Network Regularity and the Influence of Asynchronism on the Evolution of Cooperation

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Abstract. In a population of interacting agents, the update dynamics defines the temporal relation between the moments at which agents update the strategies they use when they interact with other agents. The update dynamics is said to be synchronous if this process occurs simultaneously for all the agents and asynchronous if this is not the case. On the other hand, the network of contacts defines who may interact with whom. In this paper, we investigate the features of the network of contacts that play an important role in the influence of the update dynamics on the evolution of cooperative behaviors in a population of agents. First we show that asynchronous dynamics is detrimental to cooperation only when 1) the network of contacts is highly regular and 2) there is no noise in the strategy update process. We then show that, among the different features of the network of contacts, network regularity plays indeed a major role in the influence of the update dynamics, in combination with the temporal scale at which clusters of cooperator agents grow.

1 Introduction

Why cooperative behaviors do exist in nature? How can we promote this type of behaviors in human and artificial societies? In light of the evolution theory, there seems to exist a contradiction between the existence of altruistic behaviors in nature and the fact they seem apparently less advantageous from an evolutionary point of view [21], hence the first question. The second question is more relevant in social sciences and informatics, for example. In these cases, besides explaining observed phenomena, the goal is to identify mechanisms that promote the emergence and maintenance of cooperative behaviors.

Evolutionary games [21] have been one of the main tools used to help answering these questions. In these models there is a population of agents interacting with each other during several time steps through a given game that is used as a metaphor for the type of interaction that is being studied. The structure which defines who may interact with whom is called the *network of contacts*. On each iteration, the agents may update the strategy they use to play the game using a so called *transition rule*. The *update dynamics* defines the temporal relation between the moments at which agents update their strategy. If this process is modeled as if occurring simultaneously for all the agents for all time steps, one says that the system is under a *synchronous dynamics*. If only a subset of the agents (simultaneously) update their strategies, the system is under an *asynchronous dynamics*. In this paper we analyze the features of the network of contacts that are determinant on the influence of the update dynamics on the evolution of cooperation.

There are several aspects whose influence on the evolution of cooperation has been studied, among which, the network of contacts [12, 1, 7, 19], the presence of noise in the strategy update process [22] and direct and indirect reciprocity [2, 13], to name just a few. The update dynamics is also among the studied aspects. The results reported in most of previous studies vary with the conditions used [9, 11, 25, 10, 17, 18, 5]. That is, depending on the conditions, asynchronous dynamics can be beneficial, detrimental or innocuous. In [5] we tested a broad number of conditions, namely different networks of contacts, transition rules with tunable noise levels and intermediate levels of asynchronism. We have confirmed the results of previous works where the conditions coincide. However, the broad number of conditions allowed us to show that asynchronous dynamics is detrimental to cooperation only when there is no noise involved in the strategy update process and only for highly regular networks. That is, in general, an asynchronous dynamics supports more cooperation than the synchronous counterpart. We identified also the features of the transition rules that play an important role in the influence of the update dynamics. On the other hand, the variety of the networks we used is not enough to completely identify network features that also play a relevant role in this influence. Identifying these features is the subject of this paper.

The paper is structured as follows: in Section 2 we describe the model used in the simulations and the experimental setup. In Section 3 we show the influence of the update dynamics when the games are played on different networks of contacts and with two transition rules which model distinct noise levels. In Section 4 we analyze the network features that determine the type of influence of the update dynamics. Finally, in the last section some conclusions are drawn and future work is proposed.

2 The Model

2.1 The Games

Symmetric 2-player games are among the most studied games in evolutionary game theory. These games can be described by the payoff matrix

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} \tag{1}$$

where C (cooperate) and D (defect) are the possible actions for each player. Each element of the matrix represents the payoff received by the row-player when it plays the game against the column-player. Let us consider $R = 1$ and $P = 0$, and restrict S and T to the intervals $-1 < S < 1$, $0 < T < 2$ [6, 17]. The $S > 0$, $T < 1$ region corresponds to the Harmony game where the rational action for both players is to play C in a one shot game. The famous Prisoner's Dilemma game [2] corresponds to the region $S < 0$, $T > 1$. In this game there is a strong temptation to play D, which is the rational choice. However, if both players play D, they receive a smaller payoff than if they both play C, hence the dilemma. In the Snowdrift game [7], $S > 0$, $T > 1$, the best action depends on the opponent's decision: it is better to play C if the other player plays D and vice-versa. Finally, the region $S < 0$, $T < 1$ corresponds to the Stag-Hunt game [20]. In this game there is a dilemma between playing the potentially more profitable but risky action C and the less profitable but less risky action D.

2.2 Network of Contacts

We use two types of network models: the *small-world networks* model of Watts-Strogatz [27] and the *scale-free networks* model of Barabási-Albert [3]. In order to build small-world networks, first a toroidal regular 2D grid is built so that each node is linked to its 8 surrounding neighbors by undirected links (this is called the *Moore neighborhood*); then, with probability ϕ, each link is replaced by another one linking two randomly selected nodes. Self links, repeated links and disconnected graphs are not allowed. Two measures are often used to characterize networks of contacts: The *average path length* L and the *clustering coefficient* C. L measures the average (smallest) distance between two nodes in the network. C measures the average probability that the neighbors of a node are also connected. In general, regular networks ($\phi = 0$) have both large L and C. Random networks ($\phi = 1$) have both very small L and C. Since L decreases at a faster rate with ϕ than C, there are networks with low L and large C between regular and random networks ($0.01 \leq \phi \leq 0.1$). These are the so called small-world networks. The values used in the simulations are $\phi = \{0, 0.01, 0.05, 0.1, 1\}$.

Scale-free networks are built in the following way: the network is initialized with m fully connected nodes. Then, new nodes are added, one at a time, until the network has the desired size. Each added node is linked to m already existing nodes so that the probability of creating a link to some existing node i is equal to $k_i / \sum_j k_j$, where k_i is the degree of i, which is defined as the number of nodes to which it is connected. This method of link creation, named *preferential attachment*, leads to a power law degree distribution $P(k) \sim k^{-\gamma}$ that is very common in real social networks. Scale-free networks built with this model have very low L and C values. Unless stated otherwise, all the networks for which results are presented have average degree $\overline{k} = 8$ (equivalent to $m = 4$ in scale-free networks).

2.3 Dynamics

In the synchronous model, at each time step all the agents play a one round game with all their neighbors and collect the payoffs resulting from these games, forming an *aggregated payoff*. After this, they all simultaneously update their strategies using the transition rule (see below). In the asynchronous model, at each time step, one agent x is randomly selected; x and its neighbors play the game with their neighbors and, after this, x updates its strategy. This is an extreme case of asynchronism, named *sequential dynamics*, in which only one agent is updated at each time step. In our opinion, both synchronous and sequential dynamics are artificial ways of modeling the update dynamics of real social systems. Synchronous dynamics is artificial because there is no evidence that behavior updating occurs simultaneously for all individuals in a population. We consider that sequential dynamics is artificial because it presupposes that 1) events are instantaneous, 2) events never occur simultaneously and 3) the information resulting from an event becomes immediately accessible by other members of the population. In [5] we used an update method which allows us to model intermediate levels of asynchronism. We verified that, in general, the results change monotonically as we go from synchronous to sequential dynamics. This means that we can evaluate the maximum influence of the update dynamics by using these two extreme methods and that is why we use them here.

We use two imitation transition rules: the *best-takes-over* rule and the *Moran* rule. With the *best-takes-over* rule each agent x always imitates its most successful neighbor y, provided y's payoff is larger than x's payoff. The Moran rule is defined in the following way: let G_x be the aggregated payoff earned by agent x in the present time-step; let $N_x^* = N_x \cup x$, where N_x is the set of x's neighbors, with $k_x = |N_x|$. According to this rule, the probability that an agent x, with strategy s_x, imitates agent y, with strategy s_y, is given by

$$p(s_x \to s_y) = \frac{G_y - \Psi}{\sum_i (G_i - \Psi)}, \quad y, i \in N_x^*. \tag{2}$$

The constant Ψ is subtracted from G_i because payoffs in the Stag-Hunt and the Prisoner's Dilemma games can be negative. If G_x is set to the accumulated payoffs gained by agent x in the games played in one time step, then $\Psi = \max_{i \in N_x^*}(k_i)\min(0, S)$. If the average of the payoffs gained in one time step is considered instead, then $\Psi = \min(0, S)$. For small-world networks, the results obtained with the two approaches are similar since all the agents have approximately the same k (k is the same for all agents when $\phi = 0$). Major differences appear for scale-free networks due to the large degree heterogeneity. Average payoffs are intended to model the fact that agents have limitations in the number of interactions they can sustain simultaneously and also that relationships are costly [26, 24]. Finally, we note that this rule models the presence of noise in the decision process since it allows agents to imitate neighbors less successful than themselves, contrary to what happens with the *best-takes-over* rule.

2.4 Experimental Setup

The charts presented were obtained with populations of $n = 10^4$ agents. We let the system run during 10^4 time steps for the synchronous model and $10^4 \times n$

time steps for the sequential model, which is enough for the system to converge to homogeneous populations of cooperators or defectors, or to stabilize around a ρ value (we confirmed that the results do not change if we use larger evolution periods). The steady state ρ value is computed as the average proportion of cooperators in the last 10^3 time steps for the synchronous model and in the last $10^3 \times n$ time steps for the sequential model. Populations are randomly initialized with $\rho^0 = 0.5$. Each point in the charts presented is an average of 50 independent simulations. For each simulation a new network is generated, which is kept static during the evolutionary process. As in [17], we use the average of the ρ values corresponding to the region of each game in the ST-plane as a global measure of the cooperation level obtained with that game. This average is presented next to the quadrant of each game in Figures 1-5.

3 The Influence of the Update Dynamics

The results obtained with regular grids (Fig. 1) can be summarized in the following way: sequential dynamics supports less cooperation for the best-takes-over rule, with the exception of the Stag-Hunt game; sequential dynamics supports more cooperation for the Moran rule. With the best-takes-over rule, the main differences appear in the Snowdrift game, while there are no large differences in the Stag-Hunt and Prisoner's Dilemma games. The main differences appear for the Moran rule, especially in the Snowdrift and Stag-Hunt games. The influence in the Prisoner's Dilemma game is limited to a small region. However, in this region, synchronous dynamics leads to uniform populations of defectors, while sequential dynamics leads to populations strongly dominated by cooperators or even states where $\rho = 1$. This is also the case of the Stag-Hunt and Snowdrift games for a noticeable portion of the space.

The main differences between the results obtained with regular and small-world networks exist for the best-takes-over rule: in the Snowdrift and Prisoner's Dilemma games, sequential dynamics becomes progressively beneficial to cooperation as ϕ is increased. For $\phi = 0.05$, sequential dynamics already supports more cooperation on average than synchronous dynamics when the best-takes-over rule is used (Fig. 2). When the Moran rule is used the results obtained with small-world networks are similar to the ones obtained with regular grids.

When scale-free networks are used there are relevant differences for both rules. The differences are larger for accumulated payoffs (Fig. 3) than for average payoffs (Fig. 4). In the first case, and for the best-takes-over rule, cooperation completely dominates the whole quadrant corresponding to the Snowdrift game when sequential dynamics is used. For the Stag-Hunt and Prisoner's Dilemma games, sequential dynamics leads to a significant increment of cooperation in large portions of the space. We note also that with these networks, when the update dynamics has some influence over ρ, sequential updating is always beneficial to cooperation when accumulated payoffs are used, with only a few exceptions to this behavior when average payoffs are used.

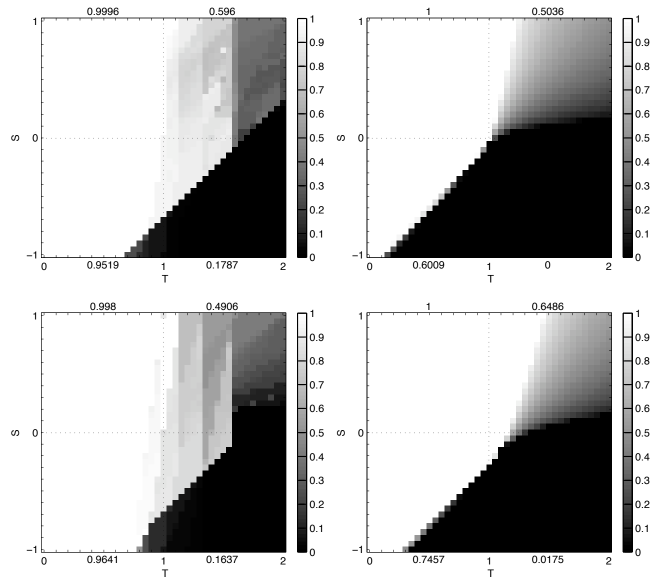

Fig. 1. Proportion of cooperators ρ in regular grids ($\phi = 0$), with synchronous dynamics (upper row) and sequential dynamics (lower row). Left column: best-takes-over rule; Right column: Moran rule. The games are the Harmony game (upper left quadrant), the Snowdrift game (upper right quadrant), the Stag-Hunt game (lower left quadrant) and the Prisoner's Dilemma game (lower right quadrant). The numbers, respectively, above the Harmony and Snowdrift games, and below the Stag-Hunt and the Prisoner's Dilemma games, are the average values of the corresponding quadrant. The S and T parameters are varied in steps of 0.05.

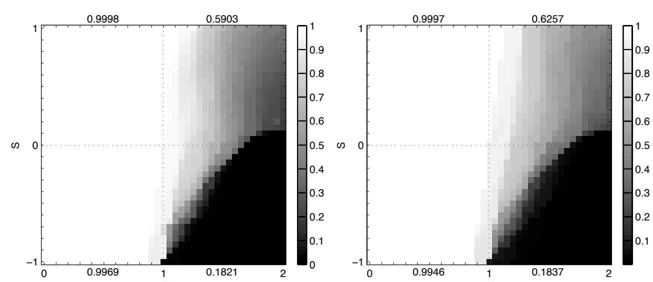

Fig. 2. Proportion of cooperators ρ on small-world networks ($\phi = 0.05$) with the best-takes-over rule. Left column: synchronous dynamics; Right column: sequential dynamics.

These results suggest that asynchronous updating is beneficial to the emergence of cooperation more often than it is detrimental. More specifically, they

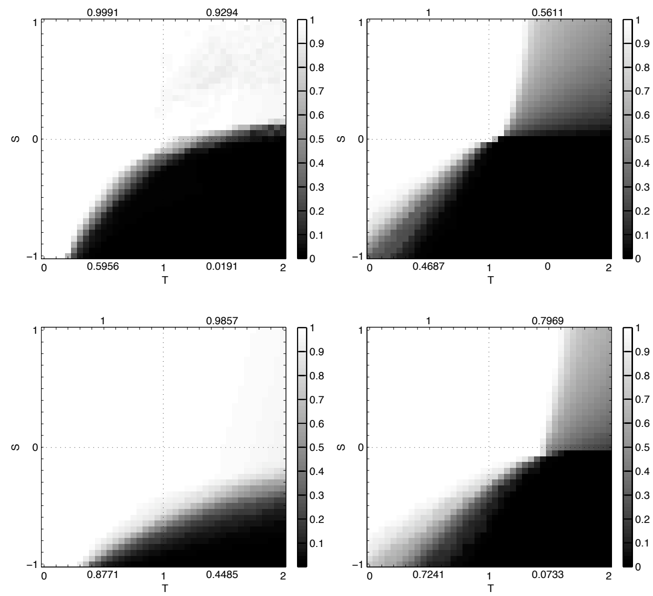

Fig. 3. Proportion of cooperators ρ in scale-free networks ($m = 4$), with synchronous dynamics (upper row) and sequential dynamics (lower row) using accumulated payoffs. Left column: best-takes-over rule; Right column: Moran rule.

suggest that asynchronism is detrimental to cooperation only for networks with a high degree of regularity and for low or no noise. We studied the role of noise in [5]. In the next section we investigate the network features that are responsible for the detrimental effect of asynchronous dynamics when the games are played on regular grids with the best-takes-over rule.

4 The Role of the Network of Contacts

The results in the previous section show that the influence of the update dynamics depends on the network of contacts, mainly when the best-takes-over rule is used. We now derive some conclusions about the network features that may determine this influence. We first recall that sequential dynamics becomes beneficial to cooperation above a certain ϕ value in small-world networks when the best-takes-over rule is used. This means that degree heterogeneity, which occurs in scale-free networks, is not a necessary condition for sequential dynamics to become beneficial to cooperation, though it may potentiate this effect.

The detrimental effect of sequential dynamics for both $\phi = 0$ (regular grid) and $\phi = 0.01$ (small-world networks) indicates that the mean path length L has no determinant role also. This conclusion comes from the significant drop of L when we change ϕ from 0 to 0.01 [27].

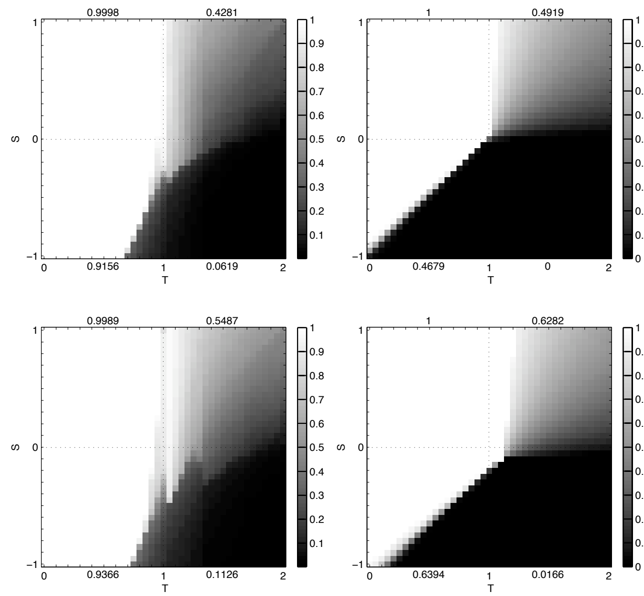

Fig. 4. As in Fig. 3 but with average payoffs.

Concerning the clustering coefficient C, we note that the regular grid with a Moore neighborhood has a large C value ($C \approx 0.428$), while it is very low for scale-free networks built with the Barabási-Albert model [3] described in Section 2.2. Considering that the influence of the update dynamics is different for these two types of network when the best-takes-over rule is used, nothing can be concluded about the role of C based on the results described in the previous section. We note that the beneficial and detrimental effect of the sequential dynamics for $\phi < 0.05$ and $\phi \geq 0.05$, respectively, indicates that this property also does not play a determinant role in the influence of the update dynamics. In order to verify this, we have done simulations with both regular grids with a von Neumann neighborhood, where $C = 0$, and with the scale-free networks model of Holme-Kim [8], which allows us to tune the value of C. These simulations were done only with the best-takes-over rule since the results obtained with the Moran rule are coherent for all networks of contacts: asynchronous dynamics benefits cooperation.

von Neumann grids are built so that each agent is linked to the four closest agents located in the four main cardinal directions. We note that, with this type of neighborhood, agents have no common neighbors. Holme-Kim networks are built in the following way: the network is initialized with m fully connected nodes. Then, new nodes are added one at a time until the network has the desired size. Each added node is linked to m already existing nodes. The first link between a new node v and an already existing node w is added using preferential attachment as in the Barabási-Albert model. The remaining $m-1$ links are created using two different processes: (i) with probability p, v is linked to a randomly chosen neighbor of w and, (ii) with probability $1-p$, preferential attachment is used. This model builds a scale-free network where the value of C depends on p: When $p=0$, it generates Barabási-Albert scale-free networks with a very low C value. For $p>0$, C grows with p.

Considering that $\bar{k}=4$ in von Neumann grids, the simulations with Holme-Kim scale-free networks were done with both $\bar{k}=4$ and $\bar{k}=8$, corresponding to $m=2$ and $m=4$, respectively. However, we only present results for $\bar{k}=8$, since the results are qualitatively similar. We used $p=0.871$ for $\bar{k}=8$ and $p=0.582$ for $\bar{k}=4$. Both values lead do networks where $C \approx 0.428$ (C value for Moore grids).

Fig. 5 shows that, concerning the influence of the update dynamics, the results obtained with the Holme-Kim model are qualitatively similar to the ones obtained with the Barabási-Albert model. This means that, independently of C, sequential dynamics benefits cooperation in scale-free networks. This is a strong evidence that the clustering coefficient plays no determinant role in the influence of the update dynamics.

Fig. 5 also shows that the results obtained with von Neumann grids qualitatively coincide with the ones obtained with Moore grids for Stag-Hunt and Prisoner's Dilemma but that they differ for Snowdrift. For this game, on average, sequential dynamics is beneficial to cooperation in von Neumann grids but detrimental in Moore grids. This result casts some doubts concerning the role of C in the influence of the update dynamics. However, as we will show next, the main role seems to be played by network regularity and less by the clustering coefficient.

We first note that, in regular grids, cooperator agents often form clusters with straight boundaries when a deterministic transition rule is used, as is the case of the best-takes-over rule. Let us start by analyzing the situations where cooperator clusters grow through their straight boundaries, which occur mainly when Moore grids are used. In this case, any mechanism that hampers the formation of straight boundaries hampers also the growth of cooperator clusters. This may happen, for example, if we use a stochastic transition rule [18] or an asynchronous dynamics. Fig. 6 shows an example for the Prisoner's Dilemma game where, under synchronous dynamics (upper row), cooperators form clusters that quickly grow through their straight boundaries, eventually forming a single large cluster which coexists with filament like defector clusters. The same figure (lower row) also shows that, when sequential dynamics is used, clusters have more irregular boundaries. Due to the local processes involved, which depend strongly on the relative value of game parameters S and T, irregular boundaries reduce the timescale at which clusters grow. This leads to equilibrium states with many small cooperator clusters that are not able to join due to the presence of defector agents between them. Fig. 7 shows an example for the Snowdrift game where cooperators are not able to form compact clusters when the population is under an asynchronous dynamics.

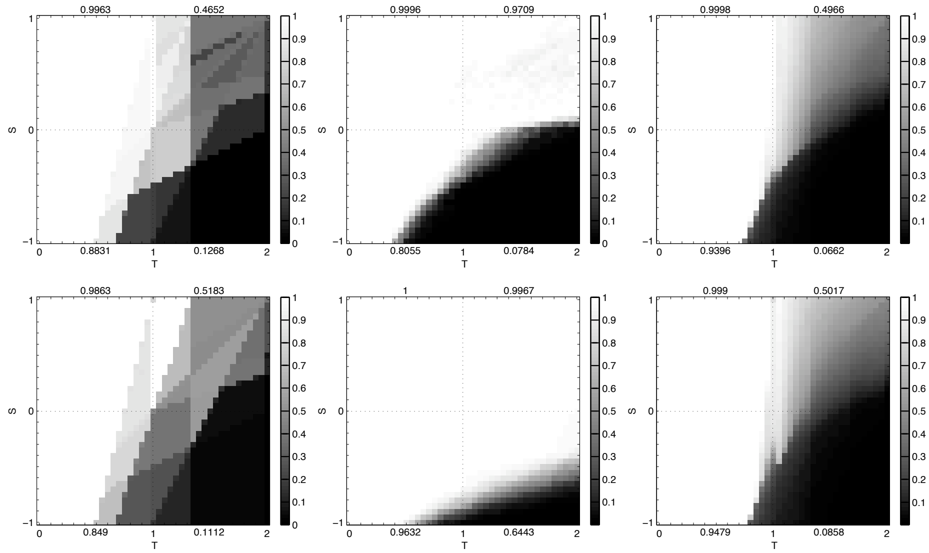

Fig. 5. Proportion of cooperators ρ in von Neumann grids (left column) and Holme-Kim networks with accumulated payoffs (middle column) and average payoffs (right column). Upper row: synchronous dynamics; Lower row: sequential dynamics. The transition rule is the best-takes-over rule.

Let us now concentrate on situations for which sequential dynamics does not prevent the formation of straight boundaries. These situations occur mainly on von Neumann grids. When the games are played on these networks, there are many combinations of S and T for which straight boundaries remain fixed once formed, unless another cluster "collides" with them. That is, contrary to what happens in Moore grids (Fig. 6), in von Neumann grids clusters grow mainly through their irregular boundaries. In these cases, the influence of the update dynamics depends on how quickly straight boundaries are formed.

Figures 8 and 9 show typical equilibrium population states for the Prisoner's Dilemma and Snowdrift games, respectively. They allow us to understand the relation between straight boundaries and the influence of the update dynamics. The left side images depict situations where sequential dynamics is beneficial for cooperation. In these cases, synchronous updating leads to a chaotic dynamics where cooperators are not able to form compact clusters. With sequential updating, the boundaries advance slowly and cluster growth is interrupted only when (diagonal) straight boundaries are finally formed, which happens when clusters have already a significant size.

The right side images depict the inverse situation. In these cases, with synchronous updating, straight boundaries are formed only when the population is already dominated by a big cluster of cooperators. On the other hand, with sequential updating, straight boundaries are formed at an early phase of the

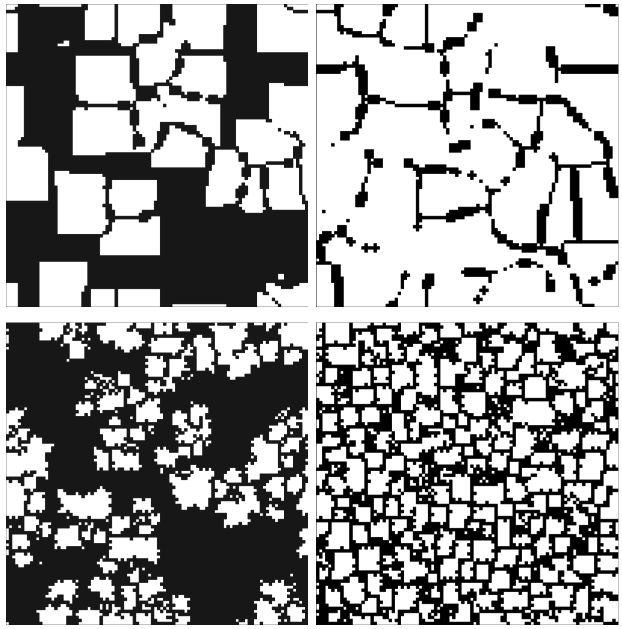

Fig. 6. Population states on Moore grids for the Prisoner's Dilemma game with $S = -0.05$ and $T = 1.35$ during the transient phase (left column) and on equilibrium (right column). Upper row: synchronous dynamics; Lower row: sequential dynamics.

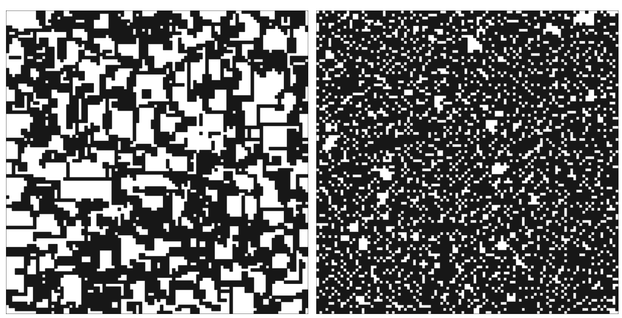

Fig. 7. Equilibrium population states on Moore grids for the Snowdrift game with $S = 0.6$ e $T = 1.6$. Left column: synchronous dynamics; Right column: sequential dynamics.

evolutionary process, preventing cooperator clusters from growing. These cases show the role that network regularity (which allows the formation of straight cluster boundaries), in combination with the timescale at which clusters grow, has on the influence of the update dynamics. They also explain why sequen-

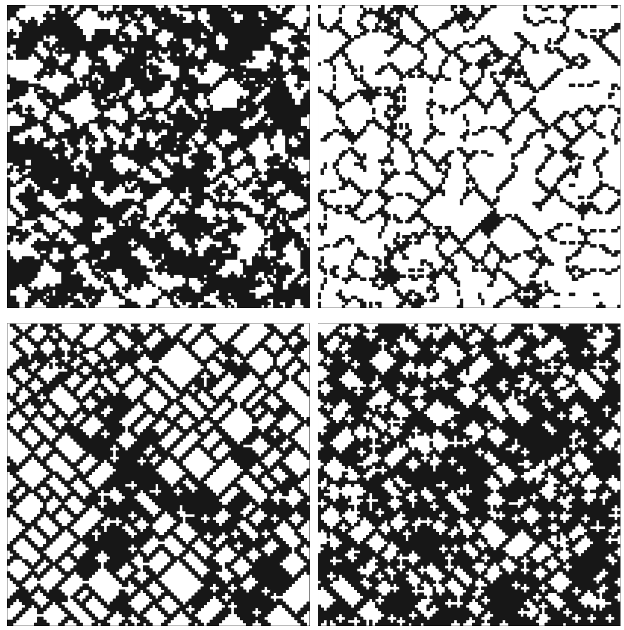

Fig. 8. Equilibrium states on von Neumann grids for the Prisoner's Dilemma with $S = -0.05$ e $T = 1.4$ (left column) and $S = -0.4$ e $T = 1.15$ (right column). Upper row: synchronous dynamics; Lower row: sequential dynamics.

tial dynamics is beneficial to the evolution of cooperation in some cases and detrimental in others when highly regular networks are used.

5 Conclusions and Future Work

We have shown that, when the update dynamics has some influence on the evolution of cooperation, asynchronous dynamics is beneficial to cooperation in the case of the Stag-Hunt game. For the Prisoner's Dilemma and Snowdrift games, asynchronous dynamics is detrimental for cooperation only when these games are played on strongly regular networks and only for the best-takes-over rule, which models the absence of noise in the strategy update process. Moreover, we verify a strong increment of cooperation on scale-free networks when we change from a synchronous to an asynchronous dynamics, mainly when accumulated payoffs are used. An analysis of these results, taking into account the features of the networks, indicates that network regularity, in combination with the temporal scale at which clusters grow, plays the main role concerning the positive or negative influence of the asynchronous dynamics. Given that both regular networks and noise free environments seldom exist in real systems, this is a strong evi-

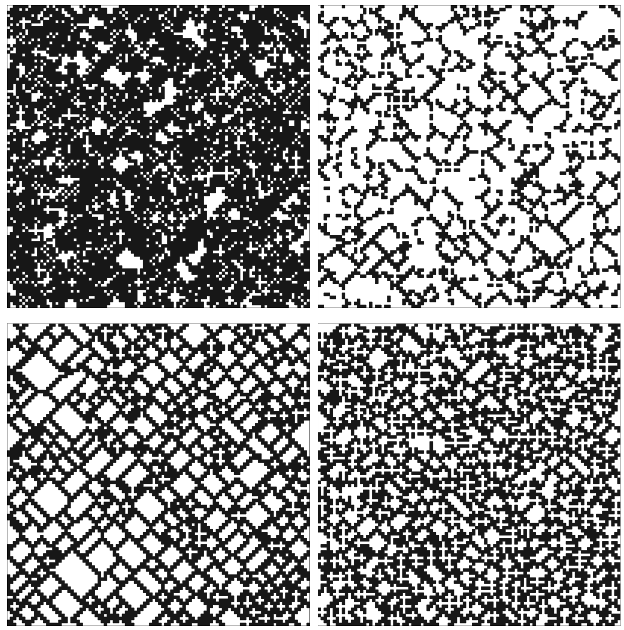

Fig. 9. Equilibrium states on von Neumann grids for the Snowdrift with $S = 0.6$ e $T = 1.7$ (left column) and $S = 0.2$ e $T = 1.2$ (right column). Upper row: synchronous dynamics; Lower row: sequential dynamics.

dence that an asynchronous dynamics is, in general, beneficial to the evolution of cooperation.

In this paper we used a stochastic asynchronous update method. A future direction for this work will be to explore deterministic asynchronous update methods in order to verify to what extent the observed behaviors are due to the stochastic nature of the update dynamics [4].

The network of contacts rarely is a static structure: agents continuously enter and leave the population; moreover, agents continuously establish new connections and break existing ones. Several works have shown that it is possible for cooperation to strive in such scenarios [14, 15, 16, 28, 23]. Another possible future direction for this work is, thus, to analyze the interplay between dynamic models of networks and the influence of the update dynamics.

Acknowledgements

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