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# Relating Statistical Image Differences and Degradation Features

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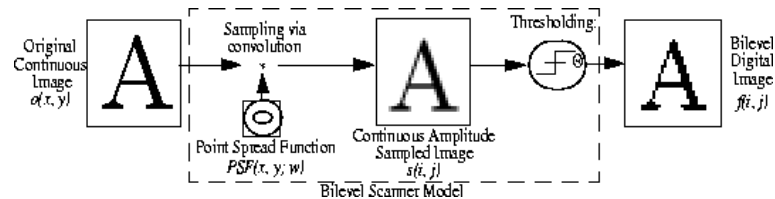
**Abstract.** Document images are degraded through bilevel processes such as scanning, printing, and photocopying. The resulting image degradations can be categorized based either on observable degradation features or on degradation model parameters. The degradation features can be related mathematically to model parameters. In this paper we statistically compare pairs of populations of degraded character images created with different model parameters. The changes in the probability that the characters are from different populations when the model parameters vary correlate with the relationship between observable degradation features and the model parameters. The paper also shows which features have the largest impact on the image.

## 1 Introduction

Document images can be degraded through processes such as scanning, printing and photocopying. This paper discusses bilevel degradations in the context of the scanning process. For the bilevel processes, two observable image degradations were described in [3]–[7]. These degradations are the amount an edge is displaced from its original location and the amount of erosion in a black or white corner. The variables that cause these image degradations can be related to the functional form of the degradation model: the PSF, the associated PSF width, and the binarization threshold.

From a calibrated model, one can predict how a document image will look after being subjected to the appropriate printing and scanning processes and, therefore, predict system performance. Large training sets of synthetic characters can be created using the model when the model parameters are matched to the source document. This can increase recognition accuracy. Models of the degradation process, along with estimates of the parameters for these models, can be combined to make a decision on whether a given document should be entered by hand or sent to an OCR routine [8, 13, 14]. A model will allow researchers to conduct controlled experiments to improve OCR and DIA performance. Knowledge of the system model parameters can also be used to determine which documents originated from the same source and, when the model includes multiple printing/scanning steps, which document was the original and which was a later generation copy.

The degradation model used for this research is convolution followed by thresholding [1]. The two most significant parameters affecting degradations of bilevel images are the point spread function (PSF) width and the binarization threshold [9]. Each pair of these values will affect an image differently. However, several combinations of these parameters will affect images in a similar fashion. The PSF accounts for the blurring caused by the optics of the scanner. Its functional form is not constrained, but needs to be specified. A form that is circularly symmetric is usually chosen so its width is determined by one parameter. The size is in units of pixels, which allows the model to be used for scanning at any optical resolution. The threshold converts the image to a bilevel image. This is often done in software, and a global threshold is assumed. The units for the threshold are absorbance. The variations in the resulting bilevel bitmaps come largely from phase effects [12].



**Fig. 1.** Scanner model used to determine the value of the pixel  $(i, j)$  centered on each sensor element.

Several methods have been proposed to calibrate this model from bilevel images [2, 4, 6, 7]. The resulting parameter estimates will never be error-free. However, not all errors are equally bad. Some will produce characters that have similar appearances and that are more likely to have the same response from an OCR system. This type of estimation error can be treated differently than estimation errors that result in a larger change in the character appearance. This paper explores the amount that characters made with different model parameters will differ as the true model parameters change.

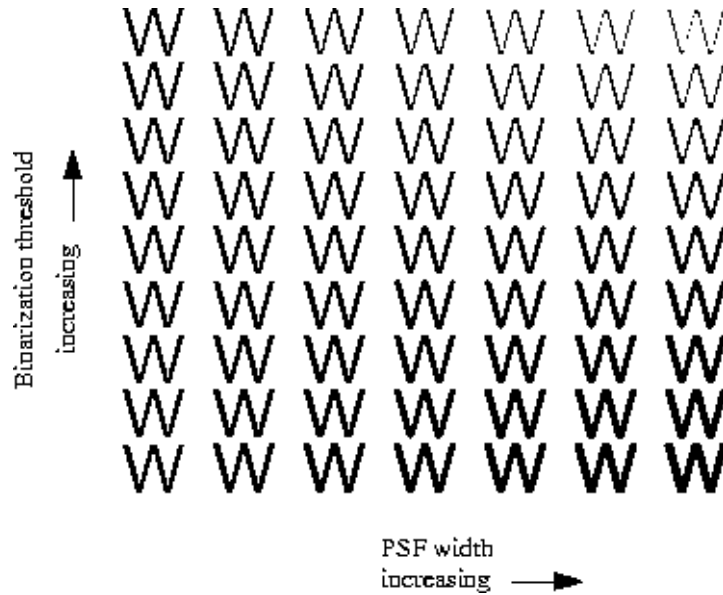
Kanungo et al. [10] proposed a method of validating degradation models. This was achieved through a nonparametric two-sample permutation test. It decided whether two images are close enough to each other to have originated from the same source, having passed through the same sequence of systems. The application he proposed was to decide whether a model of a character degradation produced characters that were "close" to a set of "real" characters generated by physical printing and scanning. This testing could validate the degradation model and the choice of model parameters. The underlying statistical method is not restricted to comparing real and synthetic characters. Consequently, the two images could also be two real images, or two synthetic images. This statistical testing procedure can also be used to determine which parameters of the degradation model created the sample of characters. Another statistical device, the power function, was used to choose between algorithm variables.

Kanungo et al. demonstrated their method using a bit flipping and morphological degradation model. Their approach of statistically comparing character populations is applied in this paper to the convolution and thresholding degradation model shown in Figure 1. The parameters in this model are the point spread function (PSF) width,  $w$ , and the binarization threshold,  $\theta$ . Both populations of character images were synthetically generated to see by how much the characters created with different model parameters will vary over the regions of the parameter space.

This paper starts by describing two image degradations and how they relate quantitatively to the degradation model parameters. It then describes the experiment conducted using Kanungo's non-parametric permutation test to mathematically illustrate the size of the difference between two sets of degraded characters created using our model with different parameters. We then describe how the difference between characters relates to the degradations.

## 2 Image Degradations:

Each model parameter set will produce a different character image. Examples of the characters that are produced for 600 dpi 12-point sans-serif font 'W' over a range of PSF widths and binarization thresholds are shown in Figure 2. Some of the degradations that are introduced are common to multiple characters, such as the final thickness of the character strokes, but each character is slightly different. Two primary image degradations associated with bilevel processes were defined in [4–6]. These are the edge displacement



**Fig. 2.** Characters after blurring and thresholding over a range of PSF widths,  $w$ , and binarization thresholds,  $\Theta$ . A broad range of character appearances can be seen, but some characters have general similarities.

and the erosion of a black or white corner. All these degradations are functions of the degradation model parameters,  $w$  and  $\Theta$ .

During scanning, the profile of an edge changes from a step to an edge spread function, ESF, through convolution with the PSF. This is then thresholded to reform a step edge, Figure 3. The amount an edge was displaced after scanning,  $\delta_c$ , was shown in [3, 4] to be related to  $w$  and  $\Theta$  by

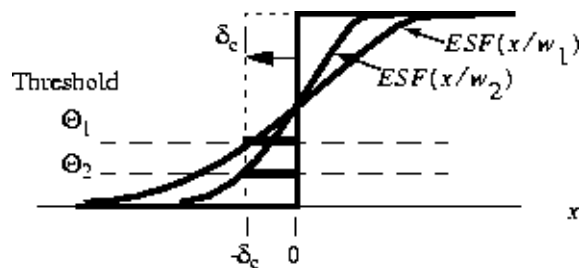
$$\delta_c = -wESF^{-1}(\Theta). \quad (1)$$

The edge spread determines the change in a stroke width after scanning. An infinite number of  $(w, \Theta)$  values could produce any one  $\delta_c$  value. Equation (1) holds when edges are considered in isolation, for example when the edges are separated by a distance greater than the support of the PSF. Figure 4 shows how the values of  $(w, \Theta)$  vary for 5 different constant  $\delta_c$  values for each of four PSF shapes. A positive threshold value will produce a negative edge displacement. The curves for  $\delta_c$  and  $-\delta_c$  are symmetric around the  $\Theta=1/2$  line. If  $\Theta=1/2$ , then  $\delta_c=0$  for all values of  $w$ .

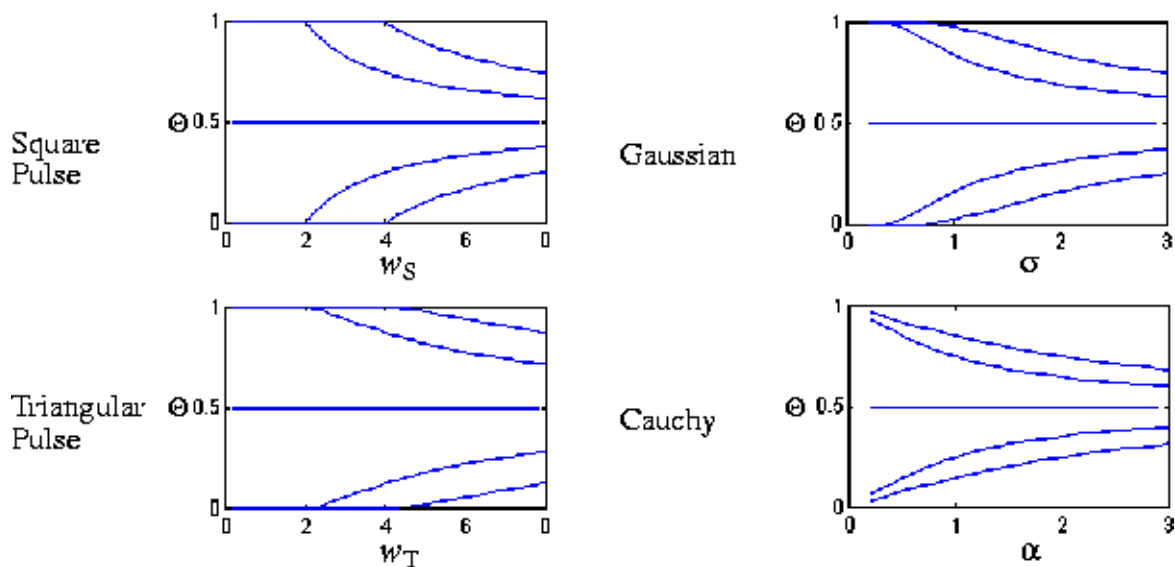
The other pair of bilevel image degradations are the amount of erosion seen in a black or a white corner after scanning [4, 6]. This degradation is caused by the interaction of the two edges, but also includes the displacement of the individual edges. The erosion of a corner can occur in any of the three forms shown in Figure 5. Point  $p_0$  is the apex of the original corner. Point  $p_2$  is the point along the angle bisector of the new rounded corner where the blurred corner equals the threshold value. Point  $p_1$  is the point where the new corner edges would intersect if extrapolated. The distance

$$d_b = \overline{p_1 p_2} \quad (2)$$

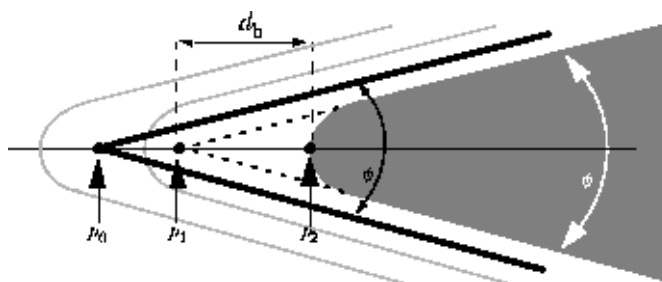
is not the erosion from the original corner location, but it does represent the degradation actually seen on the corner, and this quantity can be measured from bilevel document images. The corner erosion distance,



**Fig. 3.** Edge after blurring with a generic PSF of two widths,  $w$ . Two thresholds that produce the same edge shift  $\delta_c$  are shown.



**Fig. 4.** Contours showing constant edge spread of  $\delta_c = [-2 -1 0 1 2]$  (from top to bottom) for two PSF functions.



**Fig. 5.** The blurred corner (grey area and lines) may be displaced from the original corner position (black line) in three different ways. The visible erosion,  $d_b$ , is calculated the same for all three.

$d_b$ , depends on the threshold, the PSF width, and the functional form similar to the edge displacement above.

The corner erosion distance is a combination of the distance from the original corner to the extrapolated corner,  $\overline{p_1p_0}$ , which is based on the edge spread  $\delta_c$ , and the distance along the angle bisector from the original corner to where the amplitude of the blurred corner equals the threshold,  $\overline{p_1p_2}$ . Thus

$$\begin{aligned} d_b &= \overline{p_1p_2} = \overline{p_1p_0} + \overline{p_0p_2} \\ &= \frac{-wESF^{-1}(\Theta)}{\sin(\phi/2)} + f_b^{-1}(\Theta; w, \phi) \end{aligned} \quad (3)$$

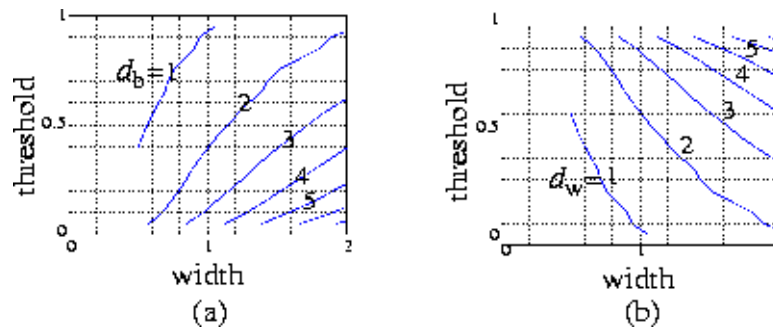
where

$$f_b(d_{0b}; w, \phi) = \int_{x=0}^{x=\infty} \int_{y=-x\tan\frac{\phi}{2}}^{y=x\tan\frac{\phi}{2}} PSF(x - d_{0b}, y; w) dy dx. \quad (4)$$

As with edge displacement, a given amount of corner erosion can also occur for an infinite number of  $(w, \Theta)$  values. The erosion of a white corner is defined similarly and results in

$$d_w(w, \Theta) = d_b(w, 1 - \Theta). \quad (5)$$

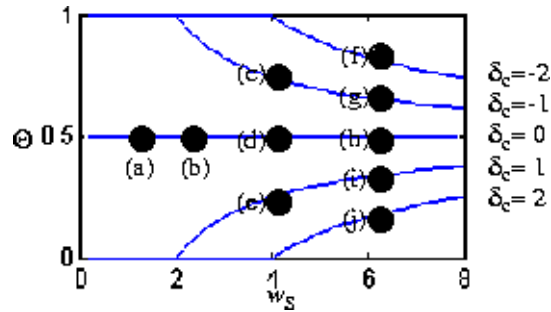
Samples of constant  $d_b$  and  $d_w$  are shown in Figure 6.



**Fig. 6.** Observable erosion contours for constant erosion on (a) a black corner,  $d_b$ , and (b) on a white corner,  $d_w$ . Loci are for a Gaussian PSF and  $\phi = \pi/4$ .

### 3 Experiment

Experiments were run to statistically compare characters in pairs of populations each made with different parameters based on the method proposed by Kanungo et al. [10]. In the experiments presented in this paper, the two populations, X and Y, are both composed of synthetically generated characters created by the blurring and thresholding model with varying phase offsets [12]. The characters in population X were created with PSF width and binarization threshold parameters  $(w_0, \Theta_0)$ , and those in population Y with  $(w_1, \Theta_1)$ . The null hypothesis that these sets of characters have been drawn from populations with



**Fig. 7.** Set of  $(w_0, \Theta_0)$  values used as null hypotheses in the sequence of experiments.

a common set of parameters was compared to the alternate hypothesis that they have been drawn from populations with different parameters:

$$H_N : (w_0, \Theta_0) = (w_1, \Theta_1) \quad (6)$$

$$H_A : (w_0, \Theta_0) \neq (w_1, \Theta_1). \quad (7)$$

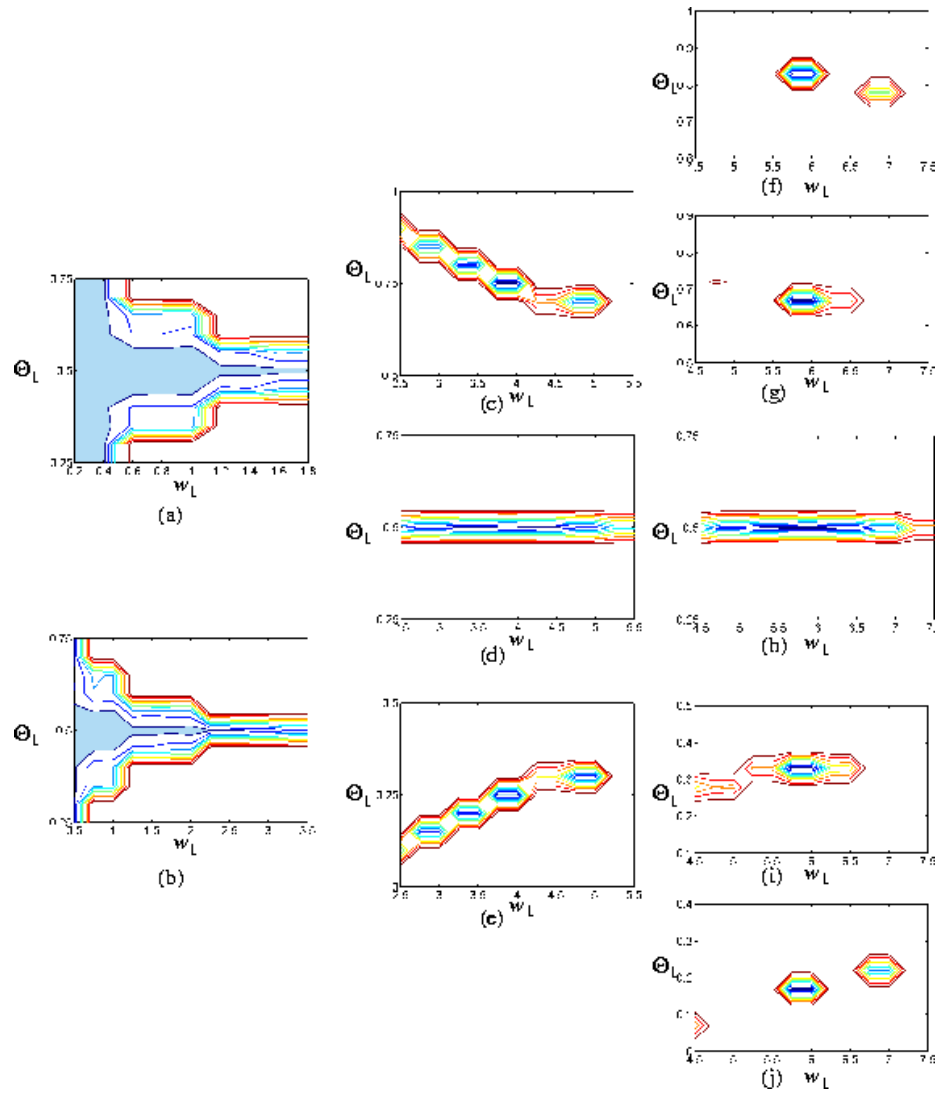
Each experiment consisted of the following steps:

1. Create a set of synthesized characters  $X = \{x_1, x_2, \dots, x_{2M}\}$  with the model parameters of  $\{w_0, \Theta_0, PSF\}$ .
2. Using the permutation test method, calculate the null distribution of the population and choose a threshold,  $d_0$ , to make the misdetection rate or significance level,  $\varepsilon$ , about 5%.
3. Create a set of synthesized degraded characters  $Y = \{y_1, y_2, \dots, y_{2M}\}$  of the same character class, using parameters  $\{w_1, \Theta_1, PSF\}$ .
4. Randomly permute the sets X and Y and select M characters from each.
5. Compute the distance  $D_k$  between the sets of  $\{x_{k1}, x_{k2}, \dots, x_{kM}\}$  and  $\{y_{kM+1}, y_{kM+2}, \dots, y_{k2M}\}$ .
6. Repeat steps (4) and (5) K times and get K distances  $D_1, D_2, \dots, D_K$ .
7. Compute the probability of  $P\{D_k > d_0\} = \#\{k | D_k \geq d_0\} / K$ .

The Hamming distance was used to calculate the distance between individual characters, and the distance between sets of characters was calculated using the truncated mean nearest-neighbor distance. K was set to 1000. Steps (3)-(7) were repeated for several parameter sets  $(w_1, \Theta_1)$  in the vicinity of  $(w_0, \Theta_0)$  to generate a two-dimensional power function. This will show how likely it is that a change in system parameters will cause the characters to differ.

Experiments were conducted around several initial parameter combinations  $(w_0, \Theta_0)$  to see how the location in the  $(w, \Theta)$  space affects the results. The combinations of initial points  $(w_0, \Theta_0)$  that were used are shown in Figure 7. These points were chosen to give a range of edge displacements  $\delta_c = \{-2, -1, 0, 1, 2\}$  for the initial characters and to fill the  $(w, \Theta)$  space. The initial character image used was a 600-dpi 12-point sans-serif 'W'. The PSF form was a square pillbox with a base width  $w_s$ .

The two-dimensional power functions for several  $(w_0, \Theta_0)$  are shown as contour images in Figure 8. These contours show the place where the probability of rejecting the null hypothesis is constant over a range of alternate parameters  $(w_1, \Theta_1)$ . The probability of rejecting the null hypothesis is less than 0.1 in the shaded region. It is 1 in the area outside of the contour lines. The blockiness in the contour shapes

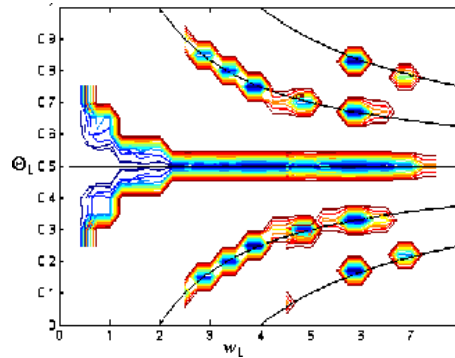


**Fig. 8.** Probability of rejecting null hypotheses with the letter 'W' (a)  $(w_0, \theta_0) = (1.0, 0.5)$ , (b)  $(2.0, 0.5)$  (c)  $(4.0, 0.75)$ , (d)  $(4.0, 0.5)$ , (e)  $(4.0, 0.25)$ , (f)  $(6.0, 0.83)$ , (g)  $(6.0, 0.67)$ , (h)  $(6.0, 0.5)$ , (i)  $(6.0, 0.33)$ , (j)  $(6.0, 0.17)$ . The shaded region has a probability of less than 0.1.



is caused by the quantization in the range of  $(w_1, \Theta_1)$  values used in the experiments and the Matlab interpretation of the contour.

The constant reject probabilities have a shape similar to the constant edge spread contours shown in Figure 4. This is more easily seen in Figure 9, where the hypothesis reject probability contours have been superimposed on the  $\delta_c$  contours. The edge spread degradation has the predominant effect on the appearance of a character visually [5] and, from these results, also statistically.



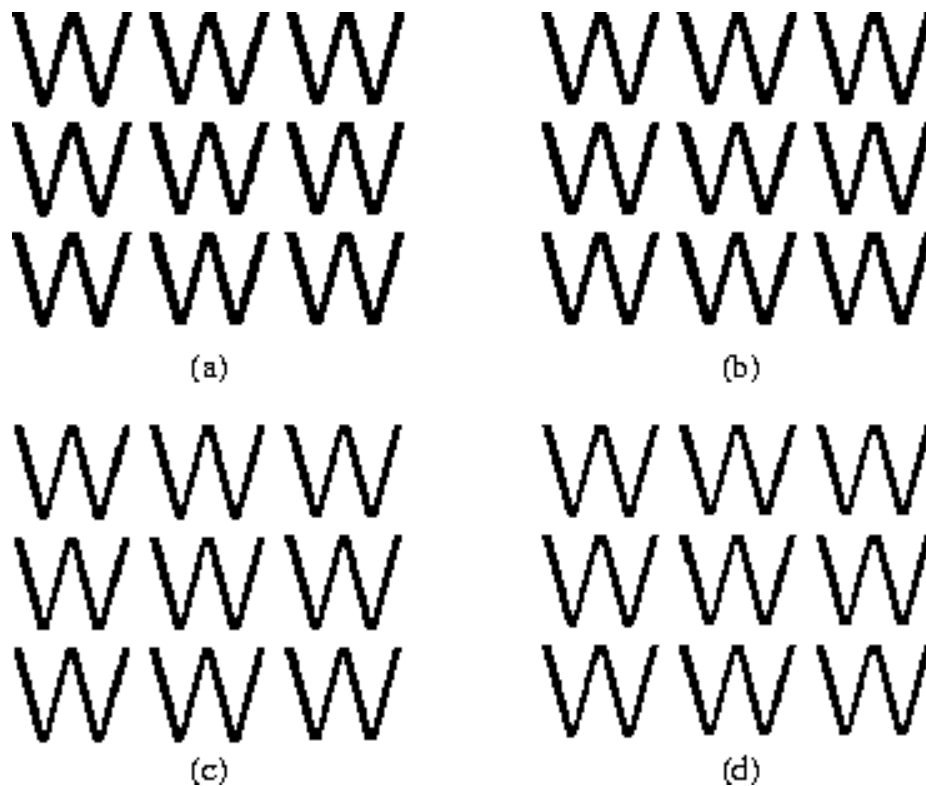
**Fig. 9.** Composite showing results from Figure 8 superimposed over constant  $\delta_c$  lines.

To show the sensitivity of this procedure, consider the corresponding sets of characters in the right and left columns of Figure 10 which are created with parameters that are very close. These appear similar, however, the null hypothesis that the populations from which these characters came were generated with the same parameters was rejected with probability equal to 1. In [5] it was proposed that characters with a common  $\delta_c$  value would appear most similar to humans, while other degradation features, such as  $d_b$  and  $d_w$ , change the character's appearance less. This similarity is now quantified through statistical testing.

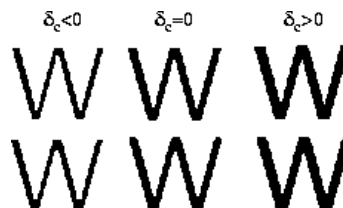
Maintaining a constant  $\delta_c$  increases the probability of the characters appearing similar, but they are only similar within a small range of  $(w, \Theta)$  values. Figure 11 shows sample characters with pairs having a common  $\delta_c$ . The first column shows  $\delta_c < 0$ , the middle  $\delta_c = 0$ , the right  $\delta_c > 0$ . For characters with a positive  $\delta_c$  (low threshold), the characters have thicker strokes, whereas with negative  $\delta_c$ , the characters have thinner strokes. The pairs of characters in Figure 11 look similar, but the differences can be easily seen because the model parameters used to create them are very different. The places where the characters with common  $\delta_c$  differ is at the corners.

While the  $\delta_c$  value has remained the same, the corner erosion and thus the character appearance is different. For  $\delta_c > 0$ ,  $(\Theta < 1/2)$  the  $d_b$  isolines are almost perpendicular to the  $\delta_c$  isolines, and for  $\delta_c < 0$   $(\Theta > 1/2)$  the  $d_w$  isolines are almost perpendicular to the  $\delta_c$  isolines. When  $w$  and  $|\Theta - 1/2|$  are large, a small change in  $(w, \Theta)$  will produce a larger change in the  $d_b$  and  $d_w$  values (see Figure 6). This causes the size of the region of low probability of rejecting the null hypothesis in Figure 8f,g,i and j to be smaller than the corresponding regions in Figure 8c and e.

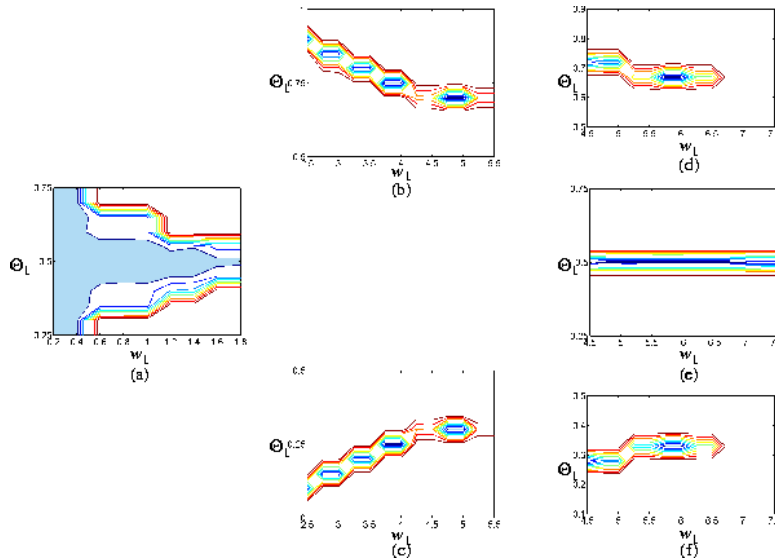
A similar set of experiments was run using a 12-point sans-serif 'O' over a subset of the cases used for the letter 'W'. This character has approximately the same stroke width for the whole character but contains no corners. The resulting power function contours are shown in Figure 12. When the plots are compared to the plots for the corresponding null hypothesis for the 'W' shown in Figure 8a,c,e,g,h and



**Fig. 10.** Synthetic characters created at two  $(w, \theta)$  combinations with varying phase offsets. (a)  $(w, \theta) = (0.4, 0.50)$ , (b)  $(w, \theta) = (0.4, 0.55)$ . The characters in the two sets look the same but are decided to be from different parameter sets with probability of 1.



**Fig. 11.** Characters degraded with  $(w, \theta)$  values to produce negative, zero and positive  $\delta_c$  values. Each character has a different  $(w, \theta)$ .



**Fig. 12.** Probability of rejecting null hypotheses with the letter ‘O’ (a)  $(w_0, \Theta_0) = (1.0, 0.5)$ , (b)  $(4.0, 0.75)$ , (c)  $(4.0, 0.25)$ , (d)  $(6.0, 0.67)$ , (e)  $(6.0, 0.5)$ , (f)  $(6.0, 0.33)$ . The shaded region has a probability less than 0.1.

i, the appearance of the same general shape can be seen. What is different, particularly for  $(w_0, \Theta_0) = (6, 0.67)$  and  $(6, 0.33)$ , is the probability of rejecting the null hypothesis being less than 1 extends for a larger range of values for the letter ‘O’. This is due to the absence of corners. The degradation seen in the characters is only due to the edge spread for a large range of  $(w, \Theta)$  values. With an absence of corners, no corner erosion is present. However, the edge spread was defined for edges that are isolated from each other, and when the PSF width is large enough, this premise is no longer valid [11]. The edges will interfere, and an effect similar to the corner erosion will occur degrading the character images. The corner erosion is a special case of two edges spreading with interference, where the overlap occurs at any PSF support width because the distance between the edges at the corners is zero.

## 4 Conclusion

A statistical test was conducted to compare the similarity between groups of characters synthetically generated with parameters  $(w, \Theta)$  varying over the parameter space. The amount of variation in the characters correlated highly with the change in the edge spread degradation. This change can be quantified in terms of the degradation system model parameters. When estimating the degradation model parameters, errors along the  $\delta_c$  isolines will not produce as large a difference in the characters generated with the model as would an error perpendicular to these isolines.

The effects of an estimation error will be less for characters with fewer corners. Characters with many corners, thin strokes, or variable width strokes will remain similar over a smaller range of  $(w, \Theta)$  values. This can be used to decide how much of an effect an error in estimating the system parameters will have when using those parameters to generate synthetic characters for choosing an OCR structure, training OCR systems, or predicting OCR performance. This can also be used to decide how to distribute model parameters if we want to experiment with characters with small differences, or larger differences that are

evenly distributed. These experiments have also given more insight on how the shape of a character will influence the variation in the resulting bitmap.

The statistical difference between characters could also be used as a metric of model parameter estimation error. Because  $w$  and  $\Theta$  are not in the same units, conventional metrics like euclidean or city block aren't reasonable for combining errors in these two estimates. Also, just adding a scaling factor won't necessarily help because we don't know how to equate width and threshold units. But if we measure error in units of character difference, that would be meaningful.

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