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APPLICATIONS TO THE PUBLIC SECTOR SPECIAL ISSUE ARTICLES

TENDER VACANCIES ALLOCATION TO THE MEMBER UNITS OF A PUBLIC RESEARCH COMPANY: A DATA ENVELOPMENT ANALYSIS APPROACH

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ABSTRACT. The Brazilian Agricultural Research Corporation is a public research company, comprised of a central management structure and forty-three research centers. Given the recent dismissal of employees, due to retirement or to promoted layoffs, and the expected announcement of a recruitment to replace the staff, it is important to define how these vacancies will be distributed among its member units. This study proposes data envelopment analysis (DEA) models to allocate the vacancies to the company's research centers, using Zero Sum Gains DEA (ZSG-DEA). We studied both production and performance modeling perspectives. Given the approaches adopted and the variables selected, the ZSG-DEA models generated allocation proposals in which all units are 100% efficient. This is the ideal scenario for a central management: maximum efficiency in the use of its resources.

Keywords: resources allocation, efficiency, data envelopment analysis.

1 INTRODUCTION

Allocation, assignment, or distribution of resources is one of the problems posed to managers of different organizations, especially when quantities are scarce or restricted. In general, allocation problems involve the optimal distribution of fixed resources among competing alternatives, to minimize total costs or maximize total return (profit or gain). The problem consists in determining how much of each resource should be assigned to each task or organizational unit, subjected to certain conditions (Hillier and Lieberman, 2017). These problems are composed of a set of

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resources available in certain quantities, a set of tasks that consume given quantities of resources and a set of costs or returns for each job and resource. The amount of resources to be allocated to each task (or organizational unit) is treated as a continuous or integer variable, depending on how the variable to be allocated is defined.

One of the traditional problems treated by Operations Research is proposing a fair distribution of resources based on several criteria, and it still deserves attention in the literature (Patriksson, 2008; Bouajaja and Dridi, 2017; Kicsiny et al., 2023). The classical approach is by formulating mathematical programming problems. Such problems can be mono- or multiobjective (with objective functions of return maximization, cost minimization, or others), linear or non-linear, with integer variables or not. Still in this context and as an alternative, there is the option of using efficiency models, as Data Envelopment Analysis (DEA), which seeks to maximize the efficiency scores of each observation under analysis.

As presented and discussed in Cooper et al. (2007), the first DEA models were proposed in the 1970s and 1980s to measure the efficiency of production units or firms (the so-called Decision-Making Units – DMUs) that consume multiple resources (inputs) and produce multiple products (outputs). The relationships between inputs and outputs are established by mathematical programming problems. Classical DEA models assume complete freedom of resources consumption or of production. However, in cases where one of the variables has a fixed total, such as in cases of resources to be (re)allocated, classical DEA models must have their formulation changed to include this additional constraint. Recent DEA literature provides alternatives for modeling this situation, referred as 'fixed costs,' 'centralized decision' or 'fixed-sum variables' DEA models (Beasley, 2003; Korhonen and Syrjänen, 2003; Lozano and Villa, 2004; Gomes and Lins, 2008; Milioni et al., 2011; Lozano, 2023, among others).

Given this context and the expectation of recruitment to replace the staff of the Brazilian Agricultural Research Corporation (Embrapa), due to recent retirement or promoted layoffs of its employees, a corporate project at the company's headquarters was set. One of the objectives of this project is "to characterize the vacancies (positions) in order to organize and prepare the recruitment To achieve such goal, it is foreseen to "specify the vacancies for the recruitment, defining the positions, areas of knowledge, and location of assignment, based on the current analysis of the staff, the dismissals that have occurred and the future needs of Embrapa." The team responsible for the Strategy Monitoring and Evaluation was asked to propose a methodology to distribute the vacancies of the recruitment among the research centers (decentralized units) of the Company. Thus, in this article, we propose DEA-based models to define possible vacancies' allocations. The idea is to use the current cost and production status of the research centers, and the results related to research activities as the basis for the allocation proposals. In this study we did not consider indicators related to professional profiles. We understand that these methods are useful decision support tools, which help to formulate the problem and to identify potential solutions.

Our paper is structured as follows. In Section 2 we describe the theoretical aspects of DEA and ZSG-DEA approaches. Section 3 is on modeling perspectives, data and structuring issues.

Results and discussion are presented in Section 4. Finally, the concluding remarks are presented in Section 5, followed by the references.

2 DEA AND ZSG-DEA MODELING

2.1 DEA Models

DEA models were designed to estimate an empirical, non-parametric efficiency frontier surrounding the data. Broadly speaking and for each unit under evaluation, this model computes an efficiency score as the ratio of the weighted sum of outputs and the weighted sum of inputs. The model allows each unit under evaluation to determine the weights for each variable (inputs and outputs) in the most benevolent way, as long as these weights applied to the other DMUs do not generate a ratio greater than one. These conditions are formulated in the fractional programming problem (1a), whose linear form is shown in (1b). In (1) Eff_o is the efficiency score of DMU o under evaluation; v_i and u_j are the weights of inputs and outputs (decision variables), respectively; x_{ik} and y_{jk} are the inputs i, $i = 1,...,r$, and outputs $j, j = 1,...,s$, of DMU k; x_{io} and y_{jo} are the inputs i and outputs j of DMU o.

$$
\begin{aligned}
&\text{Max } \; Eff_o = \left(\frac{\sum_{j=1}^{s} u_j y_{jo}}{\sum_{i=1}^{r} v_i x_{io}} \right) \\
&\text{subject to} \\
&\frac{\sum_{j=1}^{s} u_j y_{jk}}{\sum_{i=1}^{r} v_i x_{ik}} \leq 1, \quad k = 1,...,n \\
&u_j \text{ e } v_i \; \geq 0 \; \forall \; j,i \quad (a) \\
&\text{Max } \; Eff_o = \sum_{j=1}^{s} u_j y_{jo} \\
&\text{subject to} \\
&\sum_{i=1}^{r} v_i x_{io} = 1 \\
&\sum_{j=1}^{s} u_j y_{jk} - \sum_{i=1}^{r} v_i x_{ik} \leq 0, \quad k = 1,...,n \\
&u_j \text{ e } v_i \; \geq 0 \; \forall \; j,i \quad (b)
\end{aligned}
\tag{1}
$$

The simplest ways to reach the efficiency frontier are by reducing inputs while keeping outputs unchanged (input-oriented), or by increasing outputs while maintaining resource levels (output-oriented). In addition, the efficiency frontier can be convex or not, depending on the assumptions imposed and which one entails different returns to scale, variable in the first case (VRS or BCC frontier) and constant in the second (CRS or CCR frontier). Formulation (1b) is known in the literature as the input-oriented DEA-CRS model. This general formulation is known as the multipliers model, as it determines the weights or multipliers that will be used in weighting the variables.

The dual of model (1b) is the model shown in (2). In this formulation $h_o$is the efficiency and $\lambda_k$is the contribution of DMU k in calculating the target of DMU o. DMUs with nonzero $\lambda_k$are the benchmarks of DMU o. This formulation is known in the DEA literature as the envelope model because it allows for the graphical construction of the efficiency frontier which envelopes the data. DMUs that are part of the efficiency frontier are efficient. Those that are below this surface (or enveloped) are inefficient.

$$
\begin{aligned}
&\text{Min } h_o \\
&\text{subject to} \\
&h_o x_{io} - \sum_k x_{ik}\lambda_k \geq 0, \forall i \\
&-y_{jo} + \sum_k y_{jk}\lambda_k \\
&\lambda_k \geq 0, \forall k
\end{aligned} \tag{2}
$$

As already defined, the formulations presented are for input-oriented models with constant returns to scale. For models with variable returns to scale, the convexity hypothesis implies adding a restriction to model (2), that is, $\sum_k \lambda_k = 1$. Consequently, in the dual (multipliers model), there will be an additional free variable representing the type of return to scale (increasing, constant or decreasing). These formulations are described in Cooper et al. (2007).

2.2 Models with fixed-sum variable

The classical DEA models assume both total freedom of production, that is, the production of a DMU does not interfere in the production of the others, and total availability of resources, that is, the reduction of inputs in one firm does not imply reallocation to the others, when projecting onto the efficiency frontier. However, in some cases this freedom is not possible, since the variable must have a fixed, constant total. In these situations, it is necessary to change the classical DEA models to add such restrictions.

Recent DEA literature provides alternatives for modeling this situation, in general with DEA models called ‘fixed-sum variable’ or ‘centralized decision’ models.

The first work devoted to studying the allocation of a fixed cost in DEA was presented by Cook and Kress (1999)and extended by Cook and Zhu (2005), based on the idea that efficiencies should remain constant. Beasley (2003) proposed non-linear models for fixed cost allocation in DEA, considering that average efficiency should be maximized in addition to changing individual efficiencies.

When reviewing the evolution of this type of situation in the DEA context, Lozano (2023) categorized the fixed total or fixed-sum approaches into two groups. The first is Zero-Sum Gains DEA models (ZSG-DEA), originally proposed and discussed in Gomes (2003), Lins et al. (2003), Gomes et al. (2004, 2007, 2010), Gomes and Lins (2008), and later extended and applied by Gomes et al. (2008), Macedo et al. (2010), Bi et al. (2014), Pang et al. (2015), Wu et al. (2016), Feng et al. (2019), Bernardo et al. (2021), Sun et al. (2022), among others. In general, ZSG-DEA models compute a uniform efficiency frontier composed of all DMUs. To do so, for the constant sum variable there should be gains or losses in each DMU. That is, the gain in one DMU is obtained by the loss in another to keep the total unchanged (hence 'zero-sum gains'). The second group presented in Lozano (2023) is that of models based on an Equilibrium Efficient Frontier, originally proposed by Yang et al. (2011), and improved by Yang *et al.* (2014, 2015)and Fang (2016) with the concept Generalized Equilibrium Efficient Frontier. In this case, in a first stage the minimum necessary adjustment in the constant sum variable is calculated so that all DMUs are efficient, and the equilibrium frontier is obtained. In a second stage the efficiency of the original DMUs is calculated in relation to the final frontier. Lozano (2023) points out as a disadvantage of this approach the fact that the equilibrium frontier is not unique, which requires secondary objectives (Fang et al., 2016; Zhu et al., 2017, 2020). Alternatives to the original proposals can be seen, for example, in Chen et al. (2021), Zhu et al. (2021), and Li et al. (2021).

Considering the specific denomination of 'centralized decision', i.e., when DMUs are jointly projected onto the frontier and considering the fixed-sum constraints of some of the variables, one can cite, for example, the developments discussed in Korhonen and Syrjänen (2003), Lozano and Villa (2004, 2005), Lozano et al. (2004), Asmild et al. (2009), Zhou et al. (2013), Mar-Molinero *et al.* (2014), Ding et al. (2018, 2020), Ripoll-Zarraga and Lozano (2020), Yadollahi et al. (2022), Gupta et al. (2023), Lozano (2023).

It is also worth mentioning in this context, the so-called 'parametric DEA' models, in which the frontier that is constructed after the distribution of the fixed-sum variable is a parametric frontier: spherical, ellipsoidal, hyperbolic, or parabolic. This is the case of the models proposed in Milioni et al. (2011a, 2011b), Guedes et al. (2012), Milioni and Alves (2013), Alves et al. (2014), Silva et al. (2018), Silveira et al. (2019).

In our study, since there is a resource that has a constant sum (vacancies) and that must be (re)allocated based on individual efficiency scores, among the existing alternatives in the literature, we chose the approach via ZSG-DEA models. Our choice is due to the fact that, although the model in its general formulation is a multiobjective nonlinear model, it is possible to reduce it to solving only one equation. Furthermore, in ZSG-DEA models the search for efficiency changes the shape of the frontier, that is, the determination of the frontier and the search for targets are intertwined. It should also be noted that this type of model is a fixed-sum, centralized decision model, in the sense that the final efficiency frontier is obtained by shifting all DMUs, and there is a central hub that manages the member units with respect to the resource to be allocated.

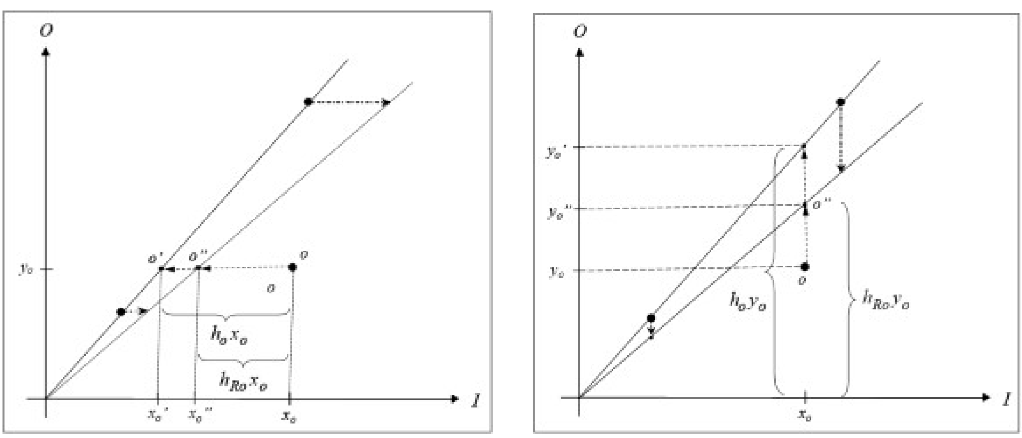
2.3 ZSG-DEA models

In ZSG-DEA models an additional restriction is imposed on classical DEA models, namely, that the sum of the variable of interest should be constant. That is, when an inefficient DMU seeks its target on the efficiency frontier, the value of this variable for the other DMUs is changed to keep the sum constant. In the ZSG-DEA paradigm, the inefficient DMUs seek the efficiency frontier cooperatively, i.e., the DMUs in this group try to take a given amount of output (or give input) only from the DMUs not belonging to the group (efficient units). After the redistribution of the variable of interest, all units will belong to the efficiency frontier, that is, the global average efficiency will be 100% (uniform frontier). This scenario meets the goal of a central administration in relation to resources, which is to promote greater efficiency in the use of resources and to have its member units operating with maximum efficiency. The interest is not in improving the performance of a specific unit, but in the performance of the organization as a whole (Afsharian et al., 2021).

The general formulation of the ZSG-DEA models is that of a multiobjective nonlinear programming problem. As discussed in Gomes (2003), Gomes et al. (2005) and Gomes and Lins (2008), theorems allow the reduction of this problem to a nonlinear equation, as presented in (3) for the input-oriented case, and in (4) for the output-oriented one. In (3) and (4), $h_{Ri}$and $h_i$are, respectively, the efficiency measures in the ZSG-DEA and classical DEA models, W is the group of cooperating units (inefficient units), $r_{ij} = \frac{h_i}{h_j}$and $q_{ij} = \frac{h_i}{h_j}$are proportionality factors resulting from the proportional frontier attainment strategy (for details see Gomes, 2003). The graphical representation of this strategy for the two-dimensional case is presented in Figure 1 for the input-oriented (a) and output-oriented (b) cases.

$$h_{Ri} = h_i \left(1 + \frac{\sum_{j \in W} \left[x_j \left(1 - r_{ij} h_{Ri} \right) \right]}{\sum_{j \notin W} x_j} \right) \tag{3}$$

$$h_{Ri} = h_i \left(1 - \frac{\sum_{j \in W} \left[y_j \left(q_{ij} h_{Ri} - 1 \right) \right]}{\sum_{j \notin W} y_j} \right) \tag{4}$$



(a) (b)

Figure 1 – Two-dimensional graphical representation of the proportional reduction strategy in input-oriented (a) and output-oriented (b) ZSG-DEA models.

Adapted from Gomes (2003).

As highlighted in Gomes et al. (2005), for the proportional strategy the value of the fixed-sum variable at the end of redistribution (final target) can be calculated by multiplying the original target (original value of the variable multiplied by the efficiency score) and the ratio between the sum of the variable and the sum of the targets. This output value after redistribution generates the same results as equations (3) and (4) (Gomes et al., 2005). It should also be noted that when there is more than one input or output variable to be reallocated, or when there is a constant sum input (or output) modeled with non-constant sum inputs (or outputs), the ZSG-DEA model remains valid, as suggested by the discussions in Gomes *et al*. (2004).

3 MODELING

3.1 Modeling Perspectives and Data

Given the availability of data per research center, adherent to both production and performance perspectives (Ramalho et al., 2010; Cook et al., 2014), in this paper we studied these two modeling possibilities. Since the number of vacancies that may be available for recruitment is not known *a priori*, we assumed in both approaches that the total number of vacancies to be allocated is one hundred.

3.2 Production perspective

The production function perspective for the research centers was grounded in the discussions of Souza et al. (1999, 2007) and Souza and Gomes (2015a, 2015b). The variables in the production function are proxies for the costs of the research centers (capital depreciation – depreciation of movable and immovable assets and amortization of immovable assets; costing expenses–expenditures for consumables, travel, and services; personnel expenses–salaries and charges), publications of articles in indexed journals (scientific production) and production of cultivars, software, and patents (technical production or research assets). Given the variation in these variables (due especially to changes in staffing), the values used refer to a five-year average (2017 to 2021; monetary values deflated to December 2021). Total staffing figures were used for the relevant normalizations (Dyson et al., 2001).

Cost data come from the company's official management and financial systems, collected, and used annually for purposes of institutional performance evaluation – productivity indicator (Embrapa, 2021). Scientific production data originate from the quantities of articles registered in the Ainfo repository (Embrapa's Documentary and Digital Collection Management System – https://www.ainfo.cnptia.embrapa.br). Data on technical production or research assets refer to the number of cultivars, software and patents registered in the Embrapa's corporate software Ideare, collected from the database of Lattes-CNPq curricula of its researchers.

Currently, Embrapa is composed of forty-three research centers. Of these, one was created in late 2018 and was disregarded in the production function perspective, due to the absence of complete data series.

3.3 Performance perspective

We used the performance-based perspective to structure DEA models with a multi-criteria outlook (Caporaletti et al., 1999; Lovell and Pastor, 1999; Gomes et al.,2012). This type of approach was applied, for example, to allocate part of the research programming budget to Embrapa's research centers using ZSG-DEA models (Gomes and Souza, 2010). Here the variables selected are the indicators of the so-called SEG Index –iSEG, as defined in Embrapa (2020). SEG is the Embrapa's Management System, which brings together Embrapa's research, development, and innovation programming, with its portfolios, programs, and research projects. iSEG is an annually calculated index that aggregates six RD&I performance indicators for Embrapa's research centers. It seeks to stimulate the alignment of research centers to Embrapa's strategic planning and to open innovation in partnership with the productive sector. In these models we used the data for the base year 2021, referring to December 2021, registered in Ideare, module SEG Index. The indicators are defined below, according to Embrapa (2020):

- Programming alignment index (Ialin): measures the alignment of the programming of research centers (projects portfolio) in relation to the composition of Embrapa's strategic planning.

- Extra SEG fundraising Index (Irext): stimulates the reduction of dependence on resources from Embrapa's own budget for project execution, by obtaining funding from the external sources.

- Open innovation index (Iinova, composed of indicators Iinova1 and Iinova2): index that measures the investment in open innovation projects in partnership with the productive sector, through funding (Iinova1) and the participation of the research center as leader of this type of project (Iinova2).

- Partnership index (Irede, composed of the indicators Irede1 and Irede2): calculates the intensity of partnerships between research centers, through the average number of internal partnerships in projects led by the center (Irede1) and the per capita number (researchers) of internal partnerships (Irede2).

3.4 Structuring the ZSG-DEA Models

For the DEA models, the DMUs are the research centers of Embrapa. For the model with a production perspective, one research center was disregarded, as already mentioned. In this model of 42 DMUs, the input is the proxy for the average total costs of each research center. The outputs are the proxies for the average total scientific production (total quantity of scientific papers published in scientific journals and registered in the Company's repository per year) and the average total research assets production of each research center (total number of cultivars, software and patents registered in Ideare per year), as described in section 3.1.1.

In DEA, it is assumed that the units under assessment are homogeneous. As discussed in Dyson et al. (2001), homogeneity may mean that DMUs undertake similar activities and produce comparable outputs, they use common technologies, the similar range of resources is available (similar inputs), and/or the units are operating in similar environments. In our paper, as for model approximation and simplification, we assume that the research centers are homogeneous: they utilize similar inputs to produce similar outputs and they have comparable conditions to produce the outputs. We also suppose that each researcher has the same potential to contribute for the outputs. We also assume that any differences due to operating environment, mission or range of inputs and outputs are managed by central administration and local managers.

In our discussion we supposed two viewpoints: allocate more vacancies at the end of the (re)allocation to the most efficient units in the initial model (i.e., take vacancies from the least efficient units and give them to the most efficient ones) or favor at the end of the (re)allocation the least efficient units in the initial model (i.e., give vacancies to the least efficient centers by taking vacancies from the most efficient ones, keeping the total unchanged). In the first viewpoint, vacancies are modeled as inputs and in the second viewpoint as outputs. We also considered reasonable that the initial allocation of vacancies can either follow a proportional distribution to the average number of personnel among the research centers, or that the initial distribution should be equal (homogeneous or uniform) among the centers.

The idea of the models under a production perspective is that with the vacancies, the research centers produce at least the average scientific production and research assets of the last five years, and that they do not exceed the average total expenditures of this period. Note that because a value of one hundred is used for the total number of vacancies, the results could be interpreted by the central administration as the percentage of vacancies that should be allocated to each research center.

For the performance-focused DEA approach we considered 43 DMUs. The variables are the six partial indicators of iSEG. As in the previous model, the variable to (re)allocate is the number of vacancies, considering as possible initial allocations the proportional and equal distributions of average personnel among the research centers (total of one hundred). This sum will be reallocated using the ZSG-DEA models, with inputs and outputs orientations, depending on the strategy to be defined. If the interest is to favor the more efficient research centers, vacancies are modeled as inputs and the six iSEG indicators are outputs. If the goal is to favor the less efficient research centers, the vacancies are modeled as outputs and the six iSEG indicators are the inputs of the DEA models.

4 RESULTS AND DISCUSSION

The DEA efficiency scores were calculated by the software SIAD v3.0 (Angulo Meza et al., 2005). Tables 1 and 2 present the results of DEA modeling with the proposed reallocation via ZSG-DEA models for the production function perspective, considering the proportional and equal distributions as starting point and the two viewpoints. Tables 3 and 4 present the results from the performance perspective, favoring the most and least efficient units and with the proportional and equal starting distributions.

Table 1 – Allocation proposed by the ZSG-DEA model for the production function perspective, proportional initial distribution, favoring higher and lower efficiency units. RC refers to 'research center'.

DMU	Favoring higher efficiency units (input-oriented model)				Favoring lower efficiency units (output-oriented model)			
	Initial		Final		Initial		Final	
	Vacancies	Efficiency	Vacancies	Efficiency	Vacancies	Efficiency	Vacancies	Efficiency
RC01	3.7130	0.4258	2.0114	1.0000	3.7130	0.7107	2.9339	1.0000
RC02	2.5662	0.1715	0.5204	1.0000	2.5662	0.5116	2.3479	1.0000
RC03	1.8552	0.5909	2.2407	1.0000	1.8552	0.7389	1.8595	0.9999
RC04	1.1676	0.8233	3.5731	1.0000	1.1676	0.4723	3.0339	1.0000
RC05	4.1541	0.5313	3.0061	1.0000	4.1541	0.9740	1.9285	1.0000
RC06	1.0145	0.4995	2.0546	1.0000	1.0145	0.5152	2.6104	1.0000
RC07	2.3352	0.5610	2.3378	1.0000	2.3352	0.7259	2.2492	1.0000
RC08	1.8682	0.3517	1.3006	1.0000	1.8682	0.5335	2.3494	1.0000
RC09	0.6435	1.0000	2.6088	1.0000	0.6435	0.5637	2.9396	1.0000
RC10	1.1287	1.0000	4.5760	1.0000	1.1287	1.0000	0.5055	1.0000
RC11	2.2496	0.5489	2.3150	1.0000	2.2496	0.7204	2.2330	1.0000
RC12	2.8334	0.3539	1.6968	1.0000	2.8334	0.6028	2.3088	1.0000
RC13	3.7208	0.3732	1.2964	1.0000	3.7208	0.7706	2.3194	1.0000
RC14	2.6544	0.6934	4.6128	1.0000	2.6544	0.7315	1.9215	1.0000
RC15	1.0327	0.1843	0.7718	1.0000	1.0327	0.2646	2.9670	1.0000
RC16	2.4416	0.3365	1.4255	1.0000	2.4416	0.5511	2.7158	1.0000
RC17	2.5739	0.8471	6.9567	1.0000	2.5739	0.6985	2.4261	1.0000
RC18	3.8402	0.4640	1.6972	1.0000	3.8402	0.8387	2.2014	1.0000
RC19	1.9772	0.3468	1.5118	1.0000	1.9772	0.5055	2.7771	1.0000
RC20	2.4442	0.3523	1.3353	1.0000	2.4442	0.5928	2.4065	1.0000
RC21	3.6949	1.0000	14.9797	1.0000	3.6949	1.0000	1.6548	1.0000
RC22	2.6544	0.4806	2.6728	1.0000	2.6544	0.6189	2.3428	1.0000
RC23	1.5205	0.3433	1.6445	1.0000	1.5205	0.4336	3.1597	1.0000
RC24	2.0135	0.3523	1.2892	1.0000	2.0135	0.5520	2.3235	1.0000
RC25	3.0851	0.2262	0.7834	1.0000	3.0851	0.6307	2.3625	1.0000
RC26	4.9144	0.4826	3.2349	1.0000	4.9144	1.0000	2.2010	1.0000
RC27	0.6409	0.5612	1.3442	1.0000	0.6409	0.1864	2.9259	1.0000
RC28	3.9206	0.3834	1.4837	1.0000	3.9206	0.7819	2.4116	1.0000
RC29	1.6035	0.2181	0.9454	1.0000	1.6035	0.3544	2.9668	1.0000
RC30	1.1858	0.3540	1.5347	1.0000	1.1858	0.4343	2.7500	1.0000
RC31	1.7955	1.0000	7.2795	1.0000	1.7955	0.7913	1.9699	1.0000
RC32	1.3726	0.3166	1.2772	1.0000	1.3726	0.4296	2.6171	1.0000
RC33	3.3108	0.2853	0.8674	1.0000	3.3108	0.7753	2.0089	1.0000
RC34	0.9367	0.5562	2.1120	1.0000	0.9367	0.5088	2.6582	1.0000
RC35	1.7462	0.3336	1.0876	1.0000	1.7462	0.5290	2.0935	1.0000
RC36	1.4738	0.2858	1.1446	1.0000	1.4738	0.4142	2.6453	1.0000
RC37	2.5662	0.2902	1.1144	1.0000	2.5662	0.5474	2.5296	1.0000
RC38	4.1152	0.4934	1.5652	1.0000	4.1152	1.0000	1.8431	1.0000
RC39	5.9004	0.1923	0.7342	1.0000	5.9004	1.0000	2.6426	1.0000
RC40	1.8786	0.4902	1.8797	1.0000	1.8786	0.6409	2.2656	1.0000
RC41	1.4764	0.3189	1.1128	1.0000	1.4764	0.4673	2.2607	1.0000
RC42	1.9798	0.5048	2.0641	1.0000	1.9798	0.6598	2.3325	1.0000
Total	100	-	100	-	100	-	100	-
Mean	-	0.4744	-	1.0000	-	0.6376	-	1.0000

Table 2 – Allocation proposed by the ZSG-DEA model for the production function perspective, equal initial distribution, favoring higher and lower efficiency units. RC refers to 'research center'.

DMU	Favoring higher efficiency units (input-oriented model)				Favoring lower efficiency units (output-oriented model)			
	Initial		Final				Initial	
	Vacancies	Efficiency	Vacancies	Efficiency	Vacancies	Efficiency	Vacancies	Efficiency
RC01	2.3810	0.4539	2.6510	1.0000	2.3810	0.7050	2.9371	1.0000
RC02	2.3810	0.1715	0.8165	1.0000	2.3810	0.8803	2.1515	1.0000
RC03	2.3810	0.5909	2.8596	1.0000	2.3810	0.9196	2.1303	1.0000
RC04	2.3810	0.5977	3.4908	1.0000	2.3810	0.6902	2.9074	1.0000
RC05	2.3810	0.5313	2.6132	1.0000	2.3810	1.0000	1.8940	1.0000
RC06	2.3810	0.4656	2.6221	1.0000	2.3810	0.7707	2.6568	1.0000
RC07	2.3810	0.5610	3.1336	1.0000	2.3810	0.8272	2.4638	1.0000
RC08	2.3810	0.3517	1.6598	1.0000	2.3810	0.8386	2.3045	1.0000
RC09	2.3810	0.7020	4.0997	1.0000	2.3810	0.7047	3.0756	1.0000
RC10	2.3810	1.0000	5.8400	1.0000	2.3810	1.0000	1.8940	1.0000
RC11	2.3810	0.5489	2.9545	1.0000	2.3810	0.8350	2.4335	1.0000
RC12	2.3810	0.3463	1.9113	1.0000	2.3810	0.8863	2.1370	1.0000
RC13	2.3810	0.3732	1.8098	1.0000	2.3810	0.8487	2.2734	1.0000
RC14	2.3810	0.6455	3.4428	1.0000	2.3810	0.9834	1.9294	1.0000
RC15	2.3810	0.1799	0.9850	1.0000	2.3810	0.6982	2.7126	1.0000
RC16	2.3810	0.3365	1.8193	1.0000	2.3810	0.7400	2.6516	1.0000
RC17	2.3810	0.8351	4.8767	1.0000	2.3810	0.8017	2.4501	1.0000
RC18	2.3810	0.4640	2.3701	1.0000	2.3810	0.8895	2.1644	1.0000
RC19	2.3810	0.3468	1.9294	1.0000	2.3810	0.7245	2.7200	1.0000
RC20	2.3810	0.3523	1.7042	1.0000	2.3810	0.8208	2.3611	1.0000
RC21	2.3810	1.0000	5.8400	1.0000	2.3810	1.0000	1.8940	1.0000
RC22	2.3810	0.4584	2.6100	1.0000	2.3810	0.8559	2.2285	1.0000
RC23	2.3810	0.3770	2.2019	1.0000	2.3810	0.6548	3.0804	1.0000
RC24	2.3810	0.3523	1.6453	1.0000	2.3810	0.8468	2.2796	1.0000
RC25	2.3810	0.2262	0.9998	1.0000	2.3810	0.8646	2.1905	1.0000
RC26	2.3810	0.4826	2.6651	1.0000	2.3810	0.9276	2.0534	1.0000
RC27	2.3810	0.2359	1.3774	1.0000	2.3810	0.7031	2.6937	1.0000
RC28	2.3810	0.3834	1.8936	1.0000	2.3810	0.8141	2.3890	1.0000
RC29	2.3810	0.2181	1.2066	1.0000	2.3810	0.6937	2.7412	1.0000
RC30	2.3810	0.3540	1.9586	1.0000	2.3810	0.7299	2.6995	1.0000
RC31	2.3810	0.8302	4.8481	1.0000	2.3810	0.9268	2.0677	1.0000
RC32	2.3810	0.3166	1.6300	1.0000	2.3810	0.7672	2.5400	1.0000
RC33	2.3810	0.2853	1.1071	1.0000	2.3810	0.9849	1.9231	1.0000
RC34	2.3810	0.4690	2.6954	1.0000	2.3810	0.7611	2.7086	1.0000
RC35	2.3810	0.3336	1.3881	1.0000	2.3810	0.9339	2.0423	1.0000
RC36	2.3810	0.2858	1.4608	1.0000	2.3810	0.7640	2.5331	1.0000
RC37	2.3810	0.2902	1.4222	1.0000	2.3810	0.7948	2.4285	1.0000
RC38	2.3810	0.4934	1.9976	1.0000	2.3810	1.0000	1.8940	1.0000
RC39	2.3810	0.1923	0.9370	1.0000	2.3810	0.7844	2.4145	1.0000
RC40	2.3810	0.4902	2.4718	1.0000	2.3810	0.8539	2.3301	1.0000
RC41	2.3810	0.3189	1.4202	1.0000	2.3810	0.8743	2.1956	1.0000
RC42	2.3810	0.5048	2.6342	1.0000	2.3810	0.8327	2.4245	1.0000
Total	100	-	100	-	100	-	100	-
Mean	-	0.4465	-	1.0000	-	0.8317	-	1.0000

Table 3 – Allocation proposed by the ZSG-DEA model for the performance perspective, initial proportional distribution, favoring higher and lower efficiency units. RC refers to 'research center'.

DMU	Favoring higher efficiency units (input-oriented model)				Favoring lower efficiency units (output-oriented model)			
	Initial		Final				Initial	
	Vacancies	Efficiency	Vacancies	Efficiency	Vacancies	Efficiency	Vacancies	Efficiency
RC01	3.6850	0.1981	2.1894	1.0000	3.6850	1.0000	2.2342	1.0000
RC02	0.4063	1.0000	1.2185	1.0000	0.4063	0.2117	1.1638	1.0000
RC03	2.6762	0.2346	1.8824	1.0000	2.6762	0.9502	1.7076	1.0000
RC04	1.9336	0.3485	2.0208	1.0000	1.9336	0.5390	2.1748	1.0000
RC05	1.2470	1.0000	3.7397	1.0000	1.2470	0.3754	2.0139	1.0000
RC06	4.2175	0.1631	2.0634	1.0000	4.2175	0.9120	2.8036	1.0000
RC07	1.0929	0.7425	2.4336	1.0000	1.0929	0.2217	2.9894	1.0000
RC08	2.3960	0.2317	1.6651	1.0000	2.3960	1.0000	1.4527	1.0000
RC09	1.8075	0.4954	2.6851	1.0000	1.8075	0.5079	2.1576	1.0000
RC10	0.7146	0.1500	0.3214	1.0000	0.7146	1.0000	0.4333	1.0000
RC11	1.2050	1.0000	3.6136	1.0000	1.2050	0.4199	1.7399	1.0000
RC12	2.2138	0.2431	1.6142	1.0000	2.2138	0.7735	1.7353	1.0000
RC13	2.8303	0.2357	2.0002	1.0000	2.8303	0.4463	3.8450	1.0000
RC14	3.6430	0.1817	1.9847	1.0000	3.6430	1.0000	2.2087	1.0000
RC15	2.5921	0.1900	1.4767	1.0000	2.5921	0.8238	1.9076	1.0000
RC16	1.1349	0.5987	2.0376	1.0000	1.1349	0.2249	3.0601	1.0000
RC17	2.5501	0.3852	2.9460	1.0000	2.5501	0.5573	2.7741	1.0000
RC18	2.5501	0.2104	1.6091	1.0000	2.5501	0.5786	2.6722	1.0000
RC19	3.7130	0.1928	2.1468	1.0000	3.7130	0.7977	2.8220	1.0000
RC20	1.9756	0.2297	1.3608	1.0000	1.9756	0.7927	1.5111	1.0000
RC21	2.3820	0.2879	2.0567	1.0000	2.3820	1.0000	1.4442	1.0000
RC22	3.7551	0.1376	1.5497	1.0000	3.7551	0.8289	2.7467	1.0000
RC23	2.1438	0.8673	5.5756	1.0000	2.1438	0.7076	1.8370	1.0000
RC24	1.6534	0.4797	2.3786	1.0000	1.6534	0.4055	2.4718	1.0000
RC25	1.9896	0.2885	1.7214	1.0000	1.9896	0.5372	2.2454	1.0000
RC26	3.0125	0.1615	1.4589	1.0000	3.0125	1.0000	1.8265	1.0000
RC27	5.1702	0.7116	11.0335	1.0000	5.1702	1.0000	3.1347	1.0000
RC28	0.6445	1.0000	1.9329	1.0000	0.6445	0.1239	3.1552	1.0000
RC29	3.6290	0.4038	4.3940	1.0000	3.6290	0.6606	3.3307	1.0000
RC30	1.5553	0.3010	1.4039	1.0000	1.5553	0.5868	1.6069	1.0000
RC31	1.2050	0.3546	1.2812	1.0000	1.2050	0.6913	1.0568	1.0000
RC32	1.8075	0.5934	3.2164	1.0000	1.8075	0.3648	3.0044	1.0000
RC33	1.3311	0.5457	2.1783	1.0000	1.3311	0.4450	1.8136	1.0000
RC34	3.2226	0.1597	1.5432	1.0000	3.2226	0.8097	2.4132	1.0000
RC35	0.9107	0.7361	2.0105	1.0000	0.9107	0.1761	3.1357	1.0000
RC36	1.7514	0.4177	2.1938	1.0000	1.7514	0.3386	3.1365	1.0000
RC37	1.4572	0.4329	1.8916	1.0000	1.4572	0.2550	3.4642	1.0000
RC38	2.5781	0.3747	2.8973	1.0000	2.5781	0.5565	2.8090	1.0000
RC39	4.0073	0.1365	1.6400	1.0000	4.0073	0.8007	3.0344	1.0000
RC40	5.6747	0.0931	1.5847	1.0000	5.6747	1.0000	3.4405	1.0000
RC41	1.9196	0.1969	1.1332	1.0000	1.9196	1.0000	1.1638	1.0000
RC42	1.5273	0.4503	2.0624	1.0000	1.5273	0.3815	2.4270	1.0000
RC43	2.0877	0.2960	1.8535	1.0000	2.0877	0.6680	1.8947	1.0000
Total	100	-	100	-	100	-	100	-
Mean	-	0.4060	-	1.0000	-	0.6388	-	1.0000

Table 4 – Allocation proposed by the ZSG-DEA model for the performance perspective, equal initial distribution, perspectives of favoring higher and lower efficiency units. RC refers to 'research center'.

DMU	Favoring higher efficiency units (input-oriented model)				Favoring lower efficiency units (output-oriented model)			
	Initial		Final				Initial	
	Vacancies	Efficiency	Vacancies	Efficiency	Vacancies	Efficiency	Vacancies	Efficiency
RC01	2.3256	0.9320	2.3476	1.0000	2.3256	0.6787	2.4859	1.0000
RC02	2.3256	1.0000	2.5189	1.0000	2.3256	1.0000	1.6873	1.0000
RC03	2.3256	0.9071	2.2849	1.0000	2.3256	1.0000	1.6873	1.0000
RC04	2.3256	0.9050	2.2796	1.0000	2.3256	0.6813	2.4766	1.0000
RC05	2.3256	1.0000	2.5189	1.0000	2.3256	0.7378	2.2870	1.0000
RC06	2.3256	0.9376	2.3618	1.0000	2.3256	0.7087	2.3807	1.0000
RC07	2.3256	0.9585	2.4143	1.0000	2.3256	0.5709	2.9555	1.0000
RC08	2.3256	0.8686	2.1880	1.0000	2.3256	1.0000	1.6873	1.0000
RC09	2.3256	0.9500	2.3931	1.0000	2.3256	0.6986	2.4154	1.0000
RC10	2.3256	0.2638	0.6644	1.0000	2.3256	1.0000	1.6873	1.0000
RC11	2.3256	1.0000	2.5189	1.0000	2.3256	0.8475	1.9909	1.0000
RC12	2.3256	0.8515	2.1449	1.0000	2.3256	0.8608	1.9602	1.0000
RC13	2.3256	0.9966	2.5104	1.0000	2.3256	0.4980	3.3881	1.0000
RC14	2.3256	0.8878	2.2363	1.0000	2.3256	1.0000	1.6873	1.0000
RC15	2.3256	0.9039	2.2768	1.0000	2.3256	1.0000	1.6873	1.0000
RC16	2.3256	0.9905	2.4950	1.0000	2.3256	0.7048	2.3939	1.0000
RC17	2.3256	0.9659	2.4330	1.0000	2.3256	0.5910	2.8547	1.0000
RC18	2.3256	0.9208	2.3195	1.0000	2.3256	0.7410	2.2769	1.0000
RC19	2.3256	0.9469	2.3852	1.0000	2.3256	0.5660	2.9813	1.0000
RC20	2.3256	0.9008	2.2692	1.0000	2.3256	1.0000	1.6873	1.0000
RC21	2.3256	0.9250	2.3299	1.0000	2.3256	1.0000	1.6873	1.0000
RC22	2.3256	0.9533	2.4014	1.0000	2.3256	0.4816	3.5032	1.0000
RC23	2.3256	1.0000	2.5189	1.0000	2.3256	1.0000	1.6873	1.0000
RC24	2.3256	1.0000	2.5189	1.0000	2.3256	0.6212	2.7160	1.0000
RC25	2.3256	0.9703	2.4441	1.0000	2.3256	0.9075	1.8594	1.0000
RC26	2.3256	0.9230	2.3251	1.0000	2.3256	1.0000	1.6873	1.0000
RC27	2.3256	1.0000	2.5189	1.0000	2.3256	0.5762	2.9284	1.0000
RC28	2.3256	0.9104	2.2932	1.0000	2.3256	0.7153	2.3588	1.0000
RC29	2.3256	0.9810	2.4710	1.0000	2.3256	0.5757	2.9306	1.0000
RC30	2.3256	0.8651	2.1792	1.0000	2.3256	0.7319	2.3053	1.0000
RC31	2.3256	0.9756	2.4575	1.0000	2.3256	1.0000	1.6873	1.0000
RC32	2.3256	1.0000	2.5189	1.0000	2.3256	0.4224	3.9944	1.0000
RC33	2.3256	1.0000	2.5189	1.0000	2.3256	0.8064	2.0924	1.0000
RC34	2.3256	0.9647	2.4301	1.0000	2.3256	0.6173	2.7333	1.0000
RC35	2.3256	0.8567	2.1581	1.0000	2.3256	0.5891	2.8643	1.0000
RC36	2.3256	0.8682	2.1868	1.0000	2.3256	0.7504	2.2486	1.0000
RC37	2.3256	0.9287	2.3394	1.0000	2.3256	0.6521	2.5876	1.0000
RC38	2.3256	0.9410	2.3704	1.0000	2.3256	0.8131	2.0752	1.0000
RC39	2.3256	0.9221	2.3227	1.0000	2.3256	0.6428	2.6247	1.0000
RC40	2.3256	0.8532	2.1493	1.0000	2.3256	0.7468	2.2594	1.0000
RC41	2.3256	0.9126	2.2989	1.0000	2.3256	1.0000	1.6873	1.0000
RC42	2.3256	0.9526	2.3996	1.0000	2.3256	0.6308	2.6750	1.0000
RC43	2.3256	0.9083	2.2879	1.0000	2.3256	0.7852	2.1490	1.0000
Total	100	-	100	-	100	-	100	-
Mean	-	0.9232	-	1.0000	-	0.7663	-	1.0000

Another modeling alternative for the vacancy allocation problem would be to use single- or multi-objective integer linear programming models. From the perspective of a production function, the alternatives we discussed in team were: (a) minimize costs, with restrictions on minimum acceptable scientific production and assets, total vacancies equal to 100; (b) maximize scientific production, with restrictions on maximum cost and minimum acceptable assets production, total vacancies equal to 100; (c) maximize production of assets, with restrictions on maximum cost and minimum acceptable scientific production, total vacancies equal to 100; (d) simultaneously maximize scientific production and production of assets, with restrictions on maximum cost and total vacancies equal to 100. We could also add restrictions on the minimum allocation of vacancies to each research center. The hypothesis of minimum or maximum acceptable value of costs, scientific production and assets would be the ones in force in each research center.

We tested such models using mean values from the period 2017-2021. We used the OPTMODEL procedure of the SAS 9.4 software (SAS, 2017). In our internal discussions, the results of such models were not perceived as being suitable, since the vacancies were concentrated in five research centers; the others would receive one vacancy (minimum allocation constraint). The non-dominated solutions of the multiobjective problem (a total of six) presented the same assignment pattern. For this reason, the team's choice was for the distribution via ZSG-DEA models. The constraints of these integer programming problems could be changed to reflect the Company's management understanding of what the minimum and maximum acceptable values are, as well as individual and global vacancy limits. Adjustments in the objective functions would also be possible.

As expected, the results in Tables 1 to 4 ratify that the efficiency score after reallocation is 100% for all units, that is, given the adopted standpoint the allocation proposed by the ZSG-DEA models generates the maximum efficiency scenario for the system as a whole. It is worth noting that in the case of the models under the production-based perspective it was necessary more than one round for the efficiencies of all DMUs to be 100%. As observed in Gomes *et al*. (2004), this is because the variable 'vacancies' was not the only input or output. In addition, as Gomes et al. (2005) discuss, some approximation errors introduced by the algorithm to calculate the DEA linear programming problems used in the SIAD software can generate efficiency values slightly below 100%.

In Tables 1 to 4, the value of the vacancies allocated at the end of the distribution is not an integer value. The results obtained are a first approximation to the solution of our problem. Indeed, the allocation of job positions may require the use of integer variables in DEA models. The issues regarding integer values in DEA can be seen, for instance, in Kordrostami et al. (2019), Kuosmanen and Matin (2009), Matin and Kuosmanen (2009), Lozano and Villa (2006). In practical terms, the percentages of job positions presented in Tables 1 to 4 would be rounded to integer numbers. In such cases, the efficiency scores could change and theoretical improvements in the ZSG-DEA models would be needed. Alternatively, more precise algorithms, such as those proposed by Soares de Mello et al. (2006) or Gomes and Soares de Mello (2009), should be used so that the obtained value is an integer. It is also important to observe that the initial distribution of the variable to be reallocated influences the efficiency results and, consequently, the targets and the final value assigned. The decision to favor the most efficient or the least efficient units also impacts the results. This means that central administration managers (decision makers) must keep in mind the assumptions that will guide the allocation of vacancies, so that the models and their results represent faithfully such assumptions guiding the decision.

Regarding the results of the ZSG-DEA models, the team's perception is that the models that consider the production perspective would be the most appropriate. This is because production models have been used at Embrapa since 1996 to evaluate research centers from efficiency (via DEA) and productivity points of view (for details see Souza *et al.*, 1999, 2007; Souza and Gomes, 2015a, 2015b). The iSEG, an aggregate index composed of the six partial indicators used in the performance perspective models, was implemented in the Company in 2019 and the perception is that it still lacks further consolidation with managers and teams of the research centers.

As for the model orientation, the understanding is that favoring research centers with lower efficiency would be the strategy that would potentially bring greater benefit to the institution in fulfilling its public function. In fact, in the ZSG-DEA paradigm, more efficient research centers could lose vacancies allocated to them without changing their scores or improving them (for those DMUs with efficiency scores close to 100%). And, at the same time, boost the overall performance of less efficient centers.

Regarding the initial distribution, we verified that the proportional distribution is the one that produces the greatest variations (positive and negative) between the initial and final allocations, whether from the production or performance perspectives. Thus, the team understood that the homogeneous distribution would be the most appropriate, given the smaller disruption it would cause at the end of the process (more conservative solution).

The demand received was to propose a quantitative/objective approach to allocate the new vacancies based on the production structure of the research centers. In parallel, a project is underway at Embrapa to define how the recruitment will be formalized with regard to the regional distribution of vacancies, professional profiles and positions to be filled (the current job titles are assistant, analyst and researcher). At Embrapa, the number of vacancies is not fixed by location, job title or area of activity. In this context, our study considered vacancies in the general sense, without assumptions on regional or other issues. The variables we used refer to the research center as a production unit, without defining the activities performed.

Research centers do not have a priori fixed capacity. The "size" of the research center in terms of number of employees is a function of the demands presented by the productive sector and the research projects carried out there. In theory, it is possible to redistribute employees among the research centers and to allocate new employees to meet the needs that arise. Aspects related to the employment contracts and the company's career plan should be considered in such situation.

It is expected that the practical contribution of the study is to be one of the decision-making aids that company managers will have at their disposal. It is not intended to be the decision itself. In addition to the distribution derived from this study, the Company should consider the studies underway in other teams, especially on current and future professional profiles required. At this point, it should be highlighted that Embrapa's VII Master Plan (Embrapa, 2020) identified priority themes for agricultural research in the main production chains and defined strategic goals on innovation and organizational themes. Gaps were then observed in terms of professional profiles, which must be filled to consolidate the so-called Vision of the Future (Embrapa, 2022) and fulfill the actual research programming (by the assessment of the potential risk to research programming, in the short term, with the retirement of employees highly engaged in the research programming).

It should be added that an alternative of applying the model proposed here would be after a first definition by the Executive Board on the distribution of vacancies (based, for instance, on the profile and risk studies already mentioned), instead of using the proportional or equal distribution as we proposed in this paper. The distribution given by the Executive Board could then be submitted to the DEA-GSZ model for the (re)allocation.

It is worth mentioning that the DEA-GSZ model was already proposed for the reallocation of CNPq grants in the Company. At that time, the results of the model were approved by the Executive Board and were effectively implemented, as described in Gomes et al. (2007).

Finally, environmental variables were not considered in the DEA models. Human Development Index (UNDP, 2022), for instance, may have an impact on staff recruitment. It would also be possible that a research center that would receive more vacancies is located in a region with a history of low quantity and quality of research. The allocation of more researchers could show an inverse effect, reducing productivity indicators, similar to congestion assumptions (Cooper et al., 2007). This type of situation and improvement should be considered in further studies.

5 CONCLUDING REMARKS

The optimization models structured, formulated, and solved here allowed us to propose the allocation of vacancies to Embrapa's research centers, both from production function and from performance-based perspectives. Each proposed model has its own advantages and pitfalls that influence its results. This is a first approach to the demand that we received and that should be discussed and potentially improved with the aid of the decision makers.

Given the approaches adopted and the variables selected, the ZSG-DEA models generated allocation proposals in which all units are 100% efficient. This is the idealized scenario for central management: maximum efficiency in the use of resources. It is important to note that the results are influenced by the initial suggested allocation. In this context, we suggested as alternatives homogeneous and proportional distribution of vacancies among the research centers. Another option would be the distribution of vacancies based on managers' judgment or perception, especially when there is a definition as to the total number of vacancies that will be made available to the Company by the Ministry of Public Service Management and Innovation.

Likewise, the choice of other variables to compose the DEA model (inputs and outputs) and different assumptions regarding the frontier have an influence on the efficiency scores (values and interpretation) and, therefore, on the final allocation. In this sense, it is worth noting that in the institutional project there is a team engaged in mapping profiles and studying the potential risk to the Company's research programming due to the departure of researchers that are project leaders and engaged in key research activities. We believe that it is feasible to model via ZSG-DEA the risk indicators under study by this team. We have recently started discussions in this direction.

Models are decision support tools. The proposition and use of mathematical models to allocate or reallocate resources is important for a central administration to communicate to its member units that such a decision is based on objective, quantifiable criteria that reflect the management of these units. This allows the decision process to be potentially less vulnerable to criticisms and interferences. Finally, we emphasize that the choice for one or another approach and the respective solution is up to the decision makers.

References

AFSHARIAN M, AHN H & HARMS S. 2021. A review of DEA approaches applying a common set of weights: The perspective of centralized management. *European Journal of Operational Research*, **294**: 3–15.

ALVES L, MILIONI A & SOMA N. 2014. Input allocation with the Ellipsoidal Frontier Model. *Pesquisa Operacional*, **34**: 39–48.

ANGULO MEZA L, BIONDI NETO L, MELLO SOARES JCCB & GOMES E. 2005. ISYDS - Integrated System for Decision Support (SIAD - Sistema Integrado de Apoio à Decisão): a software package for Data Envelopment Analysismodel. *Pesquisa Operacional*, **25**: 493–503.

ASMILD M, PARADI J & PASTOR J. 2009. Centralized resource allocation BCC models. *Omega*, **37**: 40–9.

BEASLEY J. 2003. Allocating fixed costs and resources via data envelopment analysis. *European Journal of Operational Research*, **147**: 198–216.

BERNARDO M, SOUZA M, MOREIRA RL & RODRIGUES L. 2020. University library performance management: applying zero-sum gains DEA models to resource allocation. *Socioeconomic Planning Science*, **100808**.

BI G, FENG C, DING J, LIANG L & CHU F. 2014. The linear formulation of the ZSG-DEA models with different production technologies. *Journal of the Operational Research Society*, **65**: 1202–11.

BOUAJAJA S & DRIDI N. 2017. A survey on human resource allocation problem and its applications. *Operational Research*, **17**: 339–369.

CAPORALETTI L, DULÁ J & WOMER N. 1999. Performance evaluation based on multiple attributes with nonparametric frontiers. *Omega*, **27**: 637–645.

CHEN L, GUO M, LI Y, LIANG L & SALO A. 2021. Efficiency intervals, rank intervals and dominance relations of decision-making units with fixed-sum outputs. *European Journal of Operational Research*, **292**: 238–49.

COOK W & KRESS M. 1999. Characterizing an equitable allocation of shared costs: A DEA approach. *European Journal of Operational Research*, **119**: 652–661.

COOK W, TONE K & ZHU J. ????. Data envelopment analysis: Prior to choosing a model. *Omega*, **44**: 1–4.

COOK W & ZHU J. 2005. Allocation of shared costs among decision making units: A DEA approach. *Computers and Operations Research*, **32**: 2171–2178.

COOPER W, SEIFORD L & TONE K. 2007. *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software*. 2nd ed. Springer.

DING T, CHEN Y, WU H & WEI Y. 2018. Centralized fixed cost and resource allocation considering technology heterogeneity: a DEA approach. *Annals of Operations Research*, **268**(1-2): 497–511.

DING T, LI F & LIANG L. 2020. Fixed cost and resource allocation considering technology heterogeneity in two-stage network production systems. *International Series in Operations Research and Management Science*, **290**: 227–249.

DYSON R, ALLEN R, CAMANHO A, PODONOVSKI VV SC & SHALE E. 2001. Pitfalls and protocols in DEA. *European Journal of Operational Research*, **132**: 245–259.

EMBRAPA. 2020a. VII Plano Diretor da Embrapa: 2020–2030. Available at: https://www.embrapa.br/vii-plano-diretor.

EMBRAPA. 2020b. Índice SEG (iSEG): uso e cálculo para fins da Descentralização Direta de Recursos para as Unidades Descentralizadas. In: *Nota Técnica. Secretaria de Pesquisa e Desenvolvimento, Gerência de Inteligência e Planejamento da Programação*.

EMBRAPA. 2021. Metodologia de Avaliação de Desempenho Institucional - Período 2021/2022. Anexo 1 da Deliberação no 3, de 02.03.2021. *BCA no*, **12**.

EMBRAPA. 2022. Visão de Futuro do Agro Brasileiro. Available at: https://www.embrapa.br/visao-de-futuro.

FANG L. 2016. A new approach for achievement of the equilibrium efficient frontier with fixed-sum outputs. *Journal of the Operational Research Society*, **67**: 412–20.

FENG C, CHU F, ZHOU N, BI G & DING J. 2019. Performance evaluation and quota allocation for multiple undesirable outputs based on the uniform frontier. *Journal of the Operational research Society*, **70**: 472–86.

GOMES E. 2003. *Modelos de Análise Envoltória de Dados com Ganhos de Soma Zero*. Ph.D. thesis. Rio de Janeiro.

GOMES E, ABREU U, MELLO J, CARVALHO T & ZEN S. 2012. Unitary input DEA model to identify beef cattle production systems typologies. *Pesquisa Operacional*, **32**: 389–406.

GOMES E & LINS M. 2008. Modelling undesirable outputs with zero sum gains data envelopment analysis models. *Journal of the Operational Research Society*, **59**: 616–623.

GOMES E & MELLO J C C B S. 2009. Distribuição de bolsas de iniciação científica com algoritmo híbrido baseado em eficiências DEA. *Pesquisa Operacional para o Desenvolvimento*, **1**: 169–177.

GOMES E, MELLO SOARES JCCB & ANGULO MEZA L. 2008. Large discreet resource allocation: A hybrid approach based on DEA efficiency measurement. *Pesquisa Operacional*, **28**(3): 597–608.

GOMES E, MELLO SOARES JCCB & LINS M. 2005. Uniformização da fronteira eficiente em modelos de análise de envoltória de dados com ganhos de soma zero e retornos constantes de escala. *Pesquisa Operacional*, **25**: 261–277.

GOMES E & SOUZA G. 2010. Allocating financial resources for competitive projects using a Zero Sum Gains DEA model. *Engevista*, **12**: 4–9.

GOMES E, SOUZA G, LIMA S & FONSECA C. 2007. Alocação de bolsas de iniciação científica às unidades da Embrapa com modelos de Análise Envoltória de Dados com Ganhos de Soma Zero. *Engevista*, **9**: 14–21.

GOMES EG, MELLO SOARES JCCB & LINS MPE. 2004. Redistribuição de inputs e outputs em modelos de análise envoltória de dados com ganhos de soma zero. *Pesquisa Operacional*, **24**: 269–284.

GUEDES E, MILIONI A, AVELLAR JVG & SILVA R. 2012. Adjusted spherical frontier model: allocating input via parametric DEA. *Journal of the Operational Research Society*, **63**: 406–417.

GUPTA A, PACHAR N, JAIN A, GOVINDAN K & JHA PC. 2023. Resource reallocation strategies for sustainable efficiency improvement of retail chains. *Journal of Retailing and Consumer Services*, **73**(103309).

HILLIER B & LIEBERMAN N. 2017. *Introduction to Operations Research.* 10th ed. Mc-Graw Hill.

KICSINY R, HUFNAGEL L & VARGA Z. 2023. Allocation of limited resources under quadratic constraints. *Annals of Operations Research*, **322**: 793–817.

KORDROSTAMI S, AMIRTEIMOORI A & NOVEIRI M. 2019. Inputs and outputs classification in integer-valued data envelopment analysis. *Measurement*, **139**: 317–325.

KORHONEN P & SYRJÄNEN M. 2003. Resource allocation based on efficiency analysis. *Management Science*, **50**: 1134–1144.

KUOSMANEN T & MATIN R. 2009. Theory of integer-valued data envelopment analysis. *European Journal of Operational Research*, **192**(2): 658–667.

LI Y, HOU W, ZHU W, LI F & LIANG L. 2021. Provincial carbon emission performance analysis in China based on a Malmquist data envelopment analysis approach with fixed–sum undesirable outputs. *Annals of Operations Research*, **304**: 233–61.

LINS M, GOMES E, MELLO SOARES JCCB & MELLO A. 2003. Olympic ranking based on a zero sum gains DEA model. *European Journal of Operational Research*, **148**: 312–22.

LOVELL C & PASTOR J. 1999. Radial DEA models without inputs or without outputs. *European Journal of Operational Research*, **118**: 46–51.

LOZANO S. 2023. Bargaining approach for efficiency assessment and target setting with fixed–sum variables. *Omega*, **114**(102728).

LOZANO S & VILLA G. 2004. Centralized resource allocation using data envelopment analysis. *Journal of Productivity Analysis*, **22**: 143–161.

LOZANO S & VILLA G. 2005. Centralized DEA models with the possibility of downsizing source. *Journal of the Operational Research Society*, **56**: 357–364.

LOZANO S & VILLA G. 2006. Data envelopment analysis of integer-valued inputs and outputs. *Computers & Operations Research*, **33**(10): 3004–3014.

LOZANO S, VILLA G & ADENSO–DIAZ B. 2004. Centralized target setting for regional recycling operations using DEA. *Omega*, **32**: 101–110.

MACEDO MA, MELLO SOARES JCCB & GOMES E. 2010. Modelo para redistribuição de cotas de emissão de gases do efeito estufa baseado em medidas de eficiência técnica. *Gestão & Produção*, **17**: 512–524.

MAR–MOLINERO C, PRIOR D, SEGOVIA M & PORTILLO F. 2014. On centralized resource utilization and its reallocation by using DEA. *Annals of Operations Research*, **221**: 273–83.

MATIN R & KUOSMANEN T. 2009. Theory of integer-valued data envelopment analysis under alternative returns to scale axioms. *Omega*, **37**(5): 988–995.

MELLO SOARES JCCB, GOMES E, MELLO M & LETA SOARES FR. 2006. Algoritmo de alocação de recursos discretos com análise de envoltória de dados. *Pesquisa Operacional*, **26**: 225–239.

MILIONI A & ALVES L. 2013. Ten years of research on parametric data envelopment analysis. *Pesquisa Operacional*, **33**: 89–104.

MILIONI A, AVELLAR J, GOMES E & MELLO J. 2011a. An ellipsoidal frontier model: allocating input via parametric DEA. *European Journal of Operational Research*, **209**: 113–121.

MILIONI A, AVELLAR J, RABELLO T & FREITAS G. 2011b. Hyperbolic frontier model: a parametric DEA approach for the distribution of a total fixed input. *Journal of the Operational Research Society*, **62**: 1029–1037.

PANG R, DENG Z & CHIU Y. 2015. Pareto improvement through a reallocation of carbon emission quotas. *Renewable and Sustainable Energy Reviews*, **50**: 419–430.

PATRIKSSON M. 2008. A survey on the continuous nonlinear resource allocation problem. *European Journal of Operational Research*, **185**: 1–46.

RAMALHO E, RAMALHO JJS & HENRIQUES PD. 2010. Fractional regression models for second stage DEA efficiency analyses. *Journal of Productivity Analysis*, **34**: 239–255.

RIPOLL–ZARRAGA AE & LOZANO SA. 2020. A centralized DEA approach to resource reallocation in Spanish airports. *Annals of Operations Research*, **288**: 701–732.

SAS. 2017. *SAS® Optimization 8.1: Mathematical Optimization Procedures*. Cary, NC: SAS Institute Inc.

SILVA RC, MILIONI AZ & TEIXEIRA JE. 2018. The general hyperbolic frontier model: establishing fair output levels via parametric DEA. *Journal of the Operational Research Society*, **69**: 946–958.

SILVEIRA J, MELLO SOARES JCCB & ANGULO MEZA L. 2019. Input redistribution using a parametric DEA frontier and variable returns to scale: The parabolic efficient frontier. *Journal of the Operational Research Society*, **70**: 751–759.

SOUZA GS, AVILA AFD & ALVES ERA. 1999. Technical efficiency of production in agricultural research. *Scientometrics*, **46**: 141–160.

SOUZA GS & GOMES EG. 2015a. Management of agricultural research centers in Brazil: A DEA application using a dynamic GMM approach. *European Journal of Operational Research*, **240**: 819–824.

SOUZA GS & GOMES EG. 2015b. A performance measure to support decision–making in agricultural research centers in Brazil. *Procedia Computer Science*, **55**: 405–414.

SOUZA GS, GOMES EG, MAGALHÃES MC & AVILA AFD. 2007. Economic efficiency of Embrapa's research centers and the influence of contextual variables. *Pesquisa Operacional*, **27**: 15–26.

SUN J, WEN W, WANG M & ZHOU P. 2022. Optimizing the provincial target allocation scheme of renewable portfolio standards in China. *Energy*, **250**(123699).

UNDP. 2022. Human Development Report 2021-22: Uncertain Times, Unsettled Lives: Shaping our Future in a Transforming World. Available at: https://hdr.undp.org/informe-sobre-desarrollo-humano-2021-22.

WU X, TAN L, GUO J, WANG Y, LIU H & ZHU W. 2016. A study of allocative efficiency of PM2.5 emission rights based on a zero sum gains data envelopment model. *Journal of Cleaner Production*, **113**: 1024–31.

YADOLLAHI AH, EBRAHIMNEJAD A & KAZEMI-MATIN R. 2022. Centralized resource reallocation in two–stage network structures with undesirable products. *Computational and Applied Mathematics*, **41**(228).

YANG F, WU DD, LIANG L & O'NEILL L. 2011. Competition strategy and efficiency evaluation for decision making units with fixed–sum outputs. *European Journal of Operational Research*, **212**: 560–9.

YANG M, LI Y, CHEN Y & LIANG L. 2014. An equilibrium efficiency frontier data envelopment analysis approach for evaluating decision-making units with fixed–sum outputs. *European Journal of Operational Research*, **239**: 479–89.

YANG M, LI YJ & LIANG L. 2015. A generalized equilibrium efficient frontier data envelopment analysis approach for evaluating DMUs with fixed–sum outputs. *European Journal of Operational Research*, **246**: 209–17.

ZHOU Z, SUN L, YANG W, LIU W & MA C. 2013. A bargaining game model for efficiency decomposition in the centralized model of two–stage systems. *Computers & Industrial Engineering*, **64**(103108).

ZHU Q, LI X, LI F, WU J & SUN J. 2021. Analyzing the sustainability of China's industrial sectors: a data–driven approach with total energy consumption constraint. *Ecological Indicators*, **122**(107235).

ZHU Q, SONG M & WU J. 2020. Extended secondary goal approach for common equilibrium efficient frontier selection in DEA with fixed–sum outputs. *Computers & Industrial Engineering*, **144**(106483).

ZHU Q, WU J, SONG M & LIANG L. 2017. A unique equilibrium efficient frontier with fixed–sum outputs in data envelopment analysis. *Journal of the Operational Research Society*, **68**: 1483–90.

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