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# Entanglement properties of optical coherent states under amplitude damping

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**Abstract.** Through concurrence, we characterize the entanglement of optical coherent-state qubits subject to an amplitude damping channel. We investigate the distillation capabilities of known error correcting codes and obtain upper bounds on the entanglement depending on the non-orthogonality of the input states and the channel damping parameter, thus quantitatively analysing these photon-loss codes which are naturally reminiscent of the standard qubit codes against Pauli errors.

**Keywords:** Error Correction, Concurrence, Entanglement, Coherent state superpositions

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## INTRODUCTION

Entanglement is a necessary resource for many useful quantum tasks, but must be protected against undesired environmental interactions which lead to decoherence of the quantum system. Two proven ways of safeguarding exist: Quantum Error Correction (QEC) codes protect the information by encoding in a larger Hilbert space; Entanglement Purification (EP) protocols distill entanglement from a number of identical copies. QEC codes can be recast as EP schemes and vice-versa; the connection for discrete-variable (DV) systems was established by Bennett *et al* [1], whereas the bridge for continuous-variables (CV) has been demonstrated in [2]. In the optical context, amplitude damping, or photon loss, is a dominant source of decoherence. It consists of a *Gaussian* error, and as such it is known that Gaussian resources - the set of operations implemented through Gaussian ancillae, beam splitters, phase shifters, squeezers and homodyne measurements - are of no use in protecting the signal state [2–5], and more elaborate non-Gaussian operations must be accounted for. Here we revisit such a QEC scheme as proposed by Glancy *et al.*[6], which protects coherent-state superpositions (CSS, or “cat states”) by employing non-Gaussian encoding operations.

A coherent state  $|\alpha\rangle$  is defined as an eigenstate of the annihilation operator ( $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ ). Following [6], we identify the logical qubit basis as  $|0\rangle_L = |-\alpha\rangle$  and  $|1\rangle_L = |\alpha\rangle$ . An arbitrary qubit superposition is represented as

$$|Q_\alpha\rangle = \frac{1}{\sqrt{N(\alpha)}}(a|-\alpha\rangle + b|\alpha\rangle), \quad (1)$$

where  $|a|^2 + |b|^2 = 1$  and  $N(\alpha) = 1 + e^{-2|\alpha|^2}(ab^* + a^*b)$ . For sufficiently large  $\alpha$ ,  $|-\alpha\rangle$  and  $|\alpha\rangle$  are approximately orthogonal; however, present-day technologies only achieve limited sizes (“Schrödinger Kittens”), and significant overlaps must be considered.

## AMPLITUDE DAMPING

We model photon loss by combining the signal with a vacuum mode  $|0\rangle_l$  at a beam splitter of transmissivity  $\eta$ , then integrate over the loss mode. For a single coherent state, this integration amounts to an amplitude contraction, but for a superposition, the resulting state is now mixed, and one obtains

$$\rho = (1 - p_e)|Q_{\alpha\sqrt{\eta}}\rangle\langle Q_{\alpha\sqrt{\eta}}| + p_e Z|Q_{\alpha\sqrt{\eta}}\rangle\langle Q_{\alpha\sqrt{\eta}}|Z, \quad (2)$$

where  $p_e = \frac{1}{2}(1 - e^{-2(1-\eta)|\alpha|^2})$  is the probability that the Pauli Z operator was applied. Photon loss can be seen as having a two-fold effect: first, the amplitude is unconditionally reduced from  $\alpha$  to  $\alpha\sqrt{\eta}$ ; second, with probability  $p_e$ , the qubit suffers a phase flip.

## ERROR CORRECTION

Having identified the effect of amplitude damping as a phase flip, a 3-mode error-correcting code can be used to protect the qubit. Such can be implemented [6] in the optical setting by sending the input state through three beam splitters followed by Hadamard gates which implement, up to a normalization constant,  $|0\rangle_L \rightarrow |0\rangle_L + |1\rangle_L$ ,  $|1\rangle_L \rightarrow |0\rangle_L - |1\rangle_L$ . The resulting (unnormalized) encoded state is

$$a(|-\alpha\rangle + |\alpha\rangle)^{\otimes 3} + b(|-\alpha\rangle - |\alpha\rangle)^{\otimes 3}. \quad (3)$$

After transmission, another Hadamard gate is applied to each of the modes, which are then recombined through an inverted sequence of beam splitters. The ancilla modes are measured to provide syndrome information, from which the appropriate correcting operation can be applied to return the signal to its "unflipped" state. Finally, teleporting the state into an appropriately prepared Bell state, the amplitude can also be restored.

The three-way redundant encoding outlined above can correct up to one error; therefore, an error-free transmission is achieved with probability  $1 - 3p_e^2 + 2p_e^3$ . This can be further increased by encoding the input state with a higher number of repetitions. Here, we will analyze codes with 3, 5, 11 and 51 repetitions.

## ENTANGLEMENT

The overall performance of the scheme above in its original proposal was not quantified except for certain success probabilities or the fidelities of some involved operations. As such, we will explore the fact that QEC codes can be recast as EP protocols and, employing the entanglement as a figure of merit, provide quantitative benchmarks for this codification. For this task, we will employ initial cat states of the form

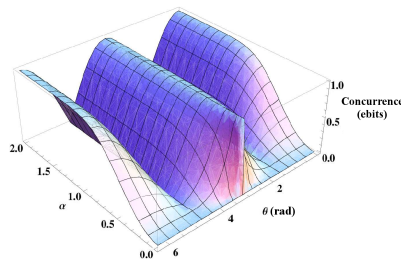
$$|\chi_{\alpha_1, \alpha_2}\rangle = \frac{1}{\sqrt{\tilde{N}(\alpha_1, \alpha_2)}} \left( \sqrt{w}|\alpha_1, \alpha_2\rangle + e^{i\theta}\sqrt{1-w}|-\alpha_1, -\alpha_2\rangle \right), \quad (4)$$

with  $\tilde{N}(\alpha_1, \alpha_2) = 2 + 2\cos\theta\sqrt{w(1-w)}e^{-2|\alpha_1|^2 - 2|\alpha_2|^2}$  and  $|\alpha_1, \alpha_2\rangle$  a shorthand notation for  $|\alpha_1\rangle|\alpha_2\rangle$ . The first mode is kept, while the second is sent through the photon-loss

channel, either directly or encoded using the repetition code. We then adopt Wootters' concurrence to quantify the entanglement, given, for a bipartite system, by[7]

$$C = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (5)$$

where  $\lambda_i$  are the eigenvalues, listed in decreasing order, of  $\rho\tilde{\rho}$ .  $\tilde{\rho}$  is the time-reversed density operator,  $\tilde{\rho} = (\sigma_{y,1} \otimes \sigma_{y,2})\rho^*(\sigma_{y,1} \otimes \sigma_{y,2})$ , where  $\sigma_{y,i}$  is the Pauli  $Y$  operator in the  $i$ -th mode. We make use of an orthogonal basis  $\{|u_\alpha\rangle, |v_\alpha\rangle\}$  in order to cast the density matrices for the input and output states in different transmission scenarios. With those, one can calculate the concurrence as per (5). Fig. 1 plots the entanglement of the initial state with  $w = 1/2$ , corresponding to "genuine" Bell cat states. It is seen that  $\theta = 0$  and  $\theta = \pi$  (dubbed "even" and "odd" cats due to the parity of the number states in the superposition) result in maximum entanglement, the former asymptotically, the latter independently of size.

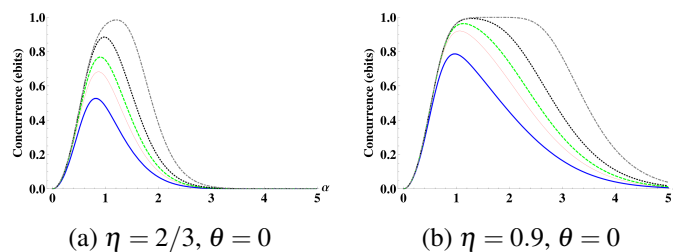


**FIGURE 1.** Concurrence for the initial state as a function of the size of the CSS  $|\alpha|$  and the phase  $\theta$ .

The above method is not convenient for calculating the entanglement of different levels of encoding, requiring the eigenvalues to be computed for each variation. A more efficient way can be found by employing an evolution equation [8], which equates the entanglement of the final state to the product of the concurrence of a maximally entangled state  $|\phi^+\rangle$  subjected to the same channel times the concurrence of the initial state  $|\chi\rangle$ . The problem is reduced to the calculation of two simpler concurrences, cf.

$$C[(1 \otimes \$) |\chi\rangle\langle\chi|] = C[(1 \otimes \$) |\phi^+\rangle\langle\phi^+|] C[|\chi\rangle]. \quad (6)$$

This method matches the results found in the previous subsection, but allows for a computationally significant speed-up in calculation times. The concurrences for different encodings for a particular choice of input states are then compared (the interested reader is invited to see [9] for additional calculations and other examples):



**FIGURE 2.** Concurrence after transmission for direct transmission (blue, thick line) compared to encoding with 3 (red line), 5 (green, dashed), 11 (black, dotted) and 51 (grey, dash-dotted) qubits.

The non-orthogonality of the basis states prevents the pair from achieving high entanglement for low values of  $\alpha$ ; higher encodings are of no advantage in this regime. Equally, for sufficiently large sizes of  $\alpha$ , the flip probability approaches 0.5 and dephases the qubit entirely. However, for a certain range, the encoding helps achieve and sustain higher entanglement between the shared pairs.

## DISCUSSION AND CONCLUSIONS

In this work, we have studied the decoherence process of entangled coherent-state superpositions in the amplitude damping channel, exploring QEC codes originated from the discrete-variable regime in an optical, continuous-variable setting. We have observed a trade-off between the orthogonality of the coherent state basis and the probability of a phase-flip error. The former imposes a necessity for a maximum size of coherent-state superposition, while the latter prevents the use of arbitrarily large superpositions, thus hinting at an optimal regime depending on channel parameters.

We conclude that experimentally-viable odd-parity cat states, combined with linear optics, non-Gaussian Hadamard operations and non-linear measurements, can provide an efficient error correction and entanglement distillation strategy.

## ACKNOWLEDGMENTS

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