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ANALYSIS OF AN INVENTORY MODEL FOR TIME-DEPENDENT LINEAR DEMAND RATE THREE LEVELS OF PRODUCTION WITH SHORTAGE

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ABSTRACT

Objective: The objective of this study is to investigate the continuous-production inventory problem for a single product at a three-level facing a constant deterioration rate. It is anticipated that the customer will make renewal arrangements in accordance with the established and extensively utilized order-up-to policy, and may place orders from two providers with differing prices and levels of consistency.

Theoretical Framework: In this study demand is a linear function of time, and the deterioration rate is persistent. In this model, the shortage is allowed, and partial backlogging. It is expected that the backlog rate is influenced by how long it takes to get the next replenishment. It involves aspects from various areas of scientific knowledge to understand the different perceptions contained in the scientific literature on the phenomenon of Industry 4.0.

Method: The algorithms for this model have been developed in Mathematica Software 9.0. We examine the effects of several difficulty criteria on the best options regarding manufacturing time and replenishment at the store using analytical analysis.

Results and Discussion: Thus, our model offers a novel management insight that aids in achieving the ideal profit level for a production system or business. In reality, we obtain a promising outcome from the numerical case. To compare various solutions, we also carry out very comprehensive numerical experiments.

Research Implications: This research aims to detect the best course of action for reducing overall inventory costs and increasing overall income. We also verified that the total cost and commonly optimal time used to interrupt geometric production systems could be analyzed with a constant production model. This approach is particularly relevant for organizations seeking to improve operational efficiency and customer satisfaction through better risk management.

Originality/Value: This study contributes to our understanding of how and with whom to collaborate by highlighting the relationships among inventory management, innovation performance, and logistics performance. The value of the study is the contribution it makes to the literature on the cost analysis issues. Therefore, the article can be of benefit to the scientific community with an interest in the study of the subject.

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ANÁLISE DE UM MODELO DE ESTOQUE PARA TAXA DE DEMANDA LINEAR DEPENDENTE DO TEMPO TRÊS NÍVEIS DE PRODUÇÃO COM ESCASSEZ

RESUMO
Objetivo: O objetivo deste estudo é investigar o problema de estoque de produção contínua para um único produto em um nível triplo que enfrenta uma taxa de deterioração constante. Prevê-se que o cliente fará acordos de renovação de acordo com a política de pedido até, estabelecida e amplamente utilizada, e poderá fazer pedidos de dois fornecedores com preços e níveis de consistência diferentes.
Estrutura Teórica: Neste estudo, a demanda é uma função linear do tempo, e a taxa de deterioração é persistente. Nesse modelo, a escassez é permitida e o acúmulo parcial de pedidos. Espera-se que a taxa de backlog seja influenciada pelo tempo que leva para obter o próximo reabastecimento. Envolve aspectos de várias áreas do conhecimento científico para entender as diferentes percepções contidas na literatura científica sobre o fenômeno da Indústria 4.0.
Método: Os algoritmos para esse modelo foram desenvolvidos no Mathematica Software 9.0. Examinamos os efeitos de vários critérios de dificuldade sobre as melhores opções em relação ao tempo de fabricação e ao reabastecimento na loja usando análise analítica.
Resultados e Discussão: Assim, nosso modelo oferece uma nova visão de gerenciamento que ajuda a atingir o nível de lucro ideal para um sistema de produção ou empresa. Na realidade, obtivemos um resultado promissor com o caso numérico. Para comparar várias soluções, também realizamos experimentos numéricos bastante abrangentes.
Implicações da Pesquisa: Esta pesquisa visa a detectar o melhor curso de ação para reduzir os custos gerais de estoque e aumentar a receita geral. Também verificamos que o custo total e o tempo ideal comumente usado para interromper sistemas de produção geométricos podem ser analisados com um modelo de produção constante. Essa abordagem é particularmente relevante para organizações que buscam melhorar a eficiência operacional e a satisfação do cliente por meio de um melhor gerenciamento de riscos.
Originalidade/Valor: Este estudo contribui para nossa compreensão de como e com quem colaborar, destacando as relações entre gerenciamento de estoque, desempenho de inovação e desempenho de logística. O valor do estudo é a contribuição que ele faz para a literatura sobre as questões de análise de custos. Portanto, o artigo pode ser útil para a comunidade científica com interesse no estudo do assunto.

Palavras-chave: EPQ, Estoque, Item em Deterioração, Escassez, Demanda Dependente do Tempo, Produção.

ANÁLISIS DE UN MODELO DE INVENTARIO PARA UNA TASA DE DEMANDA LINEAL DEPENDIENTE DEL TIEMPO TRES NIVELES DE PRODUCCIÓN CON ESCASEZ

RESUMEN
Objetivo: El objetivo de este estudio es investigar el problema de inventario de producción continua para un único producto a tres niveles que se enfrenta a una tasa de deterioro constante. Se anticipa que el cliente realizará acuerdos de renovación de acuerdo con la política establecida y ampliamente utilizada de pedidos hasta el momento, y puede realizar pedidos a dos proveedores con precios y niveles de consistencia diferentes.
Marco Teórico: En este estudio la demanda es una función lineal del tiempo, y la tasa de deterioro es persistente. En este modelo se permite la escasez, y el backlogging parcial. Se espera que la tasa de atrasos esté influida por el tiempo que se tarda en obtener la siguiente reposición. Involucra aspectos de diversas áreas del conocimiento científico para comprender las diferentes percepciones contenidas en la literatura científica sobre el fenómeno de la Industria 4.0.
Método: Los algoritmos para este modelo se han desarrollado en Mathematica Software 9.0. Examinamos los efectos de varios criterios de dificultad sobre las mejores opciones en cuanto a tiempo de fabricación y reposición en tienda mediante análisis analítico.
Resultados y Discusión: Así, nuestro modelo ofrece una novedosa visión de gestión que ayuda a alcanzar el nivel de beneficio ideal para un sistema de producción o negocio. En realidad, obtenemos un resultado prometedor a partir del caso numérico. Para comparar varias soluciones, también llevamos a cabo experimentos numéricos muy completos.
Implicaciones de la Investigación: Esta investigación pretende detectar la mejor línea de actuación para reducir los costes totales de inventario y aumentar los ingresos totales. También comprobamos que el coste total y el tiempo comúnmente óptimo utilizado para interrumpir sistemas de producción geométricos pueden analizarse con un modelo de producción constante. Este planteamiento es especialmente pertinente para las organizaciones que pretenden mejorar la eficacia operativa y la satisfacción del cliente mediante una mejor gestión del riesgo.
Originalidad/Valor: Este estudio contribuye a nuestra comprensión de cómo y con quién colaborar poniendo de relieve las relaciones entre la gestión de inventarios, el rendimiento de la innovación y el rendimiento logístico. El

valor del estudio es la contribución que hace a la bibliografía sobre las cuestiones de análisis de costes. Por lo tanto, el artículo puede beneficiar a la comunidad científica interesada en el estudio del tema.

Palabras clave: EPQ, Inventario, Artículo Deteriorable, Escasez, Demanda Dependiente del Tiempo, Producción.

1 INTRODUCTION

There are three sections in the introduction. The purpose of the research is covered in the first section, and a reported literature review is covered in the second. The description of the research contributions appears in the third and final section.

1.1 MOTIVATION (GENERAL OBJECTIVE OF RESEARCH)

In the past, production policies for raw material lot-sizing choices were frequently kept outside of production control systems to simplify and lessen the degree of teamwork and complexity in functional areas such as output, scheduling, inventory management, and procurement. When there are weak links between functional domains, this type of treatment can help to clarify the management of misbehaving individuals. The issue of inventory in fabrication and operation management is interesting. There is a lot of competition in the industry nowadays. Effective inventory management is essential for any agency or creative organization that aims to get maximum benefits at the lowest possible cost. To accomplish these goals, inventory management has been enhancing business education. Inventory management is defined as "the art and science of regulating to have the right product, at the best possible price, in the precise right volume, at the right/perfect time and location."

Administration Scientists and researchers Industrial Engineers have provided more significance to measurements of stock management. The primary analysis, utilization, determines the effectiveness with which companies utilize their assets. It is operational by list hypothesis, a measure of the level of units fabricated to stock. The greater extent of units, the more noteworthy the theory along with the lower proficiency with which the assembling associations satisfy clients' requests, worked to stock. The second execution measurement, effectiveness, catches the nature of the procedure yield. As clients judge the products by cost and quality, value is frequently an essential factor. The third measurement, productivity, measures change proficiency and is accounted for as the stock turnover proportion. The greater turnover causes expanded efficiency with which an industry/firm utilizes its inventories. In most assembling associations, analysts and professionals have perceived four components: (i) OPT (Optimum Production Technology), (ii) JIT (Just-in-Time), (iii) MRP (Material Requirement Planning), and (iv) FMS (Flexible Manufacturing System). Volume adaptability is a significant segment of FMS that takes care of the market– demand at an ideal level. Various endeavors are made to characterize and measure fabricating adaptability.

1.2 LITERATURE REVIEW

We also include promises in our article that industry experts may use, including important administrative details that need to be configured correctly. The professional must choose boundaries that take into account the following factors: (i) Deterioration rate; (ii) Time-Dependent Demand (iii) Production model; as well as (iv) Linear demand. Only then can they decide on the traditional request amount choice and the quality (deterioration rate) control choice and (v) Shortages. We have reconstructed mathematical tests that show the components of decisions under various circumstances and boundary choices.

Deterioration is an organic phenomenon, especially for inventory. Deterioration is defined as damage, decay, spoilage, or loss of a material's marginal value, which lowers its initial usefulness. Blood, fish, capsules and vegetables, alcohol, fuels, radioactive compounds, medicines, and so on lose their advantage concerning time. This time, the manufacturers of such products implemented a price reduction strategy to increase sales. Shaikh [1] developed a stock model, together with other deterioration-related researchers, for a continuously deteriorating product with selling cost as well as recurrence of promotion subordinate interest under the combined kind monetary exchange credit approach. Bai et al. [2] examined the purchaser merchant EOQ stock paradigm for disintegrating products with fixed shipping costs. A few inventory frameworks from 2001 to 2010 that have a different kind of degradation rate are discussed by Bakker et al. [3]. Banerjee and Agrawal [4] provided an explanation of a stock framework in which the selling cost of an item is the basis for the request rate for breaking it down, followed by the item's newness condition. A stock model under imperfect multiplication with time-differing holding costs and time-subordinate interest was developed by Mishra et al. [5]. A stock model with problematic inventory was studied by Tiwari et al. [6], wherein each component may include irregular portions of harmful objects with the known conveyance. Pervin et al. [7] developed a diminishing inventory paradigm with a defective artifact and fluctuating demand. Singh and Mukherjee [8] discussed how giving top-notch solid items and adjusting the stock accessibility to keep up with consumer loyalty can be critical in administering an organization's stock administration, expanding its piece of the pie, efficiency, and productivity. Singh et al. [9] investigated an inventory model with undependable supply where they established that a lot might have a random fraction of defective items with known distribution.

At the present age, everything depends on time. Time is an important factor for an effective business, and without proper time management, we cannot go ahead. Some of the scholars discussed time-dependent demand few are; Nevertheless, the demand-stimulating function of retail inventories is not taken into account by the majority of existing models in the inventory of joint pricing literature with reference price effects. In contrast to constant or deterministic demand rates, the displayed stock level positively impacts sales and profits for specific items. An inventory model for degrading commodities with time-dependent demand and holding costs under partial backlog has been covered by Mishra and Sahib [10]. An EMQ (economies, manufacturing quantity) model with price & time-dependent demand below the inflation rate and dependability was created by Sarkar et al. [11]. A portion of the similar works is introduced in Table 1.

Table 1

Recent and related work related to inventory literature.

<table>
<tr><th>Authors</th><th>EPQ/EOQ models</th><th>Demand Pattern</th><th>Level of Production</th><th>Deterioration Rate</th><th>Shortage</th></tr>
<tr><td>Sivashankari and Panayappan [12]</td><td rowspan="6">EPQ</td><td rowspan="4">Constant</td><td>Two</td><td>Constant</td><td rowspan="4">Allowed</td></tr>
<tr><td>Mishra et al. [13]</td><td rowspan="3">Three</td><td>Controllable Deterioration</td></tr>
<tr><td>Viji and Karthikeyan [14]</td><td>Weibull distribution</td></tr>
<tr><td>Krishnamoorthi and Sivashankari [15]</td><td>Constant</td></tr>
<tr><td>Singh et al. [16]</td><td>Time-Dependent</td><td>No</td><td>Weibull Distribution</td><td>No</td></tr>
<tr><td>Muniappan et al. [17]</td><td>Stock dependent</td><td>one</td><td>constant</td><td>Allowed</td></tr>
<tr><td>This Paper</td><td>EPQ</td><td>Linear</td><td>Three</td><td>Constant</td><td>Allowed</td></tr>
</table>

Tripathi and Mishra [18] examined an economic order quantity (EOQ) model including distinct holding cost functions and a linear time-dependent demand. An ideal course of action for degrading commodities under time-proportional degradation and time-dependent linear demand rates was stated by Singh et al. [19]. In this way, the involvement of stock inspires those in its immediate vicinity. Additionally, there might be intermittent inventory shortages for a variety of reasons.

A one-period discount factor rediscounts cash flows that occur in the following periods. All sale revenues and related inventory costs are stated in the beginning-of-period cash units. In the advertisement, manufacturers and suppliers often disclose information about their products, most notably the introduction of a new item or a modified version of an existing one. Clients are informed about the product and its use with the help of this data. Consequently, the impact of production directly affects the demand for any given good or service. Production-related issues were examined by a few researchers. Production inventory methods for broken items with shortages that integrated the rise and time value of currency were covered by Sarkar [20]. The demand rate was carefully taken into account to be a quadratic diminishing function. A moral creation steering problem was examined by Lee et al. [21]. A plant involves planning, production, stock, and stock management activities to minimize overall costs, and it distributes and appropriates a single commodity to several customers in a condensed amount of time. In addition, presents new difficulties in production and stock arranging production the board stance, and freedom to abuse the distinction in the circumstance of the selling season between geologically scattered business sectors for decaying things are imperative to working on a firm's productivity. Singh [22] defined a production stock model by taking into account the item substitution office of the disappointed item inside ensures periods to their clients. This work additionally drives two imperative presumptions: (1) clients' interest relying upon the substitution time frame, stock level, as well as selling cost of the item and (ii) The rate at which the maker's capital is lost through replacement is a nonlinear capacity that is dependent on the duration of substitution. De et al. [23] investigated an inventory management strategy for a store dealing with a consistent demand from customers for a certain item. When making replenishment decisions, this merchant is presumed to consult the well-known and extensively utilized order-up-to policy. Baqleh and Alateeq [24] have discussed multiple layers comparing streamlining issue is enunciated numerically and consequently settled by utilizing supply chain. Gupta et al. [25] fostered a defective production framework under the accompanying four distinct cases. The first is blemished production with modification and without disturbance. The second one is flawed production with revamping and interruption. The third one is defective production with revision, interruption, and without accumulating; the fourth one is defective production with adjusting, interruption, and interruption multiplying.

In the business world, it is a typical situation that the client's interest is the most significant factor which is subject to numerous critical factors like cost of things, stock level, dependability of items, and numerous others, and at the same time, it assumes a huge part to push up or push back the market economy. In this respect, each maker needs to grab the clients' eye momentarily. Assembling organizations consult different impetuses with their separate items to their clients. Grocery stores rely on volume to make profits, yet mass merchandisers and storeroom clubs have become low-price leaders in the grocery retailing market by purchasing in large quantities. Declining merchandise in a store might quickly lose money due to poor pricing and inventory choices. In addition, there is strong empirical evidence that the quantity of inventory exhibited and the reference price have an impact on demand; however, not many researchers have concurrently examined both impacts.

This study integrates the reference price impact into an inventory model of optimal pricing for degrading products when the demand rate is based on both inventory and selling price. Our goal is to find out how managerial pricing decisions, in the context of repeat business and long-term relationships between customers and the store, are influenced by stock level and consumer price expectations. The relationships between pricing, degrading rate, stock level shape parameter, and reference price are also examined, and the impact of the initial reference price on the retailer's ideal pricing strategy is examined. If an organization runs out of stock, then a customer gives an order and does not receive the order correctly, called backlogging. The term "backlog" is used in finance/ production. A few works on shortage and backlogging are discussed; Kumar et al. [26] highlighted how inventory models incorporating time-dependent demand and partial backlog are utilized to ensure profit-maximizing output. Sarkar and Sarkar [27] addressed a time-varying degradation rate and stock-dependent demand with partial backlogging in an enhanced inventory model. Tai [28] addressed an inventory model where items are reviewed through multiple transmission processes on various quality characteristics previously delivered to customers. Each screening method on a single quality distinctive has an independent screening rate and defective percentage. Shortage back-ordering is also permissible in the model. Wu et al. [29] assigned two inventory systems with partial backlog, time-dependent deterioration rates, and demand for the trapezoidal type. Sharma [30] focused on the study of partial backlogging inventory problems.

1.3 RESEARCH GAP AND CONTRIBUTION

Panayappan and Sivashankari [12] examined the production stock model with degrading elements, wherein it is possible for two distinct production rates to have started at the same rate. Starting at a slow speed of production creates an enticing situation where it may eventually be swapped for another rate. Three extraordinary production levels, together with the flaws that are permitted and finally result in a delayed buy, are reported by Mishra et al. [13], Viji and Karthikeyan [14], Sivashankari and Krishnamoorthi [15], among other writers. A decaying entity that resembles 3-boundary Weibull dissemination crumbling was proposed by Singh et al. [16]. This paradigm allows for deficiencies. Based on the results and affectability analysis, it was determined that the overall inventory cost will increase in proportion to the level of demand and the degree of product deterioration. Nobody developed the inventory model by taking into account points EPQ, linear demand, 3-level production, and shortage and continuous deterioration in the same format.

This work adds to two rivers of existing literature: (1) the finite planning limit inventory model for degrading products with time-dependent demand that is both linear and direct, and (2) shortage is permitted pricing with impacts on reference prices. According to the first stream of literature, demand is only dependent on the present price and the linear trend level, with ending inventory levels constrained to zero. The effect of the degradation rate upon request is usually ignored by the second stream. This is how the article is organized: Section 2 represents the fundamental symbols and assumptions of the suggested inventory model. The mathematical models for the inventory system are covered in Section 3. Theoretical outcomes for ideal solutions are obtained in Section 4. A numerical example to validate the proposed model is revealed in Section 5. Section 6 offers a sensitivity analysis of the relevant parameter along with a graphical analysis. Section 7 concludes with recommendations for future research projects utilizing this study.

2 ASSUMPTIONS AND NOTATIONS

2.1 THE FOLLOWING ARE THE ASSUMPTIONS OF AN INVENTORY MODEL

- the rate of production is constant and well-known;
- 3 rates of production are taken into account;
- the demand rate is a linear function of time and nonnegative;
- the total demand rate is always equal to or greater than the rate of production;
- the lead time assumes zero or negligible;
- the item is a solo product; it is not associated with any other inventory items;
- a limited replenishment rate exists;

- shortages are allowed and partial backlogging.

2.2 FOLLOWING NOTATIONS HAS BEEN SHOWN IN TABLE-2

Table 2

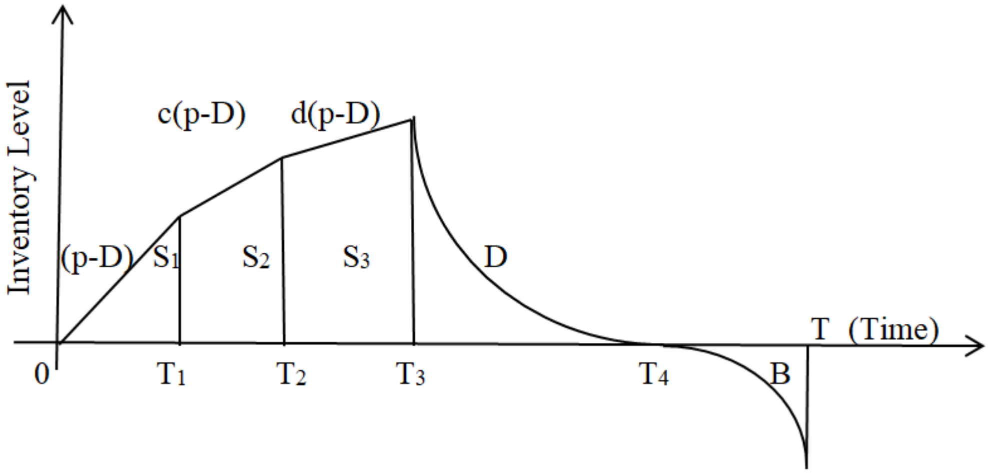
p	Production rate is per unit's time.
D	Demand rate per unit time$D = (a + bt)$, where a and b are a constant parameter and non-negative.
θ	Deterioration rate without preservation technique.
S_1, S_2 and S_3	Maximum inventory level at a time T_1, T_2 and T_3 respectively.
B	Maximum shortage level.
c_p	Production cost per unit.
h_c	Holding cost per unit time.
C_0	Set up cost per production cycle.
$\theta_1(\xi)$	Deterioration rate after investing in PT (Preservation Technology).
ξ	Cost of PT per unit time where $\xi > 0$.
ϕ	The sensitive parameter of investment to the deterioration rate.
s_c	Shortage cost per unit/per unit time.
l_c	Lot sale cost per unit time.
T	Length of the inventory cycle.
T_i	Unit time in periods $i = 1,2,3\&4$.
TC	Total cost.

3 MATHEMATICAL MODEL AND SOLUTION

Every production starts with the first opening/initial stage and stops with the last closing stage. The demand rate is time-dependent. Consider that the production began at the time $t = 0$ and was completed at the time $t = T$. Let during the time interval $[0, T_1]$, the demand rate be ' $D = (a + bt)$' and the production rate be' here a, b indicates positive value and D is l < p. The stock attains a maximum inventory level S_1at a time $t = T_1$. Again, during the time intervals $[T_1, T_2]$ & $[T_2, T_3]$, the growth rate is to be deemed as $c(p - D)$ and $d(p - D)$ here c and d represents constants. The inventory level achieves maximum inventory levels S_2 and S_3 at the time T_2 and T_3 respect. The product becomes outdated in terms of technology during this phase of decline. Demand causes the inventory level to start dropping at a rate of D.

Figure 1

Geometry of production inventory model with shortage



The shortage will reach its peak levels when inventory drops to zero for the entirety of a cycle $t = T$, and the manufacturer will essentially use a subcontract to backlog the insufficient demand. Let $I(t)$ indicate the system's current inventory level at the time T. Graphical representation is shown in figure-1.

The following DEs controlling the inventory system in the time interval $[0,T]$ given are as follow:

3.1 PHASE-1: WHEN SHORTAGE IS NOT ALLOWED

$$\frac{dI(t)}{dt}+\theta I(t)=p-(a+bt)\ ; \qquad 0\le t\le T_1 \tag{1}$$

$$\frac{dI(t)}{dt}+\theta I(t)=c\{p-(a+bt)\}; \qquad T_1\le t\le T_2 \tag{2}$$

$$\frac{dI(t)}{dt}+\theta I(t)=d\{p-(a+bt)\}\ ; \ T_2\le t\le T_3 \tag{3}$$

$$\frac{dI(t)}{dt}+\theta I(t)=-(a+bt)\ ; \qquad T_3\le t\le T_4 \tag{4}$$

3.2 PHASE-2: WHEN SHORTAGE ALLOWED

$$\frac{dI(t)}{dt} = -\frac{(a+bt)}{1+\delta(T-t)}; \qquad T_4 \le t \le T \tag{5}$$

With boundary conditions

$$I(0)=0\ ,\ I(T_1)=S_1\ ,I(T_2)=S_2\ ,I(T_3)=S_3\ ,I(T_4)=0\ \&\ I(T)=-B \tag{6}$$

A solution of the phase-1 from differential equations (1), (2), (3) & (4) using boundary condition (6) are as follow:

$$\text{I(t)}=\frac{-e^{-t\theta}\left[b-be^{t\theta}-a\theta+a\theta e^{t\theta}+p\theta-p\theta e^{t\theta}+bt\theta e^{t\theta}\right]}{\theta^2} \tag{7}$$

$$\text{I(t)}=\frac{-ce^{-t\theta}\left[b-be^{t\theta}-a\theta+a\theta e^{t\theta}+p\theta-p\theta e^{t\theta}+bt\theta e^{t\theta}\right]}{\theta^2} \tag{8}$$

$$\text{I(t)}=\frac{-de^{-t\theta}\left[b-be^{t\theta}-a\theta+a\theta e^{t\theta}+p\theta-p\theta e^{t\theta}+bt\theta e^{t\theta}\right]}{\theta^2} \tag{9}$$

$$\text{I(t)}=\frac{-e^{-t\theta}\left[-be^{t\theta}+be^{T_4\theta}+a\theta e^{t\theta}-a\theta e^{T_4\theta}+bt\theta e^{t\theta}-bT_4\theta e^{T_4\theta}\right]}{\theta^2} \tag{10}$$

A solution of the phase-2 DEs (5) with boundary condition (6) is as follow:

$$\text{I(t)}=\frac{1}{\delta}\left[bt-bT-B\delta^2+(b+a\delta+bT\delta)(T_4-t)+b(T-t)\right] \tag{11}$$

Maximum inventory level S_1 **:** The inventory level during time T_1 is solved as follow from equation (6) & (7), $I(T_1)=S_1$; $S_1=\dfrac{-e^{-T_1\theta}\left[b-be^{T_1\theta}-a\theta+a\theta e^{T_1\theta}+p\theta-p\theta e^{T_1\theta}+bT_1\theta e^{T_1\theta}\right]}{\theta^2}$

When we increase the exponential term for a small value of θ and ignore the third term and higher power of θ, we obtain

$$S_1=\left[\frac{bT_1^2}{2}+aT_1-\frac{a\theta T_1^2}{2}-pT_1+\frac{p\theta T_1^2}{2}\right] \tag{12}$$

Maximum inventory S_2: The maximum inventory level during time T_2 is solve as from equation (6) & (8), $I(T_2)=S_2$

$$S_2=\frac{-ce^{-T_2\theta}\left[b-be^{T_2\theta}-a\theta+a\theta e^{T_2\theta}+p\theta-p\theta e^{T_2\theta}+bT_2\theta e^{T_2\theta}\right]}{\theta^2}$$

When we increase the exponential term for a small value of θ and ignore the third term and higher power of θ, we obtain

$$S_2=c\left[\frac{bT_2^2}{2}+aT_2-\frac{a\theta T_2^2}{2}-pT_2+\frac{p\theta T_2^2}{2}\right] \tag{13}$$

Maximum inventory S_3: The maximum inventory level during time T_3 is solve as follow from equation (7) & (9), $I(T_3)=S_3$; $S_3=\frac{-de^{-T_3\theta}\left[b-be^{T_3\theta}-a\theta+a\theta e^{T_3\theta}+p\theta-p\theta e^{T_3\theta}+bT_3\theta e^{T_3\theta}\right]}{\theta^2}$

Expanding the exponential term and neglecting the third span and higher power of θ for the small value of θ then we get

$$S_3=d\left[\frac{bT_3^2}{2}+aT_3-\frac{a\theta T_3^2}{2}-pT_3+\frac{p\theta T_3^2}{2}\right] \tag{14}$$

4 COST CALCULATION OF INVENTORY MODEL

1. Production cost $PC=Dc_p$ (15)

2. Set up cost $TS_c = C_0$ (16)

3. Holding cost of per unit time TH_c

$$TH_c = h_c \int_0^{T_4} I(t)dt = h_c \left[\int_0^{T_1} I(t)dt + \int_{T_1}^{T_2} I(t)dt + \int_{T_2}^{T_3} I(t)dt + \int_{T_3}^{T_4} I(t)dt \right]$$

$$= h_c \left[\begin{array}{l} \frac{e^{-T1\theta}\left\{-2\theta(a-p)\left\{1+e^{T_1\theta}(-1+T_1\theta)\right\}+b\left\{2-e^{T_1\theta}(2+T_1\theta(-2+T_1\theta))\right\}\right\}}{2\theta^3}+ \\ \frac{e^{-(T_1+T_2)\theta}\left\{2c(e^{T_1\theta}-e^{T_2\theta})(b-a\theta+p\theta)+ce^{(T_1+T_2)\theta}(T_1-T_2)\theta\left(2a\theta-2p\theta-2b+b(T_1+T_2)\theta\right)\right\}}{2\theta^3}+ \\ \frac{e^{-(T_2+T_3)\theta}\left\{2d(e^{T_2\theta}-e^{T_3\theta})(b-a\theta+p\theta)+de^{(T_2+T_3)\theta}(T_2-T_3)\theta\left\{2a\theta-2p\theta-2b+b(T2+T3)\theta\right\}\right\}}{2\theta^3} \\ +\frac{2a\theta(-1+e^{(-T_3+T_4)\theta}+T_3\theta-T_4\theta)+b(2-2T_3\theta+(T_3-T_4)(T_3+T_4)\theta^2+2e^{(-T_3+T_4)\theta}(-1+T_4\theta))}{2\theta^3} \end{array} \right]$$

Expanding the exponential functions and ignoring second and higher power of θ then

$$TH_c = h_c \left[\frac{bT_1^2 - c(T_1^2 - T_2^2)\left(b-2a\theta+2p\theta\right) - d(T_2^2 - T_3^2)\left(b-2a\theta+2p\theta\right) + b\left(T_3 - T_4\right)^2}{2\theta} \right] \quad (17)$$

4. Deterioration cost of finished products TD_c

$$TD_c = c_d\theta \int_0^{T_4} I(t)dt = c_d\theta \left[\int_0^{T_1} I(t)dt + \int_{T_1}^{T_2} I(t)dt + \int_{T_2}^{T_3} I(t)dt + \int_{T_3}^{T_4} I(t)dt \right]$$

Expanding the exponential functions and overlooking second and higher power of θ then

$$TD_c = \theta c_d \left[\frac{bT_1^2 - c(T_1^2 - T_2^2)\left(b-2a\theta+2p\theta\right) - d(T_2^2 - T_3^2)\left(b-2a\theta+2p\theta\right) + b\left(T_3 - T_4\right)^2}{2\theta} \right] \quad (18)$$

5. Preservation Cost $PT_c = \xi T$ (19)

6. Shortage Cost $TS_c = -s_c \int_{T_4}^{T} I(t)dt$

$$TS_c = -s_c\left[BT - \frac{aT^2}{2} - \frac{bT^3}{2} - BT_4 + aTT_4 + bT^2T_4 - \frac{aT_4^2}{2} - \frac{bTT_4^2}{2}\right] \tag{20}$$

7. Lot sale cost $TL_s = l_s \int_{T_4}^{T} (a+bt)\left(1 - \frac{1}{1+\delta(T-t)}\right)dt$

$$TL_s = \frac{l_s\delta}{6}\left[(T-T_4)^2(3a+bT+2bT_4)\right] \tag{21}$$

The total cost of this model $TC = \left[PC + TS_c + TH_c + TD_c + PT_c + TS_c + TL_s\right]$

$$TC = \begin{bmatrix} C_0 + c_p(a+bT_4\delta) + \dfrac{T_4^2(h_c+c_d\theta)}{2\theta}\left\{bT_1^2 - c(T_1^2-T_2^2)\left(b-2a\theta+2p\theta\right) - d(T_2^2\right. \\ \left. -T_3^2)\left(b-2a\theta+2p\theta\right) + b\left(T_3-T_4\right)^2\right\} + \xi T + \dfrac{\delta l_s(T-T4)^2}{6}(3a+bT+2bT_4) \\ -c_s\left(BT - \dfrac{aT^2}{2} - \dfrac{bT^3}{2} - BT_4 + aTT_4 + bT^2T_4 - \dfrac{aT_4^2}{2} - \dfrac{bTT_4^2}{2}\right) \end{bmatrix} \tag{22}$$

Let $T_1 = \alpha T_4;\ T_2 = \beta T_4;\ T_3 = \gamma T_4\ \&\ T = \lambda T_4$ therefore the whole cost will be

$$TC = \begin{bmatrix} C_0 + c_p(a+\lambda bT_4) - + \dfrac{T_4^2(h_c+c_d\theta)}{2\theta}\left\{b\left(c(\beta^2-\alpha^2) + d(\gamma^2-\beta^2) + (1-\alpha^2)\right.\right. \\ \left.\left. +\gamma(2-\gamma)\right) + 2(p-a)\left(c(\beta^2-\alpha^2) + d(\gamma^2-\beta^2)\right)\theta\right\} + \xi\lambda T_4 + \dfrac{\delta l_s(\lambda-1)^2}{6} \\ (3aT_4^2 + 2bT_4^3 + \delta bT_4^3) - \dfrac{T_4(\lambda-1)c_s}{2}\left\{-2B + (\lambda-1)(aT_4 + b\lambda T_4^2)\right\} \end{bmatrix} \tag{23}$$

4.1 OBJECTIVE

The objective of the study is to regulate the optimal value of preservation cost ξ^* for this model that minimizes the total cost TC is as follows Put $\theta = \theta_1 e^{-\phi\xi}$ then equation reduces to TC follow as;

$$TC = \begin{bmatrix} C_0 + c_p(a + bT_4\lambda) + \dfrac{T_4^2(h_c + c_d\theta_1 e^{-\phi\xi})}{2\theta_1 e^{-\phi\xi}}\Big\{b\Big(c(\beta^2-\alpha^2) + d(\gamma^2-\beta^2) + (1-\alpha^2) \\ \gamma(2-\gamma)\Big) + 2(p-a)\Big(c(\beta^2-\alpha^2) + d(\gamma^2-\beta^2)\Big)\theta_1 e^{-\phi\xi}\Big\} + \xi\lambda T_4 + \dfrac{\delta l_s(\lambda-1)^2}{6} \\ (3aT_4^2 + 2bT_4^3 + \lambda bT_4^3) - \dfrac{T_4(\lambda-1)c_s}{2}\left\{-2B + (\lambda-1)(aT_4 + b\lambda T_4^2)\right\} \end{bmatrix} \tag{24}$$

Differentiating cost function TC concerning ξ

$$\frac{dTC}{d\xi} = \left[\frac{T_4^2 h_c\phi\theta_1 e^{\phi\xi} b}{2}\begin{pmatrix} c(\beta^2-\alpha^2) + d(\gamma^2-\beta^2) \\ +(1-\alpha^2) + \gamma(2-\gamma)\end{pmatrix} + \lambda T_4 - \phi T_4^2 c_d\theta_1(p-a)e^{-\phi\xi}\right] \tag{25}$$

Again, differentiating cost function concerning ξ

$$\frac{d^2TC}{d\xi^2} = \phi^2 T_4^2\theta_1\left\{\frac{h_c e^{\phi\xi} b}{2}\begin{pmatrix} c(\beta^2-\alpha^2) + d(\gamma^2-\beta^2) \\ +(1-\alpha^2) + \gamma(\gamma-2)\end{pmatrix} + c_d(p-a)e^{-\phi\xi}\right\} \tag{26}$$

$$\frac{d^2TC}{d\xi^2} > 0\, if\, ,(p-a),(\beta^2-\alpha^2),(\gamma^2-\beta^2),(1-\alpha^2),(2-\gamma);$$

The optimal value ξ^* will be calculated using Mathematica-software-9 from equation (25). The next objective of the study is to determine the optimal value of T_4^*, therefore the value of T_4^*, which minimizes TC as surveys then differentiate concerning T_4.

$$\frac{dTC}{dT_4}=\begin{bmatrix} c_p b\lambda+\frac{T_4(h_c+c_d\theta)}{\theta}\left\{b\left(c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2)+(1-\alpha^2)+\gamma(2-\gamma)\right)+\right. \\ \left.2(p-a)\left(c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2)\right)\theta\right\}+\frac{(\lambda-1)^2 l_s\delta}{6}(6aT_4+3bT_4^2(2+\lambda)) \\ +\xi\lambda-\frac{(\lambda-1)c_s}{2}\left\{-2B+(\lambda-1)(2aT_4+3b\lambda T_4^2)\right\} \end{bmatrix} \tag{27}$$

Again, differentiate cost function concerning T_4 then equation becomes

$$\frac{d^2TC}{dT_4^2}=\begin{bmatrix} \frac{(h_c+c_d\theta)}{\theta}\left\{b\left(c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2)+(1-\alpha^2)+\gamma(2-\gamma)\right)\right. \\ \left.+2(p-a)\left(c(\beta^2-\alpha^2)+d(\gamma^2-\beta^2)\right)\theta\right\}+(\lambda-1)^2 a(l_s\delta-c_s) \\ +(\lambda-1)^2 bT_4\left\{(\lambda+2)l_s\delta-3\lambda c_s\right\} \end{bmatrix} \tag{28}$$

$$\frac{d^2TC}{dT_4^2}>0\, if\ (p-a),(\lambda-1),\left\{(\lambda+2)l_s\delta-3\lambda c_s\right\},(l_s\delta-c_s)$$

5 NUMERICAL ANALYSIS

To validate the suggested model, the following data was used as an illustration. The production rate is \$200, the backlogging cost is \$2.2 lot sale parameter is \$1.6, $\alpha=0.6, \beta=0.7, \gamma=0.8 \& \lambda=2.1$, the holding cost is \$0.3 per unit, the set-up cost is \$200, the deterioration cost is \$2, the production cost is \$10, the unit, deterioration rate is \$0.2, the demand rate of parameter $a=5, b=4, c=2 \& d=2.5$. The optimal value of the preservation technique ξ^*, the optimal value of $T_1^*, T_2^*, T_3^*, T_4^* \& T^*$, the maximum inventory of $S_1^*, S_2^* \& S_3^*$, and the optimal total cost TC^*, with preservation technology, correspondingly were computed with the support of eq. (25, 27,12, 13,14 & 24), shown in Table 3:

Table 3

(Optimal Value of Different Parameters)

p	ξ^*	T_1^*	T_2^*	T_3^*	T_4^*	T^*	S_1^*	S_2^*	S_3^*	TC^*
200	5.11	12.04	14.04	16.05	20.06	42.13	768.28	3004.32	6022.85	65635.3
400	6.29	23.63	27.57	31.51	39.39	82.72	1258.99	5276.48	10878.9	78497.4

6 SENSITIVITY ANALYSIS

The impact of alterations in the different parameters of the suggested model is examined by sensitivity analysis, wherein increments and decreases of 15 and 30 percent are assigned to each of the aforementioned parameters while maintaining the same values for the remaining parameters. It is conducted by altering the specific parameter $p, a, b, c, d, c_p, c_d, c_s \& l_s$ Table 4 displays the sensitiveness of the different parameters on the optimal value of $\xi^*, T_1^*, T_2^*, T_3^*, T_4^*, T^*, S_1^*, S_2^* \& S_3^*$ as well as a total cost TC^* study manifested the following facts:

Table 4

Sensitivity Analysis of Different Parameter

Parameter		ξ^*	T_1^*	T_2^*	T_3^*	T_4^*	T^*	S_1^*	S_2^*	S_3^*	TC^*
p	30	8.81	29.09	29.09	29.02	29.09	-22.53	19.16	22.69	24.19	16.36
	15	4.70	14.55	14.57	14.52	14.59	-12.73	9.58	11.34	12.09	8.18
	-15	-5.47	-14.69	-14.70	-14.81	-14.72	17.27	-9.58	-11.34	-12.09	-8.18
	-30	-11.93	-29.56	-29.58	-29.63	-29.57	42.01	-19.16	-22.69	-24.19	-16.36
a	30	-0.26	-2.89	-2.88	-2.92	-2.86	2.94	-0.48	-0.57	-0.60	0.79
	15	-0.13	-1.40	-1.38	-1.49	-1.41	1.43	-0.24	-0.28	-0.30	0.40
	-15	0.12	1.43	1.47	1.38	1.43	-1.42	0.24	0.28	0.30	-0.40
	-30	0.25	2.84	2.82	2.75	2.82	-2.54	0.48	0.57	0.60	-0.79
b	30	-15.65	-20.34	-20.32	-20.42	-20.35	25.55	11.64	8.26	6.82	12.12
	15	-8.21	-11.45	-11.49	-11.51	-11.48	12.96	5.98	4.32	3.61	5.79
	-15	9.79	15.47	15.42	15.39	15.43	-13.36	-5.34	-3.56	-2.80	-6.85
	-30	21.34	33.40	31.35	30.31	32.36	-27.21	-11.00	-7.50	-6.01	-13.18
c	30	-1.56	12.23	12.22	12.21	12.24	-10.90	34.97	30.00	43.84	21.52
	15	-0.78	6.08	6.10	6.05	6.11	-5.76	17.42	14.97	21.03	11.72
	-15	0.79	-6.13	-6.15	-6.22	-6.15	6.56	-16.55	-15.00	-19.42	-13.52
	-30	1.77	-12.36	-12.35	-12.39	-12.33	14.06	-33.16	-30.00	-38.18	-28.61
d	30	-2.34	17.63	17.63	17.57	17.63	-14.98	66.71	66.16	50.00	-31.68
	15	-1.17	8.82	8.87	8.79	8.85	-8.13	32.94	34.06	25.05	-14.03
	-15	1.18	-8.87	-8.86	-8.96	-8.89	9.75	-33.74	-31.76	-25.00	14.08
	-30	2.55	-17.84	-17.83	-17.87	-17.81	21.67	-65.18	-59.14	-51.00	30.66
c_p	30	8.62	-0.73	-0.74	-0.74	-0.72	0.73	-3.60	-2.72	-2.35	0.04
	15	4.51	-0.40	-0.38	-0.43	-0.37	0.37	-1.85	-1.40	-1.21	0.02
	-15	-5.47	0.35	0.33	0.26	0.33	-0.32	1.68	1.27	1.09	-0.03
	-30	-11.54	0.68	0.68	0.63	0.68	-0.68	3.46	2.60	2.25	-0.06
	30	0.00	19.21	19.20	19.12	19.17	-16.09	0.00	0.00	0.00	9.41
	15	0.00	9.57	9.58	9.53	9.60	-8.76	0.00	0.00	0.00	4.67

c_d	-15	0.00	-9.70	-9.64	-9.71	-9.64	10.67	0.00	0.00	0.00	-4.67
	-30	0.00	-19.34	-19.32	-19.42	-19.36	20.00	0.00	0.00	0.00	-9.35
c_s	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-42.39
	15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-20.55
	-15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	21.19
	-30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	42.39
l_s	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	55.15
	15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	27.57
	-15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-27.57
	-30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-55.15

The finest value of ξ^* the slight change in the value of parameters $a, c \& d$, moderately $c_p \& p$ and highly with b.

The finest value of $T_1^*, T_2^*, T_3^*, T_4^* \& T$ slightly change in the value of parameters $a \& c_p$, moderately c highly with $p, b, d \& c_d$.

The optimal value of $S_1^*, S_2^* \& S_3^*$ slightly change in the value of parameters $a \& c_p$, moderately b highly with $p, c \& d$.

The optimum value of TC^* moderately deviations in the value of parameters $p, b \& c_d$ highly with the value of $c, d, c_s \& l_s$ whereas slightly with $a \& c_p$.

6.1 GRAPHICAL ANALYSIS

Figures 2 and 3 depict the graphical depiction of the ideal total cost for the preservation technology, which is the convexity TC^* of concern and an ideal time ξ^*.

Figure 2

Total cost and preservation technique

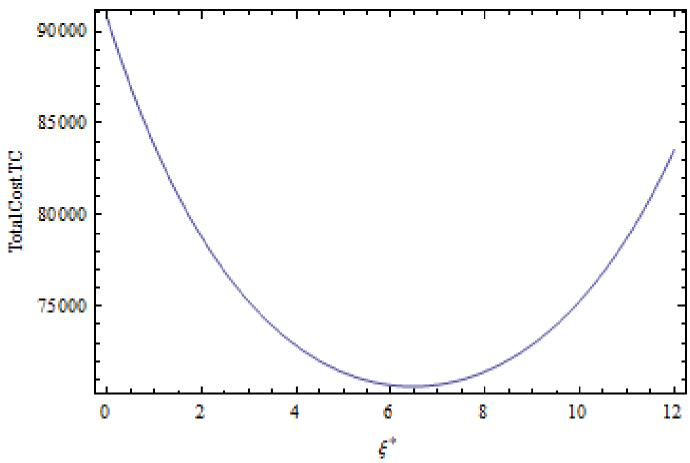
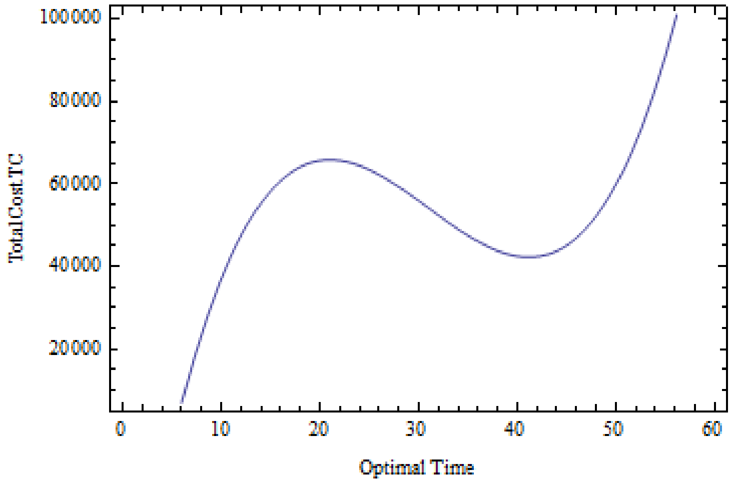


Figure 3

Total cost and optimal time T_4^*



7 CONCLUSIONS

For a recently introduced product with consistent demand up to a time-dependent point, the suggested approach is ideal. The approach is ideal in this scenario because it allows for a significant quantity of manufactured goods to be stockpiled at a low pace of production. As a result, we will see increased customer satisfaction and possible profit. Here, we developed a mathematical framework and a fix for this. A sensitivity analysis and numerical example of the model are provided to exemplify it. The appropriate order amount, total inventory cost, and cycle time may be precisely determined by the producer and retailer with the use of the suggested inventory model. The model for the time worth of money might be expanded upon. We may also classify as time-dependent the unit purchase cost, the time-dependent inventory holding cost, and other costs. This model may be expanded in many ways for use in future studies. We may, for example, see a rise in the desire for a wide variety of designs. The model for the time worth of money might be expanded upon. Thus, our model offers a novel management insight that aids in achieving the ideal profit level for a production system or business. In reality, we obtain a promising outcome from the numerical case.

FOR FURTHER RESEARCH

There are several methods to extend this model with varied demand rates, including rework of defective items, time discounting, and constant, cubic, quadratic, and Weibull degradation with 3 or more parameters. A conceivable future study problem is to examine the effect of timeshifting disintegration on the ideal arrangement. Another intriguing and testing element is considering a multi-thing EOQ display.

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