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Intuitionistic fuzzy $\overline{X}$-R control charts based on IF-WABL defuzzification method

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Abstract. Control charts are one of the most important tools used for process monitoring. Traditional control charts use crisp data, but in the real world the processes to be monitored have uncertainties, due to human subjectivity, measurement instruments among other factors. In this case Shewhart charts may not be adequate and fuzzy control charts should be used. In this paper $\bar{X}$-R fuzzy control chart for triangular intuitionistic fuzzy numbers (TIFNs) using Intuitionistic Fuzzy - Weighted Averaging Based on Levels (IFWABL) defuzzification method is proposed. An interesting aspect in the defuzzification method that is considered, which differs from others, is the property of flexibility. The method is considered flexible because users can adjust the coefficients according to the problem being solved. Therefore, it is possible to obtain different defuzzification values for the same triangular intuitionistic fuzzy number. This is completely in accordance with the principles of fuzzy set theory. Intuitionistic fuzzy control charts are able to represent both uncertainty and hesitation present at various stages of the process. An illustrative example is developed in order to understand and show the use of the proposed control chart and finally the results are interpreted.

Keywords: control charts, intuitionistic fuzzy number, uncertainty.

1. Introduction

Control charts, developed by Shewhart [1], have as their main objective to control process variability. Due to its easy implementation and interpretation, it can be applied in many areas to improve the productivity and quality of production processes.

When the data involves a certain level of uncertainty and hesitation coming from both human judgments and measuring instruments the traditional Shewhart control charts may not be as effective for monitoring the process, since they are constructed using precise data. For this purpose, intuitionistic fuzzy control charts should be used.

The intuitionistic fuzzy set is a way to represent not only the uncertainty in the data, but also the hesitation of the decision makers. According to existing studies, using this set allows quality engineers to represent decision making hesitation in control charts [2].

Regarding intuitionistic fuzzy control charts, three studies are found in the literature, [3] obtained the c-control chart for trapezoidal intuitionistic fuzzy numbers and performed a comparison of defuzzification methods, [4] developed a control chart for trapezoidal intuitionistic fuzzy numbers using the defuzzification and probability methods and [5], proposed a type of intuitionistic fuzzy control chart using a ranking method for triangular intuitionistic fuzzy numbers.

The aim and innovation of this study is to introduce intuitionistic fuzzy $\bar{X}$-R control charts for triangular intuitionistic fuzzy number (TIFNs), using the Intuitionistic Fuzzy - Weighted Averaging Based on Levels (IF-WABL) defuzzification method proposed by [6].

2. Intuitionistic fuzzy sets

In this section, basic definitions about triangular intuitionistic fuzzy sets, fuzzification of crisp observations, and finally the defuzzification method used for deciding the control state of the proposed control chart are given.

2.1. Intuitionistic fuzzy definitions

In the real world there may be circumstances where an object belongs to a set to a certain degree, but it is also possible that one is not so sure about this. There may be a hesitation about the degree of membership. In classical fuzzy sets there is no possibility to include such hesitation. One can say that intuitionistic fuzzy sets are a generalization of classical fuzzy sets.

Developed by [7], the intuitionistic fuzzy set considers the degree of the membership and the degree of non-membership. It can be represented by Eq. (1).

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \upsilon_{\tilde{A}}(x)\rangle\}, x \in E, \mu_{\tilde{A}}: E \rightarrow [0,1] \text{ e } \upsilon_{\tilde{A}} \rightarrow [0,1] \quad (1)$$

where $\mu_{\tilde{A}}$ is defined as the degree of the membership and $\upsilon_{\tilde{A}}$ as the degree of the non-membership. For each x, the sum of the membership and non-membership degrees must be between 0 and 1 and can be expressed as $0 \leq \mu_{\tilde{A}} + \upsilon_{\tilde{A}} \leq 1$.

The degree of hesitancy of the intuitionistic fuzzy set $\tilde{A}$ can be represented by Eq. (2).

$$\pi_{\tilde{A}} = 1 - \mu_{\tilde{A}} - \upsilon_{\tilde{A}} \quad (2)$$

TIFN can be described by Eq. (3).

$$\tilde{A} = (a_1, a_2, a_3,; a'_1, a_2, a'_3) \quad (3)$$

Its membership function and non-membership function is represented according to Eq. (4) and Eq. (5) respectively.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & if\ a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2} & if\ a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases} \quad (4)$$

$$\upsilon_{\tilde{A}}(x) = \begin{cases} \frac{a_2 - x}{a_2 - a'_1} & if\ a'_1 \leq x \leq a_2 \\ \frac{x - a_2}{a'_3 - a_2} & if\ a_2 \leq x \leq a_3 \\ 1 & otherwise \end{cases} \quad (5)$$

where $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$. Figure 1 illustrates a representation of the triangular intuitionistic fuzzy number.

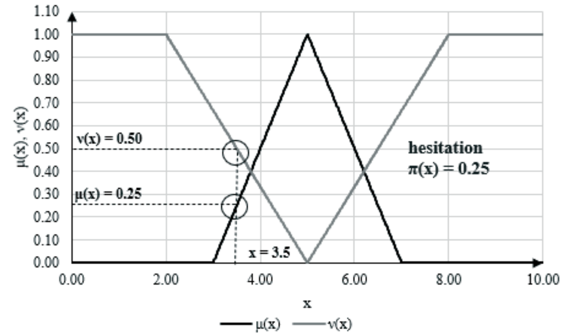

Figure 1 Triangular intuitionistic fuzzy number

2.2. Fuzzification Method

In this paper, the fuzzifications of the crisp observations for triangular intuitionistic fuzzy number can be obtained using Eq. (6), Eq. (7), Eq. (8), Eq. (9) and Eq. (10).

$$a_{1j} = a_{2j} - U \times runif_j \quad (6)$$

$$a_{3j} = a_{2j} - U \times runif_j \quad (7)$$

$$a_{2j} = a_{2j} \quad (8)$$

$$a'_{1j} = a'_{1j} - U \times runif_j \quad (9)$$

$$a'_{3j} = a'_{3j} - U \times runif_j \quad (10)$$

where is given by the expanded uncertainty and is a random number generated according to a uniform distribution.

The expanded uncertainty of direct measurements can be obtained by multiplying the combined uncertainty by the coverage factor k. The combined uncertainty is given by the quadratic sum of all uncertainties involved in the measurement process. In this work we considered uncertainties of type A and the resolution of the instrument as type B uncertainty. For the coverage factor k, the value corresponding to the number of effective degrees of freedom with 95.45% probability was adopted [8].

An instrument resolution of 0.001 mm was established in this work in view of the standard of the measurements presented. For calculation of the type A uncertainty an experimental standard deviation of 0.0005 mm between repeated measurements was established. It was considered that five repeated measurements were made until the value of the experimental standard deviation was obtained.

2.3. Defuzzification Method

The proposed defuzzification method is very useful for obtaining a defuzzified value for intuitionistic fuzzy numbers. The main idea of the Intuitionistic Fuzzy - Weighted Averaging Based on Levels (IF-WABL) method is to insert all the information contained in the intuitionistic fuzzy numbers into the defuzzification. In order to decide whether the process is in control or out of control, the defuzzification method proposed by [6] was used.

The method is considered flexible since the coefficients can be adjusted according to the needs of the problems to be solved by the users. Each decision maker can customize the IF-WABL method according to the parameter values appropriate to their problem.

Let $\tilde{A} = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ be a TIFN. The IF-WABL method in its essence can be defined by Eq. (11).

$$\begin{aligned} &\mathrm{IF-WABL}(\tilde{A})\ \theta\left[(c_R a_3 + c_L a_1)\left(1 - E(\alpha)\right) + a_2 E(\alpha)\right] \\ &+ (1-\theta)\left[a_2\left(1 - E(\beta)\right) + (c_L a'_1 + c_R a'_3)E(\beta)\right] \end{aligned} \quad (11)$$

where $\theta, c_L, c_R, p(\alpha), s(\beta)$ are IF-WABL coefficients and $E(\alpha) = \int_0^1 \alpha p(\alpha) d\alpha$, $E(\beta) = \int_0^1 \beta s(\beta) d\beta$.

Some conditions in relation to normality and non-negativity must be respected when the coefficients are chosen by the user of the method.

$$\theta \in [0,1], c_L, c_R \geq 0, c_L + c_R = 1, \int_0^1 p(\alpha) d\alpha = 1, \int_0^1 s(\beta) d\beta = 1$$

The coefficient θ shows the user's preference between the membership function and the non-membership function. The coefficients C_L and C_R are scalars that represent weights for the left and right sides for both the membership function and the non-membership function. The functions p(α) and s(β) are proposed by [6] as a way to assign weights to α-levels and β-levels respectively.

In this paper in order to give equal importance to both membership and non-membership functions used the value of θ equal to 0.5. In relation to C_L and C_R was used $C_L = 0.4$ and $C_R = 0.6$. According to [6] was adopted in this study $p(\alpha) = 2\alpha$ and $s(\beta) = 1$.

3. Intuitionistic fuzzy $\overline{\mathrm{X}}$-R control charts

In this section, definitions about the intuitionistic fuzzy $\overline{X}$-R control chart for TIFNs are presented.

3.1. Intuitionistic $\overline{\mathrm{X}}$-R control charts for triangular fuzzy intuitionistic number

The upper limit (UCL), center line (CL) and lower limit (LCL) for the intuitionistic fuzzy $\overline{\mathrm{X}}$ control chart for TIFNs can be obtained by Eq. (12). Eq. (13) and Eq. (14).

$$\widetilde{UCL_{\overline{X}}} = \left(\overline{\overline{X}}_{a1} + A_2\overline{R}_{a1}, \overline{\overline{X}}_{a2} + A_2\overline{R}_{a2}, \overline{\overline{X}}_{a3} + A_2\overline{R}_{a3}; \overline{\overline{X}}_{a'1} + A_2\overline{R}_{a'1}, \overline{\overline{X}}_{a2} + A_2\overline{R}_{a2}, \overline{\overline{X}}_{a'3} + A_2\overline{R}_{a'3}\right) \quad (12)$$

$$\widetilde{CL_{\overline{X}}} = \left(\overline{\overline{X}}_{a1}, \overline{\overline{X}}_{a2}, \overline{\overline{X}}_{a3}; \overline{\overline{X}}_{a'1}, \overline{\overline{X}}_{a2}, \overline{\overline{X}}_{a'3}\right) \quad (13)$$

$$\widetilde{LCL_{\overline{X}}} = \left(\overline{\overline{X}}_{a1} - A_2\overline{R}_{a3}, \overline{\overline{X}}_{a2} - A_2\overline{R}_{a2}, \overline{\overline{X}}_{a3} - A_2\overline{R}_{a1}; \overline{\overline{X}}_{a'1} - A_2\overline{R}_{a'3}, \overline{\overline{X}}_{a2} - A_2\overline{R}_{a2}, \overline{\overline{X}}_{a'3} - A_2\overline{R}_{a'1}\right) \quad (14)$$

where $\overline{\overline{X}}$ expresses the means of the mean of the TIFN observations and can be calculated by Eq. (15), Eq. (16), Eq. (17), Eq. (18) and Eq. (19).

$$\overline{\overline{X}}_{a1} = \frac{\sum_{i=1}^{m} \overline{X}_{a1}}{m} \quad (15)$$

$$\overline{\overline{X}}_{a2} = \frac{\sum_{i=1}^{m} \overline{X}_{a2}}{m} \quad (16)$$

$$\overline{\overline{X}}_{a'1} = \frac{\sum_{i=1}^{m} \overline{X}_{a'1}}{m} \quad (17)$$

$$\overline{\overline{X}}_{a'1} = \frac{\sum_{i=1}^{m} \overline{X}_{a'1}}{m} \quad (18)$$

$$\overline{\overline{X}}_{a'3} = \frac{\sum_{i=1}^{m} \overline{X}_{a'3}}{m} \quad (19)$$

The range can be obtained by Eq. (20), Eq. (21), Eq. (22), Eq. (23) and Eq. (24).

$$R_{a1} = X_{max\ a1} - X_{min\ a3} \quad (20)$$

$$R_{a2} = X_{max\ a2} - X_{min\ a2} \quad (21)$$

$$R_{a3} = X_{max\ a3} - X_{min\ a1} \quad (22)$$

$$R_{a'1} = X_{max\ a'3} - X_{min\ a'1} \quad (23)$$

$$R_{a'3} = X_{max\ a'3} - X_{min\ a'1} \quad (24)$$

where ($X_{max\ a1}$, $X_{max\ a2}$, $X_{max\ a3}$; $X_{max\ a'1}$, $X_{max\ a'3}$), is the maximum intuitionistic fuzzy number of the sample and ($X_{min\ a1}$, $X_{min\ a2}$, $X_{min\ a3}$; $X_{min\ a'1}$, $X_{min\ a'3}$) is the minimum intuitionistic fuzzy number of the sample.

The mean of range can be calculated by Eq. (25), Eq. (26), Eq. (27), Eq. (28) and Eq. (29).

$$\overline{R}_{a1} = \frac{\sum_{i=1}^{m} R_{a1}}{m} \quad (25)$$

$$\overline{R}_{a2} = \frac{\sum_{i=1}^{m} R_{a2}}{m} \quad (26)$$

$$\overline{R}_{a3} = \frac{\sum_{i=1}^{m} R_{a3}}{m} \quad (27)$$

$$\overline{R}_{a'1} = \frac{\sum_{i=1}^{m} R_{a'1}}{m} \quad (28)$$

$$\overline{R}_{a'3} = \frac{\sum_{i=1}^{m} R_{a'3}}{m} \quad (29)$$

The upper limit (UCL), center line (CL) and lower limit (LCL) for the intuitionistic fuzzy R control chart for TIFNs can be obtained by Eq. (30), Eq. (31) and Eq. (32).

$$\widetilde{UCL_R} = (D_4\overline{R}_{a1}, D_4\overline{R}_{a2}, D_4\overline{R}_{a3}; D_4\overline{R}_{a'1}, D_4\overline{R}_{a2}, D_4\overline{R}_{a'3}) \quad (30)$$

$$\widetilde{CL_R} = (\overline{R}_{a1}, \overline{R}_{a2}, \overline{R}_{a3}; \overline{R}_{a'1}, \overline{R}_{a2}, \overline{R}_{a'3}) \quad (31)$$

$$\widetilde{LCL_R} = (D_3\overline{R}_{a1}, D_3\overline{R}_{a2}, D_3\overline{R}_{a3}; D_3\overline{R}_{a'1}, D_3\overline{R}_{a2}, D_3\overline{R}_{a'3}) \quad (32)$$

4. Illustrative example

In this section, to illustrate the construction of an intuitionistic fuzzy control chart -R we used the application example according to [9] which has measurements of the internal diameters (mm) of automobile engine piston rings. Table 1 illustrates the data.

Table 1. Measurements of the internal diameters.

Sample number	X_1	X_2	X_3	X_4	X_5	Sample number	X_1	X_2	X_3	X_4	X_5
1	74.030	74.002	74.019	73.992	74.008	14	74.006	73.967	73.994	74.000	73.984
2	73.995	73.992	74.001	74.011	74.004	15	74.012	74.014	73.998	73.999	74.007
3	73.988	74.024	74.021	74.005	74.002	16	74.000	73.984	74.005	73.998	73.996
4	74.002	73.996	73.993	74.015	74.009	17	73.994	74.012	73.986	74.005	74.007
5	73.992	74.007	74.015	73.989	74.014	18	74.006	74.010	74.018	74.003	74.000
6	74.009	73.994	73.997	73.985	73.993	19	73.984	74.002	74.003	74.005	73.997
7	73.995	74.006	73.994	74.000	74.005	20	74.000	74.010	74.013	74.020	74.003
8	73.985	74.003	73.993	74.015	73.988	21	73.982	74.001	74.015	74.005	73.996

Sample number	X_1	X_2	X_3	X_4	X_5	Sample number	X_1	X_2	X_3	X_4	X_5
9	74.008	73.995	74.009	74.005	74.004	22	74.004	73.999	73.990	74.006	74.009
10	73.998	74.000	73.990	74.007	73.995	23	74.010	73.989	73.990	74.009	74.014
11	73.994	73.998	73.994	73.995	73.990	24	74.015	74.008	73.993	74.000	74.010
12	74.004	74.000	74.007	74.000	73.996	25	73.982	73.984	73.995	74.017	74.013
13	73.983	74.002	73.998	73.997	74.012	-	-	-	-	-	-

For the fuzzification process expanded uncertainty of U=0.0008 mm was obtained as explained in the previous section, which was used to construct the TIFNs.

The upper limit (UCL), center line (CL) and lower limit (LCL) for the intuitionistic fuzzy control chart R are calculated using Eq. (30), Eq. (31) and Eq. (32).

$$\widetilde{UCL_R} = (0.0475, 0.0491, 0.0507; 0.0458, 0.0491, 0.0522)$$

$$\widetilde{CL_R} = (0.0224, 0.0232, 0.0240; 0.0217, 0.0232, 0.0247)$$

$$\widetilde{LCL_R} = (0, 0, 0; 0, 0, 0)$$

Subsequently the IF-WABL defuzzification method given by Eq. (11) is used to defuzzify the control limits. The defuzzified control limits can be given by:

$$UCL_R = 0.0493$$

$$CL_R = 0.0233$$

$$LCL_R = 0$$

The ranges were calculated and then defuzzified using Eq. (11), can be seen in the Table 2. Figure 2 illustrates the intuitionistic fuzzy control chart R. It is observed that the process is under statistical control considering the valid control limits for monitoring future processes.

Table 2. Range defuzzified.

Sample number	R defuzzified	Sample number	R defuzzified
1	0.0380	14	0.0391
2	0.0190	15	0.0161
3	0.0362	16	0.0210
4	0.0220	17	0.0260
5	0.0261	18	0.0182
6	0.0241	19	0.0212
7	0.0120	20	0.0199
8	0.0299	21	0.0331

Sample number	R defuzzified	Sample number	R defuzzified
9	0.0140	22	0.0190
10	0.0171	23	0.0250
11	0.0081	24	0.0221
12	0.0112	25	0.0354
13	0.0290	-	-

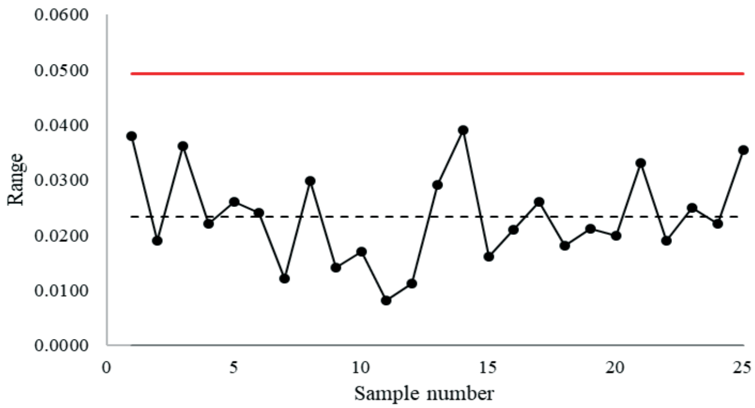

Figure 2 Intuitionistic R control chart

The upper limit (UCL), center line (CL) and lower limit (LCL) for the intuitionistic fuzzy control chart are calculated using Eq. (12), Eq. (13) and Eq. (14).

$$\widetilde{UCL_{\bar{X}}} = (74.0137, 74.0146\ 74.0154; 74.0129, 74.0146, 74.0162)$$

$$\widetilde{CL_{\bar{X}}} = (73.0008, 74.0012, 74.0016; 74.0004, 74.0012, 74.0020)$$

$$\widetilde{LCL_{\bar{X}}} = (73.9869, 73.9878, 73.9886; 73.9861, 73.9878, 73.9895)$$

Subsequently the IF-WABL defuzzification method given by Eq. (11) is used to defuzzify the control limits. The defuzzified control limits can be given by:

$$UCL_{\bar{X}} = 74.0147$$

$$CL_{\bar{X}} = 74.0012$$

$$LCL_{\bar{X}} = 73.9879$$

The means were calculated and then defuzzified using Eq. (11) can be seen in the Table 3. Figure 3 illustrates the intuitionistic fuzzy control chart . It is observed that the process is under statistical control considering the valid control limits for monitoring future processes.

Table 3. Mean defuzzied.

Sample number	defuzzified	Sample number	defuzzified
1	74.0102	14	73.9902
2	74.0007	15	74.0061
3	74.0081	16	73.9967
4	74.0030	17	74.0009
5	74.0034	18	74.0074
6	73.9956	19	73.9982
7	74.0001	20	74.0093
8	73.9968	21	73.9998
9	74.0043	22	74.0017
10	73.9980	23	74.0025
11	73.9943	24	74.0052
12	74.0015	25	73.9982
13	73.9985	-	-

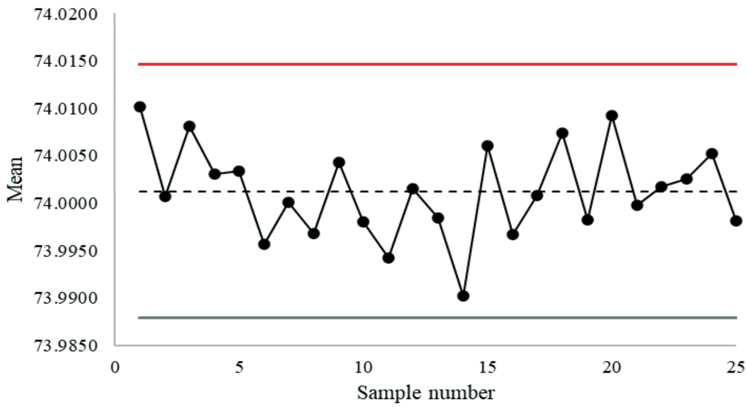

Figure 3 Intuitionistic control chart

5. Conclusion

In this study a new intuitionistic fuzzy $\overline{X}$-R control chart for triangular intuitionistic fuzzy numbers based on the IF-WABL defuzzification method has been developed. The proposed intuitionistic fuzzy control chart is interesting since it considers the uncertainty of the measurements and the hesitation of the operator at the time of measurement and the expert when deciding the control state of the process.

The fuzzification of observations using the expanded uncertainty as a limiting factor to perform the conversion of crisp observations in intuitionistic triangular fuzzy numbers comes as a way to facilitate and make the monitoring of processes closer to the reality experienced in organizations.

The defuzzification method used in this study is very useful for obtaining a representative value for intuitionistic fuzzy numbers. Its use is important for maintaining the standard format of the control charts and facilitating the plotting of sample observations.

As a result of the numerical example, it was observed that the proposed chart is effective for monitoring the process under the conditions described.

In future studies it is suggested to analyze the performance of the charts developed here by means of measures available in the literature such as Average Run Length (ARL), Extra Quadratic Less (EQL) and Standard Deviation Run Length (SDRL) among others.

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