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Introducing teacher students to digital fabrication to support children's mathematical learning

Henrik Stigberg [a], Susanne Stigberg [b,*], Marianne Maugesten [a]

[a] Faculty of Teacher Education and Languages, Østfold University College, 1757, Halden, Norway
[b] Department of Computer Science and Communication, Østfold University College, 1757, Halden, Norway

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ABSTRACT

While digital fabrication has been closely linked to narratives of science, technology, engineering, and mathematics (STEM) education and design thinking, it can provide a gateway to reimagining mathematical teaching and deepening the understanding of both mathematical content and pedagogy. In this paper, we present a workshop series that engages teacher students in the creation of their own mathematical manipulatives using digital fabrication tools. Manipulatives are tangible objects that can be used to support children's learning of mathematical concepts, such as numbers, fractions, or geometry. The workshops included activities for finding, adapting, creating, and sharing manipulatives (FACS) using 2D and 3D modeling and fabrication techniques. Our in-depth analysis of video recordings presents how teacher students successfully acquire digital fabrication skills and reflect on the use of customized manipulatives to support children's mathematical learning. The findings suggest that introducing digital fabrication in teacher education programs can shift the focus from consuming ready-made manipulatives to creating customized materials that better suit the teaching context. The authors propose FACS as a specific approach for introducing digital fabrication to teacher students.

1. Introduction

We are undergoing an ongoing digitalization of teaching and learning practices. Children's access to and use of digital tools impacts and fundamentally transforms our educational practices (Lund et al., 2014). As emphasized by policymakers, researchers, school leaders, and teachers alike, there is a need for teacher educators to adapt and integrate digital competence into the curriculum (Bourgeois et al., 2019; Gudmundsdottir & Hatlevik, 2018; Ottestad & Gudmundsdottir, 2018).

Despite living in an era of technology, children still require concrete materials and firsthand experience to comprehend abstract concepts (Furner & Worrell, 2017). Manipulatives are concrete objects used in mathematical teaching, reifying mathematical concepts to support children's learning, and are one way of representing a mathematical concept. They are used to help children understand abstract concepts and are common in education of children and youth (K–12). Manipulatives are advocated among researchers and teachers (Brown et al., 2009; Holmes, 2013; Sowell, 1989). Teachers need to help children connect manipulatives to representational and abstract ideas in mathematics and deeply understand the mathematics they are learning and need to apply them to their everyday life (Furner & Worrell, 2017). So far, teachers have been regarded as consumers and implementers of manipulatives created by others, restricted by the limited options available for purchase and constrained by school budgets (Marshall & Swan, 2008). However, by having access to digital fabrication technologies, teachers can create new opportunities that challenge this assumption and disrupt the resulting denial of agency. Adopting the mindset of teachers as makers is a perspective that may be embraced more fully than ever (Akuom & Greenstein, 2021; Greenstein et al., 2020; Harron et al., 2022; Läufer & Ludwig, 2023). Therefore, we propose that digital fabrication can bridge teachers' digital competence and children's access to manipulatives in mathematics education in the digital era.

Digital fabrication (DF) is "the process of translating a digital design developed on a computer into a physical object" (Berry et al., 2010, p. 168). DF technologies, such as 3D printers, laser cutters, and vinyl cutters, have become increasingly popular and can be found at Makerspaces and FabLabs worldwide (Fab Lab Network, 2023). DF and making have been integrated into education centered on science, technology, engineering, and mathematics projects (STEM). Papavlasopoulou et al. (2017) reviewed current research in making and found studies suggesting that learning through "making in art, design, and

* Corresponding author.
E-mail address: susannks@hiof.no (S. Stigberg).

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technology practice can provide fertile ground for developing STEM in education" (p. 61). The DF research community has made considerable efforts to support teachers in acquiring DF skills, focusing on how teachers can integrate DF and design thinking in a STEM classroom (Hjorth et al., 2016; Milara et al., 2020). There has been little focus on how teachers can utilize DF to produce materials that aid learning, such as manipulatives, in mathematics education (Stigberg, 2022).

Greenstein et al. (2019) demonstrated that DF in teacher education deepens teacher students' understanding of mathematics content, curriculum, and pedagogy. The ability to incorporate technology into education requires a set of generic skills that are useful in all situations, both personal and professional, as well as specific teaching-related skills. This is commonly known as *professional digital competence* for teachers (Lund et al., 2014). Teacher educators have a twofold responsibility. They must not only be proficient in using technology for their own teaching but also contribute to cultivating teacher students' digital competencies. In a multifaceted domain such as teacher education, where teacher preparation occurs both on campus and in field practice schools, the issue of how technology is integrated into each of these arenas becomes particularly significant (Instefjord & Munthe, 2017). In this paper, we report on our efforts to explore one approach to integrating DF into mathematics teachers' education, focusing on making manipulatives for teaching mathematics. We reviewed previous research on manipulatives in mathematics education and DF for mathematics teachers to inform how technology and digital competencies can bridge the need for concrete materials when learning mathematics. Synthesizing previous work, we proposed a "find-adapt-create-share" (FACS) framework describing how DF can be introduced in mathematics teacher education to make artifacts that aid children's learning. We conducted a series of four workshops with five students enrolled in our teacher education program, in the following called teacher students, to answer the following two research questions:

(1) How do teacher students acquire DF skills in workshops guided by the FACS framework?
(2) How does the FACS framework enable the integration of digital fabrication in a mathematical–pedagogical teaching context?

In the following section, we present previous work on manipulatives in mathematics education alongside research on DF in teacher education, resulting in the design of the FACS framework. We continue with a description and results from the workshops and discuss the FACS framework as one approach to integrating DF into mathematics teacher education.

2. Background

2.1. Manipulatives in mathematics education

Manipulatives in mathematics education are physical objects used to reify abstract concepts, such as arithmetic, algebra, geometry, and fractions (Sowell, 1989). They are commonly used in K–12 classrooms and are supported by research (Brown et al., 2009; Holmes, 2013; Sowell, 1989) and national policies (e.g., Common Core State Standards - National Council of Teachers of Mathematics, 2017). The popularity of manipulatives stems from the belief that children require multiple embodiments to understand concepts (Montessori, 2013; Wilson, 2002), that concrete experiences are necessary for learning (Piaget, 1952) and that physical objects form the basis for later abstract learning (Skemp, 1987). Manipulatives are objects designed to explicitly and concretely represent abstract mathematical concepts. They have both visual and tactile features and can be manipulated by learners through hands-on experience. However, manipulatives are not carriers of meaning or knowledge on their own and must be used as tools to gain insight. To use manipulatives effectively, children must know them well enough to use them automatically. The significance of manipulatives as potential tools depends on the task for which they are used. To draw maximum benefit from children's use, teachers must continually situate their activities and the material based on children's previous experiences and the teaching context (Brown et al., 2009; Sarama & Clements, 2016; Uttal et al., 1997). Successful teachers repeatedly reuse the same manipulative, so that they become familiar and are no longer viewed as toys or objects in themselves but transparent for the mathematical concept they are supposed to reify (Uttal et al., 1997). Nührenbörger and Steinbring (2008) highlighted the need to train teacher students in how to use manipulatives, both theoretically and practically. Here, we explore an approach for introducing teacher students to DF, with the goal of enabling them to find, adapt, create, and share manipulatives and design classroom activities that encourage children's mathematical learning.

2.2. Digital fabrication in teacher education

Previous research in the field of DF and mathematics teacher education has primarily focused on how teachers can integrate DF and design thinking into STEM classrooms (Stigberg, 2022). Andersen and Pitkänen (2019) presented an overview of nine initiatives in the field. However, there is a lack of research on how DF can be used to create manipulatives specifically for mathematics education. Although there are scattered examples of using DF to create mathematical models (Hart, 2005; Knill & Slavkovsky, 2013; Rainone et al., 2014), few studies have explored DF's potential for making manipulatives to reimagine mathematical teaching. Greenstein and colleagues are an exception (Akuom & Greenstein, 2022; Greenstein et al., 2020; Greenstein & Olmanson, 2018; Greenstein & Seventko, 2017), as their work has focused on how DF technologies can be used to create manipulatives and enhance teacher students' understanding and engagement with mathematics. However, they did not present how these workshops were designed. Inspired by their work, we inquire how to introduce DF to enable teacher students to develop a deeper mathematical knowledge by creating manipulatives and reflecting on children's mathematical learning when designing customized learning activities.

Lassiter et al. (2013) identified six critical categories of knowledge that an educator needs to successfully integrate DF into a learners' formal educational experiences: digital design and fabrication techniques, engineering fundamentals, application of the design process, project design and management, strategies to align student learning to benchmarks and to leverage standards for assessment, partnership, and asset building and alignment, and the larger context of DF in the making, tinkering, and fabbing communities, as well as the interests of industry and national economy. However, they did not specify how these categories could be implemented in a DF intervention. Hjorth et al. (2016) described a framework for educating educators based on a design studio approach, including three types of activities: workshops and lectures, peer collaboration, and in-school practice. A mixture of literature on DF in education and pragmatist design literature was taught through lectures, group exercises, and pre-work in a series of workshops. However, the authors did not specify in more detail how these topics were introduced in the workshops. Ulbrich et al. (2020) reported on DF workshops for teachers. They divided workshops into two parts: first, they presented examples and demonstrations to inspire and motivate teachers; then, they focused on providing teachers with hands-on experience in 3D modeling and 3D printing, as well as finding and downloading free online models. Peterson and Scharber (2018) advocated the "Focus, Fiddle, and Friends" approach (Frank et al., 2011) for innovation knowledge diffusion within schools. In their professional development initiative, they provided information about makerspaces and DF (focus), time to tinker and make (fiddle), and time to reflect with peers (friends).

In summary, we found sporadic examples of mathematical concepts produced using DF (Hart, 2005; Knill & Slavkovsky, 2013; Rainone et al., 2014) and sparse research into how DF can be introduced to teachers and teacher students. We identified three main concepts: learn DF tools and techniques through hands-on experience (Andersen &

Pitkänen, 2019; Peterson & Scharber, 2018; Ulbrich et al., 2020), design thinking through long-term projects, and integration into a teaching context through in-school practice (Hjorth et al., 2016) and sharing with colleagues (Andersen & Pitkänen, 2019; Peterson & Scharber, 2018). In our project, we built on these concepts to implement a DF workshop series for mathematics teacher students. We adopted a situated learning approach when introducing DF in the context of making manipulatives for mathematics education, merging DF skills with pedagogical thinking and mathematical concepts in authentic activities, as discussed in more detail in the next section.

3. Find-adapt-create-share framework

The objective of developing DF workshops is to familiarize teacher students with DF technologies, enabling them to create their own manipulatives and supporting their ability to reflect on children's mathematical learning while using manipulatives, as requested by prior research (Moyer, 2001; Nührenbörger & Steinbring, 2008). Our workshops are grounded in a situated learning perspective, where learning is "a process in which students actively reorganize their ways of participating in classroom practices" (Cobb & Bowers, 1999, p. 9). Learning is inherent in the process of engaging in activities that extend what one knows beyond the immediate situation (Lave, 2009), e.g., "actively participate in the collective decision of planning, negotiating, and reflecting on these processes to achieve the goal of the group" (Ryu & Lombardi, 2015, p. 72). From this standpoint, we do not focus on individual cognitive skills but view learning as an integral and inseparable aspect of social practice (Lave & Wenger, 1991). Korthagen (2010) proposed an integrative view in teacher education that emphasizes the importance of reflection and critical inquiry in the learning process. He argued that teacher education programs should provide opportunities for teachers to engage in meaningful, situated learning experiences and reflect on those experiences to develop a deeper understanding of their own practice. Hence, teacher educators need to provide suitable learning experiences that are realistic and provide opportunities for reflection. Similar, Herrington and Oliver (1995) suggested that to provide authentic context and authentic activities that reflect the way the knowledge will be used in real life is necessary for instructional design. Furthermore, they (Herrington & Oliver, 1995) proposed that these experiences need to provide access to expert performances, support collaborative construction of knowledge, coaching, and scaffolding at critical times, and reflection and articulation to enable abstractions to be formed and tacit knowledge to be made explicit. According to Mishra and Koehler (2006), teacher students should learn technological, pedagogical, and mathematical skills integrated to be able to teach in a contemporary classroom using available resources.

Based on previous work, we developed a four-component framework describing how DF can be introduced to teacher students to make artifacts that aid learning. As illustrated in Fig. 1, we imagine three levels when making manipulatives for teaching mathematics, arranged by increasing complexity from finding to creating manipulatives. Technology use often involves finding available resources online. For example, Ulbrich et al. (2020) suggested finding and downloading 3D models as an important task when introducing teachers to 3D printing. Teacher students learn how to use online resources and reflect on how to adapt them to classroom activities with various levels of sophistication, from choosing a filament color or printing size to changing the digital model. Teacher students need to acquire skills to modify or copy available manipulatives using different types of DF software at the adapt level. Finally, teacher students can apply acquired DF skills to create new manipulatives for mathematical concepts inspired by available designs. At this create level, teacher students use DF tools and techniques to ideate new manipulatives in their design processes. Creating manipulatives almost always builds on teacher students' previous experiences of finding and adapting manipulatives. Each level is essentially a superset of what has come before. Finally, teacher students are

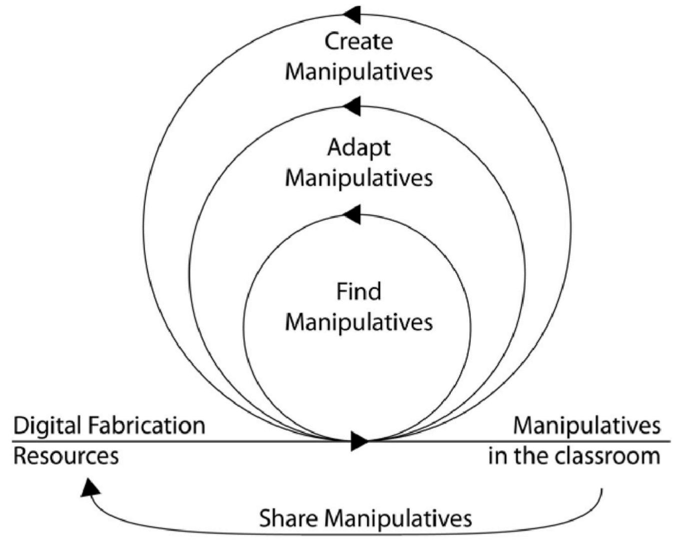


Fig. 1. Find-Adapt-Create-Share framework for learning digital fabrication skills.

encouraged to participate in a maker culture and share their work with others to increase the number of available DF resources, support peer-to-peer learning, and thrive in this maker culture, as illustrated in Fig. 1 by an arrow pointing from the created manipulatives for the classroom back to DF resources. Each level is presented by a circle in the framework, highlighting the iterative process and an opportunity for reflection on the produced manipulative and its use in the classroom. While we advocate using the FACS framework to describe how teacher students can acquire DF skills and competencies, it is not intended to depict three distinct steps. In practice, we see no clean breakpoints among finding, adapting, and creating but gradual metamorphoses from one level to another. When designing the workshops, we need to pay attention to teacher students' previous knowledge and experiences to select the appropriate entry level. Each level should focus on collaborative, hands-on activities where teacher students can explore DF technologies and receive coaching and scaffolding from DF experts and teacher educators at critical times. The fourth component in the framework, sharing, should be part of each level to aid in learning DF skills, provide an opportunity for reflection and articulation of tacit knowledge, and enable participation in a DF community. In the following section, we describe how we operationalized the framework in four workshops for mathematics teacher students.

4. Four workshops on digital fabrication

We piloted a series of four workshops with five teacher students (four male, one female) in their fourth year of study to explore the proposed FACS model. The teacher students had a sound mathematical background and experiences from in-school practice in 5th–10th grade classes. They completed a course in basic information and communications technology (ICT) and joined a field trip to the local FabLab. They were recruited in a mathematics course where we presented the project, and the interested teacher students signed up voluntarily. As compensation, they got lunch during the workshops and received two cinema tickets and a certificate of participation. The workshops were designed and conducted by an interdisciplinary team of researchers as follows: one associate professor in computer science with expertise in DF and design thinking (DF expert); one professor and one associate professor in mathematic didactics with expertise in subject didactics, teacher education development, and professional development research (teacher educator); and one Ph.D. fellow with extensive experience as a mathematics teacher in primary and secondary education (teacher educator).

The workshops were conducted once a week over four weeks, each lasting 4 h. We video-recorded the workshops to help us analyze, reflect on, and plan them. In addition, the research group held weekly meetings between workshops to share reflections and discuss workshop implementations and results. Meeting minutes were stored on an online collaboration platform. To elicit teacher students' feedback from the workshops, we conducted final individual interviews with the five teacher students, as previously reported in Stigberg et al. (2022). Here, we focus on what happened during the workshops by analyzing the video recordings.

4.1. Research questions, data collection, and data analysis

To guide our inquiry and assess the FACS framework, we formulated the following research questions:

RQ1: How do teacher students acquire DF skills in workshops guided by the FACS framework?
RQ2: How does the FACS framework enable the integration of DF in a mathematical–pedagogical teaching context?

We collected 27 h and 4 min of video data. Cameras were positioned within each student group to record their discussions and interactions with their computers and the content shown on the main screen mirrored from a group member's computer. Additionally, we placed a camera with 3D printers in the room. We used another camera to capture our lectures and teacher students' presentations.

The analysis process followed several iterative steps. Initially, the first author immersed himself in the data, segmenting the raw information into activity sequences and constructing a comprehensive outline of these activities, complete with timestamps denoting the instances when teacher students were engaged, actively participating in the collective decision (Ryu & Lombardi, 2015), e.g., teacher students negotiating, discussing, modeling, or planning. Sections that contained lectures or tutorials and teacher students were supposed to listen, were *not* included as instances. Subsequently, all authors performed individual inductive coding, based on technological, pedagogical, and content-related activities within the dataset, inspired by Mishra & Koehler's (2006) TPACK model. Activities were coded as technological if participants focused on technological aspects, e.g., model in Tinkercad. Pedagogical activities include aspects of how to use manipulatives for teaching, e.g. plan a classroom activity. Activities that are content-related focus on mathematical concepts, e.g., discussing the Pythagoras theoreom. In the third step, we systematically pursued thematic elements aligned with each category identified in the previous step. In the fourth step, we addressed the discrepancies in the analysis together. Our discussions prompted us to revisit the data and achieve consensus on the emerging themes. To address RQ1, we focused on the technological category and established a shared interpretation of the coding. In addressing RQ2, we emphasized the interrelationships across all three domains: technological, pedagogical, and content-related. To ensure anonymity, we assigned pseudonyms to the students, as detailed in Table 1, in the coding process. Our analysis of the video recordings provides insights into workshop activities, but we acknowledge that videos may influence participants and that our interpretation shapes the descriptions presented (Knoblauch & Schnettler, 2012).

Table 1
Teacher student pseudonyms used in the coding process.

Group 1	Group 2
Ingrid	Magnus
Lars	Anders
	Kristian

4.2. Implementation of workshops

Each workshop was comprised of four parts: reflections, lecture, group work, and presentation (Fig. 2) inspired by the "Focus, Fiddle, and Friends" approach (Frank et al., 2011). We started with an opportunity for student reflections to connect to previous experiences and to make tacit knowledge explicit (Friends). Then, we provided lectures on DF, teaching mathematics, and example manipulatives to demonstrate a mathematical concept (Focus). All manipulatives presented during the lectures are illustrated in Fig. 3. The third and largest part of the workshop consisted of group work, which provided students with the opportunity for hands-on collaborative inquiry (Fiddle). We divided teacher students into two groups, as shown in Table 1. Finally, the teacher students presented at the end of each workshop to share their manipulatives and classroom activities (Friends). In the following sections, we present a description of each workshop before discussing the results of the analysis of the workshops. An overview of the content categorized by DF, teaching mathematics, mathematical concepts, and sharing activities in each workshop can be found in Table 2.

4.2.1. Workshop 1: finding manipulatives

We started the workshop by introducing the project and invited the teacher students to reflect on their rationales for participating, and their expectations, first individually, then in pairs, and lastly, share their thoughts in an open discussion. We continued with a lecture on representations in mathematics and manipulatives as one type of representation. We discussed pedagogical aspects that teachers should be aware of when working with manipulatives in the classroom; for example, manipulatives need to be used long term to be effective, and mathematical concepts can be taught starting with tangible manipulatives and gradually increasing the abstraction level to symbols. We concluded the presentation with an example task: What object does not fit? Using a set of 3D-printed solids, teacher students discussed which solids should be removed based on geometrical characteristics (Fig. 3a). Subsequently, we introduced DF tools and techniques and Thingiverse as an online repository for DF resources. For the group activity, the teacher students were tasked with exploring Thingiverse and finding a manipulative they could use in the classroom. We did not specify a mathematical concept or school context to allow for an open exploration of the DF tool. We provided the groups with two 3D printers, a laser cutter, a vinyl cutter, and tutorials on how to use them. We assisted the groups in making their manipulatives. Finally, we gave the groups a pedagogical task to design a K–12 classroom activity with the manipulative. At the end of the workshop, the groups presented their produced manipulative, including their rationale for choosing it, how they would integrate the manipulative in a classroom activity, and their experiences of using Thingiverse for finding manipulatives.

4.2.2. Workshop 2: adapting manipulatives

In the second workshop, teacher students were asked to reflect on what went well, problems that occurred, and their thoughts on using 3D printing for making manipulatives. We asked them to share tips for using the 3D printer on our local learning platform. Finally, they were asked to produce the same manipulative from the previous workshop using each other's tips for 3D printing. We continued with a lecture on problem-solving tasks with a low threshold and high sealing, exemplified by a classroom activity on fractions retrieved from a national mathematical resource page (Brøkstaver | Mattelist, 2022). We created fractional bars for this exercise using a laser cutter and a vinyl cutter (Fig. 3b). We presented the laser cutter and vinyl cutter as DF tools and provided a

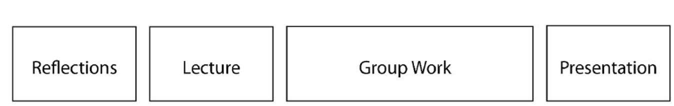
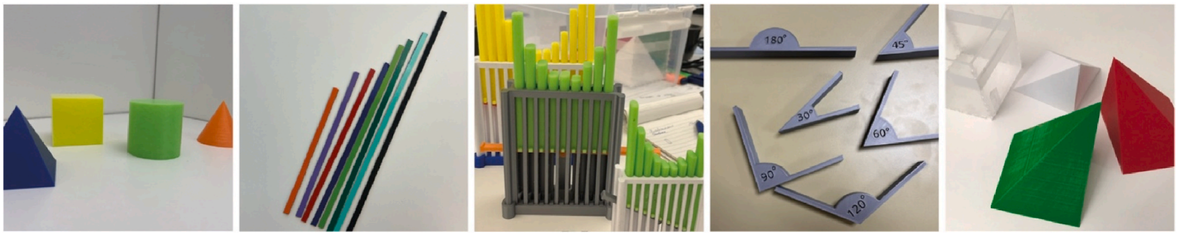


Fig. 2. General workshop layout consisting of four activities.



(a) (b) (c) (d) (e)

Fig. 3. Example manipulatives included in the lectures.

Table 2
Overview of workshop content categorized by technology, pedagogy, content, and sharing.

	DF Skills	Teaching mathematics	Mathematical concepts	Sharing
Find **Workshop 1**	Thingiverse Prusa Slicer 3D printer	Representations of mathematical concepts Four principles for working with manipulatives (Svingen, 2018)	Geometrical concepts and properties *What object does not fit?*	Presentation of found manipulative and planned classroom activity
Adapt **Workshop 2**	Inkscape Laser cutter Cricut Design Space Vinyl cutter	Low threshold–high sealing tasks	Fractions MatteList (Brøkstaver | Mattelist, 2022)	Tips for 3D printing Presentation of adapted manipulative and planned classroom activity
Workshop 3	Tinkercad Prusa Slicer 3D printer	Four stage framework for using manipulatives in teaching (Malmer, 1997)	Parabola *Explore the relationship between manipulative and quadratic functions*	Tips for laser cutting Presentation of adapted manipulative and classroom activity
Create **Workshop 4**	Design thinking Thingiverse Remix	Curriculum: different representations and core elements	Pyramid Algebra, Probability	Role play activity Share on Thingiverse Feedback on workshops

tutorial to Inkscape and Cricut Design Space for creating 2D models. The groups were asked to produce the manipulative from the first workshop using a laser cutter and a vinyl cutter, adapting the manipulative to a 2D model. We asked them to sketch the 2D model on paper and add measurements before creating the digital model in Inkscape. We assisted them using the laser cutter. At the end of the workshop, the groups shared their manipulatives and adjustments, and they discussed utilizing a laser cutter to make manipulatives.

4.2.3. *Workshop 3: adapting manipulatives*

The third workshop started with a group discussion on the use of manipulatives in teaching. Then, we asked them to share tips for using the laser cutter. In the lecture, we introduced a four-stage framework (Malmer, 1997) for using manipulatives in the classroom. As an example task, we produced a manipulative reifying of a quadratic function (Fig. 3c). The task was twofold: first, to solve mathematical problems, e.g., how and why the parabola is moving when changing the b-coefficient in a quadratic function $y = ax^2+bx + c$; second, to reflect on how to improve the manipulative to reify the mathematical concept of a graph representing the quadratic function better. For DF skills, we focused on 3D modeling in the third workshop. We introduced Tinkercad as modeling software to modify or create manipulatives for 3D printing. Teacher students followed the built-in tutorial to learn the basic functionality of Tinkercad. In the group work, teacher students could select from several manipulatives available on a table (including their previously produced manipulatives and some new ones). The task was to discuss how the manipulative should be adapted to reify the mathematical concept better and design a suitable classroom activity. The groups presented what manipulative they selected, how they wanted to adapt it, and how they planned to integrate it into a classroom activity. During the remaining time, they worked on their manipulative in Tinkercad. The teacher students did not finish their manipulatives in this workshop, and we continued the work in the next workshop.

4.2.4. *Workshop 4: creating manipulatives*

The fourth workshop started with a reflection task about the manipulative that the teacher students started modeling during the third workshop. The teacher students were asked to describe how the manipulative would be used in the classroom and how the proposed changes would better support the classroom activity. The groups had time to finish their manipulatives and perform their designed classroom activity as role play, where we researchers acted as pupils. In the second part of the workshop, we presented a design thinking process for creating new manipulatives proposed by Hjorth et al. (2016) and how they could share their new manipulatives as remixes or novel artifacts in Thingiverse. For example, we presented a created manipulative reifying the relationship between the volume of the pyramid and the cube. The workshop concluded with the teacher students sharing their experiences from the workshops, their thoughts on how DF could be integrated into mathematics teacher education, and their suggestions for improving the conducted workshop series.

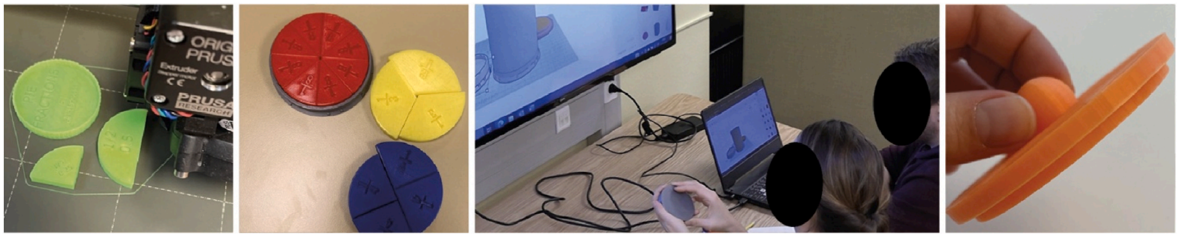
5. Results

In this section, we share the results of our analysis of the video data recorded during the four workshops. These findings provide a detailed response to our research questions. We refer to the participation of two student groups in these workshops. Group 1 concentrated on a fractions manipulative, as shown in Fig. 4a–d, while Group 2 explored multiples manipulatives, as seen in Fig. 5a–e.

5.1. *How do teacher students acquire DF skills in workshops guided by the FACS framework?*

5.1.1. *Find activities that are engaging and accessible*

The students found the FIND activities introducing Thingiverse and 3D printing to be engaging and accessible. They quickly familiarized themselves with the Thingiverse platform, searching for manipulatives related to math education. Their search approach began with general



(a) (b) (c) (d)

Fig. 4. Design process of a pie fraction manipulative in group 1.

(a) (b) (c) (d) (e)

Fig. 5. Manipulatives produced by group 2.

terms and became more specific as they interacted with their groups. Ingrid mentioned that they were interested in seeing what appeared when searching for "manipulatives" in general. They narrowed down their selection by reframing their search criteria and gaining an overview of the available options. They experimented with different search terms, including "manipulatives math," "concrete materials," and "teaching." The students were forced to identify English terminology, as searching with Norwegian words yielded no results. A teacher educator suggested using Google Translate to find appropriate English terminology. Printing time was important when selecting a manipulative from Thingiverse, as students were keen on having a 3D printed manipulative at the end of the first workshop. Some manipulatives were perceived as time-intensive and "possibly very advanced" (Anders), and both student groups chose manipulatives that were feasible to print.

After deciding on a mutual manipulative within the groups, each student individually downloaded the model into the slicing program to gain hands-on experience. In this phase, minimal teacher educator assistance was necessary, mainly to assist in unpacking .zip files and clarifying the necessity of the .stl format for printing. The students explored functions that impacted print time, such as scaling and infill, within the slicing program. Both groups focused on reducing print time to ensure completion before the workshop's end. Technical support was needed for transferring files to the printer, adjusting filament settings, and initiating the printing process. One student in each group oversaw the actual printing process. Group 1 began printing after 34 min, while group 2, faced with a malfunctioning 3D printer, started printing after 49 min.

5.1.2. Adapt activities support flexible inquiry into DF technologies

During the ADAPT activities, students engaged with 2D modeling using Inkscape and 3D modeling using Tinkercad. The two groups followed divergent paths in adapting their manipulatives. Group 2 chose to adapt a binomial theorem manipulative (Fig. 5b), switching DF technology from a 3D printer to a laser cutter to construct a larger version more suitable for use in demonstrations in the classroom. The initial 3D model dimensions were 10 cm × 10 cm. The group decided to make the side edges twice as long because "the [laser cutter] is very fast" (Magnus), and they added handles and labels to enhance usability. They explored Inkscape independently, including program functionalities such as align, rotate, and text tools for engraving text. The DF expert informed them that lines under 0.025 mm would be cut, while those above this measurement would be engraved.

Group 1 adapted the pie fraction manipulative (Fig. 4a-d) to create a student kit, including a container, lid, and additional pie fraction components. They demonstrated confidence in navigating Tinkercad; for example, they were able to combine objects to create a hollow cylinder for the container. They had problems creating additional fraction pieces with the correct angle of the circular sector. They tried cutting cylinders unsuccessfully but found a tool in the menu that enabled them to create circular sectors. They continued to expand their repertoire and familiarity with the 3D modeling program.

In summary, our video analysis revealed that ADAPT activities facilitated flexible inquiry into DF technologies. The groups started with pre-existing objects and autonomously explored 2D and 3D modeling programs, identifying suitable functions to adapt the manipulatives. They displayed endurance in their inquiry and relied little on coaching from teacher educators and DF experts.

5.1.3. Students' design collaboration and exploration in create activities

During CREATE activities, students were tasked with designing a new manipulative for teaching mathematics, with the option of drawing inspiration from existing manipulatives. While group 1 continued to adapt their manipulative in Workshop 4, group 2 embraced the CREATE activities. They selected the concept of angles within mathematics as their focus and discussed various ideas. Each member within the group individually modeled their version, and at the end, they collectively decided which prototype to demonstrate (Fig. 5e). Video recordings displayed their collaborative efforts when making decisions regarding the mathematical concept, discussing alternative ideas, and choosing a prototype for presentation.

Furthermore, the video data exposed students' varying levels of DF skills during CREATE activities. However, they exchanged insights into the functionalities they had discovered and asked each other for help when encountering challenges. Frequent exchanges, such as "Could you help me now. I need these holes here. Ten degrees, twenty degrees ..." (Kristian) and "I've found a highly useful function; pressing that button there zooms in on the object you're working on" (Magnus), reflect on their collaboration exploring Tinkercad. They were comfortable with the slicing program and 3D printing and experimented with techniques, such as rotation to optimize printing surface usage, minimizing the need for additional support material, or considering factors such as infill. They managed the 3D printer, from loading the .stl file to precisely adjusting the settings and initiating the printing process. Magnus chose to print a scaled-down prototype, explaining, "I just want to try it out. I suspect it might not work as intended." Their collaboration with DF technologies suggests a maker mindset, a willingness to explore tools and techniques without the fear of making mistakes.

5.2. How does the FACS framework enable the integration of DF in a mathematical–pedagogical teaching context?

The FACS framework provided a meaningful objective for acquiring DF skills, and at the same time, engaging with DF activities forced students to reflect on what they wanted and why, empowering them to a reflective use of manipulatives for children's mathematical learning.

When using Thingiverse in FIND activities, the goal of finding manipulatives guided students to explore the DF sharing platform, how it is structured, and how they can find relevant resources. At the same time, engaging with Thingiverse provided them with an overview of existing manipulatives created by other mathematics teachers, offering opportunities for both content and pedagogical discussions on what the manipulative is supposed to reify and how. For example, one group discussed the binomial theorem manipulative (content) they found on Thingiverse and reflected on how they could use the manipulative in their future teaching (pedagogical). The video recordings revealed that students paused and reflected on manipulatives that they initially found interesting, e.g., they found a manipulative that could help draw the sinus function, but they concluded that the manipulative was not suitable for their K-12 classroom.

Similarly, the teacher students had clear objectives for adapting their manipulatives while learning 2D and 3D modeling. They explored the modeling software to make manipulatives that fit their pedagogical needs, e.g., changing size or materials, adding labels and handles, and making student kits. However, the modeling software empowered students to explore the manipulatives in more depth and provided a way to reflect on and discuss how manipulatives can be used in the classroom. The video recordings revealed that the teacher students discussed technology, as well as mathematical and pedagogical perspectives.

During CREATE activities, students were faced with the complexity of designing new manipulatives, revisiting the introduced DF technologies and manipulatives to dive deeper into DF, and reflecting on how the manipulative supports children's learning of the chosen mathematical concept. For example, when creating the angles manipulative, group 2 discussed mathematical concepts, such as angle sums and acute and obtuse angles, as well as how children could use the manipulative, e.g., searching for angles in the classroom.

Students shared their manipulatives along with a proposed teaching activity in each workshop, requiring them to reflect on how the manipulatives should be used, both because they needed to articulate their thoughts and because fellow students and teacher educators posed critical questions about the manipulatives and the activities related to them. This, in turn, led to discussions about changes and manipulative adaptations made possible by DF skills and mathematics pedagogical skills.

6. Discussion

The objective of the FACS framework is to scaffold DF skills for teacher students in a mathematical teaching context, enabling them to create their own manipulatives and supporting their ability to reflect on children's mathematical learning while using manipulatives, as requested by previous research (Moyer, 2001; Nührenbörger & Steinbring, 2008). This was accomplished by providing teacher students with 1) activities to find, adapt, and create manipulatives using DF tools, and 2) sharing activities to encourage them to reflect on these manipulatives and how they can be used by children in their future mathematics classroom. We believe that this approach might address known challenges such as limited access to manipulatives and limited school budgets (Marshall & Swan, 2008), adopting a mindset of teachers as makers as described by Akuom and Greenstein (2021) and Greenstein et al. (2020).

6.1. Scaffolding DF skills and making

Previous research on DF and making in teacher education has emphasized the importance of hands-on experience, peer collaboration, workshops, and lectures (e.g., Hjorth et al., 2016). Andersen and Pitkänen (2019) presented a 1:1:1-model for developing professional practice in DF and design thinking, including inspiration in courses, immersion in the field, and development of professional practice. Similarly, Peterson and Scharber (2018) advocated their "Focus, Fiddle, and Friends" approach. However, much of this existing research has concentrated on professional development programs at large and the broader perspective of creating a community of practice around DF and making (Andersen & Pitkänen, 2019; Hjorth et al., 2016; Milara et al., 2020). In contrast, our approach had a narrower focus, delving into how to scaffold teacher students to acquire DF skills within these programs (RQ1). Surprisingly, there is limited research available on the didactics and practical implementation of DF workshops. Ulbrich et al. (2020) reported on DF workshops for teachers, where they first presented examples to inspire and then focused on providing teachers with hands-on experiences, such as finding and downloading free online models, which is a crucial task in this context. In this paper, we propose the FACS framework as a specific approach for scaffolding DF skills among teacher students. This framework starts with low complexity, guiding students from producing ready-made objects to becoming makers skilled in creating their own objects.

Hjorth et al. (2016) propose the use of a design process model as well as long-term projects, and integration into a teaching context through in-school practice to strengthen teacher students' design literacy. This has not been integrated into our workshops; however, we intend to address this limitation as part of our future research.

6.2. DF skills as a tool for teacher professionalism

Research on DF and making in teacher education often investigates how professional development programs can empower teachers to implement DF and design thinking activities in schools (e.g., Andersen & Pitkänen, 2019; Hjorth et al., 2016; Milara et al., 2020). Instead, we focus on how DF skills can empower teachers in their own professionalism—in our case, making manipulatives for teaching. Greenstein and colleagues (Greenstein & Olmanson, 2018; Greenstein & Seventko, 2017) pointed out the potential of making activities contribute to students' increased mathematical and pedagogical skills. The FACS framework enables the integration of DF in a mathematical teaching context (RQ2). In line with (Lave, 2009), teacher students engage in DF activities that extend what one knows and allow them to inquire into how manipulatives can be used in the classroom. Further, teacher students engage in this inquiry, integrating technology, pedagogy, and content to prepare them to teach in contemporary classrooms, as proposed by Mishra and Koehler (2006). However, this does not come naturally, and teacher educators need to prompt this reflection by asking targeted questions and including explicit sharing activities, e.g., providing opportunities for role play, to train these skills. Designing workshops to include activities for both acquiring DF skills and reflecting on mathematical and pedagogical contexts is a complex endeavor. Pitkänen et al. (2019) highlighted the need for collaboration between FabLab facilitators familiar with DF and educators with teacher domain knowledge for instructional scaffolding. In our study, the research team comprised four people, three teacher educators and one DF expert, to provide coaching and scaffolding at critical times and enable students' flexible inquiry into DF tools. We have reported on technological, pedagogical and workshop design tensions that arose during the last workshop (Stigberg et al., 2022), in line with previous research in teacher professional development (Nipper et al., 2011). During the analysis, we discussed our role as workshop leaders and the tensions between letting teacher students inquire freely into what manipulatives to make and how to use them or guiding them to make appropriate manipulatives for the classroom and design good pedagogical tasks (workshop design tensions).

Previous studies have focused on sharing and collaborating in a community of practice (Andersen & Pitkänen, 2019; Milara et al., 2020; Peterson & Scharber, 2018). Similarly, in our study, the teacher students shared their manipulatives in the workshops with each other. However, they did not upload their manipulatives to Thingiverse and they were not able to evaluate their manipulatives through in-school practice. This is a limitation within our project and requires attention in future research endeavors.

6.3. Methodological considerations and limitations

The FACS framework embraces a situated learning perspective, viewing learning as "a process in which students actively reorganize their ways of participating in classroom practices" (Cobb & Bowers, 1999). This perspective led us to conceptualize the acquisition of DF skills through participating in workshop activities. To gain insights into this practice, we opted for group work and the use of video recordings as our primary observational tool. Nevertheless, we acknowledge the inherent biases in video data, the discomfort associated with being recorded, and the possibility of events occurring beyond the video frame (Knoblauch & Schnettler, 2012). To obtain a more comprehensive perspective, we conducted interviews with the teacher students, as presented in Stigberg et al. (2022), and their responses aligned with the results presented in this article.

Our primary objective was to inquire into FACS rather than rigorously evaluate its effectiveness. Therefore, while our study provides insights into students' activities and experiences, it does not offer an assessment of FACS' performance or impact. Furthermore, the study involved five participants, which may not fully capture students' diversity in teacher education in general. As a qualitative study, the findings are subjective and context-dependent and do not provide an exhaustive assessment of FACS' effectiveness or generalizability. Further research with larger and more diverse student groups, as well as a focus on evaluation, is necessary to build upon the insights gained in this study. For this, we plan to integrate the FACS framework into our teacher education program and invite others to adopt the framework and share their experiences.

7. Conclusion

In this paper, we describe a workshop series that introduces teacher students in making manipulatives. Manipulatives are often mass-produced, and teachers are usually encouraged to think like consumers regarding how these materials can support children's understanding and learning of mathematics. We explored an opportunity for teacher students to create their own manipulatives using DF tools to open new learning experiences and support reflected mathematical teaching. The workshops included inquiry activities to find, adapt, create, and share mathematical manipulatives using DF tools, techniques, and platforms. Teacher students explored 2D and 3D modeling and fabrication using a 3D printer, laser cutter, and vinyl cutter. Our in-depth analysis of the video recordings indicates that teacher students successfully acquired DF skills and reflected on the use of customized manipulatives to support children's mathematical learning. We see that introducing DF into teacher education opens the possibility for future teachers to go from being consumers of ready-made manipulatives to being able to create materials that fit their situations and students. We propose FACS as a specific approach to introducing DF to teacher students for making manipulatives, an aspect that has been missing from previous research.

8. Selection and participation

The participants were teacher students (between 20 and 30 years old) at a Norwegian higher education institution. The study took place at the institution's makerspace. Data related to the study were collected after approval from the National Data Protection Official for Research, following the regulations and recommendations for research with people. A researcher contacted the teacher students to obtain written consent permitting data collection. The participants were informed about the data collection process, and their participation in the study was completely voluntary. In addition, the participants were able to withdraw their consent for data collection at any time without affecting their participation in the activity.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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