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Quantification of corrosion-like defect in pipelines using multi-frequency identification of non-dispersive torsional guided waves

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Abstract

Pipeline guided wave inspection is an efficient tool for determining a defect location. However, quantifying the defect size remains a challenging task. This paper proposes a quantification method for corrosion-like defects in pipelines based on the multi-frequency identification of non-dispersive torsional guided waves. Firstly, a theoretical scattering model describing the T(0,1) wave's interaction with simplified corrosion-like damage is introduced. Subsequently, a multi-frequency identification method is proposed, enabling the inverse quantification of defect parameters by a defined spectral defect index (SDI). To implement this approach, a pseudo pulse-echo configuration is devised, which contains two rings of piezoelectric transducers attached on the pipeline's out surface. Finite element (FE) models are employed to test the performance of the proposed method for both axisymmetric and non-axisymmetric defects, and an analysis of the robustness of the method is also conducted. The

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28 results show that this method has good accuracy even for signals with a very low signal to noise (SNR)
29 ratio. Furthermore, a FE model is developed to validate the feasibility of this method for long-distance
30 detection considering attenuation effect. Finally, experimental validation of the proposed method
31 demonstrates close agreement between predicted and actual defect sizes, showing its potential for
32 practical applications.

33 **Keywords**

34 Ultrasonic guided wave; Defect quantification; Multi-frequency identification; Reciprocity
35 theorem; Spectral defect index.

36

37 **Introduction**

38 Pipeline systems are widely used in transportation of petrochemical products (Lu et al.
39 2023)(Peng et al. 2023). However, due to the harsh working conditions, corrosion frequently occurs
40 and affects the pipeline integrity (Xu et al. 2021). Guided wave detection technology is an efficient
41 tool for non-destructive testing (NDT) (Zhang et al. 2022)(Zhang et al. 2023)(Wang et al. 2023) and
42 structural-health monitoring (SHM) (Croxford et al. 2007)(Liu et al. 2021). For pipeline defect
43 detection, transducers are usually attached equally around the circumference on the pipe surface to
44 generate axisymmetric plane guided waves. Since the plane guided wave does not have beam
45 divergence (Chua et al. 2019), a long range (over 10 m) inspection effect can be achieved at relatively
46 low frequencies. As such , pipeline guided wave inspection usually serves as a screening tool for defect
47 localization (Zang et al. 2023), while the distance-amplitude correction (DAC) lines is applied to assess

48 the severity of defect qualitatively (Catton et al. 2008). However, more precise quantification of defect
49 is still a challenging task in a guided wave detection approach (Mitra and Gopalakrishnan 2016).

50 The scattering model describes the interaction of fundamental guided wave with various types of
51 defects, and it is a basis for defect quantification. In order to examine the theoretical basis of pipeline
52 guided wave scattering, Ditri (Ditri 1994) developed a theoretical model to describe the pipeline guided
53 wave scattering by circumferential cracks with S-parameter formalism. Lee et al. (Lee et al. 2018)
54 employed a wave field superposition technique along with the reciprocity theorem to simplify the
55 solution of the T(0,1) scattering problem.

56 In order to reveal characteristics of the guided wave when interacting with more complex defects,
57 finite element (FE) simulation has been carried out. Demma et al. (Demma et al. 2004) employed FE
58 simulation to study the interaction of guided waves with rectangular notch defect in pipelines, and
59 summarized the effects of pipe size, defect size, guided-wave mode, and frequency on reflection from
60 the notches. Carandente et al. (Carandente and Cawley 2012) further studied the reflection of T(0,1)
61 mode with a simulated tapered step notch. They analyzed the effects of different contours and depths
62 on the reflection coefficients, and argued that the corresponding findings could be used to evaluate the
63 defect depth according to the defect circumferential range and the maximum reflection coefficient.
64 Lovstad and Cawley (Løvstad and Cawley 2011) analyzed the effect of circular holes on T(0,1)
65 reflection and proposed a relationship between pit cluster shape and the reflection coefficient. Zhu et
66 al. (Zhu et al. 2022) studied the interaction between L(0,2), T(0,1) wave with dent defect, and
67 demonstrated the difference in the scattering mechanisms as compared with notch defect. Considering
68 the fact that using finite element modelling is relatively time-consuming to obtain a complete solution,

69 Duan and Kirby (Duan and Kirby 2015) introduced a semi-analytical method to solve the guided wave
70 scattering problem. They introduced a weighted residual formulation based on the Galerkin's principle
71 to account for arbitrarily shaped defects in pipelines. The method was shown to be a highly efficient
72 numerical approach.

73 Based on the scattering features from defects, several signal processing methods have been
74 proposed to decompose the reflected wave and quantify defects. Wang et al. (Wang et al. 2010) used
75 the Hilbert–Huang transform to decompose the reflection signals from front and rear edges of the
76 defect, so the axial length of the defect can be obtained. Tse and Wang (Tse and Wang 2013) proposed
77 an optimized matching pursuit (MP) method by analyzing the interference between the reflection
78 components, which can extract axial length information by separated reflection signal. However, such
79 methods are mainly applied in axisymmetric defects.

80 Some researchers have found that there exists a certain relationship between scattering amplitude
81 and the wavelength-to-defect length ratio in various types of defects, such as notch (Demma et al.
82 2004), hole (Løvstad and Cawley 2011) and dent (Zhu et al. 2022). Therefore, there is a potential
83 solution that utilizes the relationship between the amplitude and the wavelength-to-defect length ratio
84 parameter to quantify defect, and this has attracted much research interest recently. Wang et al. (Wang
85 et al. 2019) used lamb wave inspection at multiple frequencies, and the method was shown to improve
86 the probability of detection as compared with single-frequency excitation. Xu et al.(Xu et al. 2023)
87 used frequency-dependent scattering of wide band laser-generated Rayleigh waves to characterize
88 vertical surface crack. Shan et al.(Shan et al. 2022) proposed a defect quantification algorithm based
89 on the wave reflections from the defect and wave-guided end, and used it to quantify geometric

90 parameters of the defects in cylindrical structures. Generally speaking, these methods are in the initial
91 phase of development, and the corresponding research and exploration of potential applications are
92 still insufficient.

93 This paper proposes a multi-frequency identification approach for quantifying pipeline defect
94 using a scattering model between T(0,1) waves and corrosion-like defects. The organization of the
95 paper is as follows. Firstly (Section 2), a scattering model is introduced, which describes the scattering
96 wave from a T(0,1) wave interacting with simplified corrosion-like defect in pipeline. In Section 3, a
97 methodology for inversely quantifying the defect by a multi-frequency method is proposed, and a
98 corresponding inspection configuration is designed. In Section 4, the proposed method is validated by
99 FE simulation, and the robustness is also tested. An experimental validation is presented in Section 5.
100 Finally, Section 6 summarizes the main conclusions.

101

102 **Basic theory**

103 **Characterization of T (0,1) mode**

104 For the guided wave propagation in an elastic isotropic hollow cylinder, three types of wave
105 modes are mainly studied, namely, longitudinal mode, torsional mode, and flexural mode. The particle
106 displacements can be described as (Rose 2014):

$$u_r(r, z, t) = U_r(r) \cos(m\theta) e^{i(kz - \omega t)}, \quad (1a)$$

$$u_\theta(r, z, t) = U_\theta(r) \sin(m\theta) e^{i(kz - \omega t)}, \quad (1b)$$

$$u_z(r, z, t) = U_z(r) \cos(m\theta) e^{i(kz - \omega t)}. \quad (1c)$$

107 where, $U_r(r)$, $U_\theta(r)$, $U_z(r)$ are the displacement profiles of radial displacement, tangential
 108 displacement, and axial displacement in the radial direction, respectively. r and z denote radial and
 109 axial locations respectively. m is the circumferential order of a wave mode. k , ω and t denote wave
 110 number, angular frequency and time respectively.

111 T(0,1) mode offers several advantages, such as nondispersive and no leakage to the surrounding
 112 fluid, making it particularly attractive for long-range inspection (Niu et al. 2019)(Alleyne et al. 2009).
 113 For this reason, only T(0,1) mode is considered here, and the wave displacement can be simplified to
 114 only include the tangential displacement shown in Eq. (1b).

115 The tangential displacement equation of motion is:

$$\frac{d^2 U_\theta}{dr^2} + \frac{1}{r} \frac{dU_\theta}{dr} + \left[\left(\frac{\omega^2}{c_T^2} - k^2 \right) - \frac{1}{r^2} \right] U_\theta = 0 \quad (2)$$

116 For the zero mode torsional wave, $\frac{\omega^2}{c_T^2} - k^2 = 0$. Therefore, $U_\theta(r)$ can be expressed as $U_\theta(r) = Ar$,
 117 with A being the wave amplitude.

118 Consider an incident T(0,1) wave propagation forward in a pipe. The incident wave can be
 119 described as:

$$u_\theta^{in}(z, t) = A^{in} U_\theta(r) e^{i(kz - \omega t)}, \quad (3a)$$

$$\tau_{z\theta}^{in}(z, t) = ik\mu u_\theta^{in}(z, t). \quad (3b)$$

120 where, the superscript "in" indicates incident wave.

121 Scattering model

122 In this paper, we adopt the elastodynamic reciprocity theorem (Balogun and Achenbach
 123 2020)(Pau et al. 2016) to model the scattering wave from a simplified corrosion-like defect as shown

124 in Fig. 1. The scattered torsional wave is the response due to the shear stress on the defect introduced
 125 by the incident T(0,1) wave. Since the two surfaces of defect are simplified as parallel to radial
 126 direction, only the shear stress $\tau_{z\theta}$ on the defect surface is considered. The forward scattered torsional
 127 wave generated by the rear surface can be described as:

$$u_{\theta}^{sc+}(z, t) = A^{sc+}U_{\theta}(r)e^{ikz}, \quad (4a)$$

$$\tau_{z\theta}^{sc+}(z, t) = i\mu k A^{sc+}U_{\theta}(r)e^{ikz}. \quad (4b)$$

128 The backward scattered torsional wave generated by the front surface can be described as:

$$u_{\theta}^{sc-}(z, t) = A^{sc-}U_{\theta}(r)e^{-ikz}, \quad (5a)$$

$$\tau_{z\theta}^{sc-}(z, t) = -i\mu k A^{sc-}U_{\theta}(r)e^{-ikz}. \quad (5b)$$

129 where, A^{sc+} and A^{sc-} represents amplitude of the forward and backward scattered wave, respectively.

130 The elastodynamic reciprocity theorem establishes a connection between two distinct
 131 elastodynamic states within the same body V and its corresponding surface S (Phan et al. 2013). Since
 132 no body force is assumed to exist in each state (Kubrusly and Dixon 2021)(Pau et al. 2015), it provides:

$$\iint_S (\tau_{ij}^B u_j^A - \tau_{ij}^A u_j^B) n_i ds = 0 \quad (6)$$

133 where, τ_{ij} and u_i denote stress and displacement, respectively. n_i is the unit outward normal to the
 134 surface S . The superscripts A and B denote two elastodynamic states. In this case, state A corresponds
 135 the scattering of T(0,1) wave generated by defect and propagating in both forward and backward
 136 directions. State B denotes a virtual incident T(0,1) wave propagating in the forward direction without
 137 defect, as (Lee et al. 2018):

$$u_{\theta}^{vi}(z, t) = A^{vi}U_{\theta}(r)e^{ikz}, \quad (7a)$$

$$\tau_{z\theta}^{vi}(z, t) = i\mu k A^{vi}U_{\theta}(r)e^{ikz}. \quad (7b)$$

138 where, the subscript vi indicates virtual incident wave.

139 The contour S can be decomposed to the $S_1, S_2 \dots S_8$ as shown in Fig. 2. The integration on each
 140 surface is called $J_1, J_2 \dots J_8$, respectively. The integrations on the upper and bottom surfaces are zero
 141 since the $\tau_{z\theta}$ is zero on these boundary free surfaces, yielding $J_2 = J_4 = J_6 = J_8 = 0$. The integrations
 142 on S_3 is zero since the forward scattered wave and incident wave are in the same direction on state A
 143 and B. Therefore, the integrations can be reduced to:

$$\iint_S (\tau_{ij}^B u_j^A - \tau_{ij}^A u_j^B) n_i ds = J_1 + J_5 + J_7 = 0 \quad (8)$$

144 For J_1, J_5 and J_7 substitution of Eqs. (4), (5) and (7) to Eq. (6), and integrating over r from a to b
 145 and over θ from 0 to 2π yields:

$$J_1 = \int_a^b (\tau_{z\theta}^{vi} u_{\theta}^{sc-} - \tau_{z\theta}^{sc-} u_{\theta}^{vi}) * (-1) r dr = 4i\pi\mu k A^{vi} A^{sc-} \int_a^b U_{\theta}^2(r) r dr \quad (9)$$

146 The shear stress generated by incident torsional wave can be considered as uniform due to the
 147 small ratio of defect depth to incident wavelength. The generated shear stress can be described as:

$$\tau'_{z\theta} = i\mu k A^{in} U_{\theta}(r) e^{ikz} \quad (10)$$

148 and the displacement generated by the $\tau'_{z\theta}$ is u'_{θ} .

149 For the integrations on the defect surfaces:

$$J_5 = 2\pi \int_a^b (\tau_{z\theta}^{vi} u_{\theta}'^+ - \tau_{z\theta}'^+ u_{\theta}^{vi}) * (-1) r dr, \quad (11)$$

$$J_7 = 2\pi \int_a^b (\tau_{z\theta}^{vi} u_{\theta}'^- - \tau_{z\theta}'^- u_{\theta}^{vi}) r dr. \quad (12)$$

150 Since the displacements are opposite and shear stresses are the same on the two surfaces of the
 151 defect in state A, it yields:

$$J_5 + J_7 = -2\pi \int_{b-d}^b \tau_{z\theta}^{vi} (u_{\theta}'^+ - u_{\theta}'^-) dr \quad (13)$$

152 Since u_{θ}^{sc+} and u_{θ}^{sc-} are generated by $\tau_{z\theta}^{in}$ on the defect surface, the displacement can be
 153 approximated as:

$$u_{\theta}^{\prime\pm} = u_{z\theta}^{in} e^{\pm ikle/2}, \quad (14)$$

154 therefore:

$$\begin{aligned} J_5 + J_7 &= - \int_{b-d}^b \tau_{z\theta}^{vi} u_{z\theta}^{in} \left(e^{\frac{ikle}{2}} - e^{-\frac{ikle}{2}} \right) dr \\ &= 4i\pi\mu k A^{vi} A^{in} \sin(kle) \int_{b-d}^b U_{\theta}^2(r) r dr. \end{aligned} \quad (15)$$

155 Substitution of Eqs. (9) and (15) to Eq. (8) yields:

$$\left| \frac{A^{sc}}{A^{in}} \right| = \sin(kL_e) \frac{b^4 - (b-d)^4}{b^4 - a^4}, \quad (16)$$

156 where d is the depth of wall-thinning defect.

157

158 **Scheme of multi-frequency identification method**

159 **Method**

160 Section 2 has introduced the theoretical model describing a scattering model for a wall-thinning
 161 defect. The quantification of the defect severity is an inverse problem (Liu et al. 2023) (Gao et al.
 162 2024). For this purpose, we introduce a spectral defect index (SDI), which measures the scattering
 163 amplitude in the frequency domain. The frequency dependent index SDI is subsequently compared
 164 with the SDI calculated by the theoretical model, and the severity of the defect can be estimated by
 165 minimizing the difference between the measured and theoretical SDI results.

166 For the sake of enhancing the precision and robustness in the defect quantification, it is necessary
 167 to obtain more SDI values across varying frequencies. Therefore, we propose a multi-frequency
 168 identification (MFI) method that provides more information related to the size of defect. We use two
 169 different approaches to get the SDI values as follows. 1) Single excitation: here a pulse signal is excited
 170 at one central frequency which is below the cut-off frequency of T(0,2), and the SDI is calculated at 3

171 dB bandwidth of the central frequency. 2) Multiple excitation: here different pulse signals are excited
 172 within the frequency range below the cut-off frequency of T(0,2) separately, and the SDI is calculated
 173 at the central frequencies of each pulse signal. The performance of these two approaches will be
 174 compared in the following test.

175 Fig. 3 shows a schematic of the MFI method. First we measure the SDIs at a frequency range
 176 which is below the T(0,2) cutoff frequency. The excited signal from the transducers and reflected signal
 177 from the defect are measured separately. Then, the spectral defect index is calculated as:

$$SDI(f) = \frac{S^s(f)}{S^i(f)}, \quad (17)$$

178 where, $S^s(f)$ and $S^i(f)$ are the spectral responses of the scattering wave and incident wave calculated
 179 by fast Fourier transform (FFT), respectively.

180 In order to quantify the defect, the SDIs are compared with the results calculated from the
 181 theoretical model. Eq. (18) is used to calculate the difference between the measured results and
 182 theoretical results in terms of root mean square error (RMSE):

$$RMSE(d, l) = \sqrt{\frac{1}{n} \sum_{i=1}^n (SDI_m(f_i) - SDI_t(f_i)|_{d,l})^2}, \quad (18)$$

183 where n is the number of SDI, $SDI_m(f_i)$ is the measured SDI at frequency f_i , and $SDI_t(f_i)|_{d,l}$ is the
 184 theoretical SDI at frequency f_i with a defect of depth d and length l . The RMSE is used to compare
 185 the difference between the measured defect SDI and the theoretically calculated defect SDI of depth d
 186 and length l . The defect sizes d and l are selected within a certain possible range, and the maximum
 187 $RMSE_{\max}$ and the minimum $RMSE_{\min}$ within this range can be obtained. In order to determine the
 188 sizes of the defect, a nominal probability function is proposed as,

$$P(d, l) = \frac{(1/\text{RMSE}(d, l) - 1/\text{RMSE}_{\max})}{(1/\text{RMSE}_{\min} - 1/\text{RMSE}_{\max})}. \quad (19)$$

189 Where $P(d, l)$ denotes the possibility that the measured defect depth is d and the length is l . If $P(d, l)$
 190 is close to 1, it means that the RMSE for defect with depth d and length l is close to the minimum
 191 RMSE calculated in the possible size range, indicating a high probability of defect to be of that size.
 192 Conversely, if $P(d, l)$ is close to 0, indicating that the measured defect is least likely to be of that size.
 193 Using Eq. (19), we can plot a probability map to predict the defect sizes.

194 **Inspection configuration**

195 In the traditional pulse-echo method, the transducer acts as both the exciter and the sensor.
 196 However, the incident wave signal is difficult to obtain, and the SDI of the defect cannot be directly
 197 measured. To deal with this problem, in this paper we propose a pseudo pulse-echo method that
 198 requires two rings of transducers. One ring is used for T(0,1) wave generation, and another ring is used
 199 for measuring the incident wave and reflection wave from defect, as shown in Fig. 4. The whole
 200 process of the inspection can be described by Eqs. (20), (21) below:

$$T(f) = F(V_{in}(t)) \cdot E(f) \cdot e^{-\alpha z_1} \cdot R(f), \quad (20)$$

201 where $T(f)$ is the transmission wave measured by the transducer ring 2 in frequency domain, $F()$ is
 202 Fourier transform function, $V_{in}(t)$ is the input signal in time domain, $E(f)$ is frequency response of
 203 transducer ring 1 for T(0,1) wave excitation, and $R(f)$ is frequency response of transducer ring 2 for
 204 T(0,1) wave reception.

$$S(f) = F(V_{in}(t)) \cdot E(f) \cdot e^{-\alpha z_1} e^{-2\alpha z_2} \cdot \text{SDI}(f) \cdot R(f), \quad (21)$$

205 where $S(f)$ is the scattered wave measured by the transducer ring 2 in frequency domain, α is the
 206 attenuation coefficient for T(0,1) wave propagating in the pipe.

207 Eq. (21) represents the frequency response function of the T(0,1) wave throughout the entire
208 detection process. It describes the process in which (i) the T(0,1) wave is firstly excited by ring 1,
209 $E(f)$, through the input signal $V_{in}(f)$, (ii) it then propagates to the defect considering attenuation
210 $e^{-\alpha(z_1+z_2)}$, (iii) the interaction between incident T(0,1) wave and the defect, denoted as SDI(f), (iv)
211 the reflected wave considering attenuation $e^{-\alpha z_2}$, and finally (v) measured by the ring 2 by $R(f)$.

212 The SDI can be obtained by dividing Eq. (21) by Eq. (20), as:

$$\text{SDI}(f) = \frac{S(f)e^{2\alpha(f)z_2}}{T(f)}. \quad (22)$$

213 If $\alpha(f)$ has no significant change in the frequency range, and z_2 is not very long, the attenuation
214 term $e^{2\alpha(f)z_2}$ can be ignored. Otherwise, the attenuation needs to be considered, and the effect of
215 attenuation to the quantification accuracy will be discussed in Section 4.4.

216 In the following sections, we will realize the proposed method through FE simulation and
217 laboratory experiment.

218

219 **FE validation**

220 **FE set-up**

221 To validate the proposed method, a three-dimensional FE simulation is carried out using
222 commercial FE analysis software, ABAQUS/Explicit 6.14-3. Fig. 5 shows the configuration of the FE
223 model setup. The pipe is made of aluminum, which has Young's modulus $E = 69.0$ GPa, Poisson's
224 ratio $\nu = 0.33$ and density $\rho = 2700$ kg m⁻³. The pipe has a length of 600 mm, an outer diameter of
225 90 mm, and a wall thickness of 3 mm. In the FE model, the pipe is modeled using C3D8R elements.
226 To ensure the accuracy of the simulation results, the size of elements is 1 mm which is less than 1/20

227 the shortest wavelength, and the analysis time step is $0.1 \mu\text{s}$ which is less than $1/(20f_c)$, with f_c being
228 central frequency of the applied pulse signal. The excitation signal is a 5-cycle Hanning window-
229 modulated sinusoid tone-burst, and it is imposed by applying tangential displacement around the outer
230 circumference at the pipe end to generate T(0,1) mode. The reflection signals are measured by a circle
231 of 16 points at a distance of 200 mm from the excitation end in a typical way (Xu et al. 2019).

232 **FE realization**

233 Axisymmetric defect

234 An axisymmetric defect scenario is firstly simulated. The defect is simulated by a reduction of
235 the thickness throughout 1 mm, over a length of 30 mm at a distance of 400 mm from the excitation
236 end, as indicated in Fig. 5.

237 For the single-excitation method, a central frequency of 100 kHz is selected for the excitation
238 pulse signal, while for the multiple-excitation method a frequency range from 50 kHz to 150 kHz with
239 10 kHz step is selected. The incident wave and reflected wave are measured at this frequency range as
240 can be seen in Fig. 6.

241 The SDI are calculated by dividing the incident wave amplitude to the reflection wave amplitude
242 in frequency domain, and the results are shown in Fig. 7 (a) for single-excitation method and Fig. 8 (a)
243 for multiple-excitation method. By calculating the RMSE between the calculated SDI with the
244 theoretical model, the probability map of defect size can be obtained as can be seen in Fig. 7 (b) for
245 single-excitation method and Fig. 8 (b) for multiple-excitation method. The defect sizes of maximum
246 probability are 1.02 mm in depth and 30.6 mm in length for the single excitation, and 1.11 mm in depth

247 and 30.3 mm in length for the multiple-excitation method. Both sets of the results are very close to the
248 actual defect sizes that are set in the FE model.

249 Non-axisymmetric defect

250 Non-axisymmetric defects are more common in pipeline. For the non-axisymmetric cases, Eq.
251 (16) should be modified as:

$$\left| \frac{A^{sc}}{A^{in}} \right| = \sin(kL_e) \frac{b^4 - (b - D)^4}{b^4 - \alpha^4} \frac{\theta_d}{2\pi}, \quad (23)$$

252 where θ_d is the circumferential degree which can be seen in Fig. 5.

253 Based on Eq. (23) the amplitude of the SDI is linear to the circumferential degree, which means
254 if an unknown defect was considered as an axisymmetric case, the axial length of defect could still be
255 quantified accurately, but the depth would be underestimated. In fact, the circumferential extent of the
256 defect may be judged by using mode conversion ratio (Zhu et al. 2022). Since the circumferential
257 degree is not the main focus in this paper, we assume that the circumferential extent is a known
258 parameter. In the FE simulation we created two non-axisymmetric models with defect of 1 mm in
259 depth and 30 mm in axial length, while the circumferential extent is set at $\pi/2$.

260 The defects are quantified as 1.12 mm in depth and 30.3 mm in axial length using the multiple
261 excitation method as can be seen in Fig. 9 (a). The results are quite accurate comparing to the actual
262 defect in the FE models, demonstrating the feasibility to be applied in a non-axisymmetric defect
263 situation. Even if the circumferential extent parameter is not known and the defect is treated as an
264 axisymmetric case, the depth of the defect is underestimated while the axial length of defect can still
265 be quantified as the same, as can be seen in Fig. 9 (b).

266

267 **Robustness testing**

268 The axisymmetric defect case with 1-mm depth and 30-mm length is used to test the robustness
269 of the proposed multi-frequency method. To simulate a real inspection situation, Gaussian white noise
270 is added to the FE simulation results. The signal-to-noise ratio (SNR) ranges from 5 dB to 30 dB.

271 Fig. 10 (a) shows the received signal with 10 dB SNR as an example. Fig. 10 (b) shows the
272 variation of the percentage error in the predicted defect size with the SNR. It can be observed that even
273 for the 5-dB SNR case, the percentage errors of depth and length are just between -10% to 20%, which
274 may be regarded as acceptable. For cases with SNR over 10 dB, the percentage errors for single
275 excitation are only between -5% to 5%. The percentage error for the depth prediction using multiple
276 excitation detection is around 10%, which is slightly higher than other results. In general, both the
277 single excitation method and the multiple excitation method exhibit high accuracy and strong
278 robustness. In terms of predicting defect depth, the single excitation method demonstrates greater
279 accuracy.

280 **Attenuation effect**

281 In order to demonstrate that the proposed method can be applied under long-distance and
282 attenuation conditions, the distance between the defect and the sensor is set to be 10 m, and the
283 Rayleigh damping model is introduced to simulate attenuation effect in FE model. The damping ratio
284 ζ is defined as:

$$\zeta = \frac{1}{2} \left(\frac{\alpha}{\omega_n} + \beta \omega_n \right), \quad (24)$$

285 where, ω_n is the natural frequency. α and β are two parameters of the damping model, where $\alpha = 5$
286 and $\beta = 0$ are set to simulate the attenuation in pipeline guided wave propagation (Mei and Giurgiutiu

287 2019). Fig. 11 shows the time-domain signals and corresponding Hilbert envelopes collected by nodes
288 located at distances of $d_1 = 0.5 \text{ m}$ and $d_2 = 2.5 \text{ m}$ from the excitation nodes. The attenuation
289 coefficient $\alpha(f)$ is calculated by Eq. (25) at each central frequency.

$$\alpha(f) = \frac{-20 \log_{10}(A_2/A_1)}{d_2 - d_1}, \quad (25)$$

290 where, A_1 and A_2 are the peak amplitudes of the Hilbert envelopes of the time-domain signals
291 collected at $d_1 = 0.5 \text{ m}$ and $d_2 = 2.5 \text{ m}$ respectively. Fig. 12 shows the attenuation coefficients at
292 different frequencies, where the tendency is similar with the theoretical results (Marzani 2008). It can
293 be seen that as the frequency increases, the amplitude and change of attenuation coefficients increase.

294 In the present example, the single-excitation method at 50, 100, 150 kHz and the multi-excitation
295 method with 50-150 kHz central frequencies are all used for the defect quantification, the results are
296 shown in Table 1. It can be observed that attenuation has a certain impact on quantification accuracy.
297 For the single-excitation method, the quantification accuracy decreases as the center frequency
298 increases. For the multi-excitation method, the quantization accuracy is also reduced since the
299 attenuation coefficient changes significantly in the 100-150 kHz range. These results show that, if a
300 relatively low frequency ($< 100 \text{ kHz}$ in this case) is used, the proposed method maintains satisfactory
301 accuracy in long-distance detection considering attenuation effect.

302

303 **Experimental validation**

304 **Experimental set-up**

305 In order to validate the proposed method for pipeline defect quantification in a physical
306 environment, a series of laboratory experiments have been conducted on an aluminum pipe (Al-6061).

307 The pipe has the same dimensions as used in the FE simulation, and two transducer rings are used to
308 excite and receive $T(0,1)$ guided wave respectively. As seen from the experimental setup in Fig. 13,
309 each transducer ring contains 16 rectangle PZT strip transducers, which are equally bonded along the
310 pipe's circumferential direction, for $T(0,1)$ wave generation and reception. The distance between the
311 excitation ring and reception ring is 200 mm along the axial direction of the pipe. PZT transducers are
312 type PZT-5H ceramic with length 8 mm, width 4 mm, and thickness 1 mm. These PZT transducers are
313 mounted on the surface of the pipe using epoxy.

314 The defect is made as axisymmetric wall-thinning defect with 40-mm axial length and 1-mm
315 depth, located at 400 mm away from the excitation end, and it is created by a computer numerical
316 control (CNC) cutting machine to ensure the accuracy of the defect sizes.

317 The piezoelectric wafers are driven by a five-cycle Hanning window-modulated sinusoidal tone-
318 burst signal generated by an arbitrary function generator (Tektronix AFG3022) and amplified by a
319 high-voltage power amplifier (Pintech HA-205). The transmission and reflection signals were
320 measured by the ring of piezoelectric sensors and superimposed collected using a digital oscilloscope
321 (Rohde & Schwarz, RTB-2002).

322 **Experimental realization**

323 Fig. 14 (a), (b) show the time domain signal for 50 and 150 kHz excitation, respectively. It can be
324 seen that the transmission wave is the first wavepacket and reflection wave from defect is the second
325 wave packet in the time domain and easy to extract. These signals are then calculated in the frequency
326 domain, and subsequently the SDI can be obtained. Fig. 15 (a), 16 (a) and 17 (a) show the results for

327 single excitation at 50, 100, 150 kHz , and the results for multiple excitation method are shown in Fig.
328 **18** (a). The dashed line is the best fitted theoretical model. By comparing the RMSE between the
329 experimental and theoretical results with different sizes, we can draw the probability map to predict
330 the defect sizes, as shown in Fig. **14** (b), **16** (b) and **17** (b) for single excitation at 50, 100, 150 kHz and
331 Fig. **18** (b) for the multiple excitation method.

332 In order to avoid false predictions in using the multiple excitation method, an additional parameter,
333 i.e. the number of peaks of SDI curve, can be utilized to narrow the possible axial length range firstly.
334 There are three peaks in the measured SDI curve from 50 kHz to 150 kHz as can be seen in Fig. **18** (a),
335 and according to the theoretical model, the axial length should be between 370 and 470 mm. Then, the
336 probability map is narrowed as can be seen from a red frame in Fig **18** (b) to avoid false predictions.

337 The calculated defect results are summarized in Table **2** and **3**. The percentage errors for axial
338 length prediction is around 7% except for the 150 kHz cases. Excluding a false prediction in the 150
339 kHz case, the percentage error from the other prediction result is around 5%. The percentage errors for
340 the depth prediction are around 30%. In other words, for the actual defect of 1-mm depth and 40-mm
341 axial length, the predicted defect length is around 42.5 mm, and the predicted defect depth is around
342 1.35 mm. These results demonstrate that the proposed method maintains satisfactory accuracy in a
343 physical laboratory testing environment.

344 The percentage errors for the defect depth are higher than those for the axial length, which is
345 understandable and maybe attributed primarily to two reasons: 1) The axial length of the defect is
346 significantly larger than its depth in terms of the absolute dimension, thus for the same magnitude of
347 error in sizes, the percentage error in depth is obviously larger. 2) As the scattering amplitude directly

348 influences the predicted depth value without directly affecting the predicted length value, there are
349 more factors influencing the depth prediction in practical detection. The results also show the effect of
350 central frequency on the predicted defect parameters, and this is more evident in the depth prediction.
351 The percentage errors are 40% and 22% corresponding to the central frequency of 50 kHz and 150
352 kHz, respectively. This trend suggests that a higher central frequency can effectively improve the
353 prediction accuracy, especially for the depth prediction.

354

355 **Conclusions**

356 This paper proposes a new approach for pipeline defect quantification based on T(0,1) guided
357 wave. A forward theoretical model describing the interaction between T(0,1) wave with wall-thinning
358 defect is introduced, and an inverse quantification process is realized by the proposed multi-frequency
359 identification method.

360 The theoretical scattering model, which is based on the reciprocity theorem, considers the T(0,1)
361 wave interaction with a wall-thinning defect in that the scattering amplitude is a superposition of the
362 front and back edges of the defect. Thus, the defect depth and axial length are represented, and their
363 effect on the model is expressed as frequency dependent amplitudes. On this basis, a spectral defect
364 index (SDI) is introduced which is associated with defect sizes and therefore allows quantifying the
365 defect through an inverse procedure. An inspection configuration is proposed to actually measure SDI.
366 Two specific methods for obtaining SDI are proposed, one using a single excitation method and another
367 using a multiple excitation method.

368 Finite element models are employed to test the performance and robustness of the proposed
369 method. Both axisymmetric and non-axisymmetric defect cases are considered. The predicted results
370 from the FE simulation show excellent accuracy, with a percentage error below 10% for the depth
371 prediction and below 2% for the axial length prediction. The robustness of the technique is tested by
372 adding Gaussian white noise to the received signals. The results show that for 5-dB SNR, the
373 percentage error for depth prediction is between -10% to 20%, and for axial length is between -5% to
374 10%. For 10-30 dB SNR cases, the percentage errors are generally between -5% to 10%, showing its
375 strong robustness to noise disturbance. In addition, a FE model for long-distance detection considering
376 attenuation effect is employed. The results show the proposed method can be applied within the
377 relatively low frequency range (< 100 kHz), and has achieved good results.

378 Experiments are also conducted to validate the proposed method. For the wall-thinning defect
379 with 1-mm depth and 40-mm axial length, the errors are found to be around 0.3 mm for depth and 25
380 mm for length predictions, which are satisfactory, showing its potential in real inspection. From the
381 experiment, it has also been observed that the single excitation method is more efficient comparing
382 with the multiple excitation method, and as the wave signal frequency increases, the prediction
383 accuracy also improves. False predictions occurred in both excitation methods. Since the multiple
384 excitation method obtains a wide frequency range SDI, the defect size can be narrowed based on other
385 parameters such as the number of peaks of the SDI curve. Therefore, false predictions are easier to be
386 avoided through multiple excitation method.

387 It should be noted that the corrosion defect in the present study has been simplified as wall
388 thinning defect. Future research is expected to extend the proposed method to more realistic profiles
389 of corrosion defects.

390 **Data Availability Statement**

391 All data, models, and code generated or used during the study appear in the submitted article.

392 **Declaration of competing interest**

393 The authors declare no conflicts of interest regarding the publication of this manuscript.

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1

Table 1. FE results for the axial length and depth of defect

| | Actual | 50 kHz | 100 kHz | 150kHz | 50-150 kHz |
|-------------------|--------|-------------|-------------|-------------|-------------|
| Axial length (mm) | 30 | 30.6 (30.6) | 29.1 (30.6) | 18.6 (30.7) | 31.8 (30.3) |
| Depth (mm) | 1 | 0.98 (0.96) | 0.65 (1.02) | 0.99 (1.01) | 0.63 (1.11) |

2 The results without considering attenuation are shown in parentheses.

1

Table 2. Experimental results for the axial length of defect.

| | Actual | 50 kHz | 100 kHz | 150kHz | 50-150 kHz |
|----------------------|--------|--------|---------|--------|------------|
| Axial length (mm) | 40 | 42.6 | 42.9 | 49.1 | 42.5 |
| Percentage error (%) | \ | 6.5 | 7.25 | 22.75 | 6.25 |

2

1

Table 3. Experimental results for the depth of defect.

| | Actual | 50 kHz | 100 kHz | 150kHz | 50-150 kHz |
|----------------------|--------|--------|---------|--------|------------|
| Depth (mm) | 1 | 1.40 | 0.71 | 1.22 | 1.35 |
| Percentage error (%) | \ | 40 | -29 | 22 | 35 |

2

1 **Table Caption List**

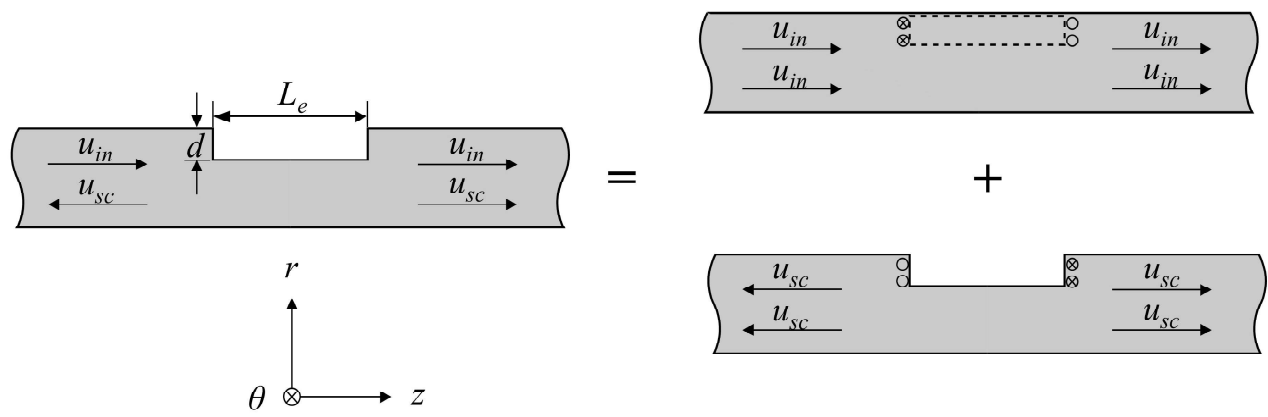
2 **Table 1.** FE results for the axial length and depth of defect

3 **Table 2.** Experimental results for the axial length of defect.

4 **Table 3.** Experimental results for the depth of defect.

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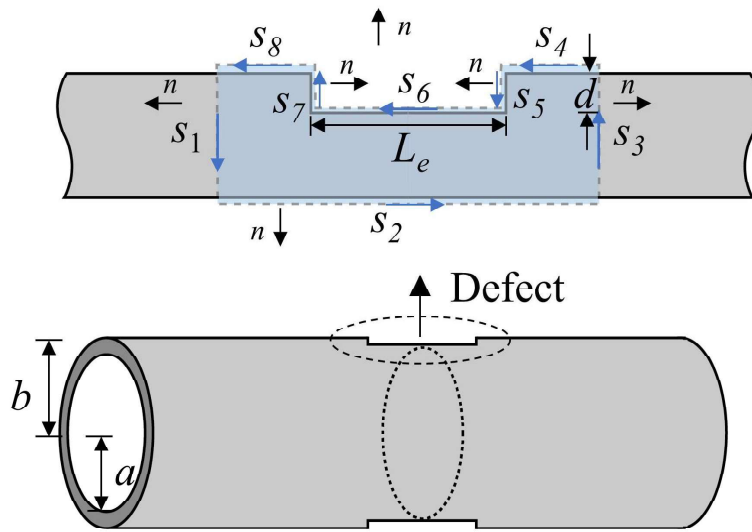


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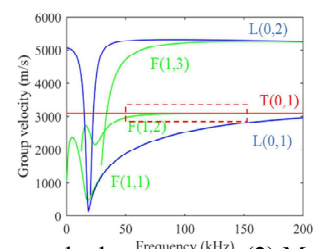
Fig. 1. Schematic of wave superposition principle.



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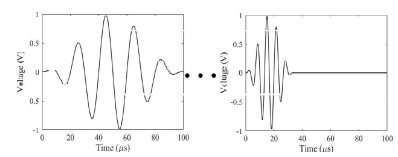
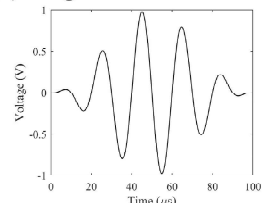
Fig. 2. Defect geometry and integration paths.

Select frequency range to generate single T(0,1) wave

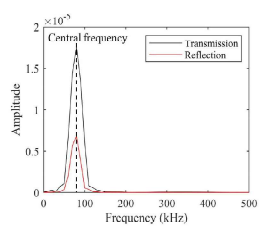
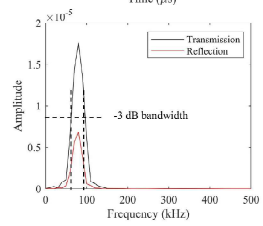


Excite T(0,1) wave at different central frequencies

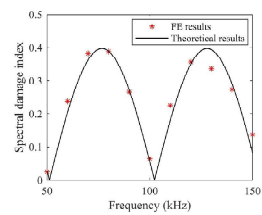
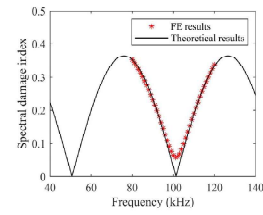
(1) Single-excitation method (2) Multiple-excitation method



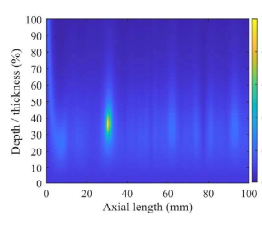
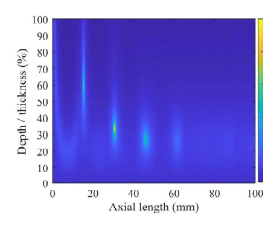
Collect transmission wave and reflection wave, then apply FFT



Calculate spectral defect index and inverse calculate defect parameters



Plot defect size probability map



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Fig. 3. Scheme of multi-frequency identification method.

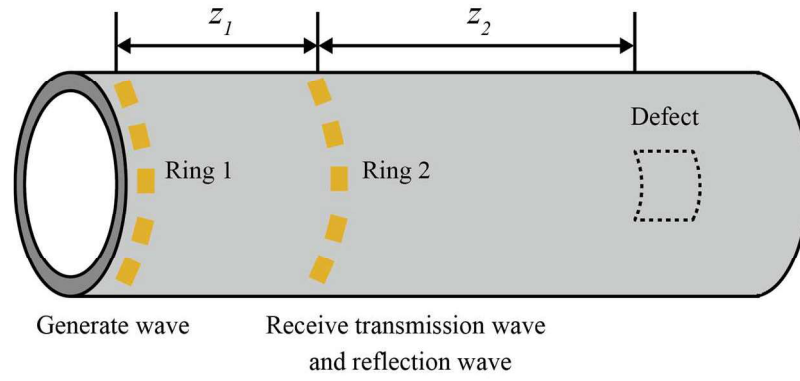
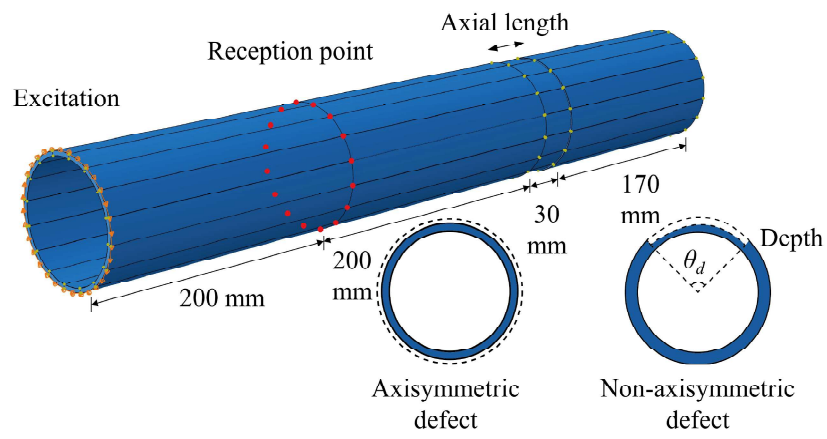
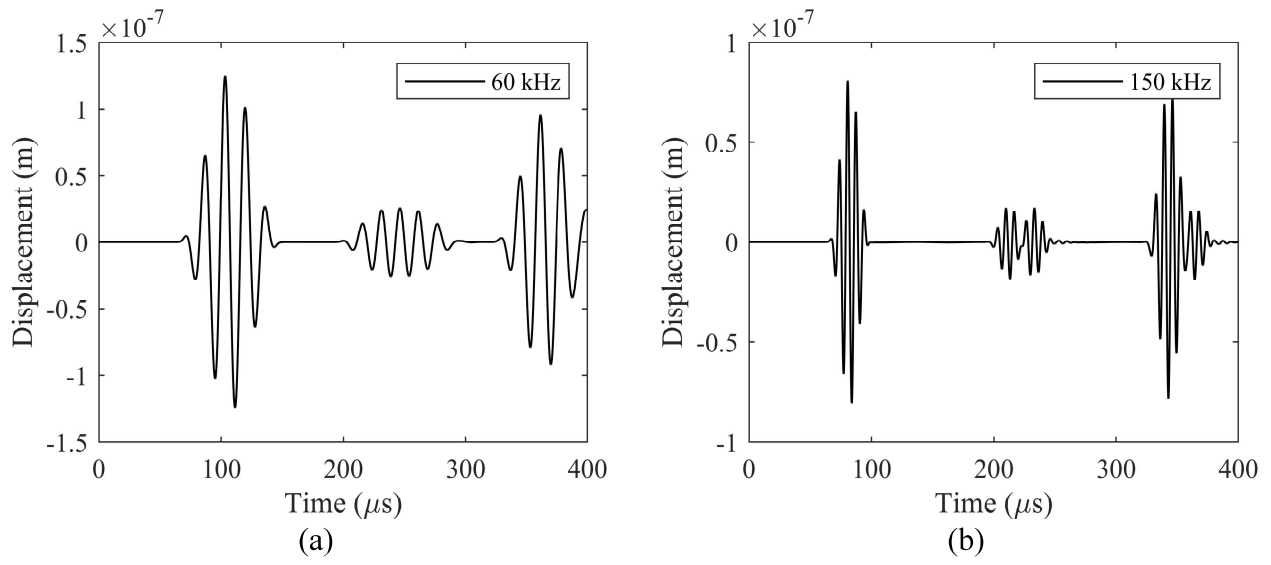


Fig. 4. Measurement configuration for multi-frequency identification method.



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Fig. 5. Schematic of FE model.

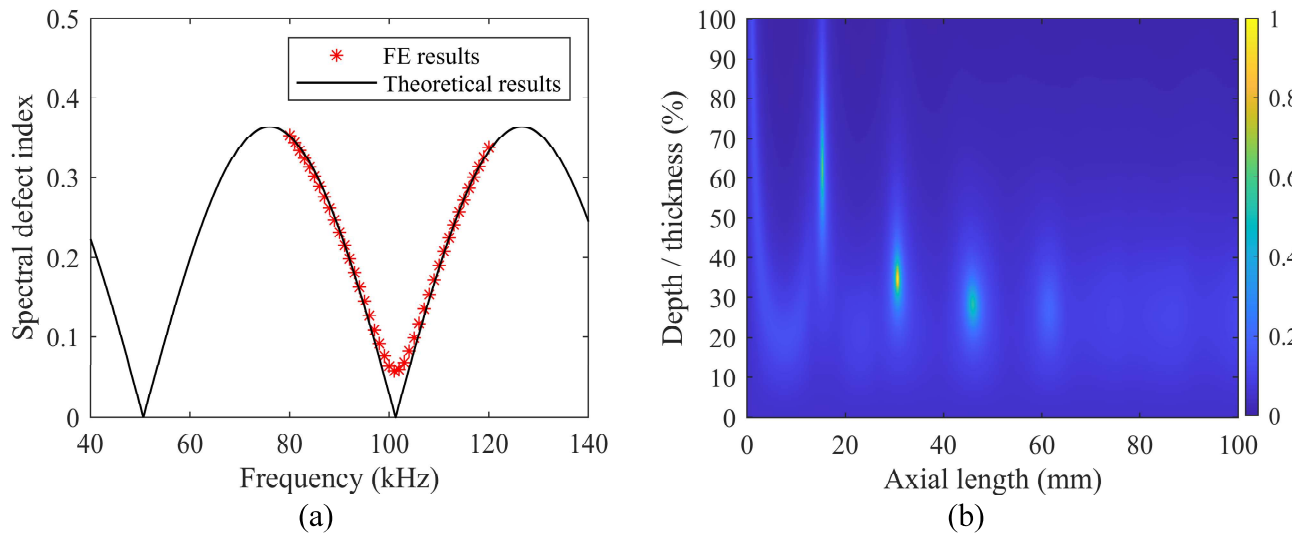


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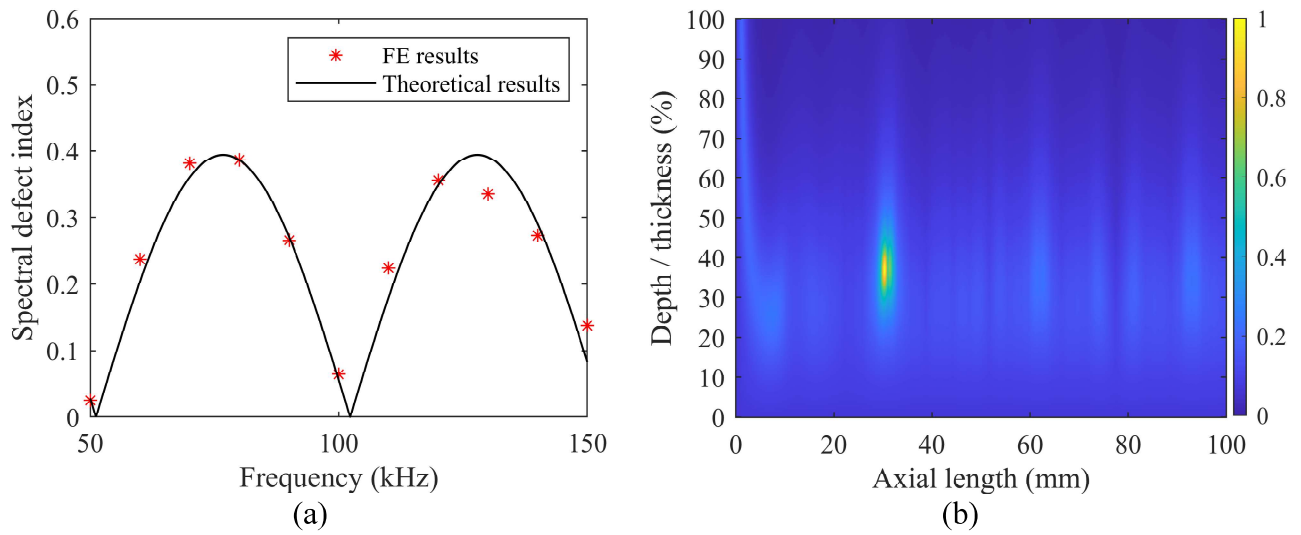
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Fig. 6. FE results of received time domain signal for (a) 60 kHz excitation, (b) 150 kHz excitation.



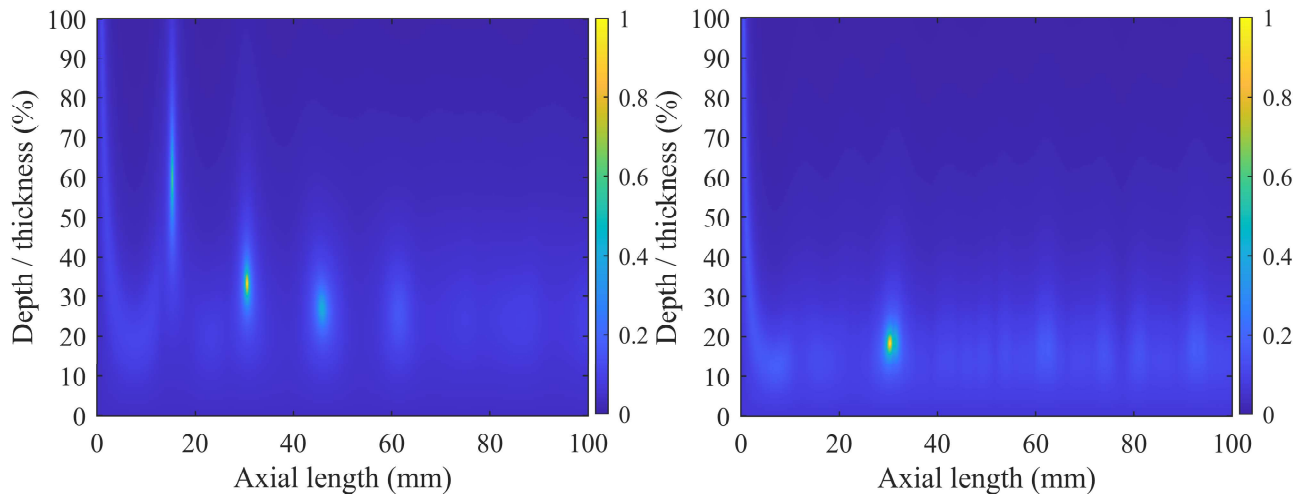
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2 **Fig. 7.** FE results of (a) Spectral defect index for 100 kHz single-excitation method, (b) Defect probability map for
3 100 kHz single-excitation method.



1

2 **Fig. 8.** FE results of (a) Spectral defect index for multiple-excitation method at 50-150 kHz, (b) Defect probability
3 map for multiple-excitation method at 50-150 kHz.



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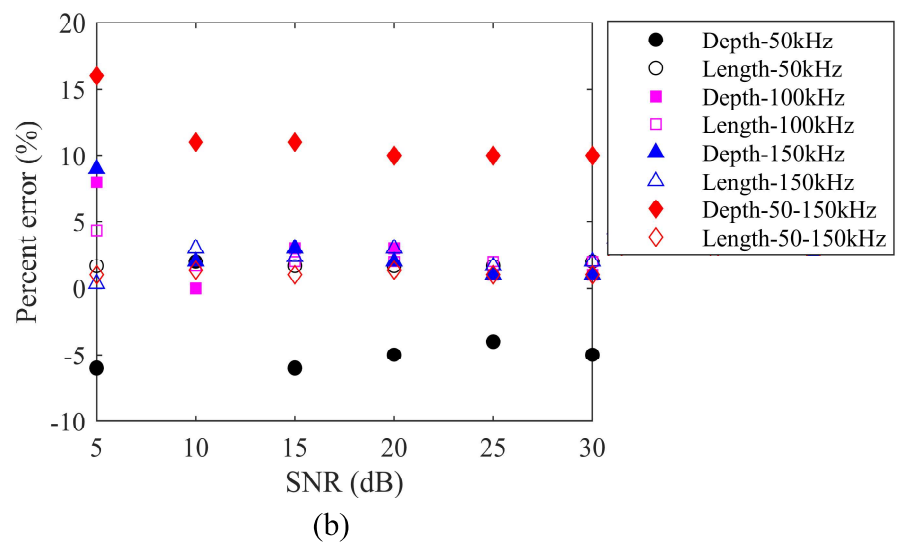
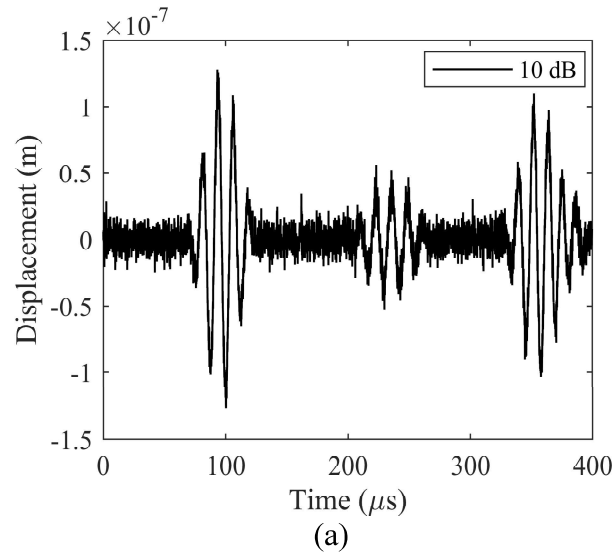
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Fig. 9. (a) Defect size probability map for multiple-excitation method at 50-150 kHz, (b) Non-axisymmetric defect size probability map for multiple-excitation method at 50-150 kHz.



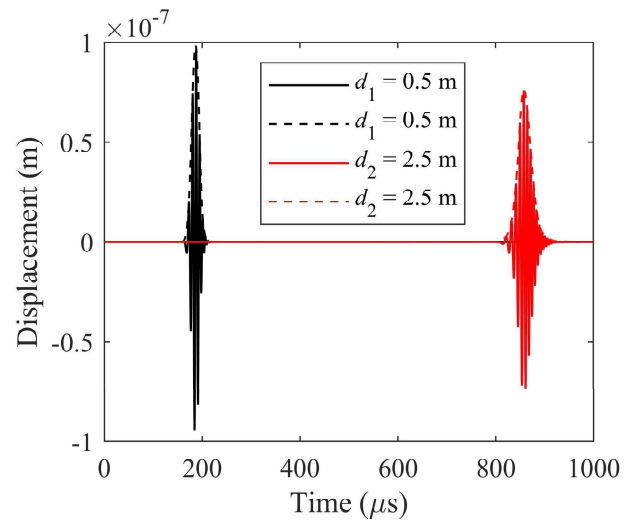
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5 **Fig. 10.** Robustness test results. (a) Received signal with Gaussian white noise, SNR =10 dB, (b) Percentage error
6 for FE results.

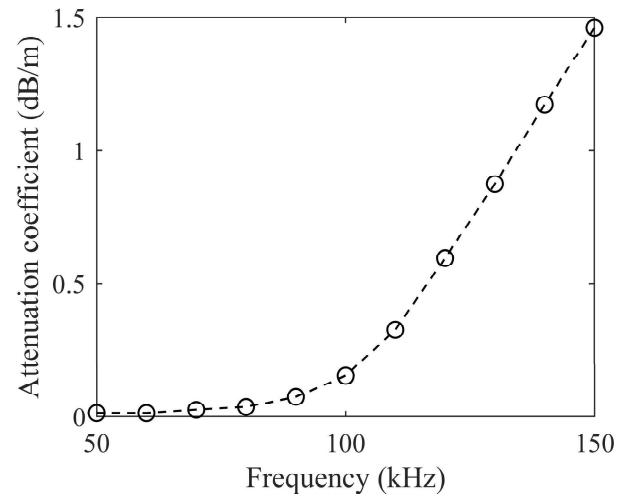


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Fig. 11. FE results of received time domain signals and corresponding Hilbert envelopes for 140 kHz excitation

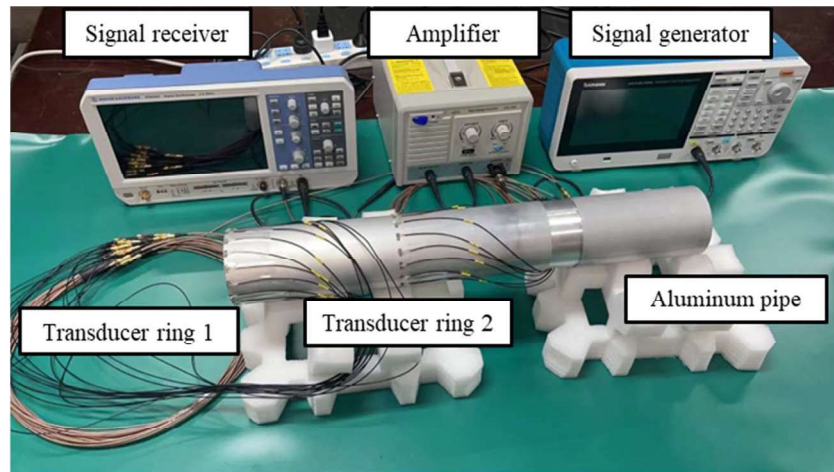


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Fig. 12. T(0,1) wave attenuation coefficients calculated by FE model

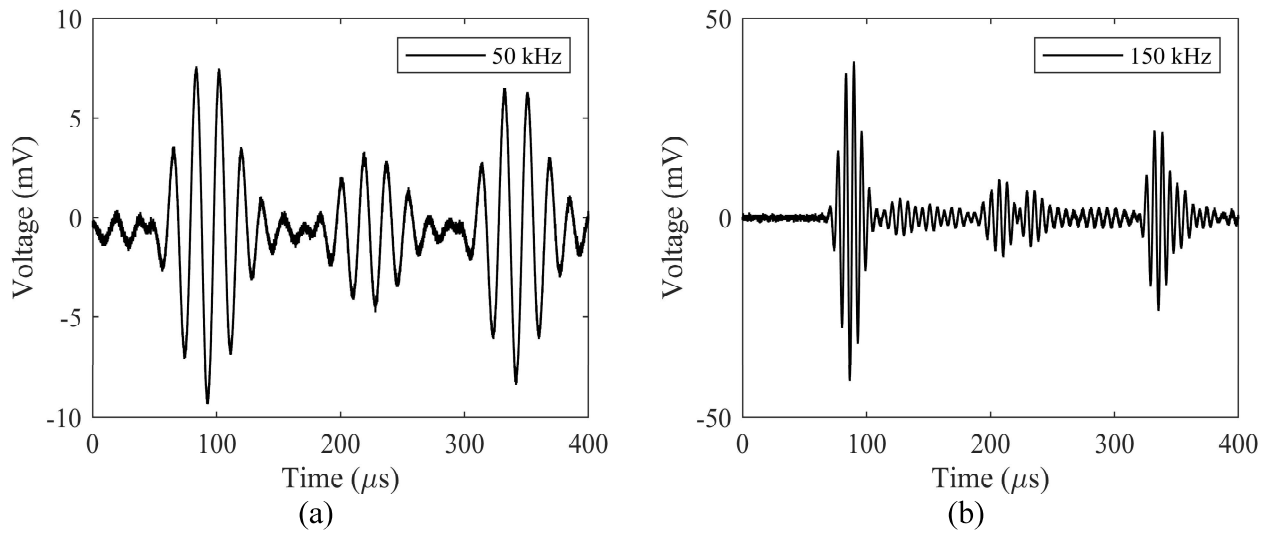


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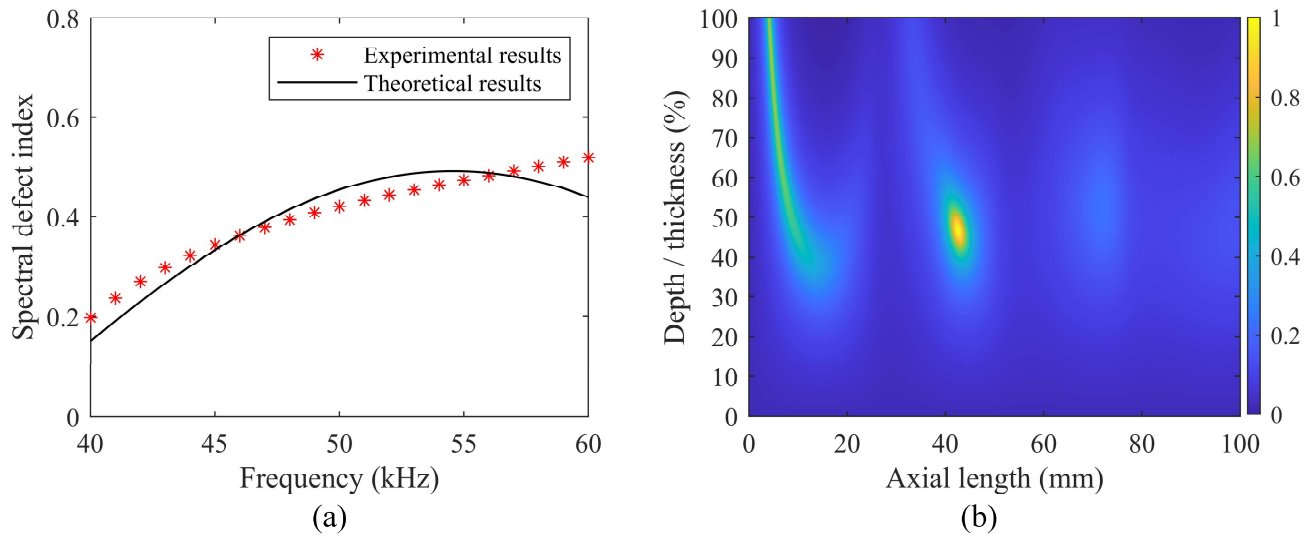
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Fig. 13. A photo of experimental setup.



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2 **Fig. 14.** Experimentally measured wave signals (a) at 50 kHz central frequency, (b) at 150 kHz central frequency.



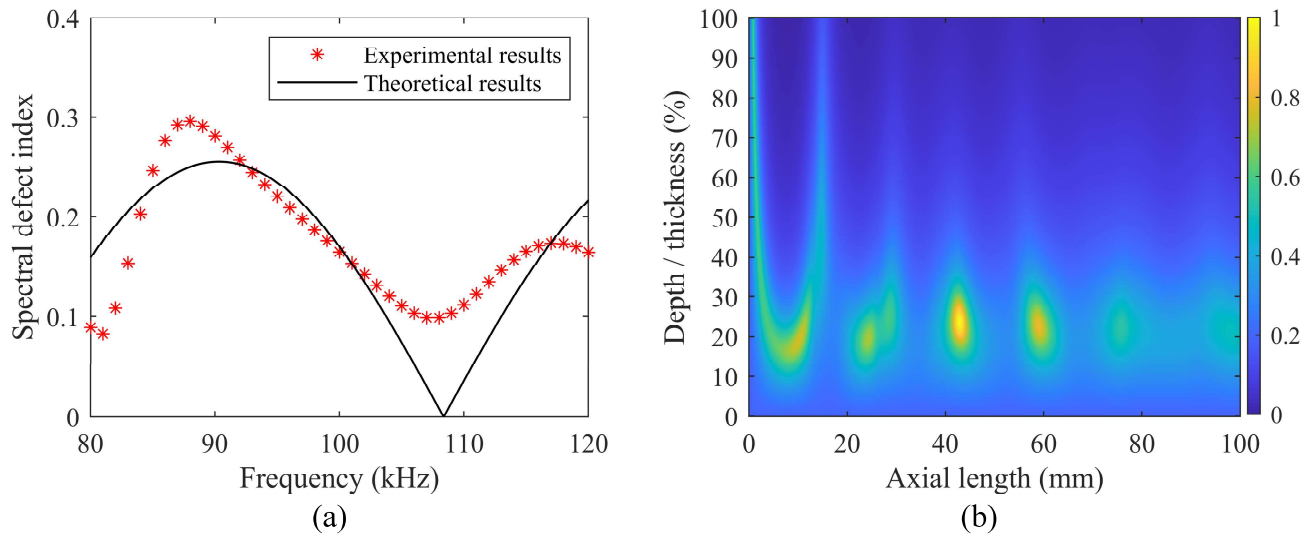
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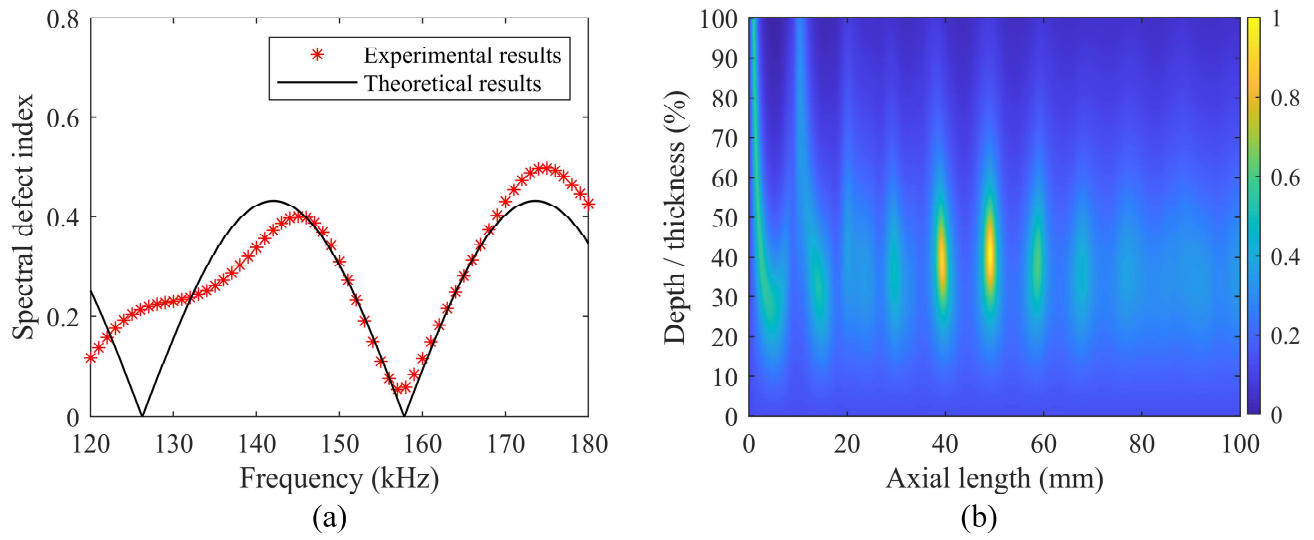
Fig. 15. (a) Spectrum defect index of experimental results and theoretical results of single-excitation method at 50 kHz center frequency, (b) Defect size probability map for single-excitation method at 50 kHz center frequency.



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3 **Fig. 16.** (a) Spectrum defect index of experimental results and theoretical results of single-excitation method at 100
4 kHz center frequency, (b) Defect size probability map for single-excitation method at 100 kHz center frequency.



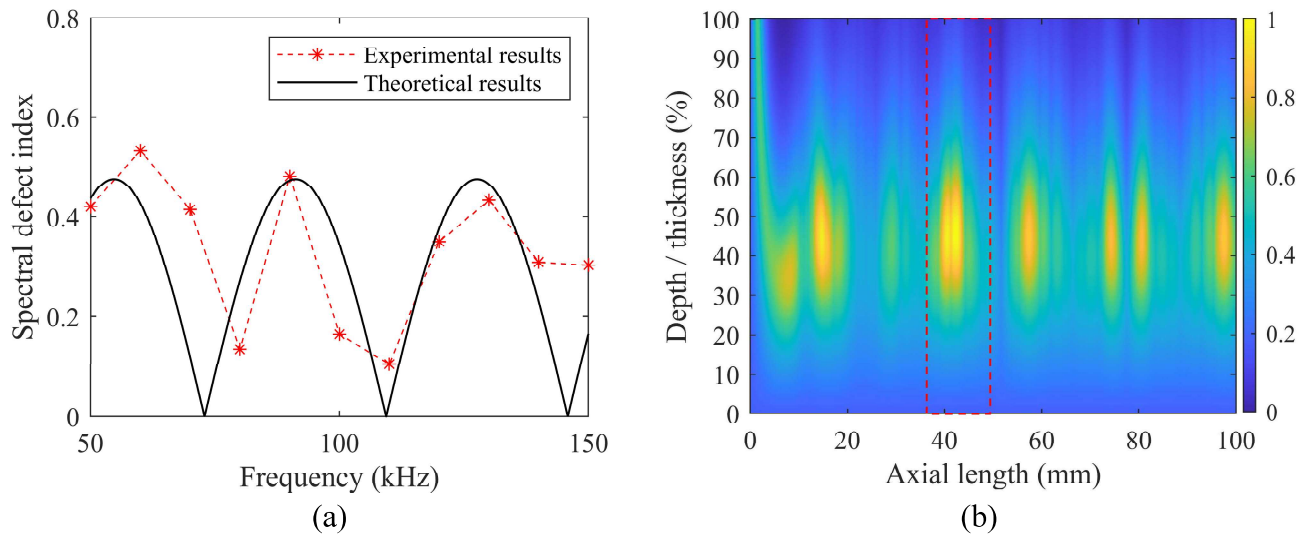
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Fig. 17. (a) Spectrum defect index of experimental results and theoretical results of single-excitation method at 150 kHz center frequency, (b) Defect size probability map for single-excitation method at 150 kHz center frequency.



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Fig. 18. (a) Spectrum defect index of experimental results and theoretical results of multiple-excitation method at 50-150 kHz frequency range, (b) Defect size probability map for multiple-excitation method at 50-150 kHz frequency range.

1 **Figure Caption List**

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18 **Fig. 12.** $T(0,1)$ wave attenuation coefficients calculated by FE model

19 **Fig. 13.** A photo of experimental setup.

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