Preprint of the paper

"A Numerical Formulation for Grounding Analysis in Stratified Soils"
I. Colominas, F. Navarrina, M. Casteleiro (2002)
IEEE Transactions on Power Delivery.

http://caminos.udc.es/gmni

A Numerical Formulation for Grounding Analysis in Stratified Soils

I. Colominas, F. Navarrina, M. Casteleiro

Abstract— The design of safe grounding systems in electrical installations is essential to assure the security of the persons, the protection of the equipment and the continuity of the power supply. In order to achieve these goals, it is necessary to compute the equivalent electrical resistance of the system and the potential distribution on the earth surface when a fault condition occurs.

In this paper we present a formulation for the analysis of grounding systems embedded in stratified soils, on the basis of the Boundary Element Method (BEM). Suitable arrangements of the final discretized equations allow to use the highly efficient analytical integration techniques derived by the authors for grounding systems buried in uniform soils. The feasibility of this approach is demonstrated by applying the BEM formulation to the analysis of a real grounding system with a two-layer soil model.

Keywords— Grounding analysis, Layered soils, BEM formulation.

I. Introduction

THE main objective of a grounding system is to provide means to dissipate electrical currents into the ground, in order to guarantee the continuity of the power supply and the integrity of the equipment, and to ensure that a person in the vicinity of the grounded installation is not exposed to a critical electrical shock. To achieve these targets, the apparent electrical resistance of the grounding system must be low enough to guarantee that fault currents dissipate mainly through the earthing electrode into the ground, while the potential gradients between close points that can be connected by a person must be kept under certain maximum safe limits [1], [2].

Since the sixties, several methods and procedures for the analysis and design of grounding systems of electrical substations have been proposed, most of them based on practice, on semi-empirical works or on intuitive ideas [2], [3], [4], [5]. Although these techniques represented an important improvement in the grounding analysis area, some problems have been reported such as large computational requirements, unrealistic results when segmentation of conductors is increased, and uncertainty in the margin of error [1], [6].

In the last years we have developed a general BEM formulation for grounding analysis in uniform soils [7], [8]. Several widespread intuitive methods and techniques can be identified as the result of introducing suitable assumptions in this general BEM approach in order to reduce the computational cost for a specific selection of test and trial functions. Furthermore, it has been possible to explain

Ignasi Colominas, Fermín Navarrina and Manuel Casteleiro work at the Department of Applied Mathematics, Civil Engineering School, University of La Coruña, Campus de Elviña. 15071 La Coruña, SPAIN. (e-mail: colominas@iccp.udc.es)

from a mathematical point of view the anomalous asymptotic behaviour of this kind of methods, and to point out the sources of error [8]. Finally, more efficient and accurate numerical formulations have been derived from this BEM approach. These formulations have been implemented in a CAD system for earthing analysis, and have been successfully applied (with a very reasonable computational cost) to large grounding systems in real cases [9].

II. Mathematical model of the physical problem

A. General statement of the problem

The physical phenomena that underlies the fault current dissipation into the ground can be studied by means of Maxwell's Electromagnetic Theory [10]. Thus, if one constrains the analysis to the obtention of the electrokinetic steady-state response and one neglects the inner resistivity of the grounding electrode (therefore, potential can be assumed constant in every point of the surface of the electrode), the 3D problem can be written as

$$\mathbf{div}(\boldsymbol{\sigma}) = 0, \quad \boldsymbol{\sigma} = -\boldsymbol{\gamma}\,\mathbf{grad}(V) \text{ in } E ;$$
$$\boldsymbol{\sigma}^t \boldsymbol{n}_E = 0 \text{ in } \Gamma_E ; \quad V = V_\Gamma \text{ in } \Gamma ;$$
$$V \to 0, \text{ if } |\boldsymbol{x}| \to \infty; \tag{1}$$

being E the earth, $\boldsymbol{\gamma}$ its conductivity tensor, Γ_E the earth surface, $\boldsymbol{n}_E$ its normal exterior unit field and Γ the electrode surface [8]. Therefore, when the earthing electrode attains a voltage V_Γ (Ground Potential Rise, or GPR) relative to a distant grounding point, the solution to problem (1) gives potential V and current density $\boldsymbol{\sigma}$ at an arbitrary point $\boldsymbol{x}$.

Furthermore, other essential parameters for grounding design, such as the leakage current density σ at an arbitrary point of the electrode surface, the total surge current I_Γ that flows into the ground, and the equivalent resistance of the earthing system R_{eq} —apparent electrical resistance of the earth-electrode circuit— can be easily obtained in terms of V and $\boldsymbol{\sigma}$:

$$\sigma = \boldsymbol{\sigma}^t \boldsymbol{n}, \qquad I_\Gamma = \iint_\Gamma \sigma \, d\Gamma, \qquad R_{eq} = \frac{V_\Gamma}{I_\Gamma}, \tag{2}$$

being $\boldsymbol{n}$ the normal exterior unit field to Γ. On the other hand, since V and $\boldsymbol{\sigma}$ are proportional to the GPR value, the normalized boundary condition $V_\Gamma = 1$ is not restrictive at all [8], and it will be used from here on.

B. Statement of the grounding problem with a multilayer soil model

Most of methods proposed up to this moment are based on the assumption that soil can be considered homogeneous

and isotropic. Hence, the conductivity tensor $\boldsymbol{\gamma}$ is substituted by an apparent scalar conductivity γ that can be experimentally obtained [1], [2]. It is widely accepted that this hypothesis does not introduce significant errors if the soil is basically uniform —both in horizontal and vertical direction— up to a distance of approximately three to five times the diagonal dimension of the grounding grid, measured from its edge. Furthermore, this uniform soil model can also be used but with less accuracy, if the resistivity varies slightly with depth [1]. Nevertheless, parameters involved in the design of grounding systems can significantly change as soil conductivity varies through the substation site. Thus, it is advisable to develop more accurate models to take into account variations of soil conductivity in the surroundings of the grounding site.

At this point, it is obvious that the development of models describing all variations of the soil conductivity in the vicinity of a grounding system would never be affordable, neither from the economical nor from the technical point of view. A more practical approach (and still quite realistic when conductivity is not markedly uniform with depth) consists of considering the soil stratified in a number of horizontal layers. Then, each layer is defined by an appropriate thickness and an apparent scalar conductivity that must be experimentally obtained. In fact, it is widely accepted that two-layer (or even three-layer) soil models should be sufficient to obtain good and safe designs of grounding systems in most practical cases [1], [11], [12].

If one considers that the soil is formed by C horizontal layers (each one with a different conductivity) and the grounding electrode is buried in the upper layer, mathematical problem (1) can be written in terms of the following Neumann exterior problem

$$
\begin{aligned}
\Delta V_1 = 0 \ \ \text{in} \ \ E_1; \ \ \dots \ ; \Delta V_C = 0 \ \ \text{in} \ \ E_C; \\
V_1 = V_2, \ \ \text{in} \ \ \Gamma_{(1,2)}; \ \ \dots \ ; V_{C-1} = V_C, \ \ \text{in} \ \ \Gamma_{(C-1,C)}; \\
\gamma_1 \frac{dV_1}{dn} = \gamma_2 \frac{dV_2}{dn} \ \ \text{in} \ \ \Gamma_{(1,2)}; \ \ \dots \ ; \\
\gamma_{C-1} \frac{dV_{C-1}}{dn} = \gamma_C \frac{dV_C}{dn} \ \ \text{in} \ \ \Gamma_{(C-1,C)}; \\
\frac{dV_1}{dn} = 0 \ \ \text{in} \ \ \Gamma_E; \ \ V_1 = 1 \ \ \text{in} \ \ \Gamma; \\
V_1 \longrightarrow 0, \ \ \dots \ , V_C \longrightarrow 0, \ \ \text{if} \ \ |\boldsymbol{x}| \to \infty; \qquad (3)
\end{aligned}
$$

being E_c each one of the soil layer $(c = 1, C)$, $\Gamma_{(c-1,c)}$ the interphase between whatever two layers $(c-1$ and $c)$, γ_c the scalar conductivity of layer c, and V_c the potential at every point of layer c [13], [14], [15], as it is shown in figure (1). Obviously, if the grounding electrode is buried in any other layer (i.e. in layer b, $V_b = 1$ in Γ), the statement of the exterior problem is analogous to (3) [15].

Grounding systems in most of real electrical substations consist of a mesh of interconnected cylindrical conductors, horizontally buried and supplemented by ground rods vertically thrust in specific places of the installation site. The ratio between the diameter and the length of the electrodes is usually small ($\sim 10^{-3}$). Obviously, in problems with this kind of geometries, it is not possible to obtain analytical solutions to problem (3), and the use of widespread numerical techniques —such as Finite Differences (FD) or
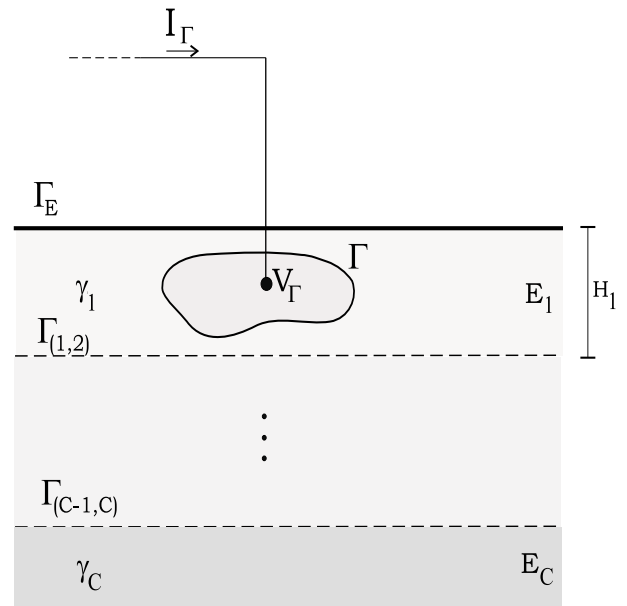

Fig. 1. Fault current disipation in a stratified soil model.

the Finite Element Method (FEM) [16]— that require the discretization of the 3D domains E_c, should involve a completely out of range computing effort [8]. Recently other numerical techniques based on meshless methods have been proposed for grounding analysis [17], [18]; these works are under development at the present moment, and their applications are still restricted to the solution of academical problems and numerical tests.

At this point, it is important to remark that computing the potential distribution is only required on the earth surface Γ_E, since from this distribution one can obtain all the other important grounding design parameters such as the "mesh", "step" or "touch" voltages [1]. Furthermore, the equivalent electrical resistance of the earthing system (R_{eq}) and the total surge current (I_Γ) can be easily obtained from the leakage current density σ that leaks at every point of the electrode surface Γ by means of (2).

Taking into account all these facts, we work to achieve an equivalent expression to problem (3) in terms of the unknown leakage current density function σ on the boundary Γ. Thus, a boundary element approach for this equivalent problem would only require the discretization of the grounding surface Γ [8], avoiding the discretization of the whole domain (i.e., the earth).

If one takes into account that the surroundings of the substation site are levelled and regularized during its construction —i.e., the earth surface Γ_E and the interphase between soil layers $\Gamma_{(c-1,c)}$ can be assumed horizontal [8], [15]—, the application of the "method of images" allows to rewrite (3) in terms of a Dirichlet exterior problem [13], [15], [19], [20]. Next, if the Green's Identity is applied to this problem [20], we obtain the following integral expressions for potential $V_c(\boldsymbol{x}_c)$ at an arbitrary point $\boldsymbol{x}_c \in E_c$ $(c = 1, C)$, in terms of the leakage current density $\sigma(\boldsymbol{\xi})$ at every point $\boldsymbol{\xi}$ of the electrode surface Γ, which is buried in

the upper layer (layer # 1):

$$V_c(\boldsymbol{x}_c) = \frac{1}{4\pi\gamma_1} \iint_{\boldsymbol{\xi}\in\Gamma} k_{1c}(\boldsymbol{x}_c, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma,$$
$$\forall \boldsymbol{x}_c \in E_c; \quad (c = 1, C); \tag{4}$$

where the integral kernel $k_{1c}(\boldsymbol{x}_c, \boldsymbol{\xi})$ is formed by an infinite series of terms corresponding to the resultant images obtained when Neumann exterior problem (3) is transformed into a Dirichlet one [13]. This weakly singular kernel depends on the inverse of the distances from the point $\boldsymbol{x}_c$ to the point $\boldsymbol{\xi}$ and to all the symmetric points of $\boldsymbol{\xi}$ (its images) with respect to the earth surface Γ_E and to the interphases $\Gamma_{(c-1,c)}$ between layers. Therefore, this kernel depends on the thickness and conductivity of each layer [13], [15].

On the other hand, if the grounding electrode is buried in any other layer (i.e., layer b), the application of Green's Identity yields to an expression that is analogous to (4), in terms of the integral kernel $k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})$:

$$V_c(\boldsymbol{x}_c) = \frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\xi}\in\Gamma} k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma,$$
$$\forall \boldsymbol{x}_c \in E_c; \quad (c = 1, C). \tag{5}$$

Although generation of electrical images in a general case is a conceptually simple well-known process [19], the final expression of the integral kernels $k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})$ can be very complicated, and its evaluation in practice may require a high computing effort. As an example, explicit expressions of the integral kernels for a two-layer soil model can be found in Appendix I.

C. Variational statement of the problem

On the other hand, the expression for potential (5) also holds on the grounding electrode surface Γ, where potential is given by the boundary condition ($V_b = 1$ in Γ). Therefore, the leakage current density σ must satisfy the Fredholm integral equation of the first kind defined on Γ :

$$1 - \frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\xi}\in\Gamma} k_{bb}(\boldsymbol{\chi}, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi})\, d\Gamma = 0, \quad \boldsymbol{\chi} \in \Gamma. \tag{6}$$

On the basis of the weighted residuals concept it must be clear that we can substitute this equation by the weaker variational form

$$\iint_{\boldsymbol{\chi}\in\Gamma} w(\boldsymbol{\chi}) \left(1 - \frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\xi}\in\Gamma} k_{bb}(\boldsymbol{\chi}, \boldsymbol{\xi}) \sigma(\boldsymbol{\xi})\, d\Gamma \right) d\Gamma = 0, \tag{7}$$

which must hold for all members $w(\boldsymbol{\chi})$ of a suitable class of weight functions defined on the surface Γ [8], [20]. At this point, it should be obvious that a boundary element formulation seems to be the right choice to solve the above stated equation.

The essential concept of methods like BEM (and FEM) is quite simple: since the exact solution $\sigma(\boldsymbol{\xi})$ to our problem (7) is unknown, we pursue to approximate it by means of numerical techniques in a finite-dimensional subspace. Thus, for a certain set of N so-called trial functions

$\{N_i(\boldsymbol{\xi})\}$, we consider discretized approximations type

$$\sigma(\boldsymbol{\xi}) \approx \sum_{i=1}^{N} N_i(\boldsymbol{\xi})\, \sigma_i. \tag{8}$$

Namely, the unknown values of the N variables $\{\sigma_i\}$ must be determined in such a way that the corresponding discretized approximation (8) is as close as possible to the exact solution $\sigma(\boldsymbol{\xi})$. Because of practical reasons, the solution domain (the electrode surface Γ) is normally divided into a finite number of subdomains (boundary elements), and the trial functions are normally chosen as piecewise functions defined over these subdomains. Since the exact solution will not be included (as a general rule) in the approximating subspace, equation (7) can not be forced to hold for any weight function $w(\boldsymbol{\chi})$. However, as it is shown in Appendix II, we can force this equation to hold for a set of N so-called test functions $\{w_j(\boldsymbol{\chi})\}$. This approach yields a system on linear equations type

$$\sum_{i=1}^{N} [R_{ji}]\, \sigma_i = \nu_j, \quad j = 1, \dots, N, \tag{9}$$

which solution gives the numerical solution (8) to our problem. The elements R_{ji} of the coefficients matrix, and the right-hand-side terms ν_j must be computed, and depend on the geometry of the electrode, and on the thickness and respective conductivities of the layers. The selection of different sets of trial and test functions leads to specific numerical approaches. The discussion and examples presented in this paper are restricted to Galerkin type weighting approaches (test functions are identical to trial functions). Thus, the coefficients matrix in linear system (9) is symmetric and positive definite [21].

TABLE I

Santiago II grounding system: Characteristics and numerical model

Data	
Number of electrodes:	534
Number of ground rods:	24
Diameter of electrodes:	11.28 mm
Diameter of ground rods:	15.00 mm
Depth of the grid:	0.75 m
Length of ground rods:	4 m
Max. dimensions of grid:	230×195 m^2
GPR:	10 kV
BEM Numerical Model	
Type of approach:	Galerkin
Type of 1D element:	Linear
Number of elements:	582
Degrees of freedom:	386

In Appendix II, we summarize our BEM numerical approach for grounding analysis. First of all, we develop a general formulation for solving variational form (7). As

stated before, grounding grids in most of electrical substations consist of a mesh of interconnected cylindrical electrodes which length is huge in comparison with its diameter. Thus, taking into account this specific geometry, we finally derive an approximated but more convenient formulation which allows to solve practical problems with an acceptable computational cost. We remark that the whole development is an extension to the one involved for uniform soil models, what can be found completely explicit in [8], [20].
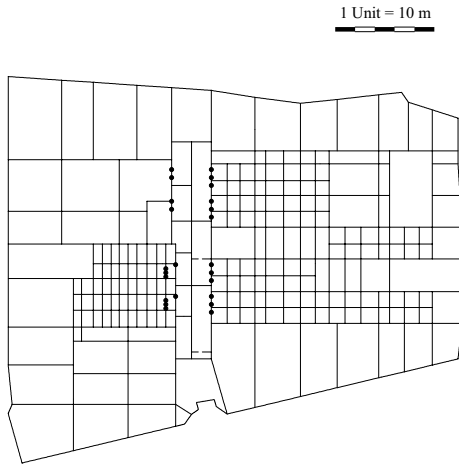

Fig. 2. Santiago II grounding system: Plan of the earthing grid (the situation of the ground rods is marked with black points).

The final layout of the numerical procedure to be implemented reminds the so-called "computer methods" for grounding analysis [6], where R_{ji} coefficients correspond to "mutual and self resistances" between segments of conductors. In fact, some particular cases of our BEM approach (e.g., a Point Collocation scheme using constant leakage current elements) can be identified with any of the very early intuitive methods that were proposed in the sixties on the basis of replacing each segment of electrode by an 'imaginary sphere'. In the case of a Galerkin type weighting with constant leakage current elements, the numerical approach can be identified with a kind of more recent methods, like the Average Potential Method [3], in which each segment of electrode is substituted by 'a line of point sources over the length of the conductor' [8], [6], [20]. Thus, in the framework of our BEM approach it is possible to explain from a mathematical point of view [8] the problems encountered by other authors with the application of these widespread methods [6], while new more efficient and accurate numerical formulations can be derived [20].

Obviously, this BEM formulation could be applied to other cases with a higher number of layers. However, it is important to take into account that the CPU time may increase considerably, mainly because of two facts that are not strictly related to the boundary element approach: first, the complexity of the kernels in the integral expression of potential (4) obtained by application of the method of images (usually a multiple infinite series); and second, the number of terms of these series that it is necessary to
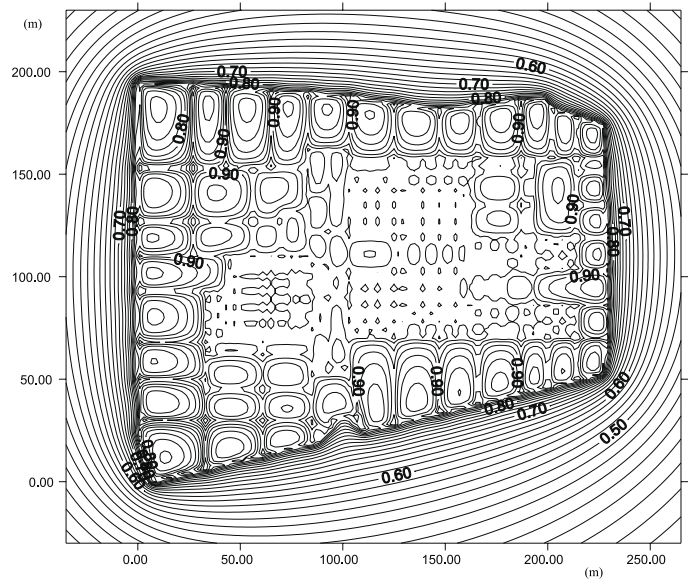

Fig. 3. Santiago II grounding system: Potential distribution ($\times 10$ kV) on ground surface obtained with a homogeneous and isotropic soil model.
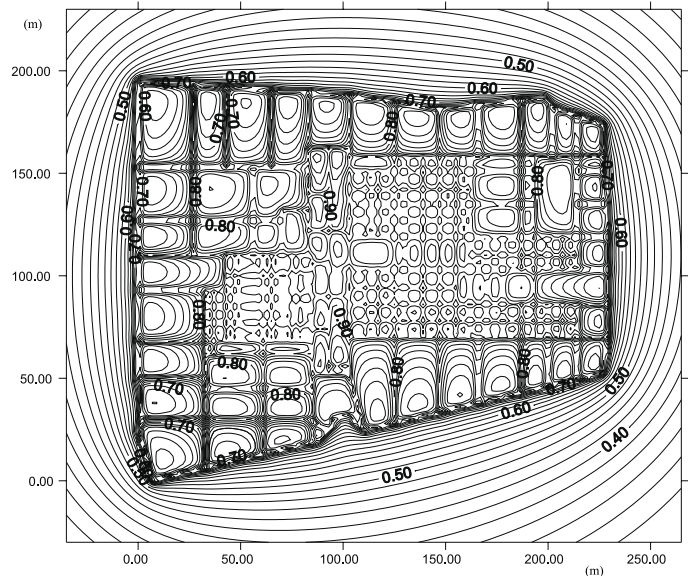

Fig. 4. Santiago II grounding system: Potential distribution ($\times 10$ kV) on ground surface obtained with a two layer soil model.

evaluate specially when conductivities between layers are very different.

III. APPLICATION OF THE BEM APPROACH TO A REAL GROUNDING SYSTEM: EXAMPLES AND DISCUSSION

Our BEM numerical approach has been applied to the grounding analysis of a real electrical installation: the Santiago II substation, close to the city of *Santiago de Compostela* in Spain. This earthing system is formed by a grid of 534 cylindrical conductors of the same diameter (11.28 mm) buried to a depth of 75 cm, supplemented with 24 ground rods of the same length (4 m) and diameter (15 mm).

One layer soil model	
Earth resistivity:	60 Ωm
Total current:	6.73 kA
Equivalent resistance:	0.149 Ω
CPU time (AXP 4000):	7.7 s.
Two layer soil model	
Upper layer resistivity:	200 Ωm
Lower layer resistivity:	60 Ωm
Thickness upper layer:	1.2 m
Total current:	5.61 kA
Equivalent resistance:	0.178 Ω
CPU time (AXP 4000):	13.35 min.

The grounding system protects a total area of 38,000 m^2. The studied area is a wider superimposed rectangular zone of 300×260 m^2 (i.e., 78,000 m^2). The Ground Potential Rise (GPR) considered in this study is 10 kV. The plan of the earthing grid (see figure 2) and the general data (see table I) were obtained from the grounding plans and specifications of the substation provided by the power company. The characteristics of the numerical model that has been used in this example can be found in table I.

Table II compares the numerical results (the equivalent resistance and the total electrical current leaked into the ground) of the analysis of the Santiago II grounding system obtained by using the homogeneous and isotropic soil model ("one layer soil model"), and the proposed two-layer soil model.

Figures 3 and 4 show the potential distributions on the earth surface (when the grounding electrode attains the GPR voltage) obtained by using the homogeneous and isotropic soil model and the proposed two-layer soil model. It is obvious that both potential distributions differ. However, it is known that noticeably different contour line drawings do not necessarily correspond to significant differences between the plotted results. For this reason, in figure 5, we compare the potential profiles computed with the two soil models along two different lines on the ground surface.

We remark that the analysis of this grounding system with the two-layer soil model is particularly difficult because the length of the ground rods (4 m) is higher than the height of the upper layer (0.75 m). Consequently, a part of the grid is buried in the upper layer while the other part is buried in the lower one. In cases like this, the final implementation of the numerical approach in a computer aided design system must be done with care, in order to apply properly the different expressions depending on the situation of the electrodes [15].

In the presented examples, we show that the results obtained by using a multiple-layer soil model can be noticeably different from those obtained by using a single layer soil model. Accordingly, the computed design parameters
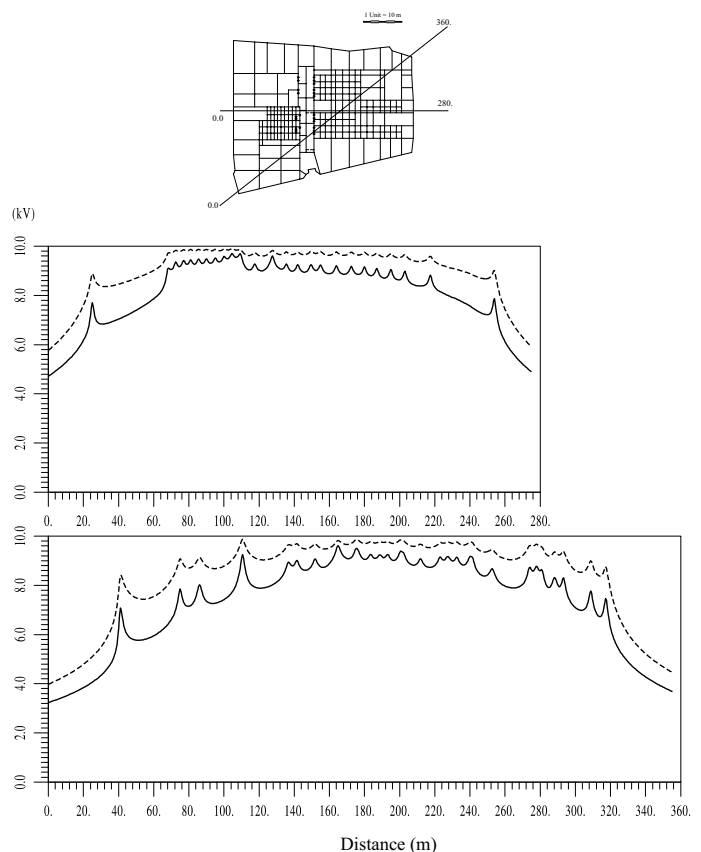

Fig. 5. Santiago II grounding system: Profiles of potential distribution on the earth surface along two lines (results obtained by using a uniform soil model are plotted in dashed style, and those obtained by using the two-layer soil model are plotted in continuous style).

of the grounding system [1], [2], [20] (such as the equivalent resistance, the touch voltage, the step voltage, the mesh voltage, etc.) do significantly vary. Therefore, it could be advisable to use efficient multi-layer soil formulations — such as the numerical approach presented in this paper— to analyze grounding systems as a general rule, in spite of the increase in the computational effort. In fact, the use of this kind of advanced models should be mandatory in cases where the conductivity of the soil changes markedly with depth.

IV. Conclusions

A Boundary Element approach for the analysis of substation earthing systems in layered soils has been presented in this paper. Taking into account the general characteristics of this kind of installations, some reasonable assumptions, and further simplifications allow to reduce the original 2D BEM approach to a much less computationally expensive 1D version. In fact, several widespread intuitive methods can be identified as particular cases of this general approach, while more efficient and accurate formulations can be derived. On the other hand, computing time can be reduced under acceptable levels by means of analytical integration techniques developed by the authors [8]. Thus, accurate results should be obtained in practical cases with

a relatively small computational cost. The proposed BEM technique has been implemented in a Computer Aided Design system developed by the authors for grounding substation design [22], extending the original capabilities of this tool (that was initially developed for uniform soil models).

The proposed approach has been applied to a practical case, and the results obtained by means of both, a single and a two-layer soil model, have been compared. As we have shown, highly accurate results can be obtained by means of the proposed techniques in the earthing analysis of real problems, and the results obtained by using a multiple-layer soil model can be noticeably different from those obtained by using a single layer soil model. Therefore, it could be advisable, or even mandatory, to use the multi-layer soil formulation as a general rule, in spite of the increase in the computational cost.

At the present moment, the study of large installations with multi-layer soil models still requires an important computing effort. In fact, single layer models can be used in real time, while the use of models with a small number of layers break off the design process (since the computing time is not contemptible) and the use of models with a high number of layers is precluded. This is due to the poor rate of convergence of the underlying series expansions, that force to evaluate a growing number of additional terms as the number of layers increases. The authors are developing new extrapollation techniques in order to accelerate the rate of convergence of the involved series expansions [15]. Thus, the proposed multi-layer BEM formulations could be used as real time design tools in a close future.

Acknowledgments

References

[1] ANSI/IEEE Std.80, *Guide for Safety in AC Substation Grounding*, IEEE, New York, (1986); and *Draft Guide for Safety in AC Substation Grounding*, IEEE, New York, (1999).

[2] J.G. Sverak *et al.*, *Safe substation grounding*, Part I: IEEE Trans. on Power Apparatus and Systems, **100**, 4281-4290, (1981); Part II: IEEE Trans. on Power Apparatus and Systems, **101**, 4006-4023, (1982)

[3] R.J. Heppe, *Computation of potential at surface above an energized grid or other electrode, allowing for non-uniform current distribution*, IEEE Trans. on Power Apparatus and Systems, **98**, 1978-1988, (1979).

[4] B. Thapar, V. Gerez, A. Balakrishnan, D.A. Blank, *Simplified equations for mesh and step voltages in AC substation*, IEEE Trans. on Power Delivery, **6**, 601-607, (1991).

[5] J.G. Sverak, *Progress in Step and Touch Voltage Equations of ANSI/IEEE Std 80. Historical Perspective*, IEEE Trans. on Power Delivery, **13**, 762-767, (1998).

[6] D.L. Garrett and J.G. Pruitt, *Problems encountered with the Average Potential Method of analyzing substation grounding systems*, IEEE Trans. on Power Apparatus and Systems, **104**, 3586-3596, (1985).

[7] I. Colominas, F. Navarrina and M. Casteleiro, *A general numerical formulation for computing and design earthing grids in large electrical substations*, (in spanish), Revista Internacional de Métodos Numéricos para Cálculo y Diseño en Ingeniería, **13**, 383–401, (1997).

[8] I. Colominas, F. Navarrina and M. Casteleiro, *A boundary element numerical approach for grounding grid computation*, Comput. Methods Appl. Mech. Engrg., **174**, 73-90, (1999).

[9] I. Colominas, F. Navarrina and M. Casteleiro, *A boundary element formulation for the substation grounding design*, Advances in Engineering Software, **30**, 693-700, (1999).

[10] E. Durand, *Électrostatique*, Masson, Paris, (1966).

[11] F.P. Dawalibi, D. Mudhekar *Optimum design of substation grounding in a two-layer earth structure*, IEEE Trans. on Power Apparatus and Systems, **94**, 252-272, (1975).

[12] H.S. Lee, J.H. Kim, F.P. Dawalibi, J. Ma *Efficient ground designs in layered soils*, IEEE Trans. on Power Delivery, **13**, 745-751, (1998).

[13] G.F. Tagg, *Earth Resistances*, Pitman Pub. Co., New York, (1964).

[14] E.D. Sunde, *Earth conduction effects in transmission systems*, McMillan, New York, (1968).

[15] J.M. Aneiros, *A numerical formulation for computing and design grounding grids of electrical substations by using two-layer soil models*, (in spanish), Research Report, Civil Engrg. Sch., Universidad de La Coruña, (1996).

[16] M. Kurtovic, S. Vujevic, *Potential of earthing grid in heterogeneous soil*, Int. J. Numer. Meth. Engng., **31**, 677-688, (1991).

[17] I. Colominas, M. Chao, F. Navarrina and M. Casteleiro, *Application of Meshless Methods to the Analysis and Design of Grounding Systems*, In: S.R. Idelsohn, E. Oñate, E. Dvorkin (Eds.), Computational Mechanics: New Trends and Applications, (in CDROM, ISBN: 84-89925-15-1); CIMNE Pub., Barcelona, (1998).

[18] I. Colominas, G. Mosqueira, M. Chao, F. Navarrina and M. Casteleiro, *Meshless methods for potential problems in electrical engineering applications*, In: J. Bento, E. Arantes e Oliveira, E. Pereira (Eds.), Computational Methods in Engineering and Science, Elsevier, Oxford, 381-390, (1999).

[19] O.D. Kellog, *Foundations of potential theory*, Springer Verlag, Berlín, (1967).

[20] I. Colominas, *A CAD system of grounding grids for electrical installations: A numerical approach based on the Boundary Element integral method*, (in spanish), Ph.D. Thesis, Civil Engrg. Sch., Universidad de La Coruña, (1995).

[21] C. Johnson, *Numerical solution of partial differential equations by the finite element method*, Cambridge Univ. Press, Cambridge, USA, (1987).

[22] M. Casteleiro, L.A. Hernández, I. Colominas and F. Navarrina, *Memory and User guide of system TOTBEM for CAD of grounding grids in electrical installations*, (in spanish), Civil Engrg. Sch., Universidad de La Coruña, (1994).

Appendix I: Integral kernels for two-layer soil models

For a two-layer soil model the integral kernels are given by [13], [15]:

$$
\begin{aligned}
k_{11}(\boldsymbol{x}_1, \boldsymbol{\xi}) =\ & \frac{1}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, \xi_z])} + \frac{1}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -\xi_z])} \\
& + \sum_{i=1}^{\infty} \frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, 2iH + \xi_z])} \\
& + \sum_{i=1}^{\infty} \frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, 2iH - \xi_z])} \\
& + \sum_{i=1}^{\infty} \frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -2iH + \xi_z])} \\
& + \sum_{i=1}^{\infty} \frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -2iH - \xi_z])};
\end{aligned}
\qquad (10)
$$

$$k_{12}(\boldsymbol{x}_2, \boldsymbol{\xi}) = \frac{1+\kappa}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, \xi_z])} + \frac{1+\kappa}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, -\xi_z])}$$
$$+ \sum_{i=1}^{\infty} \frac{(1+\kappa)\kappa^i}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, 2iH + \xi_z])}$$
$$+ \sum_{i=1}^{\infty} \frac{(1+\kappa)\kappa^i}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, 2iH - \xi_z])}; \qquad (11)$$

if the grounding electrode is completely buried in the upper layer; and

$$k_{21}(\boldsymbol{x}_1, \boldsymbol{\xi}) = \frac{1-\kappa}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, \xi_z])} + \frac{1-\kappa}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -\xi_z])}$$
$$+ \sum_{i=1}^{\infty} \frac{(1-\kappa)\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -2iH + \xi_z])}$$
$$+ \sum_{i=1}^{\infty} \frac{(1-\kappa)\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, 2iH - \xi_z])}, \qquad (12)$$

$$k_{22}(\boldsymbol{x}_2, \boldsymbol{\xi}) = \frac{1}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, \xi_z])} + \frac{1-\kappa^2}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, -\xi_z])}$$
$$+ \frac{-\kappa}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, 2H + \xi_z])}$$
$$+ \sum_{i=1}^{\infty} \frac{(1-\kappa^2)\kappa^i}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, -2iH + \xi_z])}, \qquad (13)$$

if the grounding electrode is completely buried in the lower layer. In the above expressions, $r(\boldsymbol{x}, [\xi_x, \xi_y, \xi_z])$ indicates the distance from $\boldsymbol{x}$ to $\boldsymbol{\xi} \equiv [\xi_x, \xi_y, \xi_z]$ —and to the symmetric points and images of $\boldsymbol{\xi}$ with respect to the earth surface Γ_E and to the interphase surface between layers—, H is the thickness of the upper layer, and κ is a ratio defined in terms of the layer conductivities: $\kappa = (\gamma_1 - \gamma_2)/(\gamma_1 + \gamma_2)$.

Appendix II: Boundary element numerical formulation

A. 2D general approach

The leakage current density σ that flows from the electrodes, and the electrode surface Γ can be discretized as follows

$$\sigma(\boldsymbol{\xi}) = \sum_{i=1}^{N} N_i(\boldsymbol{\xi})\,\sigma_i, \qquad \Gamma = \bigcup_{\alpha=1}^{M} \Gamma^\alpha, \qquad (14)$$

for given sets of "N" trial functions $\{N_i(\boldsymbol{\xi})\}$ defined on the surface Γ and "M" 2D boundary elements $\{\Gamma^\alpha\}$. Then, integral expression (5) for potential $V_c(\boldsymbol{x}_c)$ can also be discretized as

$$V_c(\boldsymbol{x}_c) = \sum_{i=1}^{N} V_{c_i}(\boldsymbol{x}_c)\,\sigma_i, \quad \forall \boldsymbol{x}_c \in E_c; \ \ (c = 1, C); \quad (15)$$

being

$$V_{c_i}(\boldsymbol{x}_c) = \sum_{\alpha=1}^{M} V_{c_i}^\alpha(\boldsymbol{x}_c),$$

$$V_{c_i}^\alpha(\boldsymbol{x}_c) = \frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\xi} \in \Gamma^\alpha} k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})\,N_i(\boldsymbol{\xi})\,d\Gamma^\alpha. \ (16)$$

Therefore, for a given set of "N" test functions $\{w_j(\boldsymbol{\chi})\}$ defined on Γ, variational form (7) is reduced to the following system of linear equations

$$\sum_{i=1}^{N} [R_{ji}]\,\sigma_i = \nu_j, \quad j = 1, \ldots, N, \qquad (17)$$

being the elements of the coefficients matrix

$$R_{ji} = \sum_{\beta=1}^{M} \sum_{\alpha=1}^{M} R_{ji}^{\beta\alpha},$$
$$R_{ji}^{\beta\alpha} = \frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\chi} \in \Gamma^\beta} w_j(\boldsymbol{\chi}) \iint_{\boldsymbol{\xi} \in \Gamma^\alpha} k_{bb}(\boldsymbol{\chi}, \boldsymbol{\xi}) N_i(\boldsymbol{\xi}) d\Gamma^\alpha d\Gamma^\beta,$$
$$(18)$$

and the right-hand-side terms

$$\nu_j = \sum_{\beta=1}^{M} \nu_j^\beta, \qquad \nu_j^\beta = \iint_{\boldsymbol{\chi} \in \Gamma^\beta} w_j(\boldsymbol{\chi}) d\Gamma^\beta. \qquad (19)$$

We remark that the solution to system (17) is the key to solve the problem, since it provides the values of the unknowns σ_i ($i = 1, \ldots, N$), that can be used to compute the potential at any point $\boldsymbol{x}_c$ in the earth (and, of course, on its surface), by means of expression (15), as much as the equivalent resistance R_{eq} of the grounding system by using expressions (14) and (2) [8], [20].

However, the statement of linear system (17) requires the discretization of a 2D domain (the whole surface Γ of the grounding electrodes), which involves a large number of degrees of freedom in practical cases. Besides, the matrix (17) is full and the computation of its coefficients (18) requires to perform double integration on 2D domains. On the other hand, the integral kernel $k_{bb}(\boldsymbol{\chi}, \boldsymbol{\xi})$ at (18) is given by series which computation in real problems implies a considerably high number of evaluations of its terms. For all these reasons, it is necessary to introduce some additional hypotheses in the mathematical model of this general boundary element formulation in order to decrease the computational cost.

B. 1D approximated approach

Taking into account the real geometry of grounding systems in practical cases, we can introduce an assumption that is widely used in most of theoretical developments in grounding analysis: the hypothesis of circumferential uniformity in the grid conductors. Thus, the leakage current density σ is assumed constant around the cross section of the cylindrical electrode [1], [2], [8], [6], [20]. In this way, being L the axial lines of the conductors, $\widehat{\boldsymbol{\xi}}$ the orthogonal projection (over the axis of an electrode) of a generic point $\boldsymbol{\xi}$ on the electrode surface Γ, $\phi(\widehat{\boldsymbol{\xi}})$ the electrode diameter, $P(\widehat{\boldsymbol{\xi}})$ the circumferential perimeter of the cross section of

the bar at $\widehat{\boldsymbol{\xi}}$, and $\widehat{\sigma}(\widehat{\boldsymbol{\xi}})$ the approximated leakage current density at point $\boldsymbol{\xi}$ ($\widehat{\sigma}$ is assumed constant around the cross section), expression (5) results in

$$\widehat{V}_c(\boldsymbol{x}_c) = \frac{1}{4\pi\gamma_b} \int_{\widehat{\boldsymbol{\xi}} \in L} \bar{k}_{bc}(\boldsymbol{x}_c, \widehat{\boldsymbol{\xi}}) \, \widehat{\sigma}(\widehat{\boldsymbol{\xi}}) \, dL,$$
$$\forall \boldsymbol{x}_c \in E_c; \quad (c = 1, C); \qquad (20)$$

where $\bar{k}_{bc}(\boldsymbol{x}_c, \widehat{\boldsymbol{\xi}})$ is the circumferential integral of kernel $k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})$ around the cross section of the electrode at $\widehat{\boldsymbol{\xi}}$ (the orthogonal projection of $\boldsymbol{\xi}$ over the axis):

$$\bar{k}_{bc}(\boldsymbol{x}_c, \widehat{\boldsymbol{\xi}}) = \int_{\boldsymbol{\xi} \in P(\widehat{\boldsymbol{\xi}})} k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi}) dP. \qquad (21)$$

On the other hand, since the leakage current density is not really uniform around the cross section, boundary condition $V_b(\boldsymbol{\chi}) = 1$, $\forall \boldsymbol{\chi} \in \Gamma$ can not be strictly satisfied and variational form (7) will not hold anymore. Therefore, the class of test functions must be restricted to those with circumferential uniformity, that is $w(\boldsymbol{\chi}) = \widehat{w}(\widehat{\boldsymbol{\chi}}) \, \forall \boldsymbol{\chi} \in P(\widehat{\boldsymbol{\chi}})$; thus, variational form (7) results in

$$\int_{\widehat{\boldsymbol{\chi}} \in L} \widehat{w}(\widehat{\boldsymbol{\chi}}) \left[\frac{1}{4\pi\gamma_b} \int_{\widehat{\boldsymbol{\xi}} \in L} \bar{\bar{k}}_{bb}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}}) \widehat{\sigma}(\widehat{\boldsymbol{\xi}}) dL \right] dL =$$
$$\pi \int_{\widehat{\boldsymbol{\chi}} \in L} \phi(\widehat{\boldsymbol{\chi}}) \widehat{w}(\widehat{\boldsymbol{\chi}}) dL, \qquad (22)$$

which must verify for all members $\widehat{w}(\widehat{\boldsymbol{\chi}})$ of a suitable class of test functions defined on L. The integral kernel $\bar{\bar{k}}_{bb}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$ is given by

$$\bar{\bar{k}}_{bb}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}}) = \int_{\boldsymbol{\chi} \in P(\widehat{\boldsymbol{\chi}})} \left[\int_{\boldsymbol{\xi} \in P(\widehat{\boldsymbol{\xi}})} k_{bb}(\boldsymbol{\chi}, \boldsymbol{\xi}) \, dP \right] dP. \qquad (23)$$

In contrast to the potential expression (5) and to the integral equation (7), expression (20) and integral equation (22) require the discretization of a simpler domain: the axial lines L of the grounding electrodes. Thus, the approximated leakage current density $\widehat{\sigma}$ and the axial lines L can be discretized as follows

$$\widehat{\sigma}(\widehat{\boldsymbol{\xi}}) = \sum_{i=1}^{n} \widehat{N}_i(\widehat{\boldsymbol{\xi}}) \, \widehat{\sigma}_i, \qquad L = \bigcup_{\alpha=1}^{m} L^\alpha, \qquad (24)$$

for given sets of "n" trial functions $\{\widehat{N}_i(\widehat{\boldsymbol{\xi}})\}$ defined on L, and "m" 1D boundary elements $\{L^\alpha\}$. In the same way, we can obtain a discretized form of potential expression (20):

$$\widehat{V}_c(\boldsymbol{x}_c) = \sum_{i=1}^{n} \widehat{V}_{c_i}(\boldsymbol{x}_c), \, \widehat{\sigma}_i, \quad \forall \boldsymbol{x}_c \in E_c; \quad (c = 1, C); \quad (25)$$

being

$$\widehat{V}_{c_i}(\boldsymbol{x}_c) = \sum_{\alpha=1}^{m} \widehat{V}_{c_i}^\alpha(\boldsymbol{x}_c),$$
$$\widehat{V}_{c_i}^\alpha(\boldsymbol{x}_c) = \frac{1}{4\pi\gamma_b} \int_{\widehat{\boldsymbol{\xi}} \in L^\alpha} \bar{k}_{bc}(\boldsymbol{x}_c, \widehat{\boldsymbol{\xi}}) \, \widehat{N}_i(\widehat{\boldsymbol{\xi}}) \, dL^\alpha. \quad (26)$$

Therefore, for a suitable selection of "n" test functions $\{\widehat{w}_j(\widehat{\boldsymbol{\chi}})\}$ defined on L, variational form (22) is reduced to the following system of linear equations

$$\sum_{i=1}^{n} \left[\widehat{R}_{ji} \right] \widehat{\sigma}_i = \widehat{\nu}_j, \quad j = 1, \ldots, n, \qquad (27)$$

being the elements of the coefficients matrix

$$\widehat{R}_{ji} = \sum_{\beta=1}^{m} \sum_{\alpha=1}^{m} \widehat{R}_{ji}^{\beta\alpha},$$
$$\widehat{R}_{ji}^{\beta\alpha} = \frac{1}{4\pi\gamma_b} \int_{\widehat{\boldsymbol{\chi}} \in L^\beta} \widehat{w}_j(\widehat{\boldsymbol{\chi}}) \int_{\widehat{\boldsymbol{\xi}} \in L^\alpha} \bar{\bar{k}}_{bb}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}}) \, \widehat{N}_i(\widehat{\boldsymbol{\xi}}) \, dL^\alpha dL^\beta,$$
$$(28)$$

and the right-hand-side terms

$$\widehat{\nu}_j = \sum_{\beta=1}^{m} \widehat{\nu}_j^\beta, \qquad \widehat{\nu}_j^\beta = \pi \int_{\widehat{\boldsymbol{\chi}} \in L^\beta} \phi(\widehat{\boldsymbol{\chi}}) \, \widehat{w}_j(\widehat{\boldsymbol{\chi}}) \, dL^\beta. \qquad (29)$$

As in linear system (17), the coefficients matrix of system (27) is full. Nevertheless, we can assure an important reduction of the overall computational cost by using this 1D approach, because the dimension of the system and the number of terms (28) and (29) that must be computed are significantly smaller than in the previous 2D formulation. In spite of that, the computing requirements are still excessive for practical purposes —mainly because of the need to evaluate the circumferential integrals on the perimeter of the electrodes that are involved in kernels (21) and (23)— and further simplifications must be introduced.

B.1 Integration of terms

The integrand functions in expressions (21) and (23) are kernels that can be written in the general form [13]:

$$k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi}) = \sum_{l=0}^{\infty} \frac{\psi_l(\kappa)}{r(\boldsymbol{x}_c, \boldsymbol{\xi}_l(\boldsymbol{\xi}))}; \qquad (30)$$

where the ψ_l are weighting coefficients defined in terms of a parameter κ that only depends on the conductivities of the ground layers, and $r(\boldsymbol{x}_c, \boldsymbol{\xi}_l(\boldsymbol{\xi}))$ is the euclidean distance between the points $\boldsymbol{x}_c$ and $\boldsymbol{\xi}_l$, being $\boldsymbol{\xi}_0$ the point $\boldsymbol{\xi}$ on the electrode surface ($\boldsymbol{\xi}_0(\boldsymbol{\xi}) = \boldsymbol{\xi}$) and being $\boldsymbol{\xi}_l$ ($l \neq 0$) the images of $\boldsymbol{\xi}$ with respect to the earth surface and to the interphases between layers[1]. Thus, the inner integral kernel (21) can be written as

$$\bar{k}_{bc}(\boldsymbol{x}_c, \widehat{\boldsymbol{\xi}}) = \sum_{l=0}^{\infty} \psi_l(\kappa) \left[\int_{\boldsymbol{\xi} \in P(\widehat{\boldsymbol{\xi}})} \frac{1}{r(\boldsymbol{x}_c, \boldsymbol{\xi}_l(\boldsymbol{\xi}))} dP \right]. \quad (31)$$

In general, we need to compute potential at points $\boldsymbol{x}_c$ which distance to $\widehat{\boldsymbol{\xi}}_l$ is much larger than the diameter of the conductor. Thus, if the circumferential integrals involved in (31) are expressed in terms of the angular position of $\boldsymbol{\xi}_l$

---

[1]For the case of the two-layer soil model, these kernels are given by expressions (10), (11), (12), (13)

along the perimeter of the cross section of the cylindrical conductor [8], and the resulting elliptic integrals are susbstituted by means of a simple approximation, we obtain

$$\bar{k}_{bc}(\boldsymbol{x}_c, \widehat{\boldsymbol{\xi}}) \approx \pi\, \phi(\widehat{\boldsymbol{\xi}})\, \widehat{k}_{bc}(\boldsymbol{x}_c, \widehat{\boldsymbol{\xi}}) \qquad (32)$$

being

$$\widehat{k}_{bc}(\boldsymbol{x}_c, \widehat{\boldsymbol{\xi}}) = \sum_{l=0}^{\infty} \frac{\psi_l(\kappa)}{\widehat{r}(\boldsymbol{x}_c, \widehat{\boldsymbol{\xi}}_l(\widehat{\boldsymbol{\xi}}))},$$

$$\widehat{r}(\boldsymbol{x}_c, \widehat{\boldsymbol{\xi}}_l) = \sqrt{|\boldsymbol{x}_c - \widehat{\boldsymbol{\xi}}_l|^2 + \frac{\phi^2(\widehat{\boldsymbol{\xi}})}{4}}. \qquad (33)$$

We can approximate the inner integral kernel (23) in a similar way [8], [20], giving

$$\bar{\bar{k}}_{bb}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}}) \approx \pi^2\, \phi(\widehat{\boldsymbol{\xi}})\, \phi(\widehat{\boldsymbol{\chi}})\, \widehat{\widehat{k}}_{bb}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}}) \qquad (34)$$

being

$$\widehat{\widehat{k}}_{bb}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}}) = \sum_{l=0}^{\infty} \frac{\psi_l(\kappa)}{\widehat{r}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}}_l(\widehat{\boldsymbol{\xi}}))},$$

$$\widehat{r}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}}_l) = \sqrt{|\widehat{\boldsymbol{\chi}} - \widehat{\boldsymbol{\xi}}_l|^2 + \frac{\phi^2(\widehat{\boldsymbol{\xi}}) + \phi^2(\widehat{\boldsymbol{\chi}})}{4}}. \qquad (35)$$

However, the evaluation of the integrals involved in (28) and (26) with approximations (34), (35), (32) and (33) by means of standard numerical quadratures is very costly due to the ill-conditioning of the integrands [8]. Explicit formulae have been derived by the authors to compute expressions (26) and (28) for constant, linear and parabolic leakage current elements[8], [20] in the case of the homogeneous and isotropic soil models. The cost of the integration process is now increased because the integral kernels are given by series like (33) or (35). Fortunately, the terms in these series are identical to the ones corresponding to homogeneous and isotropic soil models, and the above mentioned analytical techniques can be applied. These analytical integration techniques represent a significant improvement in the grounding analysis field, since the drastic reduction in computing time allows to obtain high accurate results for earthing systems of medium/large sizes, in real time and using low cost personal computers [9], [22].

Ignasi Colominas was born in El Prat, Barcelona, Spain, in 1966. He received the B.S. degree in Industrial Engineering from the *Instituto Químico de Sarriá* (1990), the M.Sc. degrees in Industrial Engineering from the *Universidad Autónoma de Barcelona* (1991) and in Numerical Methods in Engineering Applications from the *Universitat Politècnica de Catalunya* (1991), and the Ph.D. degree in Industrial Engineering from the *Universidad de La Coruña* (1995). He is Associate Professor of the Applied Mathematics Department in the Civil Engineering School of the University of La Coruña. Main research areas are those related with computational mechanics and the development of numerical methods in engineering applications. He has participated in different R&D projects in connection with electrical companies from Spain. In 1995 he received the "First prize *Grupo ENDESA* for the scientific research in aplications of the electricity in industrial processes" for the work of his Ph.D. thesis. He currently leads a R&D project of the spanish government with european funds for the computational design of grounding systems of electrical substations. He is member of the "Institute of Electrical and Electronics Engineers" (IEEE), the "International Association for Computational Mechanics" (IACM) and the "Spanish Society of Numerical Methods in Engineering" (SEMNI). (e-mail: colominas@iccp.udc.es)

Fermín Navarrina was born in Santiago de Compostela (La Coruña), Spain, in 1960. He received a degree in Civil Engineering from the *Universidad Politécnica de Madrid* in 1983, and a Ph.D. degree in Civil Engineering from the *Universidad Politécnica de Barcelona* in 1987. Between 1987 and 1990 he served as associate professor and full professor of Applied Mathematics at the Civil Engineering School of the *Universidad Politécnica de Cataluña*, where he also was head of the Computer Center and Vice-Dean. In 1991 he founded the Civil Engineering School of the *Universidad de La Coruña*, where he was Dean between 1991 and 1999, and serves as Professor of Applied Mathematics and Civil Engineering. Prof. Navarrina gives courses on numerical methods and optimization for engineers since more than 15 years. His research fields are related to the development and application of numerical techniques for analysis and design optimization in engineering. He has directed and/or collaborated in a number of R+D projects, both supported by public and private funds, and is the author of more than 60 papers in scientific journals, books and proceedings of international congresses. He is also member of the executive council of SEMNI and member of IACM. (e-mail: navarrina@iccp.udc.es).

Manuel Casteleiro was born in Madrid, Spain in 1946. He received a Civil Engineering degree from the *Universidad Politécnica de Madrid* (1971), a Master of Science degree from the *University of Wisconsin-Milwaukee* (1972) and a Ph.D. degree in Civil Engineering from *Northwestern University* (1974). Prof. Casteleiro came back to Spain in 1974 and for the next 18 years he served as Professor of Applied Mathematics and Civil Engineering at the Civil Engineering School of the *Universitat Politècnica de Catalunya*. In 1992 he moved to the Civil Engineering School of the *Universidade da Coruña*, where he is now Professor of Applied Mathematics. The research fields of Prof. Casteleiro are numerical methods, their application to engineering problems —more specifically by large-scale simulation techniques—, and computational mechanics. Other scientific areas of interest are statistical and risk analysis in engineering science. He has directed many research and development projects supported by public and private funds. Prof. Casteleiro has published a high number of scientific and technical papers and books. He is also member of the executive council of IACM, founder of SEMNI, and member of the most important societies in the fields of Numerical Methods and Computational Mechanics. (e-mail: casteleiro@iccp.udc.es)