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Interval type-2 fuzzy $\bar{x}$-s control charts for univariate processes

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Abstract. Statistical Process Control (SPC) is fundamental to large-scale manufacturing defect detection, being control charts the primary tool that promotes quality improvement. The $\bar{x}$-s control charts monitor the mean and standard deviation of a continuous quality characteristic that can be measured and are influenced by human subjectivity, as well as to several other sources of uncertainty. Fuzzy set theory has advantages, but there are situations that it is not sufficient to model uncertainties present in the process, requiring the application of type-2 fuzzy sets. This work proposes an interval type-2 fuzzy and s control charts using triangular fuzzy numbers. The $\bar{x}$-s control limits are obtained through mathematical equations considering the defuzzification method under study. An illustrative example is developed, in order to demonstrate the usefulness of the proposed control chart. The proposed control model is called the IT2TFN $\bar{x}$-s control chart, that is, the traditional $\bar{x}$-s control charts combined with type-2 interval triangular fuzzy numbers. The IT2TFN x-s control chart can stabilize data very uncertain or vague in the fuzzification process, it is more flexible as it adds more information regarding uncertainties and expert opinion, which improves the assertiveness of the process decisions compared to the traditional $\bar{x}$-s control charts.

Keywords: Univariate process; Type-2 fuzzy; Control chart; Uncertainty.

1. Introduction

Statistical process control (SPC) is one of the techniques used to control processes and distinguish causes of variation, thus signalling the need for corrective actions. SPC was developed by [1] in 1931 with the objective of improving quality and reducing variability in production processes [2]. Measurements are the way to record this natural variability of the process when it is subject to common or random causes [3].

Although the theory of fuzzy sets proposed by Zadeh in 1965 has great advantages, sometimes classical fuzzy sets cannot model uncertainty with clear definitions for the membership functions. Thus, type-2 fuzzy sets can be successfully used to improve the quality of uncertainty modelling. It is also known that the membership functions of classical fuzzy sets consist of two dimensions, while the membership functions of type-2 fuzzy sets are three-dimensional. Thus, type-2 fuzzy sets can successfully represent uncertainty and reduce its harmful effects [4]. In situations where there are several sources of uncertainty for the data under analysis, it is feasible to use the fuzzy type-2 approach, proposed by Zadeh in 1978, and mathematically formulated by [5].

Regarding the application of type-2 fuzzy set theory to control charts, [6] developed c control charts using a defuzzification method for interval type-2 fuzzy sets, [7] presented defuzzification and probability methods for c control charts using type-2 fuzzy sets. For variable control charts, [8] demonstrated how to create control limits for -R control charts using interval type-2 trapezoidal fuzzy numbers (IT2TraFN), [2] presented the -R, -S and I-MR control charts under a type-2 fuzzy approach, applying it to a real case study, and finally, [4] proposed a CUSUM chart for monitoring a fertilizer manufacturing process.

Variable process monitoring is based on measuring quality characteristics by means of measuring instruments used by operators in industrial environments. When it comes to variables, it is prudent to use two control charts together: one to monitor the mean and another to monitor the dispersion, which in the case of this work is the standard deviation. One realizes that there are several sources of uncertainty that can be adjusted when applying the fuzzy set theory, which in the case of applying the type-2 fuzzy set theory, considers even more uncertainties.

This work aims to present a model for applying and s control charts by an interval type-2 fuzzy approach making use of triangular fuzzy numbers (TFN).

From crisp data it is possible to transform them into interval type-2 triangular fuzzy number (IT2TFN) and then apply the defuzzification method according to the model proposed by [9], inserting more uncertainties compared to the traditional and s control charts. Equations regarding the IT2TFN control limits of and s is presented, and finally, the proposed model is applied to an illustrative example.

2. Interval type-2 fuzzy sets

In this section, the characteristics of interval type-2 fuzzy sets, the method of fuzzification and defuzzification used for the preparation of the control chart that will be proposed are presented.

2.1. Type-2 fuzzy characteristics

According to [10] the uncertainty of the primary pertinences in type-2 fuzzy sets is represented by footprint of uncertainty (FOU). If this feature is not present, the type-2 fuzzy set in question is become an ordinary fuzzy set. Its height should be consistent with the type of problem under analysis, as well as with the degree of uncertainty desired [5]. Figure 1 illustrates the FOU of the triangular interval type-2 fuzzy number.

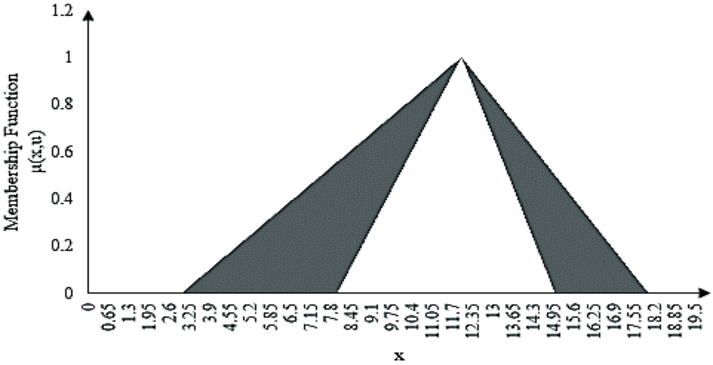

Figure 1 - FOU example for (3,25; 1,8; 12,35; 14,95; 18,2) IT2TFN.

The interval type-2 triangular fuzzy numbers can be represented mathematically by $\left(a_1^U, a_1^L, a_2^U = a_2^L, a_3^L, a_3^U\right)$, where $H(\tilde{A}^L)$ is the maximum height of the lower membership function. The upper and lower membership functions can be expressed by Eq.(1) and Eq.(2) and illustrated by Figure 1:

$$\mu_A^U = \begin{cases} \dfrac{x - a_2^U}{a_2^U - a_1^U}, if\ a_1^U \leq x \leq a_2^U \\ x = 1, if\ x = a_2^U \\ \dfrac{a_3^U - x}{a_3^U - a_2^U}, if\ a_2^U \leq x \leq a_3^U \end{cases} \quad (1)$$

$$\mu_A^L = \begin{cases} H(\tilde{A}^L)\left(\dfrac{x - a_2^L}{a_2^L - a_1^L}\right), if\ a_1^L \leq x \leq a_2^L \\ x = H(\tilde{A}^L), if\ x = a_2^L \\ H(\tilde{A}^L)\left(\dfrac{a_3^L - x}{a_3^L - a_2^L}\right), if\ a_2^L \leq x \leq a_3^L \end{cases} \quad (2)$$

Fuzzification Method

The fuzzification process takes place by transforming the crisp numbers x into IT2TFN (interval type-2 triangular fuzzy number), where IT2TFN = $\left(x_1^U, x_1^L, x_2^U = x_2^L, x_3^L, x_3^U\right)$, defined in this paper, occurs as follows:

1. identification of the crisp number x ($x_2^U = x_2^L$)
2. determination of the lower and upper bounds according to the experts

3. generation of uniformly distributed random numbers contained in the interval [0, 1]
4. calculation of the value of x_1^U from the lower bound multiplied by the random number generated, subtracting the value of x_2^U
5. the previous procedure is repeated to calculate the value of x_3^U from the upper bound factor, adding it to the value of x_2^U
6. determination of the FOU value according to experts
7. calculation of the x_1^L using the lower membership function
8. calculation of the x_3^L using the upper membership function

Although [4] have proposed $\bar{x}$ and s control charts by an interval type-2 fuzzy approach, in this paper we present a different method of fuzzification, by means of random variables.

2.2. Defuzzification Method

Unlike type-1 fuzzy logic systems, for a type-2 fuzzy number to be defuzzied, it must first undergo a type reduction, i.e., be reduced from type-2 fuzzy to type-1 fuzzy. Subsequently, the reduced number is defuzzied to the crisp output value of the type-2 fuzzy logic system [11]. The process can be seen in Figure 2.

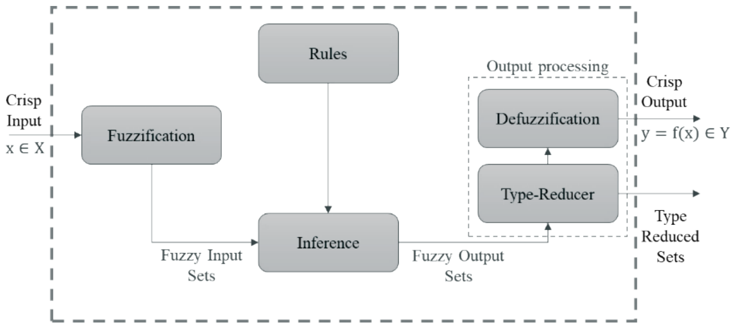

Figure 2 – Type-2 fuzzy logic system. Source: adapted from [11].

In type-2 fuzzy set theory, there are several techniques for the process of type reduction (fuzzy type-2 to fuzzy type-1) and subsequently for defuzzification. Thus, the method proposed by [9], is used for the interval type-2 fuzzy triangular numbers. The defuzzification method is used to visually compare the proposed control charts to traditional control charts, but with the insertion of more uncertainties and subjectivities arising from the process. Defuzzification method can be seen by Eq.(3).

$$D_{TriT}=\frac{\frac{(a_{i3}^{U}-a_{i1}^{U})+(a_{i2}^{U}-a_{i1}^{U})}{3}+a_{i1}^{U}+H(\tilde{A}^{L})\left[\frac{(a_{i3}^{L}-a_{i1}^{L})+(a_{i2}^{L}-a_{i1}^{L})}{3}+a_{i1}^{L}\right]}{2} \quad (3)$$

3. $\overline{\mathrm{X}}$-S control charts for a type-2 fuzzy approach

In this section, traditional $\bar{x}$-s control charts are presented and subsequently, the equations of the IT2TFN $\bar{x}$-s control limits. Subsequently, an illustrative example is presented in order to demonstrate the advantages of the proposed control charts over traditional control charts.

Traditional $\bar{x}$-s control charts

The control limits for the traditional $\bar{x}$ control chart can be obtained by Eq (4), Eq.(5) and Eq.(6):

$$UCL=\bar{\bar{x}}+A_2\bar{s} \quad (4)$$

$$CL=\bar{x} \quad (5)$$

$$LCL=\bar{\bar{x}}-A_2\bar{s} \quad (6)$$

where $\bar{\bar{x}}$ is the mean of the means and $\bar{s}$ is the mean of the standard deviations between the measurements obtained, the constant A_3 is tabulated and is related to the sample size n. For more information see [3].

The control limits for the traditional s control chart can be obtained by Eq.(7), Eq.(8) and Eq.(9):

$$UCL=B_4\bar{s} \quad (7)$$

$$CL=\bar{s} \quad (8)$$

$$LCL=B_3\bar{s} \quad (9)$$

where B_4 and B_3 are tabulated constants related to the sample size n. For more information see [3].

3.2. Interval Type-2 Triangular Fuzzy Number x-s contrōl charts

Let $\bar{x}_i=(\bar{x}_{i1}^{U},\bar{x}_{i1}^{L},\bar{x}_{i2}^{U}=\bar{x}_{i2}^{L},\bar{x}_{i3}^{L},\bar{x}_{i3}^{U})$, the IT2TFN representation for the mean of each sample i and $s_i=(s_{i1}^{U},s_{i1}^{L},s_{i2}^{U}=s_{i2}^{L},s_{i3}^{L},s_{i3}^{U})$, the IT2TFN representation for the sample standard deviation of each sample i. In order to calculate the control limits, the parameter values and must be calculated, according to Eq.(10) and Eq.(11):

$$\bar{\bar{x}}=\left(\frac{\sum_{i=1}^{m}\bar{x}_1^{U}}{m},\frac{\sum_{i=1}^{m}\bar{x}_1^{L}}{m},\frac{\sum_{i=1}^{m}\bar{x}_2^{U}}{m},\frac{\sum_{i=1}^{m}\bar{x}_3^{L}}{m},\frac{\sum_{i=1}^{m}\bar{x}_3^{U}}{m}\right) \quad (10)$$

$$\bar{s} = \left(\frac{\sum_{i=1}^{m} s_1^U}{m}, \frac{\sum_{i=1}^{m} s_1^L}{m}, \frac{\sum_{i=1}^{m} s_2^U}{m}, \frac{\sum_{i=1}^{m} s_3^L}{m}, \frac{\sum_{i=1}^{m} s_3^U}{m}\right) \quad (11)$$

Thus, we obtain the control limits for the control chart according to Eq.(12), Eq.(13) and Eq.(14):

$$UCL = \left(\bar{\bar{x}}_1^U + A_3\bar{s}_1^U, \bar{\bar{x}}_1^L + A_3\bar{s}_1^L, \bar{\bar{x}}_2^U + A_3\bar{s}_2^U, \bar{\bar{x}}_3^L + A_3\bar{s}_3^L, \bar{\bar{x}}_3^U + A_3\bar{s}_3^U\right) \quad (12)$$

$$CL = \left(\bar{\bar{x}}_1^U, \bar{\bar{x}}_1^L, \bar{\bar{x}}_2^U, \bar{\bar{x}}_3^L, \bar{\bar{x}}_3^U\right) \quad (13)$$

$$CL = \left(\bar{\bar{x}}_1^U - A_3\bar{s}_3^U, \bar{\bar{x}}_1^L - A_3\bar{s}_3^L, \bar{\bar{x}}_2^U - A_3\bar{s}_2^U, \bar{\bar{x}}_3^L - A_3\bar{s}_1^L, \bar{\bar{x}}_3^U - A_3\bar{s}_1^U\right) \quad (14)$$

The control limits for the s IT2TFN control chart are shown in Eq.(15), Eq.(16) and Eq.(17):

$$CL = \left(B_4\bar{s}_1^U, B_4\bar{s}_1^L, B_4\bar{s}_2^U, B_4\bar{s}_3^L, B_4\bar{s}_3^U\right) \quad (15)$$

$$CL = \left(\bar{s}_1^U, \bar{s}_1^L, \bar{s}_2^U, \bar{s}_3^L, \bar{s}_3^U\right) \quad (16)$$

$$CL = \left(B_3\bar{s}_3^U, B_3\bar{s}_3^L, B_3\bar{s}_2^U, B_3\bar{s}_1^L, B_3\bar{s}_1^U\right) \quad (17)$$

In order to obtain control limits capable of representing a single representative fuzzy value, the type reduction and defuzzification method proposed by [9] is applied. For the average control chart we have the control limits defined by Eq. (18), Eq.(19) and Eq.(20):

$$CL = \frac{\bar{\bar{x}}_1^U + \bar{\bar{x}}_2^U + \bar{\bar{x}}_3^U + A_3\left(\bar{s}_1^U + \bar{s}_2^U + \bar{s}_3^U\right)}{6} + H\left(\tilde{A}^L\right)\left[\frac{\bar{\bar{x}}_1^L + \bar{\bar{x}}_2^L + \bar{\bar{x}}_3^L + A_3(\bar{s}_1^L + \bar{s}_2^L + \bar{s}_3^L)}{6}\right] \quad (18)$$

$$CL = \frac{\bar{\bar{x}}_1^U + \bar{\bar{x}}_2^U + \bar{\bar{x}}_3^U}{6} + H\left(\tilde{A}^L\right)\left[\frac{\bar{\bar{x}}_1^L + \bar{\bar{x}}_2^L + \bar{\bar{x}}_3^L}{6}\right] \quad (19)$$

$$LCL = \frac{\bar{\bar{x}}_1^U + \bar{\bar{x}}_2^U + \bar{\bar{x}}_3^U - A_3\left(\bar{s}_1^U + \bar{s}_2^U + \bar{s}_3^U\right)}{6} + H\left(\tilde{A}^L\right)\left|\frac{\bar{\bar{x}}_1^L + \bar{\bar{x}}_2^L + \bar{\bar{x}}_3^L - A_3(\bar{s}_1^L + \bar{s}_2^L + \bar{s}_3^L)}{6}\right| \quad (20)$$

For the standard deviation control chart considering the process of type reduction and defuzzification, we have the control limits defined by Eq.(21), Eq.(22) and Eq.(23):

$$UCL = \frac{B_4\left(\bar{s}_1^U + \bar{s}_2^U + \bar{s}_3^U\right)}{6} + H\left(\tilde{A}^L\right)\left[\frac{B_4(\bar{s}_1^L + \bar{s}_2^L + \bar{s}_3^L)}{6}\right] \quad (21)$$

$$L = \frac{\bar{s}_1^U + \bar{s}_2^U + \bar{s}_3^U}{6} + H\left(\tilde{A}^L\right)\left[\frac{\bar{s}_1^L + \bar{s}_2^L + \bar{s}_3^L}{6}\right] \quad (22)$$

$$LCL = \frac{B_3\left(\bar{s}_1^U + \bar{s}_2^U + \bar{s}_3^U\right)}{6} + H\left(\tilde{A}^L\right)\left[\frac{B_3(\bar{s}_1^L + \bar{s}_2^L + \bar{s}_3^L)}{6}\right] \quad (23)$$

4. Illustrative example

In order to demonstrate the application of IT2TFN $\bar{x}$-s control charts, an example was simulated using as process parameters mean = 0 and standard deviation = 1. Using

RStudio, 25 samples of size 5 are simulated. From the values for the 5 measurements of each of the 25 samples, the parameters $\bar{x}$ and s that is monitored by the control charts are calculated. To calculate the control limits of traditional $\bar{x}$ control chart, we consider $\bar{\bar{x}} = 0.1290$ and $\bar{s} = 0.93544$ and parameter $A_3 = 1.427$, applied Eq.(4), Eq.(5) and Eq.(6) Similarly, using the same value of $\bar{s}$, considering $B_3 = 0$ and $B_4 = 2.089$, we calculate the control limits of the s control chart by Eq.(7), Eq.(8) and Eq.(9). Thus, we obtain LCL = -1.2058, CL = 0.1290 and UCL = 1.4639 for the $\bar{x}$ control chart and LCL = 0.0000, CL = 0.9357 and UCL = 1.9541 for the s control chart. The summary of the results can be seen in Table 1.

Table 1. Results for traditional $\bar{x}$-s control charts process monitoring.

Sample Number	$\bar{x}$	s	Decision	Sample Number	$\bar{x}$	s	Decision
1	0.027	0.96279	In control	14	-0.1688	0.82567	In control
2	0.1834	1.02433	In control	15	0.5792	0.8065	In control
3	0.6851	0.82888	In control	16	-0.1544	0.84641	In control
4	0.6679	1.23809	In control	17	0.0082	1.19756	In control
5	0.6164	0.57023	In control	18	0.2913	1.50957	In control
6	-0.4819	1.18985	In control	19	0.314	0.75684	In control
7	0.2092	1.1292	In control	20	-0.403	0.59082	In control
8	-0.0559	0.81753	In control	21	-0.1863	0.49926	In control
9	-0.1689	1.32256	In control	22	-0.27	1.07595	In control
10	0.3288	0.79327	In control	23	0.3282	0.72046	In control
11	-0.2835	1.27713	In control	24	0.245	0.46501	In control
12	0.3029	0.97455	In control	25	0.6622	0.87819	In control
13	-0.05	1.08547	In control				

From Table 2, all samples are under statistical control, for traditional -s control charts. It is important to note that no sources of uncertainty other than the expected statistical behavior were considered.

Applying the proposed fuzzification method, considering the lower and upper bounds $L_1 = L_2 = 0.1$, according to experts, FOU (footprint of uncertainty) equal to 0.20 and considering a PIT2TFN (perfectly interval type-2 triangular fuzzy number), whose $H(\tilde{A}^U) = H(\tilde{A}^L) = 1$. The crisp numbers obtained for the traditional $\bar{x}$-s control charts, was used with $x_2^U = x_2^L$ (center value) and is used as a basis for obtaining the other interval type-2 fuzzy values. After the fuzzification process, we calculate the sample means $\bar{x}_{IT2TFN} = \left(\bar{x}_1^U, \bar{x}_1^L, \bar{x}_2^U, \bar{x}_3^L, \bar{x}_3^U\right)$ and sample standard deviations $s_{IT2TFN} = \left(s_1^U, s_1^L, s_2^U, s_3^L, s_3^U\right)$, we obtain Table 2.

Table 2. Sample variables for IT2TFN $\bar{x}$-s control charts.

Sample Number	IT2TFN Sample Means					IT2TFN Sample Standard Deviations				
	$\bar{x}_1^U$	$\bar{x}_1^L$	$\bar{x}_2^U$	$\bar{x}_3^L$	$\bar{x}_3^U$	s_1^U	s_1^L	s_2^U	s_3^L	s_3^U
1	-0.0373	-0.0245	0.0270	0.0631	0.0722	0.55507	0.57183	0.63990	0.71189	0.73038
2	0.1335	0.1435	0.1834	0.2021	0.2067	0.57790	0.58970	0.63711	0.66861	0.67676
3	0.6473	0.6548	0.6851	0.7173	0.7254	0.48483	0.49838	0.55291	0.59839	0.60995
4	0.6134	0.6243	0.6679	0.6933	0.6997	0.82185	0.83441	0.88483	0.94347	0.95818
5	0.5414	0.5564	0.6164	0.6478	0.6557	0.38831	0.40179	0.45765	0.50372	0.51577
6	-0.5206	-0.5128	-0.4819	-0.4417	-0.4317	0.90068	0.91020	0.94834	1.01913	1.03693
7	0.1520	0.1634	0.2092	0.2375	0.2446	0.57846	0.59194	0.64811	0.70641	0.72132
8	-0.1195	-0.1068	-0.0559	-0.0191	-0.0099	0.56569	0.57991	0.63863	0.70824	0.72567
9	-0.2285	-0.2166	-0.1689	-0.1396	-0.1322	0.92244	0.93666	0.99357	1.03295	1.04297
10	0.2981	0.3042	0.3288	0.3658	0.3751	0.55571	0.56152	0.58761	0.61269	0.61972
11	-0.3538	-0.3397	-0.2835	-0.2330	-0.2204	0.76470	0.78092	0.84881	0.92928	0.95003
12	0.2459	0.2573	0.3029	0.3420	0.3518	0.72138	0.73176	0.77524	0.83811	0.85395
13	-0.1048	-0.0939	-0.0500	-0.0134	-0.0043	0.80354	0.81809	0.87631	0.91229	0.92155
14	-0.2259	-0.2144	-0.1688	-0.1361	-0.1279	0.46278	0.47621	0.53017	0.60507	0.62424
15	0.5219	0.5334	0.5792	0.6253	0.6368	0.46064	0.48101	0.56300	0.61267	0.62581
16	-0.2152	-0.2030	-0.1544	-0.1104	-0.0994	0.40379	0.42385	0.50467	0.58561	0.60598
17	-0.0515	-0.0395	0.0082	0.0477	0.0575	0.92657	0.93779	0.98302	1.03199	1.04439
18	0.2401	0.2503	0.2913	0.3244	0.3326	1.01374	1.02675	1.07888	1.14206	1.15792
19	0.2780	0.2852	0.3140	0.3555	0.3659	0.51437	0.52147	0.55273	0.58575	0.59455
20	-0.4505	-0.4410	-0.4030	-0.3661	-0.3569	0.33513	0.35008	0.41122	0.44753	0.45698
21	-0.2315	-0.2225	-0.1863	-0.1376	-0.1254	0.26775	0.28035	0.33554	0.39797	0.41374
22	-0.3041	-0.2972	-0.2700	-0.2287	-0.2183	0.57273	0.58624	0.64194	0.70240	0.71759
23	0.2642	0.2770	0.3282	0.3773	0.3896	0.55055	0.56609	0.62828	0.67853	0.69110
24	0.2010	0.2098	0.2450	0.2849	0.2949	0.36704	0.37649	0.41430	0.43402	0.43895
25	0.6128	0.6227	0.6622	0.7163	0.7298	0.49201	0.50772	0.57162	0.63391	0.64980

In order to be able to compare IT2TFN $\bar{x}$-s control charts with $\bar{x}$-s traditional control charts, it is of utmost importance that both the fuzzified values and the control limits of and s are defuzzied, according to Eq.(3). In addition to maintaining the same visual appearance it is possible to understand the effects of the fuzzification and de-fuzzification processes.

Based on the data presented in Table 2, one can calculate the control limits LCL, CL and UCL for the IT2TFN $\bar{x}$ control chart by through the Eq.(18), Eq.(19) and Eq. (20). Similarly, to calculate the control limits of the IT2TFN s control chart, the Eq.(21), Eq.(22) and Eq.(23). The results can be seen in Table 3.

Table 3. Results for IT2TFN $\bar{x}$-s control charts process monitoring.

Sample Number	$\bar{X}_{TriT}$	S_{TriT}	Decision	Sample Number	$\bar{X}_{TriT}$	S_{TriT}	Decision
1	0.02125	0.6415	In Control	14	-0.17365	0.53811	In Control
2	0.17545	0.6312	In Control	15	0.5793	0.55102	In Control
3	0.68583	0.54956	In Control	16	-0.15616	0.50476	In Control
4	0.66106	0.88793	In Control	17	0.00512	0.98446	In Control
5	0.60569	0.45415	In Control	18	0.28833	1.08304	In Control
6	-0.47843	0.96061	In Control	19	0.31875	0.5536	In Control
7	0.20265	0.64906	In Control	20	-0.40342	0.40203	In Control
8	-0.06118	0.64279	In Control	21	-0.18159	0.33848	In Control
9	-0.17578	0.98703	In Control	22	-0.26471	0.64381	In Control
10	0.33347	0.58748	In Control	23	0.32741	0.6238	In Control
11	-0.28567	0.85376	In Control	24	0.24676	0.40751	In Control
12	0.30049	0.78261	In Control	25	0.66768	0.57111	In Control
13	-0.05273	0.86802	In Control				

To calculate the control limits of IT2TFN $\bar{x}$ control chart, we consider parameters A_3 = 1.427 and $H(\tilde{A}^L)$ = 1, obtaining LCL = -0.8257, CL = 0.1274 and UCL = 1.0805. Similarly, considering B_3 = 0 and B_4 = 2.089 and $H(\tilde{A}^L)$ = 1, we calculate the control limits of the s control chart, obtaining LCL = 0.0000, CL = 0.6679 and UCL = 1.3952.

It can be seen from Table 3, that the control limit values for the IT2TFN $\bar{x}$-s control charts are numerically smaller than the control limit values for the graphs. This is due to the fact that the mathematical properties of fuzzy type-2 influence the calculation of standard deviations s_1^U, s_1^L, s_2^U, s_3^L and s_3^U, causing the value reduce. Besides the insertion of uncertainties occurs also a stabilization of data, which reduces the number of false alarms that the control chart can illustrate.

Visually the IT2TFN x-s control charts can be illustrated by Figure 3, with the IT-2TFN $\bar{x}$ control chart positioned on the left and the IT2TFN s control chart positioned on the right.

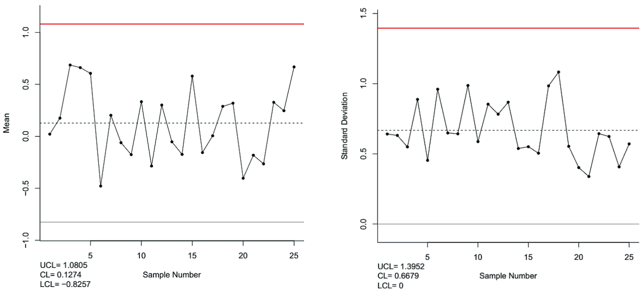

Figure 3 - IT2TFN $\bar{x}$ control chart (mean) and IT2TFN s control chart (standard deviation). Source: RStudio

Thus, it is verified that the IT2TFN $\bar{x}$-s control charts can be implemented to any production process that has several sources of uncertainty and human subjectivities involved, inserting uncertainties and, in this case, stabilizing the crisp data obtained by the measurement system. The narrowing of the control limits in relation to the traditional $\bar{x}$-s control chart, makes there is a little more sensitivity to variations in the process and the flexibility of the model is given by the choice of the L_1 and L_2 bounds, the footprint of uncertainty (FOU) in the fuzzification process and the $H(\tilde{A}^L)$ value in the defuzzification process, which adds the human factor to the model.

5. Conclusion

This paper aimed to propose a process monitoring model using IT2TFN $\bar{x}$-s control charts. We can conclude that the proposed model has advantages over the traditional $\bar{x}$-s control charts, since it is able to stabilize the input data when they are subject to uncertainty and subjectivity in decision-making by experts, which reduces the occurrence of false alarms, slightly increases the detection sensitivity to variations in the process and adds more information to the control chart through the parameters used in the methods of fuzzification and defuzzification presented.

On the other hand, the model complexity is reduced by applying interval type-2 fuzzy sets and by the equations of control limits presented, which makes its application feasible in various production systems. The visualization of IT2TFN $\bar{x}$-s control charts is exactly like traditional control charts, which assists in the interpretation of decisions about the process.

For future research it is suggested a performance analysis of the proposed control charts and the application to a real process.

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