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Identifying Similar Polygons: Comparing Prospective Teachers' Routines with a Mathematician's

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This paper reports two prospective teachers' and a mathematician's ways of identifying similar triangles and hexagons through the analysis of routines, a characteristic of geometric discourse. The findings show that visual recognition was a common approach for the mathematician as well as the two prospective teachers. However, when asked for justification, their routines of identifying similar polygons diverged. The paper also discusses the implication of classroom discourse practices to enhance prospective teachers' communication and reasoning skills while learning geometric concepts such as similarity.

Keywords: Geometry; Classroom Discourse; Teacher Knowledge

In geometry, similarity is an important concept connecting many other mathematical domains such as spatial reasoning, ratio, proportion, and transformation functions. *Similarity* is defined as a relation between figures. For example, similar figures in a plane can be seen as pre-images and images of dilations (similar transformation) that preserve their shape but not necessarily the size. Similarity is not only a cross-cutting concept in geometry for middle school students (CCSSM, 2014), but also a concept that has many applications in science and engineering (NGSS, 2011). Yet, for some teachers, similarity is still a difficult topic to teach because their prior geometry coursework made little mention of it. Therefore, it is important for prospective teachers (PSTs) to know with the topic and connect it with other concepts such as ratio and proportion. However, there is little known about PSTs' geometric thinking in the context of similarity in recent research. The purpose of the study is to address this gap.

Theoretical Framework

Sfard's (2008) framework is used to analyze participants' interviews. She has proposed that mathematical discourses differ one from another in at least four features: 1) *Word use* (mathematical vocabularies and their use), mathematical words that signify mathematical objects or process; 2) *Routines*, these are well-defined repetitive patterns characteristic of the given mathematical discourse; 3) *Visual mediator*s, these are symbolic artifacts related especially for particular communication; 4) *Endorsed narratives,* any text, spoken or written, which is framed as a description of objects, of relations between processes with or by objects, and which are subject to endorsement or rejection, that is, to be labeled as true or false. These features are interwoven with one another in a variety of ways. For example, endorsed narratives contain mathematical vocabularies and provide the context in which those words are used; mathematical routines are apparent in the use of visual mediators and produce narratives. Sfard's framework provides an analytical tool to investigate how thinking is communicated through interactions. To have a better understanding of PSTs and mathematician's geometric thinking and their ways of identifying similar polygons, this paper reports findings that address these two research questions: 1) what are PSTs' routines for identifying similar triangle and hexagons? 2) What are the mathematician's routines for identifying similar triangles and hexagons?

Method

Six PSTs who participated in the study were enrolled in a Midwestern university teacher education program in the US. The mathematician was a visiting professor at the university at the time of the study. The interviews were designed to investigate participants' geometric thinking in a 30-minute one-on-one interaction. In one task during the interviews, participants were asked to identify similar figures among fifty pre-selected geometric shapes. These shapes included triangles, quadrilaterals, hexagons, and circles; all mixed together on an 11" x 17" piece ofpaper (see Figure 1). As shown in Figure 1, all shapes on the paper are labeled with numbers from 1 to 50. Among them, some are similar figures, and some are congruent figures. All interview tasks are designed to elicit participants' thinking about similar polygons. For instance, in a task of identifying similar polygons, the interview questions such as "How do you know they are similar?" "What do you mean when you say 'they are the similar'?" were asked to capture participants' geometrical thinking in identifying similar figures. All participants were given the same tasks and were asked the same initial interview questions by the same researcher following the interview protocol. However, each interview was guided by individual participant's responses to the tasks and questions. All interviews were video recorded and transcribed. All transcripts document what participants said and did during the interviews.

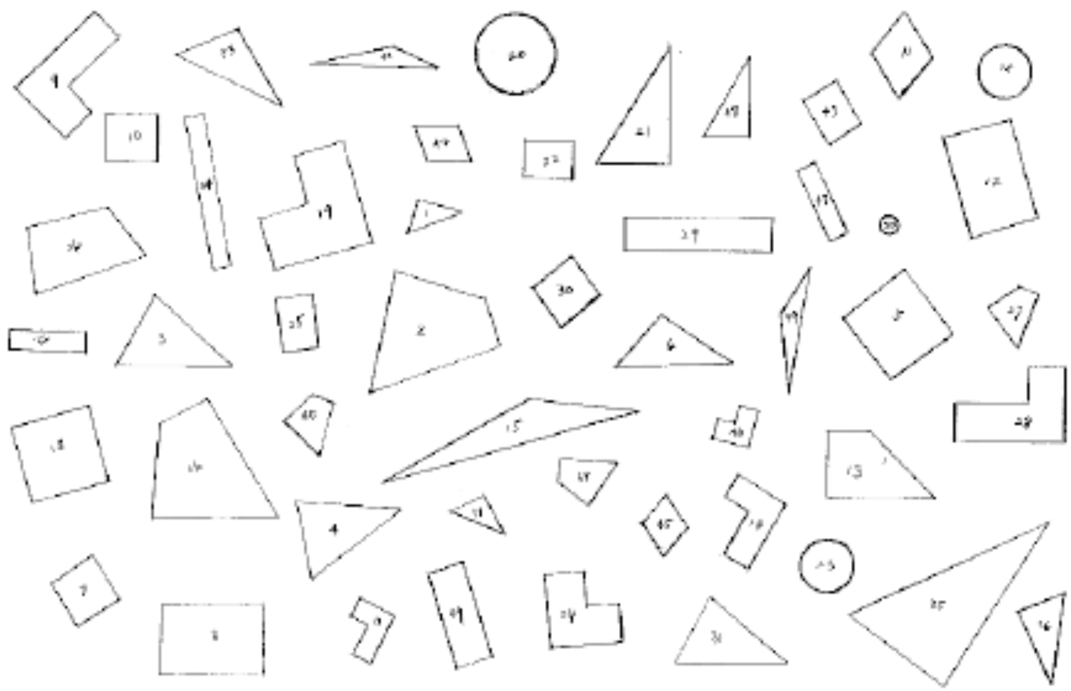


***Figure 1.* Identifying similar polygons**

Results

The results show the differences in PSTs' and mathematician's routines of identifying similar triangles and hexagons. In this paper, two PSTs' (PST1 and PST2) routines are used to illustrate these differences, and the three main differences are as follows:

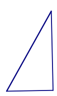
- The mathematician was aware of the abstraction of similar figures at the abstract level whereas PSTs only focused on using measurements to verify similar figures at the object level.

- The mathematician looked for two possibilities of ratios “internal versus external,” whereas the PSTs did not, but used one of them instead. That is, the mathematician explicitly explained the two possibilities of ratios, internal ratios and external rations and how they could fit together to determine similar figures.
- The mathematician considered different approaches when the first attempt did not work, whereas the PSTs refuted their conjectures and did not consider other possibilities that could show the polygons were similar.

Matching Similar Triangles

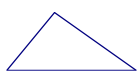
For this task, the participants were asked to identify all figures that are similar to a right triangle (#21), from the fifty figures given on the grid paper. The mathematician identified a scalene triangle (#33) as a similar triangle to this right triangle by direct recognition. When prompted by the researcher, “how do you know they are similar not based on visual?” the mathematician measured the sides of the trianglesand attempted to check for the ratio between them. She then measured the included angles of the two triangles to check for congruency. The conversation between the mathematician (M) and the researcher (I) is as follows:

1 I: Which following figures are similar to this [#21]?



#21

2M: Just looking visually, this one maybe [marked a circle on a triangle].

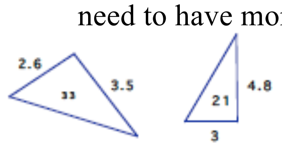


#33

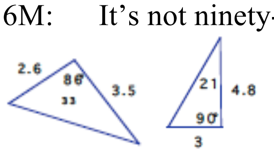
3I: If I would ask not just based on the visuals, what would you do?

4M: Can I do that [find measurements]? No, I can’t necessarily; the best I can do is that to show it definitely is not similar.

5 M: 2.6 centimetres. Is that 3.5 centimetres [#33]? I would say 3 and 4.8 on this one [#21]. I need to have more information about the angle.



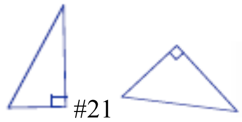
6M: It’s not ninety-degrees, maybe eighty-six. That one is ninety degrees



7M: They definitely are not similar.

The mathematician identified a triangle (#33) as a candidate, which was similar to a given right triangle (#21). After measuring the sides and angles of the two triangles, she refuted her claim and concluded, “They definitely are not similar” based on different angle measures. The mathematician chose the two sides and their included angle to check if the triangles were similar. The mathematician did not calculate the ratio of the sides but checked measurement of the included angle first. When she found the angle measure did not match, she made her claim of “they definitely are not similar.” In contrast to mathematician’s responses to the task, PST1’s routines procedures were different. She assumed the triangle (#21) was a right triangle by direct recognition. She identified another triangle (#37) that looked like a right triangle and concluded they were similar triangles:

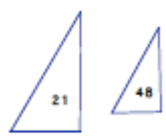
8I: Which following figures are similar to this figure [#21]?
9PST1: I am going to go with this triangle [#37] and marked right angle signs on both triangles.


#21 #37

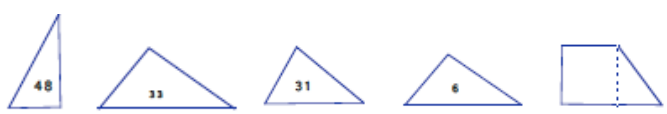
10I: How do you know this triangle[#37] is similar to this one [#21]?
11PST1: They both have a right angle.
12PST1: I am sure if you set up the proportions, they have the same angles.

PST1's routines procedures were direct recognition and it was self-evident. When prompted by the researcher, PST1 realized that two other corresponding angles of the two triangles were not congruent to each other and thenshe changed her claim about the two triangles were similar. Although PST1 did not say it explicitly, she used Angle-Angle-Angle (AAA) similarity criterion to check if the two triangles were similar. PST2's first reaction to same task was, "they should have same shape and same number of sides". That is she would consider all triangles were similar to each other:

13I: Which following figures are similar to this triangle [# 21]?
14 PST2: That is similar to it? It would be…number forty-eight.



15I: Why do you think they are similar?
16PST2: Because they have the same shape and same number of sides.
17I: Can you identify all the figures that are similar to this one [#21]?
18PST2: Circle them all?

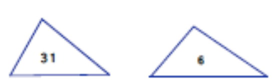


#13

19PST2: Would that be considered a triangle if I divide a shape [#13]?
20I: Maybe not. We only focus on same number of sides like you said earlier.

PST2 identified triangle #48 as a similar triangle to #21, which was correct. However, her reasoning of why the two triangles are similar was incomplete. The following excerpt [18] showed that PST2 did not understand what similar figures meant mathematically. She focused on the visual appearance of the polygons (e.g., they have same shape) by counting the number of sides (e.g., all triangles are similar if they have same numbers of sides). To explore further, the interviewer prompted for a different verification:

21I: Why do you think these two triangles are similar to this triangle [#21]?



22PST2: By measuring their sides
23I: Show me
24PST2: 1-2-3-4-5.. I don't know.. I forgot how to write

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25I: Are they similar to each other?

26PST2: They are not [similar]. They do have same number of sides, but their length measures are not the same. They are similar, but they are not the same. I am confused. It's Side-Side-Side (SSS).

Triangles #6 and #31 are not similar triangles. PST2 made a claim that the two triangles were similar because they had the same shape and the same number of sides. However, it appeared that she was confused about what "similar" meant because the two triangles had the same shape with same number of sides but different length measurements. PST2 remembered term "Side-Side-Side", but did not know how to use it in her reasoning. In addition to triangles, participants were asked to identify similar polygons to L-shaped hexagons.

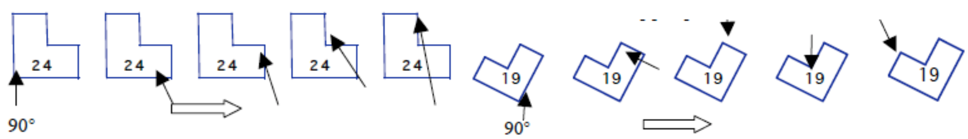
Matching Similar Hexagons

The mathematician identified an "L" shaped hexagon (#19) as a similar hexagon to the given hexagon (#24) by just looking visually again. When asked for substantiation, she measured all the angles in hexagon #24 and then in hexagon #19 accordingly, and checked if corresponding angles were congruent. After confirming the angles in the two hexagons were congruent, the mathematician proceeded to measure the sides in the hexagons, and to check the ratios between the two sides in each hexagon:

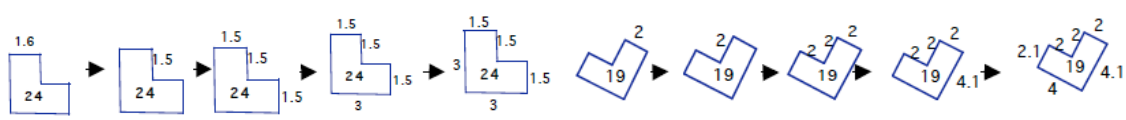
27 I: Which following is similar to this one [#24]?

28M: This one [a hexagon, #19] could be similar to [#24], just looking visually.

29M: [Used a protractor to measure the angles of the two figures].

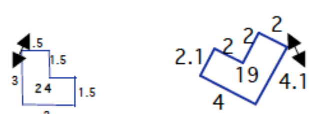


30M: Angle wise, they do seem to be all matched up. That would be not enough. I could do some measuring.



31M: [Used a ruler to measure the lengths of the two figures].

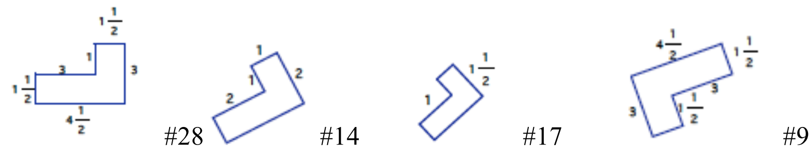
32M: This length is twice as much as that, and this is more than twice, so they can't be similar.



This excerpt highlights a set of actions describing how the mathematician substantiates the claim of "this one [#19] could be similar to [#24], just looking visually". She used direct recognition to identify a candidate that was similar to hexagon [#24]. After measuring the sides of the two hexagons, the mathematician found that the two hexagons did not share same ratios. She refuted her initial claim and concluded, "So they can't be similar." During the interview, the mathematician was concerned about the accuracy of the measurements. Her reaction to the use of measurements to determine similar figures was to "disconfirm" or to show "which is not."

In contrast, PST1 was asked to identify similar polygons to a hexagon [#28]. Her first reaction was to measure all the sides in hexagon #28, and then looked for L-shaped figures that had same shapes with hexagon #28:

33I: Which following figures are similar to this one [#28]?
34PST1: [Used a ruler to measure the sides of the given hexagon.

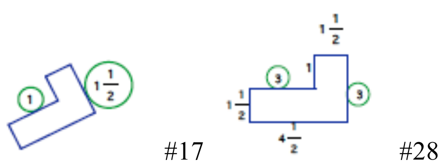


35PST1: This one [#9] is congruent to #28, so it is similar to #28 [a given hexagon].

PST1 concluded that the two hexagons (#9 and #28) were similar because they were congruent. This is a correct response as congruent figures are special cases in similar figures because the ratios between all corresponding sides in congruent figures are 1:1. However, the task was not complete as the PST1 was expected to identify all similar figures. Therefore, the researcher prompted for more information:

36I: What about #14 (a hexagon) and #17 (a hexagon) are they similar to this one [#28]?
37PST1: I measured this side and this side [in #14], and they (ratios) are equal. When you compare it to this one [in #28], they are not equal.

38PST1: And this one [#17], I have these two sides, and they (ratios) are not equal to these two sides [in #28].



39:PST1: So they're not proportional to each other, and they are not similar to #28 [the given hexagon].

When verifying similar hexagons, PST1 focused on checking the ratios between the sides in one figure (#28) to the ratio of corresponding sides in other figures (17&14). She did not check any angle measures. PST2's response to the same question was different. PST2 first identified all L-shaped figures that would be similar to the hexagon (# 28) by direct recognition. When she was asked to verify, "How do you know this one (#24) is similar to this one (#28)?"(See Figure 2).

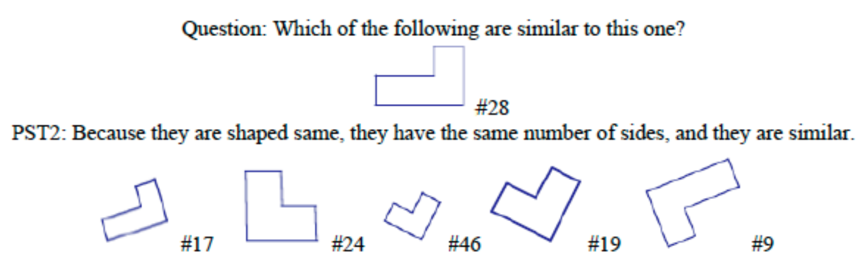


Figure 2.

PST2's responses of hexagons are similar to the given hexagon (#28). PST2 did not measure any sides and angles in the two figures, and replied by saying, "they're the same shape, but just different scale sizes," The researcher then asked, "Is this one [#9] also similar to this one [#28]?" PST2 replied, "It looks like it's the same size. Yes, I would have to say so. Yes." Figure 2 presents a collection of hexagons that PST2 considered similar to the given hexagon (#28). PST2's responses to this task showed a repeated pattern of focusing on the shape of the figures rather than checking the ratios of sides and measure of angles in the figures when identify similar hexagons.

Discussion

Findings suggest that when identifying similar figures, direct recognition is a common first approach to the PSTs and the mathematician based on the shapes of the figures and their orientations. This could be argued that geometry is one of the most intuitive areas of mathematics. However, procedures that are more rigorous are required to identify similar figures by checking the ratios of corresponding sides and the congruence of corresponding angles. Mathematically, congruence is a special case of similarity; however, findings show that one prospective teacher (PST2) did not think this is the case. When identifying similar triangles, the PSTs and the mathematician responded differently and their routines of identifying similar triangles showed the differences (see Figures 3, 4 & 5).

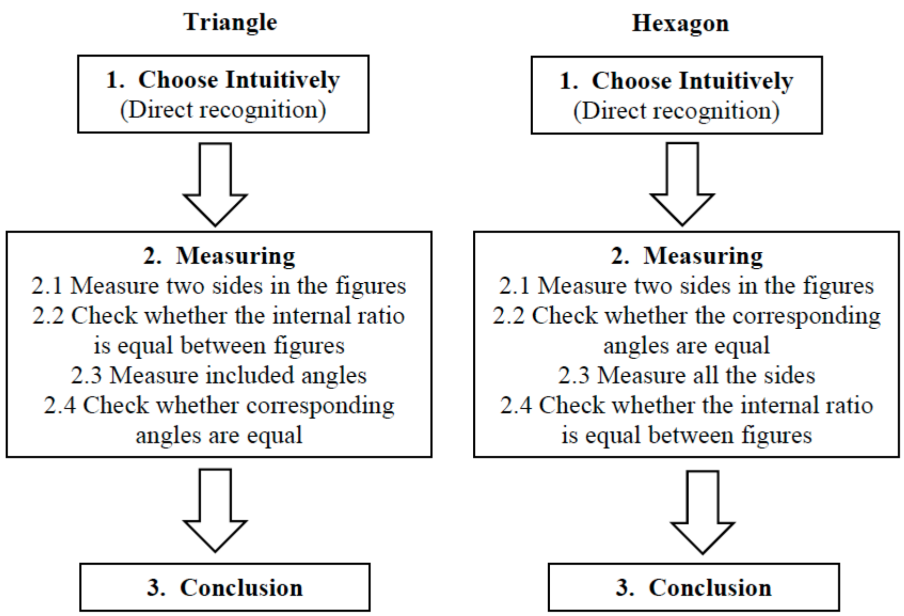


Figure 3. **The mathematician's routines of identifying similar figures**

Between the two PSTs, PST1 demonstrated a better understanding in identifying similar triangles than she did in identifying similar hexagons, whereas PST2 showed difficulties and confusion in identifying similar triangles and hexagons. The findings also suggest the vague mathematical understanding the two PSTs had through their use of the words "similar" and "same shape." Note that both of them were aware of the informal definition of similarity, but there were confusions between the colloquial and mathematical meaning of the terms *same shape* and *similar* in the context of similarity. For example, PST2 did not understand what "same shape" meant when discussing similarity, and she interpreted the term as the figures shared the same number of sides, or that figures are shaped the same, which was not correct in the classification of similar figures. To the same word, similar, the two PSTs responded very differently: PST1 focused on the conditions of similarity (e.g., corresponding angles are equal, corresponding sides are proportional), which is more of the mathematical use of the term similar, whereas PST2 focused more on the "sameness" (e.g., same

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number of sides, shaped the same) between shapes than the mathematical reasoning (see Figures 4 & 5). The vague use of mathematical words such as *similar* and *same shape* provide information on how the concept of similarity was learned and understood by PSTs to make references to the wide range of relationships between shapes.

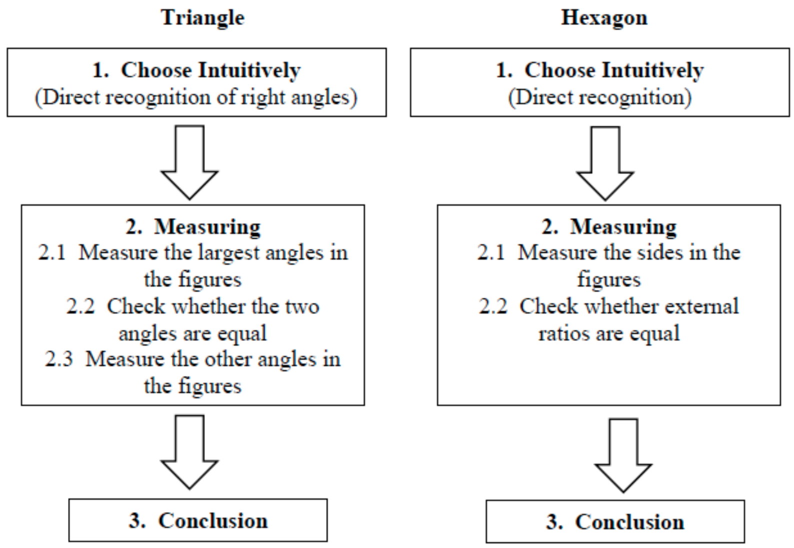

Figure 4. **The PST1s' routines of identifying similar figures**

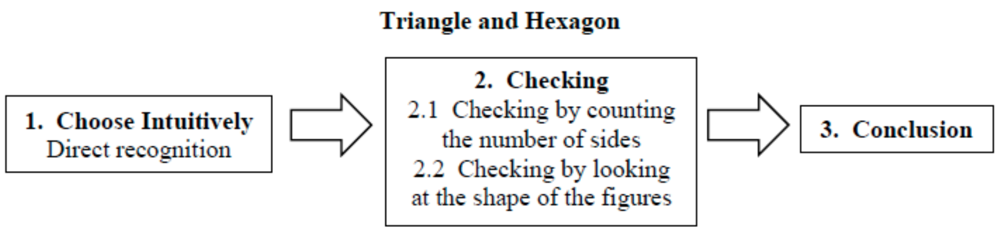

Figure 5. **PST2's routines of identifying similar polygons**

Findings of the study suggest that many misunderstandings of a concept such as similarity will likely be missed if we only focus on the product or single answer of the task. When the PSTs' were asked to explain and to justify claims they made, and to clarify what they meant when mathematical terms (e.g., similar, same, proportion, etc.) were used, we started to see the vague understanding of similarity. It was through those interactions that the misunderstandings of the concept in similarity were detected by a series of questions that were designed to elicit their thinking. Therefore, one recommendation the study could make is to infuse discourse practices in our mathematics classrooms for PSTs to enhance their explaining, clarifying, and defining skills and to shed a light on their use of mathematical terms to ensure that the mathematical concepts are developed correctly. We need to ask more questions about "why" and "how" during the classroom discussions in order to help PSTs articulate their mathematical thinking and reasoning.

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