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Changing and Unchanging the Geodetic Number: Edge Removal

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Abstract

Let S be a collection of elements in a vertex set V. If every vertex in a graph G falls on a geodesic connecting two vertices from S, then that graph is said to be a geodesic graph. g(G) is the smallest cardinality of the geodesic subset of a graph G and is known as the geodetic number. This study investigates how the removal of an edge affects the geodetic number of some unique families.

Keywords: Geodesic set, Geodetic number.

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1. Introduction

For a detailed examination of the geodetic number, see Chartrand et al.'s initial introduction of the number in [3]. What we mean by a graph is a non-trivial, finite, undirected, linked graph with many edges and no loops. As usual, V(G) represents the vertices of a graph and E represents the edges of a graph. The symbol for the edge between two vertices u and v is (u, v). The length of the shortest u-v path through a connected graph G is the distance of a vertex v from a vertex u and is denoted by d (u, v). between the vertex u and the vertex v. The minimum distance between the vertices u and v is known as the geodesic distance. The criticality and stability of the geodetic number of some particular families of graphs are being studied for the first

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time. Let us partition the edges of a graph G into four sets according to how their removal affects $g(G)$. Let $Let\ E(G) = E^{-}_{g}(G) \cup E_{g}^{0}(G) \cup E_{g}^{+}(G) \cup S_{p}(G)$

Definition 1.1 The set *S*, which is a subset of vertices *V(G)* of a graph *G* is called a geodesic set if every vertex in *G* is in a geodesic path connecting two vertices of *S*. The geodetic number of a graph *G* is the number of elements in a minimum geodetic set in *G*, denoted by *g* (*G*).

Definition 1.5 The Wagner graph has twelve edges and eight vertices, making it a three-regular graph.

Definition 1.6 For an integer n (≥3), the graph with the vertex set {(xi,xj): 0≤i,j≤n-1,i≠j} is known as the Crown graph.

Definition 1.7 Two rows of paired nodes make up a Cocktail Party graph, where all nodes other than the paired ones are connected by straight lines.

Definition 1.8 The construction of the Circular Ladder graph CLn involves either a straight connection between the four 2-degree vertices or the Cartesian product of an edge and a cycle of length n (≥3).

Definition 1.9 The Franklin graph has 18 edges and 12 vertices, making it a 3-regular graph.

Theorem 1.7 [8] states that g(G)=4 for the Wagner graph.

Theorem 1.8 [7] Considering the crown graph G, *g* (*G*) =2.

Theorem 1.9 [8] For the circular ladder graph G, *g* (*G*) =3.

2. Effects of Removal of an Edge on g(G), in Some Particular Families of Graphs

In this section, we investigate the effects of the removal of an edge on g(G) for some families of graphs like Wagner graph, crown graph, circular ladder graph, Franklin graph and cocktail party graph.

Theorem 2.1 For the Wagner graph G, $E(G) = E^{-}_{g}(G) \cup E_{g}^{0}(G)$

Proof.

Case i: An edge is incident with succeeding vertex. For every e in E(G), the geodesic set of G-{e_i} will be the set {v_{i-1},v_i,v_{i+1}}, where i = 1 to 8.

For $e \in E(G)$, the set {v_{i-1},v_i,v_{i+1}}. Hence $g(G$-$\{e_{\_i}\}) < g(G)$ and so $E(G) = E^{-}_{g}(G)$.

Case ii: An edge is not incident with succeeding vertex.

For e∈E(G), the set {v_i,v_{i+2},v_{i+4},v_{i+6}} where i= 1 and 2 will serve as a geodesic set of G-{e_i}.. Hence $g(G$-$\{e_i\}) = g(G)$ and so $E(G)= E_g^{\,0}(G)$.

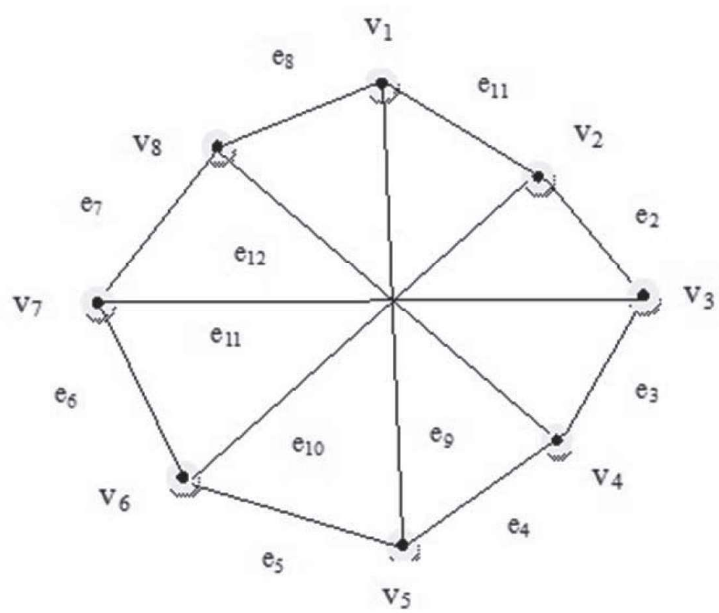

Figure 2.1. Wagner graph

Theorem 2.2 For the Crown graph G, $E(G)=E_g^{\,0}(G)$

Proof. Label the vertices and edges of G as shown in Figure 2.2. By Theorem 1.8, $g(G)$=2. Further $S_1=\{v_1,u_1\}$, $S_2=\{v_2,u_2\}$,$S_3=\{v_3,u_3\}$, $S_4=\{v_4,u_4\}$ *and* $S_5=\{v_5,u_5\}$ will be geodesic sets of G. Also, removal of any one of the edges from G does not increase the minimum geodetic number of G refer Table 2.1. Thus $E(G)=E_g^{\,0}(G)$.

Edge set	Geodesic set	Edge set	Geodesic set
$E-\{e_1\}$	S_3	$E-\{e_{11}\}$	S_2
$E-\{e_2\}$	S_2	$E-\{e_{12}\}$	S_1
$E-\{e_3\}$	S_1	$E-\{e_{13}\}$	S_1
$E-\{e_4\}$	S_3	$E-\{e_{14}\}$	S_4
$E-\{e_5\}$	S_4	$E-\{e_{15}\}$	S_1
$E-\{e_6\}$	S_1	$E-\{e_{16}\}$	S_1
$E-\{e_7\}$	S_5	$E-\{e_{17}\}$	S_5
$E-\{e_8\}$	S_1	$E-\{e_{18}\}$	S_1
$E-\{e_9\}$	S_2	$E-\{e_{19}\}$	S_4
$E-\{e_{10}\}$	S_1	$E-\{e_{20}\}$	S_5

Table 2.1

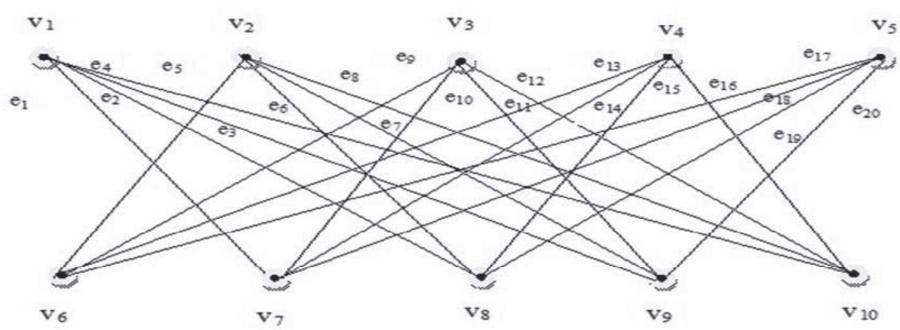

Figure 2.2 Crown graph

Theorem 2.3 For the graph G of the cocktail party, $E(G)=E_g^-(G)\cup E_g^0(G)$

Proof. Label the vertices and edges of G as shown in Figure 2.3. Clearly $g(G)=4$. Here, removal of any one of the vertices from G does not increase the geodesic number of G refer Table 2.2. Thus $E(G)=E_g^-(G)\cup E_g^0(G)$.

Edge set	Geodesic set	Edge set	Geodesic set
$E-\{e_1\}$	$\{v_1,v_2,v_4\}$	$E-\{e_{11}\}$	$\{v_1,v_3,v_5,v_7\}$
$E-\{e_2\}$	$\{v_1,v_3,v_4,v_6\}$	$E-\{e_{12}\}$	$\{v_1,v_3,v_5,v_7\}$
$E-\{e_3\}$	$\{v_1,v_3,v_4\}$	$E-\{e_{13}\}$	$\{v_2,v_4,v_6,v_8\}$
$E-\{e_4\}$	$\{v_5,v_6,v_8\}$	$E-\{e_{14}\}$	$\{v_1,v_3,v_5,v_7\}$

Edge set	Geodesic set	Edge set	Geodesic set
$E-\{e_5\}$	$\{v_2,v_4,v_5,v_8\}$	$E-\{e_{15}\}$	$\{v_1,v_4,v_5,v_8\}$
$E-\{e_6\}$	$\{v_5,v_7,v_8\}$	$E-\{e_{16}\}$	$\{v_1,v_3,v_5,v_7\}$
$E-\{e_7\}$	$\{v_1,v_3,v_4,v_7\}$	$E-\{e_{17}\}$	$\{v_2,v_4,v_6,v_8\}$
$E-\{e_8\}$	$\{v_2,v_4,v_6,v_8\}$	$E-\{e_{18}\}$	$\{v_1,v_4,v_5,v_8\}$
$E-\{e_9\}$	$\{v_2,v_4,v_6,v_8\}$		$\{v_2,v_4,v_6,v_8\}$

Table 2.2

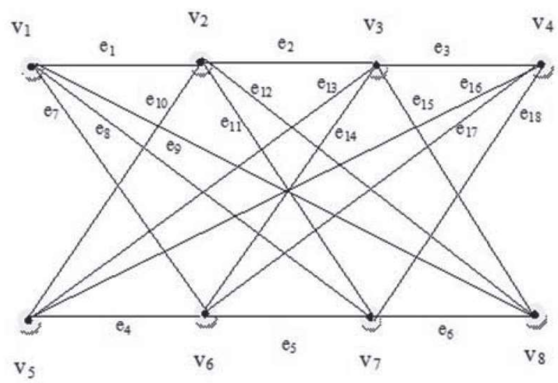

Figure 2.3 Cocktail party graph

Theorem 2.4. G is the circular ladder graph for the, $E(G)=E_g^0(G)$.

Proof. Label the vertices and edges of G as shown in Figure 2.4. By Theorem 1.9. $g(G)$=3. Further $S_1=\{v_1,u_3,u_4\}$,$S_2=\{v_2,u_4,u_5\}$,$S_3=\{v_3,u_1,u_5\}$,$S_4=\{v_4,u_1,u_2\}$,$S_5=\{v_5,u_2,u_3\}$,$S_6=\{u_1,v_3,v_4\}$,$S_7=\{u_2,v_4,v_5\}$,$S_8=\{u_3,v_1,v_5\}$,$S_9=u_4,v_1,v_2$,$S_{10}=\{u_5,v_2,v_3\}$will be geodesic sets of G. Also, removal of any one of the edges from G does not increase the geodetic number of G refer Table 2.3. Thus $E(G)=E_g^0(G)$

Edge set	Geodesic set	Edge set	Geodesic set
$E-\{e_1\}$	S_4	$E-\{e_9\}$	S_5
$E-\{e_2\}$	S_5	$E-\{e_{10}\}$	S_1
$E-\{e_3\}$	S_1	$E-\{e_{11}\}$	S_9
$E-\{e_4\}$	S_2	$E-\{e_{12}\}$	S_{10}
$E-\{e_5\}$	S_3	$E-\{e_{13}\}$	S_6
$E-\{e_6\}$	S_2	$E-\{e_{14}\}$	S_7
$E-\{e_7\}$	S_3	$E-\{e_{15}\}$	S_8

Edge set	Geodesic set	Edge set	Geodesic set
$E-\{e_8\}$	S_4		

Table 2.3

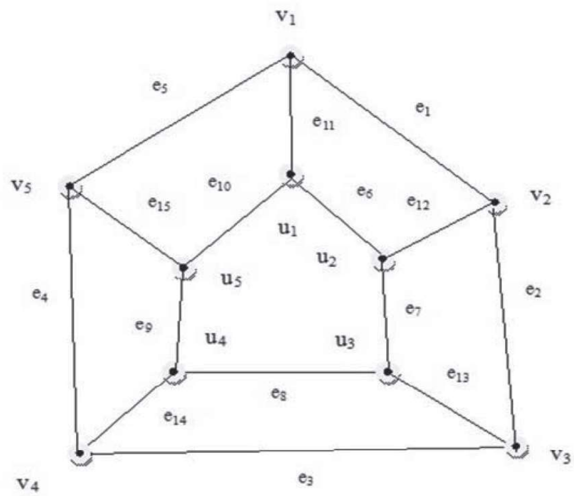

Figure 2.4. circular ladder graph

Theorem 2.5 For the Franklin graph G, $E(G)=E_g^-(G)\cup E_g^0(G)$.

Proof. Label the vertices and edges of G as shown in Figure 2.4. Clearly $g(G)=4$. Here, G's geodetic number remains unchanged if any one of its edges were removed, refer Table 2.4. Thus $E(G)=E_g^-(G)\cup E_g^0(G)$.

Edge set	Geodesic set	Edge set	Geodesic set
$E-\{e_1\}$	$\{v_1,v_2\}$	$E-\{e_{10}\}$	$\{v_3,v_7,v_{12}\}$
$E-\{e_2\}$	$\{v_3,v_6,v_8,v_9\}$	$E-\{e_{11}\}$	$\{v_4,v_7,v_8\}$
$E-\{e_3\}$	$\{v_3,v_4\}$	$E-\{e_{12}\}$	$\{v_5,v_8,v_9\}$
$E-\{e_4\}$	$\{v_2,v_5,v_{10},v_{11}\}$	$E-\{e_{13}\}$	$\{v_8,v_{11}\}$
$E-\{e_5\}$	$\{v_5,v_6\}$	$E-\{e_{14}\}$	$\{v_7,v_{10}\}$
$E-\{e_6\}$	$\{v_1,v_4,v_7,v_{12}\}$	$E-\{e_{15}\}$	$\{v_9,v_{12}\}$
$E-\{e_7\}$	$\{v_6,v_{10},v_{11}\}$	$E-\{e_{16}\}$	$\{v_2,v_7,v_{11}\}$
$E-\{e_8\}$	$\{v_1,v_{10},v_{11}\}$	$E-\{e_{17}\}$	$\{v_1,v_{10},v_{12}\}$
$E-\{e_9\}$	$\{v_1,v_7,v_{11}\}$	$E-\{e_{18}\}$	$\{v_6,v_9,v_{11}\}$

Table 2.4

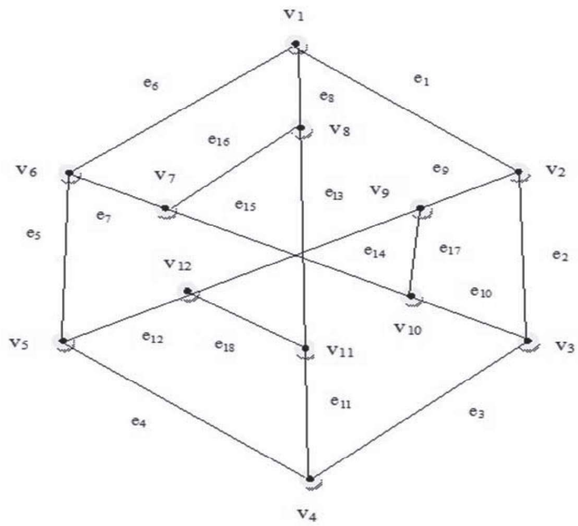

Figure 2.5 Franklin graph

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