xi}}); i = 1, \ldots, n\}$ defined on L, the whole set of axial lines of the buried conductors L and the unknown leakage current density $\widehat{\sigma}$ can be discretized as

$$L = \bigcup_{\alpha=1}^{m} L^\alpha, \qquad \widehat{\sigma}(\widehat{\boldsymbol{\xi}}) = \sum_{i=1}^{n} \widehat{\sigma}_i\, \widehat{N}_i(\widehat{\boldsymbol{\xi}}), \tag{12}$$

which leads to the following discretized form of (10)

$$\widehat{V}(\boldsymbol{x}) = \sum_{i=1}^{n} \sum_{\alpha=1}^{m} \widehat{\sigma}_i\, \widehat{V}_i^\alpha(\boldsymbol{x}); \qquad \widehat{V}_i^\alpha(\boldsymbol{x}) = \frac{1}{4\gamma} \int_{\widehat{\boldsymbol{\xi}} \in L^\alpha} \phi(\widehat{\boldsymbol{\xi}})\, \bar{k}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}}) \widehat{N}_i(\widehat{\boldsymbol{\xi}})\ dL. \tag{13}$$

Finally, for a given set $\{\widehat{w}_j(\widehat{\boldsymbol{\chi}}); j = 1, \ldots, n\}$ of test functions defined on L, variational statement (11) is reduced to the linear equations system

$$\sum_{i=1}^{n} \widehat{R}_{ji} \widehat{\sigma}_i = \widehat{\nu}_j, \quad j = 1, \ldots, n; \quad \widehat{\nu}_j = \sum_{\beta=1}^{m} \int_{\widehat{\boldsymbol{\chi}} \in L^\beta} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}_j(\widehat{\boldsymbol{\chi}})\ dL, \tag{14}$$

$$\widehat{R}_{ji} = \frac{1}{4\gamma} \sum_{\beta=1}^{m} \sum_{\alpha=1}^{m} \int_{\widehat{\boldsymbol{\chi}} \in L^\beta} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}_j(\widehat{\boldsymbol{\chi}}) \left[\int_{\widehat{\boldsymbol{\xi}} \in L^\alpha} \phi(\widehat{\boldsymbol{\xi}})\, \bar{\bar{k}}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})\, \widehat{N}_i(\widehat{\boldsymbol{\xi}})\, dL \right] dL, \tag{15}$$

SIMPLIFIED 1D BOUNDARY ELEMENT APPROACH

Extensive computing is still required to evaluate the averaged kernels $\bar{k}(\boldsymbol{x},\widehat{\boldsymbol{\xi}})$ and $\bar{\bar{k}}(\widehat{\boldsymbol{\chi}},\widehat{\boldsymbol{\xi}})$ by means of circumferential integration around cross sections at points $\widehat{\boldsymbol{\xi}}$ and $\widehat{\boldsymbol{\chi}}$. The circumferential integration can be avoided by means of the following approximations [8,9]:

$$\bar{k}(\boldsymbol{x},\widehat{\boldsymbol{\xi}}) \approx \left(\frac{1}{\widehat{r}(\boldsymbol{x},\widehat{\boldsymbol{\xi}})}+\frac{1}{\widehat{r}(\boldsymbol{x},\widehat{\boldsymbol{\xi}}')}\right), \qquad \bar{\bar{k}}(\widehat{\boldsymbol{\chi}},\widehat{\boldsymbol{\xi}}) \approx \left(\frac{1}{\widehat{\widehat{r}}(\widehat{\boldsymbol{\chi}},\widehat{\boldsymbol{\xi}})}+\frac{1}{\widehat{\widehat{r}}(\widehat{\boldsymbol{\chi}},\widehat{\boldsymbol{\xi}}')}\right); \tag{16}$$

$$\widehat{r}(\boldsymbol{x},\widehat{\boldsymbol{\xi}}) = \sqrt{\left|\boldsymbol{x}-\widehat{\boldsymbol{\xi}}\right|^2+\frac{\phi^2(\widehat{\boldsymbol{\xi}})}{4}}, \quad \widehat{\widehat{r}}(\widehat{\boldsymbol{\chi}},\widehat{\boldsymbol{\xi}}) = \sqrt{\left|\widehat{\boldsymbol{\chi}}-\widehat{\boldsymbol{\xi}}\right|^2+\frac{\phi^2(\widehat{\boldsymbol{\xi}})+\phi^2(\widehat{\boldsymbol{\chi}})}{4}}.$$

Now, for specific choices of the trial and test functions we obtain different formulations. The simplest of these can be identified with widespread previous methods based on intuitive ideas, such as superposition of punctual current sources and error averaging [1,2,3]. Thus, for constant leakage current elements, a Galerkin type choice for the trial functions lead to the Average Potential Method [1,2,3].

The unrealistic results [3] obtained with this kind of methods when segmentation of conductors is increased can be mathematically explained, since approximations (16) loose accuracy when the size of the elements is in the order of magnitude of the diameter of the bars, and linear system (14) becomes ill-conditioned. However, results obtained for low and medium levels of discretization have been proved to be sufficiently accurate for practical purposes [8,9]. On the other hand, ends and junctions of conductors are not taken into account in this formulation. Thus, slightly anomalous local effects are expected at these points, but global results should not be noticeably affected.

Computation of remaining integrals in (13) and (15) is not obvious. The cost of numerical integration is out of range, due to the undesirable behaviour of the integrands. For this reason, it has been necessary to derive specific analytical integration techniques [11].

For Galerkin type formulations the matrix of coefficients in (14) is symmetric and positive definite [10]. Because of this fact, a conjugate gradient method can be used, and the non-sparsity of the matrix can be partially overcome. At the same time, linear and parabolic leakage current elements allow to increase accuracy, and large problems can be solved with acceptable computing requirements [8,9,11].

CONCLUSIONS

A Boundary Element approach for the analysis of substation earthing systems has been presented. Some reasonable assumptions allow to reduce a general 2D BEM formulation to a simplified 1D version for practical problems. By means of new analytical integration techniques, accurate results can be obtained in real cases with acceptable computing requirements. This formulation has been implemented in a Computer Aided Design System developed during the last few years.

This approach has been applied to a real case: the *E. Balaidos II* substation grounding (close to the city of Vigo, Spain). The plan and characteristics are presented in Figure 1. Results are given in Figure 2. Each bar was discretized in one single parabolic element. The model (174 elements and 315 degrees of freedom) required only 94 seconds of cpu time on a Vax-4300 computer. At the scale of the whole grid, results are not noticeably improved by increasing discretization.

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DATA

Earth Resistivity:	$6,000\ \Omega cm$
Ground Potential Rise:	$1\ V$
Installation Depth:	$0.800\ m$
Number of Horizontal Bars:	107
Horizontal Bar Diameter:	$1.285\ cm$
Number of Vertical Bars:	67
Vertical Bar Diameter:	$1.400\ cm$
Vertical Bar Length:	$1.500\ m$

1D BEM MODEL

Type of Elements:	Parabolic
Number of Nodes:	315
Number of Elements:	174

Vertical bars marked with black points

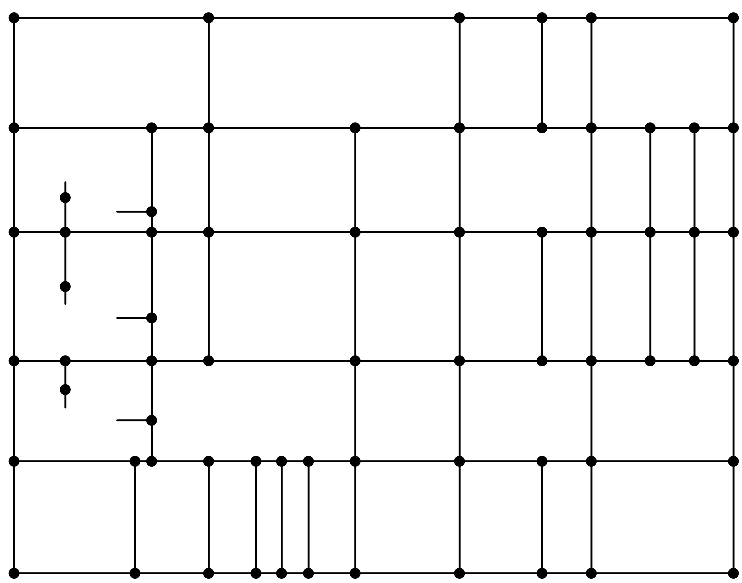

Figure 1.—E. Balaidos II Grid: Plan (Scale=1:1000), Problem Characteristics and Numerical Model (1 Parabolic Element per bar).

RESULTS

Fault Current:	2.46462 A
Equivalent Resistance:	0.40574 Ω
CPU Time:	94 sec
Computer:	VAX–4300

Surface potential contours plotted every 0.02 V. Thick contours every 0.10 V.

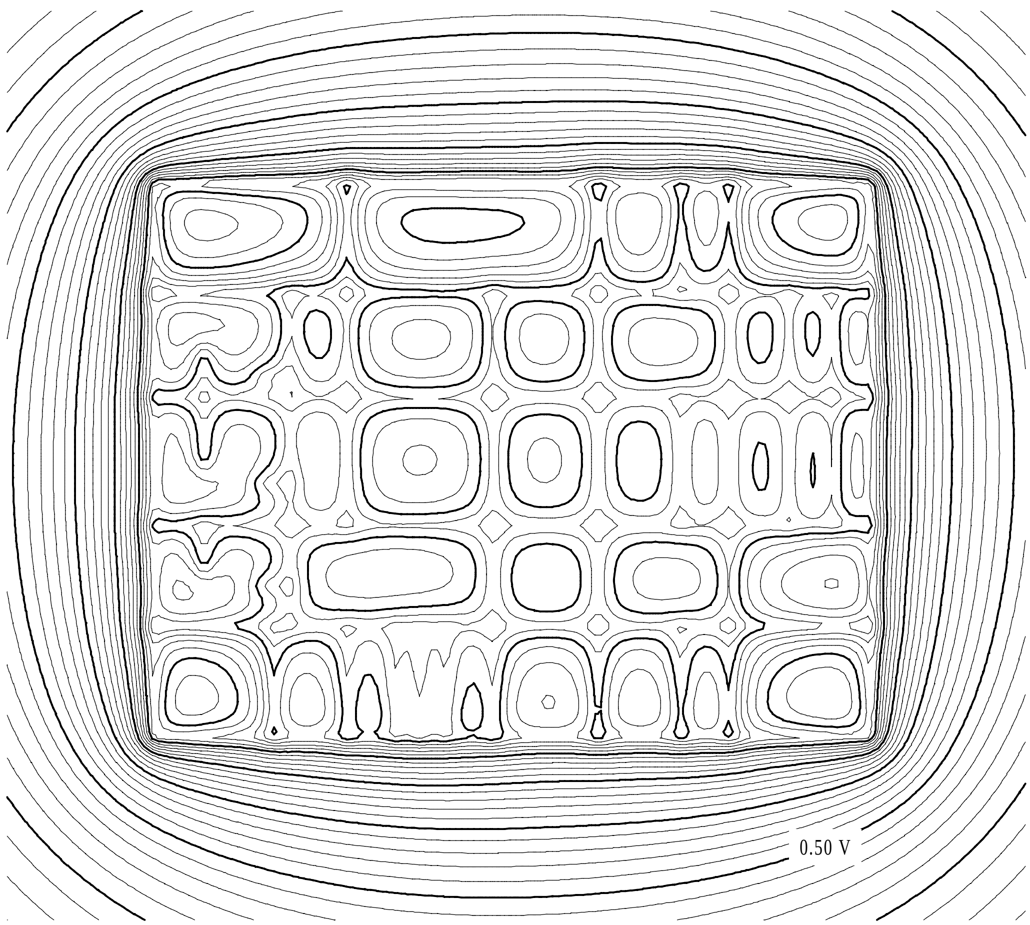

Figure 2.—E. Balaidos II Grid: Results obtained by BEM (1 parabolic element per bar). Ground surface potential distribution.