

INTEGRATION OF OBJECTIVE WEIGHTING METHODS FOR CRITERIA AND MCDM METHODS: APPLICATION IN MATERIAL SELECTION

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Abstract

Determining weights for criteria is an extremely crucial step in the process of selecting an option based on multiple criteria, also known as Multi-Criteria Decision Making (*MCDM*). This article presents the combination of five objective weighting methods for criteria with three *MCDM* methods in the context of material selection. The five objective weighting methods considered are Entropy, *MEREC* (Method based on the Removal Effects of Criteria), *LOPCOW* (Logarithmic Percentage Change-driven Objective Weighting), *CRITIC* (Criteria Importance Through Intercriteria Correlation), and *MEAN*. The three *MCDM* methods employed are *MARA* (Magnitude of the Area for the Ranking of Alternatives), *RAM* (Root Assessment Method), and *PIV* (Proximity Indexed Value). Material selection investigations were conducted in three different cases, including lubricant selection for two-stroke engines, material selection for manufacturing screw shafts, and material selection for manufacturing gears. The Spearman's rank correlation coefficient was calculated to assess the stability of ranking the alternatives using different *MCDM* methods. The combinations of objective weighting methods and *MCDM* methods were evaluated based on factors such as consistency in identifying the best material type, range, average value, and median of each set of Spearman's rank correlation coefficients. Two significant findings were identified. First, the weights of criteria calculated using *LOPCOW* method appear to be inversely related to those calculated using the Entropy method. Second, among the three *MCDM* methods used, *MARA* was identified as the most suitable for lubricant selection for two-stroke engines, *RAM* was found to be the most suitable for material selection for screw shafts and gears. The best material type in each case was also determined.

Keywords: Multi criteria decision making, weight, *MARA*, *RAM*, *PIV*, Material selection.

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1. Introduction

Multi-Criteria Decision Making (*MCDM*) methods are increasingly prevalent and effective for selecting optimal solutions across various fields such as economics, engineering, medicine, education, etc. [1, 2]. Two critical aspects in multi-criteria decision-making are the selection of methods to determine criterion weights and the choice of *MCDM* methods to be employed. These choices significantly impact the ranking of alternatives [3–5]. Weight determination methods are categorized into three groups: subjective methods, objective methods, and a combination of both, known as combined methods [6]. Among these, objective methods are the most commonly used because criterion weights remain uninfluenced by subjective judgments of decision-makers [7, 8]. Some objective weighting methods include Entropy [9], *MEREC* [10], *LOPCOW* [11], *CRITIC* [12], *MEAN* [13], *CILOS* (Criteria Impact LOSs) [14], *IDOCRIW* (Integrated Determination of Criteria Weight) [15], etc. So, one question arises: what are the differences in the weight values of the criteria

calculated by these methods? Finding the answer to this question is the first objective of this study. In this article, a comparison will be made of the weight values of the criteria when calculated by five objective weighting methods including Entropy method, *MEREC* method, *LOPCOW* method, *CRITIC* method, and *MEAN* method.

As mentioned earlier, besides selecting methods for determining criterion weights, the choice of *MCDM* methods plays a crucial role and significantly influences the ranking of alternatives. With over 200 existing *MCDM* methods, studying all of them in one research study is a massive and challenging task. Instead, in each case, only a few methods with distinct characteristics should be chosen for investigation [15, 16]. The three methods used in this article are *MARA*, *RAM*, and *PIV*. The *MARA* method, discovered in 2022, focuses on calculating the area under the chart of each alternative on important criteria. This area reflects the importance of each criterion for each alternative [6]. *RAM* is a relatively new method, first discovered in September 2023, focusing on analyzing the decision system's structure and defining the root factors influencing the decision. *RAM* prioritizes these root factors by evaluating their importance to the ultimate goal of the decision [17]. *PIV* is a well-known method with the advantage of minimizing the phenomenon of rank reversal. Although relatively new (introduced in 2018), it has attracted the attention of many scientists across various fields [18–21]. Some brief analyses above have shown different approaches when applying *MARA*, *RAM*, and *PIV* methods. So, when they are used together to make multi-criteria decisions for a specific issue, do these three methods all find the best solution? The second objective of this study is to find the answer to this question.

The five methods including Entropy, *MEREC*, *LOPCOW*, *CRITIC*, and *MEAN* each have their own characteristics in evaluating the weight of criteria. Their combination provides a solid foundation for weight determination, helping to create a comprehensive and objective evaluation table. The three *MCDM* methods including *MARA*, *RAM*, and *PIV* also have different characteristics. Decision-making when applying them not only relies on weighted criteria but also integrates multi-dimensional evaluation of multi-criteria decisions, creating transparency and efficiency. Combining weighting methods and *MCDM* methods not only proposes a comprehensive decision model but also demonstrates flexibility and practical application in selecting the best option among available options in various fields. All of these will confirm the accuracy of the answers to the two questions (corresponding to the two objectives) mentioned above.

The combination of weighting methods and *MCDM* methods is applied in material selection. The reason material selection is chosen as the problem in this study is because it is a complex task, and each type of material must be considered for many different criteria [22, 23].

This study was conducted with two aims: firstly, to compare the weight values of criteria when calculated using different methods, and secondly, to determine which *MCDM* method is suitable for selecting materials in each specific case.

2. Materials and Methods

2.1. Objective methods for weight determination of criteria

Determining the weights of criteria using the Entropy method follows the following sequence [9]:

– Step 1. Construct a decision matrix with m rows and n columns, where m is the number of alternatives to be ranked, and n is the number of criteria for each alternative. Let y_{ij} represent the value of criterion j for alternative i , with $j = 1 \div n$, $i = 1 \div m$.

– Step 2. Calculate normalized values for the y_{ij} components:

$$n_{ij} = \frac{y_{ij}}{m + \sum_{i=1}^m y_{ij}^2}. \quad (1)$$

– Step 3. Calculate the entropy measure for each criterion:

$$e_j = \sum_{i=1}^m [n_{ij} \times \ln(n_{ij})] - \left(1 - \sum_{i=1}^m n_{ij}\right) \times \ln\left(1 - \sum_{i=1}^m n_{ij}\right). \quad (2)$$

– Step 4. Calculate the weights for each criterion:

$$w_j = \frac{1 - e_j}{\sum_{j=1}^n (1 - e_j)}. \quad (3)$$

The sequence for determining weights using the *MEREC* method is as follows [10]:

– Step 1. Construct a decision matrix, similar to Step 1 of the Entropy method.

– Step 2. Calculate normalized values for the y_{ij} components. Here, B and C correspond to criteria of benefit and cost types:

$$n_{ij} = \frac{\min y_{ij}}{y_{ij}}, j \in B, \quad (4)$$

$$n_{ij} = \frac{y_{ij}}{\max y_{ij}}, j \in C. \quad (5)$$

– Step 3. Calculate the overall performance of alternatives:

$$S_i = Ln \left[1 + \left(\frac{1}{n} \sum_{j=1}^n |\ln(n_{ij})| \right) \right]. \quad (6)$$

– Step 4. Calculate the performance of alternatives:

$$S'_{ij} = Ln \left[1 + \frac{1}{n} \sum_{k, k \neq j} |\ln(n_{ij})| \right]. \quad (7)$$

– Step 5. Calculate the absolute values of deviations:

$$E_j = \sum_{i=1}^m |S'_{ij} - S_i|. \quad (8)$$

– Step 6. Calculate weights for criteria:

$$w_j = \frac{E_j}{\sum_{j=1}^n E_j}. \quad (9)$$

To calculate weights for criteria using the *LOPCOW* method, apply the following sequence [11]:

– Step 1. Construct a decision matrix, similar to Step 1 of the Entropy method.

– Step 2. Calculate normalized values for the y_{ij} components:

$$n_{ij} = \frac{y_{ij} - \min y_{ij}}{\max y_{ij} - \min y_{ij}}, j \in B, \quad (10)$$

$$n_{ij} = \frac{\max y_{ij} - y_{ij}}{\max y_{ij} - \min y_{ij}}, j \in C. \quad (11)$$

– Step 3. Calculate PV_{ij} values for each component, where σ is the standard deviation:

$$PV_{ij} = 100 \times \left| \frac{\sqrt{\sum_{i=1}^m n_{ij}^2}}{\ln \frac{m}{\sigma}} \right|. \quad (12)$$

– Step 4. Calculate weights for criteria:

$$w_j = \frac{PV_{ij}}{\sum_{j=1}^n PV_{ij}}. \quad (13)$$

The sequence for determining weights for criteria using the *CRITIC* method is as follows [12]:

– Step 1. Construct a decision matrix, similar to Step 1 of the Entropy method.

– Step 2. Calculate normalized values for the y_{ij} components:

$$n_{ij} = \frac{y_{ij} - \min y_{ij}}{\max y_{ij} - \min y_{ij}}. \quad (14)$$

– Step 3. The weight of criterion j is calculated using formulas (15) and (16):

$$w_j = \frac{C_j}{\sum_{j=1}^n C_j}, \quad (15)$$

$$C_j = \sigma_j \times \sum_{j=1}^n (1 - r_{ij}), \quad (16)$$

where σ_j is the standard deviation of criterion j ; r_{ij} is the correlation coefficient between two criteria.

Formula (17) is used to calculate weights for criteria using the MEAN method [13]:

$$w_1 = w_2 = \dots = w_j = w_n = \frac{1}{n}. \quad (17)$$

Assigning weights to criteria using these subjective weighting methods will be applied in the subsequent sections of this study using the Excel software as the tool.

2. 2. MCDM methods used

The sequence for ranking alternatives using the *MARA* method is as follows [6]:

– Step 1. Construct a decision matrix, similar to Step 1 of the Entropy method.

– Step 2. Calculate normalized values using two formulas:

$$n_{ij} = \frac{x_{ij}}{\max x_{ij}}, j \in B, \quad (18)$$

$$n_{ij} = \frac{x_{ij}}{\max x_{ij}}, j \in C. \quad (19)$$

– Step 3. Calculate normalized values considering the weights of criteria:

$$g_{ij} = w_i \cdot n_{ij}. \quad (20)$$

– Step 4. Identify elements of the optimal solution:

$$s_j = \max(g_{ij}), 1 \leq j \leq n, \forall i \in [1, 2, \dots, m]. \quad (21)$$

– Step 5. Determine the optimal solution:

$$s = \{s_1, s_2, \dots, s_j\}, j = 1, 2, \dots, n. \quad (22)$$

– Step 6. Divide the optimal solution into two subsets S^{\max} and S^{\min} .

$$S = S^{\max} \cup S^{\min}. \quad (23)$$

– Step 7. Describe the optimal solution as in (24), where k is the number of criteria of type B , l is the number of criteria of type C , $l = n - k$:

$$S = \{s_1, s_2, \dots, s_k\} \cup \{s_1, s_2, \dots, s_l\}. \quad (24)$$

– Step 8. Divide substitute alternatives into two subsets T^{\max} and T^{\min} using the same procedure as in Steps 6 and 7:

$$T_i = T_i^{\max} \cup T_i^{\min}, \quad (25)$$

$$T_i = \{t_{i1}, t_{i2}, \dots, t_{ik}\} \cup \{t_{i1}, t_{i2}, \dots, t_{il}\}, \forall i \in [1, 2, \dots, m]. \quad (26)$$

– Step 9. Determine the magnitude of each component.

+For the optimal solution.

$$S_k = s_1 + s_2 + \dots + s_k, \quad (27)$$

$$S_l = s_1 + s_2 + \dots + s_k. \quad (28)$$

+For substitute alternative i .

$$T_{ik} = t_{i1} + t_{i2} + \dots + t_{ik}, \forall i \in [1, 2, \dots, m], \quad (29)$$

$$T_{il} = t_{i1} + t_{i2} + \dots + t_{il}, \forall i \in [1, 2, \dots, m]. \quad (30)$$

– Step 10. Describe the magnitude of the constrained area of choices. This is done by constructing two linear functions.

+Linear function of the optimal solution:

$$f^{opt}(S_k, S_l) = \frac{S_l - S_k}{1 - 0}(x - S_k) + S_k = (S_l - S_k)x + S_k. \quad (31)$$

+Linear function of substitute alternative i :

$$f^i(T_{ik}, T_{il}) = \frac{T_{il} - T_{ik}}{1 - 0}(x - T_{ik}) + T_{ik} = (T_{il} - T_{ik})x + T_{ik}. \quad (32)$$

– Step 11. Calculate the area for alternatives.

+For the optimal solution:

$$F^{opt} = \int_0^1 f^{opt}(S_k, S_l) dx = \int_0^1 ((S_l - S_k)x + S_k) dx = \frac{S_l - S_k}{2} + S_k. \quad (33)$$

+For substitute alternative i :

$$F^i = \int_0^1 f^i(T_{ik}, T_{il}) dx = \int_0^1 ((T_{il} - T_{ik})x + T_{ik}) dx = \frac{T_{il} - T_{ik}}{2} + T_{ik}, \forall i \in [1, 2, \dots, m]. \quad (34)$$

– Step 12. Calculate the magnitude of the area for alternative i :

$$M_i = \int_0^1 f^{opt}(S_k, S_l) dx - \int_0^1 f^i(T_{ik}, T_{il}) dx, \forall i \in [1, 2, \dots, m]. \quad (35)$$

– Step 13. Rank substitute alternatives in increasing order of the M_i value.

To rank alternatives using the *RAM* method, the following sequence is applied [17]:

– Step 1. Construct a decision matrix, similar to Step 1 of the Entropy method.

– Step 2. Normalize the data for the y_{ij} components:

$$n_{ij} = \frac{y_{ij}}{\sum_{i=1}^m y_{ij}}. \quad (36)$$

– Step 3. Calculate normalized values considering the weights of criteria:

$$k_{ij} = w_j \times n_{ij}. \quad (37)$$

– Step 4. Calculate the total normalized score considering the weights of criteria:

$$S_{+i} = \sum_{j=1}^n k_{+ij}, j \in B, \quad (38)$$

$$S_{-i} = \sum_{j=1}^n k_{-ij}, j \in C. \quad (39)$$

– Step 5. Calculate the score for each alternative:

$$RI_i = 2^{2+S_{-i}} \sqrt{2 + S_{+i}}. \quad (40)$$

– Step 6. Rank the alternatives in decreasing order of their RI_i scores.

The sequence for multi-criteria decision making using the *PIV* method is as follows [18]:

– Step 1. Construct a decision matrix, similar to Step 1 of the Entropy method.

– Step 2. Calculate normalized values:

$$\frac{x_{ij}}{\max x_{ij}}. \quad (41)$$

– Step 3. Calculate normalized values considering the weights of criteria:

$$v_{ij} = w_j \times n_{ij}. \quad (42)$$

– Step 4. Evaluate the near-weight index:

$$u_i = v_{\max} - v_{ij}, j \in B, \quad (43)$$

$$u_i = v_i - v_{\min}, j \in C. \quad (44)$$

– Step 5. Determine the overall range of values for each alternative:

$$d_i = \sum_{j=1}^n u_i. \quad (45)$$

– Step 6. Rank the alternatives according to the principle that the best alternative is the one with the smallest d_i value.

The Excel software has also been chosen as the tool to implement these MCDM methods in ranking the types of materials.

3. Results and Discussion

3.1. Case 1: selecting lubricating oil for two-stroke engines

In this case, comparing the weight values of criteria calculated by different methods will be applied in selecting lubricating oil for two-stroke engines. The suitability or unsuitability of three MCDM methods including *MARA*, *RAM*, and *PIV* in selecting lubricating oil for two-stroke engines is also determined in this scenario. Selecting lubricating oil is particularly significant for the operation of two-stroke engines. The lubricating oil mixed with gasoline forms a mixture that reduces friction between contacting surfaces. Accurately selecting the type of lubricating oil will help equipment using two-stroke engines such as scooters, small-displacement motorcycles, portable generators, various types of saws, etc., operate most efficiently [24, 25]. In **Table 1**, data on four commonly used types of lubricating oil for two-stroke engines are summarized, denoted as options *LO1*, *LO2*, *LO3*, and *LO4*. Density, viscosity index, viscosity at 100 °C, and viscosity at 40 °C are four criteria for evaluating each option. These criteria are also denoted as *C1*, *C2*, *C3*, and *C4*, respectively. Lubricating oil with lower density is better, meaning that *C1* is a type *C* criterion. Conversely, all three criteria *C2*, *C3*, and *C4* belong to type *B*. **Table 1** summarizes data for these four types of lubricating oil [26].

Table 1

Types of lubricating oil for two-stroke engines [26]

Lubricant oil	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>C4</i>
<i>LO1</i>	0.883	95	9	71.73
<i>LO2</i>	0.953	82	8.67	75.82
<i>LO3</i>	0.9058	390	4.9	12.67
<i>LO4</i>	0.8316	166	2.67	8.04

Lubricating oil *LO1* has the smallest *C1* compared to the other three options, *C2* is the largest at 390 for option *LO3*, *C3* is the largest at 9 for option *LO1*, and *C4* is the largest at 75.82 for option *LO2*. Therefore, it is clear that to choose the best option, multi-criteria decision-making techniques must be used.

The methods for calculating weights for criteria in Chapter 2 have been applied, and the results have been calculated for the weights of criteria using five different methods as in **Table 2**.

Table 2

Weights of criteria in Case 1

Weight method	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>C4</i>	Max/min
Entropy	0.3319	0.2007	0.2583	0.2091	1.65
<i>MEREC</i>	0.8196	0.0601	0.0601	0.0601	13.64
<i>LOPCOW</i>	0.1326	0.4211	0.1371	0.3092	3.18
<i>CRITIC</i>	0.6327	0.0177	0.2453	0.1043	35.75
<i>MEAN</i>	0.2500	0.2500	0.2500	0.2500	1.00

Observing the data from **Tables 1, 2** simultaneously provides clear insights into two issues. Firstly, the level of difference in the values of criteria when calculated by the *CRITIC* method is the largest (35.75 times), followed by the degree of variation in the weights of criteria when calculated by the *MEREC* method (13.64 times), the *LOPCOW* method (3.18 times), the Entropy method (1.65 times), and naturally, when using the *MEAN* method, the weights of criteria are always equal. Secondly, the weights of criteria when calculated by the *LOPCOW* method tend to be opposite to when calculated by the Entropy method. That is, if a certain criterion, when calculated

by the *LOPCOW* method, has a large weight, then when calculated by the Entropy method, its weight will be small, and vice versa. This observation is clearly seen in **Fig. 1**.

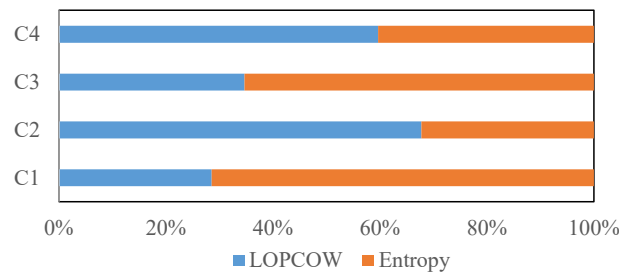


Fig. 1. Weights of criteria in Case 1 when calculated by Entropy and *LOPCOW* methods

However, this observation has only been made in one case. Whether this observation can be generalized for all cases or not requires further examination of other cases. This issue will be addressed in the subsequent examples in this article.

The formulas of the *MARA* method (from (18) to (35)) have been applied to calculate the magnitude of the M_i area for substitute options. This process is performed five times corresponding to five sets of weights for criteria. The values of M_i and the ranking of each type of lubricating oil have been summarized in **Table 3**.

Table 3

Values of M_i and rankings of lubricating oils when ranked by the *MARA* method

Lubri- cant oil	Weight method									
	Entropy		<i>MEREC</i>		<i>LOPCOW</i>		<i>CRITIC</i>		<i>MEAN</i>	
	M_i	Rank	M_i	Rank	M_i	Rank	M_i	Rank	M_i	Rank
<i>LO1</i>	0.1486	2	0.4057	2	0.0876	2	0.2765	2	0.1080	2
<i>LO2</i>	0.1454	1	0.4046	1	0.0805	1	0.2757	1	0.1033	1
<i>LO3</i>	0.2944	3	0.4435	3	0.2005	3	0.4069	3	0.2652	3
<i>LO4</i>	0.3968	4	0.4719	4	0.3573	4	0.4488	4	0.3832	4

The formulas of the *RAM* method (from (36) to (40)) have been applied to calculate the RI_i scores for each option. In **Table 4**, the scores and rankings of lubricating oils are summarized when the weights of criteria are calculated by five different methods.

Table 4

Values of RI_i and rankings of lubricating oils when ranked by the *RAM* method

Lubri- cant oil	Weight method									
	Entropy		<i>MEREC</i>		<i>LOPCOW</i>		<i>CRITIC</i>		<i>MEAN</i>	
	M_i	Rank	M_i	Rank	M_i	Rank	M_i	Rank	M_i	Rank
<i>LO1</i>	1.4627	1	1.3868	1	1.4854	2	1.4213	1	1.4749	1
<i>LO2</i>	1.4604	2	1.3834	3	1.4841	3	1.4181	2	1.4730	2
<i>LO3</i>	1.4511	3	1.3837	2	1.4978	1	1.3988	3	1.4655	3
<i>LO4</i>	1.4236	4	1.3793	4	1.4493	4	1.3922	4	1.4324	4

Using the formulas from (41) to (45) to calculate the values of d_i for each lubricating oil. **Table 5** summarizes these d_i values and ranks the methods using the *PIV* method.

Combining the data from **Tables 3–5** results in the ranking of lubricating oils in all cases examined, as shown in **Table 6**.

Table 5
Values of d_i and rankings of lubricating oils when ranked by the *PIV* method

Lubri- cant oil	Weight method									
	Entropy		MEREC		LOPCOW		CRITIC		MEAN	
	d_i	Rank	d_i	Rank	d_i	Rank	d_i	Rank	d_i	Rank
LO1	0.1516	1	0.0660	1	0.2968	2	0.0340	1	0.1837	1
LO2	0.1686	2	0.0990	4	0.3057	3	0.0612	2	0.1972	2
LO3	0.2164	3	0.0880	2	0.2317	1	0.1622	3	0.2350	3
LO4	0.3556	4	0.0969	3	0.4755	4	0.1895	4	0.4030	4

Table 6
Ranking of lubricating oils

Lubri- cant oil	MARA					RAM					PIV				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
LO1	2	2	2	2	2	1	1	2	1	1	1	1	2	1	1
LO2	1	1	1	1	1	2	3	3	2	2	2	4	3	2	2
LO3	3	3	3	3	3	3	2	1	3	3	3	2	1	3	3
LO4	4	4	4	4	4	4	4	4	4	4	4	3	4	4	4

Note: (1) = Entropy weight; (2) = MEREC weight; (3) = LOPCOW weight; (4) = CRITIC weight; (5) = MEAN weight

The chart in **Fig. 2** illustrates the ranking results of the options in this case.

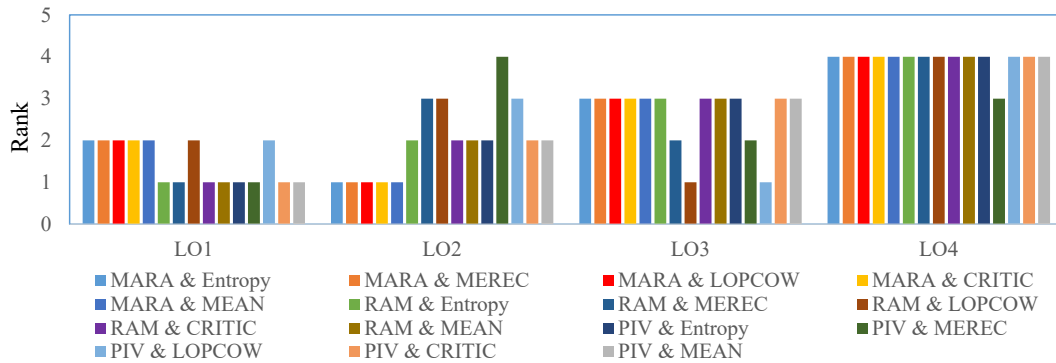


Fig. 2. Ranking of Lubricant Types

It is observed that when using five different methods to calculate weights, the rankings of lubricant types are entirely consistent when ranked by the *MARA* method. According to this, *LO2* is the best option, with lubricant types *LO1*, *LO3*, and *LO4* ranked 2nd, 3rd, and 4th, respectively. When using the *RAM* and *PIV* methods to rank the options, if the weights of the criteria are calculated using the Entropy, *MEREC*, *CRITIC*, and *MEAN* methods, all methods identify *LO1* as the best lubricant type. In the case of weighting criteria using the *LOPCOW* method, both the *RAM* and *PIV* methods indicate that *LO3* is the best option. These differences are understandable, as many studies have shown that rankings of options depend heavily on the method used for weighting and the *MCDM* method employed [3–5]. To determine which *MCDM* method is more suitable than the other two, it is necessary to test the stability of ranking options. The Spearman rank correlation coefficient (*S*) has been used for this task [27, 28]. This coefficient is calculated using formula (46), where D_i represents the difference in the ranking of option i between scenarios, and m is the number of options to be ranked:

$$S = 1 - \frac{6 \sum_{i=1}^n D_i^2}{m(m^2 - 1)} \quad (46)$$

Applying (46) has calculated the values of the S coefficient as shown in **Table 7**. After obtaining the values of the S coefficients, their distribution range, mean, and median values have been calculated for each $MCDM$ method used. These values have also been synthesized in **Table 7**.

Table 7
Spearman rank correlation coefficient (S) for case 1

Method	MARAs					RAMs					PIVs				
	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5
S1	1	1	1	1	1	1	0.8	0.4	1	1	1	0.4	0.4	1	1
S2	1	1	1	1	1	0.8	1	0.8	0.8	0.8	0.4	1	0.6	0.4	0.4
S3	1	1	1	1	1	0.4	0.8	1	0.4	0.4	0.4	0.6	1	0.4	0.4
S4	1	1	1	1	1	1	0.8	0.4	1	1	1	0.4	0.4	1	1
S5	1	1	1	1	1	1	0.8	0.4	1	1	1	0.4	0.4	1	1
Range			1					0.4÷1					0.4÷1		
Average			1					0.74					0.60		
Median			1					0.8					0.4		

When using the RAM and PIV methods, S values range from 0.4 to 1, while using the $MARA$ method, all S values are equal to 1. This is the first point showing that the $MARA$ method has an advantage over the other two methods. The mean values of S are 1, 0.74, and 0.6 for the $MARA$, RAM , and PIV methods, respectively. This is the second point showing that $MARA$ is superior to RAM and PIV . Finally, the median of the S set is 1 when using the $MARA$ method, higher when using the RAM method (0.8), and lowest when using the PIV method (0.4). This is the third point indicating that $MARA$ is more advantageous than RAM and PIV . In summary, in this case, $MARA$ is proven to be the best, and conversely, PIV is the least suitable. This also implies that LO2 is the best option among the four surveyed lubricant types.

3. 2. Case 2: selection of material for manufacturing screw shafts

In this case, the experimental subject selected is various materials for manufacturing screw shafts. The screw shaft is an indispensable component in gearboxes of worm gear screw jack, playing a crucial role in many industrial and mechanical applications. Additionally, worm gear screw jack gearboxes are also used in medical and scientific applications, where precision and accurate motion control are crucial. For example, in MRI machines or medical diagnostic equipment, worm gear screw jack gearboxes ensure smooth and reliable motion [29, 30]. Hence, the screw shaft is considered the soul of the worm gear screw jack gearbox [31]. The screw shaft, often subjected to heavy loads and high wear, requires a material with high hardness and strength to ensure stable performance and prolonged lifespan. The selection of screw shaft materials reflects not only mechanical factors but also relates to thermal resistance and dimensional stability under specific operating conditions. Six types of steel commonly used for manufacturing screw shafts are C35, C45, C50, 42CrMoS4, C15, and C10. Synthesized from various sources, six parameters were identified, each with different values for all steel types. These parameters include hardness (HB), tensile strength (kG/mm^2), yield limit (Kg/mm^2), relative elongation (%), relative contraction (%), and impact toughness (J). These criteria are denoted as C1, C2, C3, C4, C5, and C6, respectively. All these criteria belong to type B. **Table 8** presents the types of materials for manufacturing screw shafts.

The largest value for criterion C1, 431, belongs to steel type C10. The largest value for C2, 77.6, belongs to steel type 42CrMoS4. Steel type C35 has the largest C3 value of 94.9 compared to the other five types. The largest value for C4, 42, belongs to two types of steel, C35 and C45. C5's largest value, 44, belongs to steel type C50. Steel type C35 has the largest C6 value of 44 compared to the other five types. Thus, each type has only one or a few criteria that are the best compared to other steel types. This means that $MCDM$ methods need to be used to determine the best steel type.

Table 8
Types of materials for manufacturing screw shafts

Steel	C1	C2	C3	C4	C5	C6
C35	242	55.9	94.9	42	41	44
C45	232	65.7	33.9	42	12	14
C50	234	39.4	68.2	23	44	43
42CrMoS4	213	77.6	61.4	32	33	34
C15	321	37.4	38.1	31	43	34
C10	431	24.3	76.8	24	24	14

Formulas for calculating weights using five different methods (Entropy, *MEREC*, *LOPCOW*, *CRITIC*, and *MEAN*) were applied to obtain weights for criteria, as shown in **Table 9**.

Table 9
Weights of criteria in Case 2

Weight method	C1	C2	C3	C4	C5	C6	Max/min
Entropy	0.1562	0.1657	0.1638	0.1722	0.1708	0.1713	1.10
MEREC	0.0758	0.2002	0.1602	0.0953	0.2747	0.1938	3.62
LOPCOW	0.2831	0.1700	0.1818	0.1068	0.1244	0.1339	2.65
CRITIC	0.1979	0.1888	0.1261	0.2161	0.1324	0.1388	1.71
MEAN	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	1.00

From the data in **Table 9**, two issues are observed. Firstly, the weights of criteria calculated by the *MEREC* method (3.62 times) vary more than when calculated by the *LOPCOW* method (2.65 times) and the Entropy method (1.10 times). Secondly, if a certain criterion has a weight calculated by the Entropy method that is large, then when calculated by the *LOPCOW* method, that criterion will have a smaller weight. For example, when using the Entropy method, criterion *C1* has the smallest weight compared to the other five criteria, but when using the *LOPCOW* method, this criterion has the largest weight. Another example is criterion *C4*, which has the largest weight when calculated by the Entropy method but has the smallest weight when calculated by the *LOPCOW* method. Observing the graph illustrating the weights of criteria when calculated by the Entropy and *LOPCOW* methods in **Fig. 3** clarifies this observation. This issue will be further discussed in another case of this article.

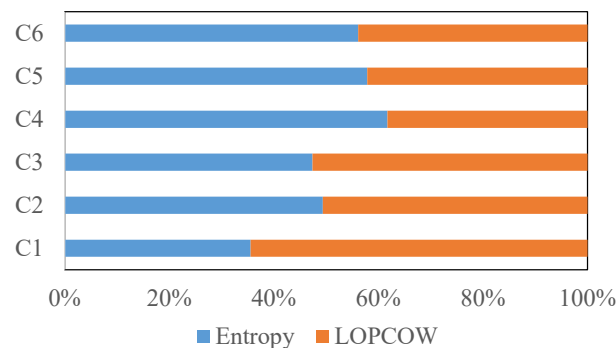


Fig. 3. Weights of criteria in Case 2 when calculated by entropy and *LOPCOW* Methods

Ranking of steel types for manufacturing screw shafts, similar to Case 1, resulted in the data presented in **Table 10**.

Table 10
Ranking of steel types for manufacturing screw shafts

Steel	MAR _A					RAM					PIV				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
C35	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C45	6	6	6	5	6	6	6	6	6	6	6	6	6	6	6
C50	3	2	3	4	3	3	3	3	4	3	3	3	3	4	3
42CrMoS4	2	3	2	2	2	2	2	2	2	2	2	2	2	2	2
C15	4	4	4	3	4	4	4	4	3	4	4	4	5	3	4
C10	5	5	5	6	5	5	5	5	5	5	5	5	4	5	5

Note: (1) = Entropy weight; (2) = MEREC weight; (3) = LOPCOW weight; (4) = CRITIC weight; (5) = MEAN weight

The chart in Fig. 4 illustrates the ranking results of the options in this case.

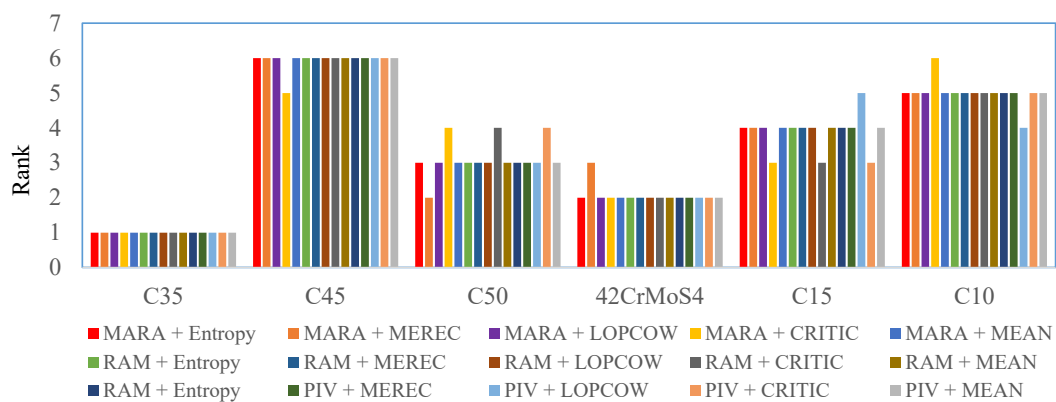


Fig. 4. Ranking of steel types for manufacturing screw shafts

All combinations of weighting methods and *MCDM* methods confirm that *C35* is the best option. This allows to confidently conclude that *C35* is the best steel type for manufacturing screw shafts among the six types surveyed. The differences in the rankings of the remaining materials are also explained by the fact that they were ranked using different *MCDM* methods [32, 33]. To compare *MCDM* methods with each other, the Spearman rank correlation coefficient is used, and the results are shown in **Table 11**.

Table 11
Spearman rank correlation coefficient (*S*) for case 2

Method	MAR _A					RAM					PIV				
	<i>S</i> ₁	<i>S</i> ₂	<i>S</i> ₃	<i>S</i> ₄	<i>S</i> ₅	<i>S</i> ₁	<i>S</i> ₂	<i>S</i> ₃	<i>S</i> ₄	<i>S</i> ₅	<i>S</i> ₁	<i>S</i> ₂	<i>S</i> ₃	<i>S</i> ₄	<i>S</i> ₅
<i>S</i> ₁	1	0.943	1	0.886	1	1	1	1	0.943	1	1	1	0.943	0.943	1
<i>S</i> ₂	0.943	1	0.943	0.771	0.943	1	1	1	0.943	1	1	1	0.943	0.943	1
<i>S</i> ₃	1	0.943	1	0.886	1	1	1	1	0.943	1	0.943	0.943	1	0.829	0.943
<i>S</i> ₄	0.886	0.771	0.886	1	0.886	0.943	0.943	0.943	1	0.943	0.943	0.943	0.829	1	0.943
<i>S</i> ₅	1	0.943	1	0.886	1	1	1	1	0.943	1	1	1	0.943	0.943	1
Range			0.771÷1						0.943÷1					0.829÷1	
Average			0.9258						0.9772					0.9487	
Median			0.9430						1					0.9430	

Comparing quantities including the range, mean value, and median of the *S* set for the *MAR_A*, *RAM*, and *PIV* methods, it is observed that the *RAM* method outperforms the other two methods.

The median of the S set is equal to 0.9430 for both the $MARA$ and PIV methods, but both the range and mean of the S set in the PIV method are larger than in the $MARA$ method. This indicates that PIV is slightly better than the $MARA$ method. In summary, in this case, RAM is confirmed to be the most suitable, while $MARA$ is considered less suitable.

3. 3. Case 3: selection of material for manufacturing gears

The selection of heavy-duty load-bearing materials has been chosen as the experimental subject in this case. The choice of materials for heavy-duty load-bearing gears is extremely important as they must withstand large forces. Heavy-duty load-bearing gears are an essential component in many mechanical systems such as industrial machinery, automotive transmissions, and construction equipment. These gears are designed to withstand strong impact forces and operate under harsh conditions, making material selection crucial to ensure durability, reliability, and performance. In industrial environments, heavy-duty load-bearing gears are used in heavy machinery such as cranes, excavators, and mining equipment, where they transmit power and bear heavy loads. Similarly, heavy-duty load-bearing gears play a crucial role in the transportation sector, operating within the gearboxes of trucks, buses, and trains to efficiently move heavy cargo. In summary, selecting the appropriate materials for heavy-duty load-bearing gears not only enhances performance but also contributes to the safety and overall success of operations in demanding applications [34]. In **Table 12**, parameters for eight commonly used materials for manufacturing gears subjected to heavy loads are synthesized. These materials are denoted as MG_i with $i = 1-8$. Seven criteria are employed to describe each material, including tensile strength (N/cm^2), percentage elongation (%), reduction in area (%), melting point (MPa), hardness (HB), impact strength (KJ/m^2), and cost (Vietnamese dong/kg), represented by corresponding letters $C1$, $C2$, $C3$, $C4$, $C5$, $C6$, and $C7$ [34]. Notably, $C7$ is a criterion of type C , while all other criteria fall under type B .

To determine the optimal material, Multi-Criteria Decision Making ($MCDM$) methods are employed. It can be stated that there is no single solution where all seven criteria are optimal. Specifically, $MG6$ excels in criteria $C1$ and $C4$, $MG1$ tops in criterion $C3$, $MG5$ leads in $C5$, $C6$'s highest value belongs to $MG7$, and $C8$ has the smallest value in $MG8$.

Weighting for the criteria using five methods: Entropy, $MEREC$, $LOPCOW$, $CRITIC$, and $MEAN$ has also been conducted, and the results are summarized in **Table 13**.

Table 12

Some materials for manufacturing heavy-duty gears [34]

Material	C1	C2	C3	C4	C5	C6	C7
$MG1$	780	18	55	635	229	880	22000
$MG2$	880	15	50	735	225	390	30000
$MG3$	930	13	45	785	269	590	31000
$MG4$	980	15	45	785	217	600	22000
$MG5$	980	12	45	835	250	950	24000
$MG6$	1080	12	50	930	220	960	22000
$MG7$	885	12	40	685	195	970	21000
$MG8$	750	12	45	400	179	940	20000

Table 13

Weights of criteria in case 3

Weight method	C1	C2	C3	C4	C5	C6	C7	Max/min
Entropy	0.1358	0.1706	0.1487	0.1360	0.1385	0.1359	0.1347	1.27
$MEREC$	0.0837	0.0536	0.0705	0.2640	0.0972	0.3077	0.1233	5.74
$LOPCOW$	0.1568	0.0652	0.0784	0.1622	0.1229	0.1674	0.2471	3.79
$CRITIC$	0.1303	0.1502	0.1297	0.0990	0.1007	0.2341	0.1561	2.36
$MEAN$	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	1.00

A chart illustrating the criteria weights using the Entropy and *LOPCOW* methods is presented in **Fig. 5**.

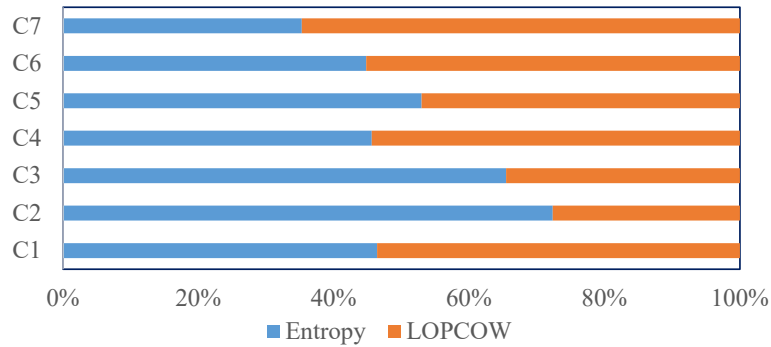


Fig. 5. Weights of criteria in case 3 using Entropy and *LOPCOW* methods

Observing **Table 13** and **Fig. 5**, it is once again noted that for a given criterion, if its weight is high when calculated by the *LOPCOW* method, then the weight becomes low when calculated by the Entropy method, and vice versa. For example, the weight of *C7* is higher than the weights of the other six criteria when using the *LOPCOW* method, but if calculated by Entropy, the weight of *C7* is lower than the weights of the other six criteria. Through comparing the weights of criteria using the Entropy and *LOPCOW* methods (review **Fig. 1, 3, and 5**), an objective observation can be made that if a criterion has a high weight when calculated by the *LOPCOW* method, then that criterion will have a low weight when calculated by the Entropy method, and vice versa.

Also, by examining the data in **Table 13**, it is evident that the weights of criteria when calculated by the *MEREC* method show a change of 5.74 times, which is higher than when calculated by the *LOPCOW* method (3.79 times) and higher than when calculated by the Entropy method (1.27 times). Thus, in all three cases studied, it is consistently observed that when the weights of criteria are calculated using the *MEREC* method, the change in their values across criteria is greater compared to the *LOPCOW* and Entropy methods.

The ranking of gear materials through the combination of five weighting methods and three *MCDM* methods has also been executed, with summarized data presented in **Table 14**.

Table 14
Ranking of heavy-duty gear steel types

Material	MARAs					RAMs					PIVs				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
MG1	2	4	3	2	2	2	4	3	2	2	2	4	3	2	2
MG2	7	8	8	8	7	7	8	8	8	7	7	8	8	8	7
MG3	6	6	7	7	6	6	6	6	7	6	6	6	7	7	6
MG4	4	5	5	5	4	4	5	5	5	4	4	5	5	5	4
MG5	3	2	2	3	3	3	2	2	3	3	3	2	2	3	3
MG6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
MG7	5	3	4	4	5	5	3	4	4	5	5	3	4	4	5
MG8	8	7	6	6	8	8	7	7	6	8	8	7	6	6	8

Note: (1) = Entropy weight; (2) = *MEREC* weight; (3) = *LOPCOW* weight; (4) = *CRITIC* weight; (5) = *MEAN* weight

The chart in **Fig. 6** illustrates the ranking results of the options in this case.

In all scenarios conducted, *MG6* is confirmed as the best option. This allows to confidently conclude that *MG6* is the best material for manufacturing gears among the eight options surveyed. The differing rankings of the remaining materials when ranked using different methods

are consistent with findings reported in recent published studies [32, 33]. The Spearman rank correlation coefficient is calculated to compare the *MCDM* methods in this case. The values of *S*, as well as its range, mean value, and median, are calculated and summarized in **Table 15**.

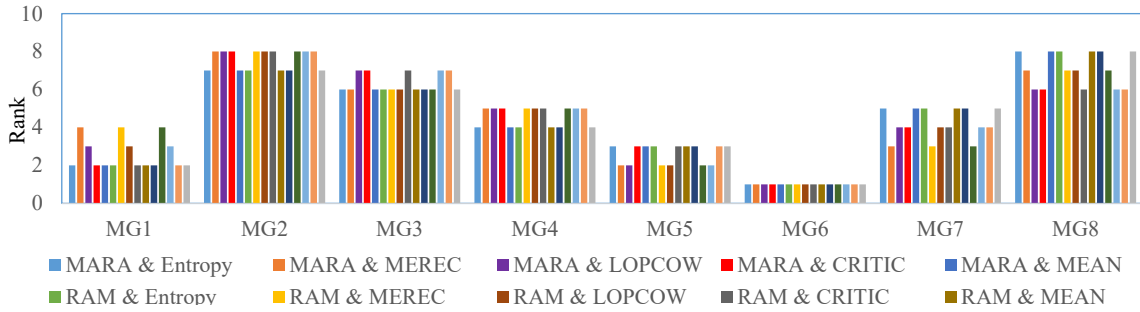


Fig. 6. Ranking of heavy-duty gear steel types

Table 15
Spearman rank correlation coefficient (*S*) for case 3

Method	<i>MARA</i>					<i>RAM</i>					<i>PIV</i>				
	<i>S</i> ₁	<i>S</i> ₂	<i>S</i> ₃	<i>S</i> ₄	<i>S</i> ₅	<i>S</i> ₁	<i>S</i> ₂	<i>S</i> ₃	<i>S</i> ₄	<i>S</i> ₅	<i>S</i> ₁	<i>S</i> ₂	<i>S</i> ₃	<i>S</i> ₄	<i>S</i> ₅
<i>S</i> ₁	1	0.857	0.881	0.905	1	1	0.857	0.929	0.905	1	1	0.857	0.881	0.905	1
<i>S</i> ₂	0.857	1	0.952	0.905	0.857	0.857	1	0.976	0.905	0.857	0.857	1	0.952	0.905	0.857
<i>S</i> ₃	0.881	0.952	1	0.976	0.881	0.929	0.976	1	0.952	0.929	0.881	0.952	1	0.976	0.881
<i>S</i> ₄	0.905	0.905	0.976	1	0.905	0.905	0.905	0.952	1	0.905	0.905	0.905	0.976	1	0.905
<i>S</i> ₅	1	0.857	0.881	0.905	1	1	0.857	0.929	0.905	1	1	0.857	0.881	0.905	1
Range	0.857÷1					0.857÷1					0.857÷1				
Average	0.9119					0.9215					0.9119				
Median	0.9050					0.9170					0.9050				

The range of *S* values falls within the range of 0.857 to 1 for all three *MCDM* methods. When using the *RAM* method, both the mean and median values of *S* are higher compared to the other two methods (*MARA* and *PIV*). Therefore, in this case, it is asserted that *RAM* is the most suitable method to use.

3. 4. Limitations and development of this research

The five methods used, including Entropy, *MEREC*, *LOPCOW*, *CRITIC*, and *MEAN*, only calculate the weights of criteria based on dry numerical values without considering the decision-maker’s opinions regarding the importance among criteria. When wanting to consider the decision-maker’s perspective on the importance of criteria while still ensuring an objective evaluation of the criteria, weights can be calculated using methods that combine subjective and objective aspects, known as combined weight methods such as the *PIPRECIA* method [35] and the *SPC* method [36].

The type of material considered best may change if additional criteria such as processing costs, recyclability, factors related to the supplier’s supply chain services, etc., are taken into account. If all these criteria are added to the list, the chosen material will ensure both economic and technical factors for the product simultaneously.

4. Conclusions

1. If a criterion has a high weight when calculated by the *LOPCOW* method, then that criterion will have a low weight when calculated by the Entropy method, and vice versa. When using

three methods including Entropy, *MEREC*, and *LOPCOW* to weigh criteria, the greatest change in weight values occurs when using the *MEREC* method, followed by the *LOPCOW* method, and lastly, the Entropy method.

2. Although five different scenarios were considered, the Spearman rank correlation coefficient consistently equals 1. This indicates that the *MARA* method is highly suitable for selecting lubricating oils for two-stroke engines. When using the *RAM* method, the average value of the Spearman rank correlation coefficient is 0.9772 in Case 2 and 0.9215 in Case 3, both higher than the average value of the Spearman rank correlation coefficient when using both the *MARA* and *PIV* methods. This indicates that the *RAM* method is most suitable for selecting materials for screw shaft fabrication and selecting materials for heavy load-bearing gear wheel fabrication.

3. Among six types of steel, including C35, C45, C50, 42CrMoS4, C15, and C10, C35 is determined to be the best for manufacturing screw shafts. Among eight material options for manufacturing gears labeled as *MG1*, *MG2*, ..., *MG8*, *MG6* is identified as the best option. *LO2* is determined to be the best among the four lubricants surveyed.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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The study was performed without financial support.

Data availability

Manuscript has data included as electronic supplementary material.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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