CONCEPT BOOKLETS: EXAMINING THE PERFORMANCE EFFECTS OF JOURNALING OF MATHEMATICS COURSE CONCEPTS

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A thesis

submitted in partial fulfillment

of the requirements for the degree of

Master of Science in Mathematics Education

Boise State University

Summer 2009

BOISE STATE UNIVERSITY GRADUATE COLLEGE

DEFENSE COMMITTEE AND FINAL READING APPROVALS

of the thesis submitted by

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Thesis Title: Concept Booklets: Examining the Performance Effects of Journaling of Mathematics Course Concepts

Date of Final Oral Examination: 12 June 2009

The following individuals read and discussed the thesis submitted by student Todd Stephen Fogdall, and they evaluated her presentation and response to questions during the final oral examination. They found that the student passed the final oral examination.

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The final reading approval of the thesis was granted by Sharon Walen, Ph.D., Chair of the Supervisory Committee. The thesis was approved for the Graduate College by John R. Pelton, Ph.D., Dean of the Graduate College.

For my family and friends,

especially my wife's encouragement and understanding,

and my daughter's remarkably infectious laugh.

ACKNOWLEDGEMENTS

I would like to thank Dr. Sharon Walen for the seemingly endless number of readings and re-readings, study advice, etc. More importantly, though, I would like to thank her for reminding me to keep the endless barrage of instructional trends and administrative mandates in perspective and always be sure to do well by my students. I would also like to thank Dr. Alan Hausrath and Dr. Kathy Rohrig for catching the necessary changes and patiently persisting until I got it right.

ABSTRACT

Journaling is an effective tool for writing about mathematics, but research is mixed about the extent to its effectiveness in writing *to learn* mathematics. This study examined the performance effects of concept booklets on curriculum assessments. Concept booklets are a hybrid style of journal-writing that include responses to journal prompts, diagramming, and traditional note-taking. Prompts were designed to sometimes investigate new concepts and, at other times, to reflect on components of previously learned concepts.

The study, of an experimental design, was carried out at the high school level in honors-level mathematics classes with the independent variable being exposure to the booklets. A unit examination consisting of two parts, the first being traditional curriculum assessment items and the second being composed of nontraditional open-ended problems, was given to both groups. Exam results were analyzed to determine any statistically significant difference between the groups' performances. Separate analyses of test results were done for sophomores and juniors. Additionally, six examination items were analyzed based on whether an accurate diagram was drawn for the problem. This analysis was performed for the control group, the treatment group, and for the pool of all students in the study.

Results showed significantly better performance by the sophomore subgroup on the entire test as well as both parts. The junior subgroup's performance reached statistical significance on only the second, open-ended part of the examination. Analysis of

diagramming showed significantly better performance on the open-ended questions by students who had provided an accurate diagram of the problem. Potential differences in age between grade levels may have contributed to different results for sophomores and juniors.

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CHAPTER 1: INTRODUCTION

The National Council of Teachers of Mathematics (NCTM) recommends in their curriculum-guiding document, *Principles and Standards for School Mathematics* (2000), that students be able to communicate clearly about mathematics and recognize the connections between mathematical concepts. More precisely, they say that students at all grade levels should be able to "organize and consolidate their mathematical thinking through communication" (p. 348), and "understand how mathematical ideas interconnect and build on one another to produce a coherent whole" (p. 354). Unfortunately, every school year starts with an inordinate amount of time devoted to teaching my students, regardless of where they are in the course sequence, how to properly write solutions to problems encountered in class. Moreover, students do not recognize the connections between concepts. An excellent example of this is the large number of students in Algebra 2 who have never connected the fact that two points are needed to write the equation of a line to the postulate in geometry that states that a unique line can be drawn between any two distinct points. These experiences, which are reinforced constantly by similar stories from my colleagues, lead me to conclude that these standards are not being met.

Teachers often complain that students too frequently exhibit an inability to retain concepts from year to year or, worse, from one instructional unit to another. According to Rich (2003), this results in long periods of review and re-teaching (to ensure that all students are ready to move on) and deprive teachers of valuable instructional time needed to cover new concepts. To make matters worse, the current education landscape of high-stakes standardized testing is creating an environment where teachers are opting to cut out important concepts simply because they are absent from topic lists, even when they are important concepts necessary for the development of the whole curriculum. By the end of the semester or school year, students have been exposed to a disjointed set of topics to be studied individually rather than the coherent curriculum they should receive (Rich, 2003). The next year begins with yet another review and re-teaching period, and the harmful cycle continues. Teachers have little control over this testing reality, so it becomes essential that they prevent further disjointing of the curriculum and help students to see mathematics as the **coherent whole** that NCTM promotes.

One practice that further disjoints the mathematics curriculum is a study and organizational habit that many of my students use. They often write any notes and diagrams from lectures or class activities at the top of the same pieces of paper that they later use to complete practice exercises. Many of these students report that they were advised to do this by previous teachers, usually with the goal of reducing the number of items to organize in their binders. Wherever they developed this habit, it has the unfortunate side effect of creating a disjointed set of notes that only serve to muddy an already disjointed curriculum. Worse yet, many students in my classes are in the habit of immediately discarding returned assignments, leaving themselves with no record of the course's main topic sequence to reference when studying for assessments.

During the 2006 - 2007 school year, I began to search for a method to help students overcome this detrimental practice. In a graduate course, I was introduced to a method for making simple booklets for organizing and displaying information that could easily be used as a way to encourage students to take notes in a separate location rather than writing them with their solutions to practice exercises. I immediately presented this method to my students and, for the rest of the year, they were required to write all of their notes and diagrams in these booklets.

I was pleasantly surprised by two consequences of this informal classroom experiment. The first consequence was that students enjoyed making the booklets, partly because there is no stapling or gluing of any kind to make them; instead, they are inventively made through several selective cuts and folds. Furthermore, they also liked how easy they were to write on, since they laid flat on writing surfaces, allowing them to use drawing tools like rulers, compasses, and protractors, which are tricky to use with a spiral notebook or loose-leaf binder. The second consequence was that many students reported that studying had become easier and more efficient, mainly because they finally had a better method for organizing and later reviewing the concepts they learned in class.

Soon after I had implemented the booklets, several students in my geometry classes commented that trigonometric ratios seemed to be disconnected to the rest of the curriculum, which revealed a lack of understanding that similar right triangles served as the foundation for these ratios. While it appeared that the booklets were serving as an effective organizational tool, this episode showed that many students still had difficulty seeing interconnections between concepts as well as how newer concepts were built on older ideas. In an attempt to remedy this, I began to modify the booklets by having students also record their responses to periodic journal prompts. Some prompts, which I referred to as investigative prompts, were designed to lead students to think about important concepts necessary to understand an upcoming topic or problem. Other reflective prompts were designed to help students think about the ways that concepts were interrelated or about common misconceptions that had arisen over time. By including these prompts, students were now explicitly forming links from previous concepts to new concepts.

Another benefit of including journal responses became apparent when students seemed more willing to provide sketches and diagrams of the situations that were presented in the prompts, especially those that were investigative in nature. Students often seemed unwilling to diagram problems on homework assignments and exams, mainly because they are so focused on the solving processes that are explicitly taught in class. In this context, however, students were responding to open-ended scenarios for which they often had no pre-packaged mathematical strategy. I started to see this as a way to get students to practice their ability to sketch diagrams for completely novel problems without immediately resorting to the strategies that they learned in class, primarily because they had yet to learn them. Several of the prompts that were used in this study were designed exactly for this purpose, especially the one for Wednesday, January 28 (see Appendix A). This prompt calls for students to consider the resultant vector of two vectors of equal magnitude. They had to consider the possible scenarios that will make the resultant vector's magnitude equal to the sum of the two forces or equal to 0, or whether it was possible for the resultant vector's magnitude to be greater than the sum of the magnitudes of the two forces. Most students' responses to this prompt consisted mainly of diagrams of each of the three scenarios. An analysis and discussion of students' uses of diagrams is included in Chapters 4 and 5.

In addition to the journal prompts, I began to have the students incorporate in the booklets their responses to the daily problems that I had always used as anticipatory sets. These problems had always been selected based on ties to the current curriculum. Prior to using these booklets, students had written their solutions to these problems on assignment pages, but as expected, they would lose the problems much like they had previously lost the notes they had written on their assignments. Consequently, it was often impossible for students to locate their work when I tried to bring attention to the connections presented in these problems. Requiring that these problems be written in the booklets made them immediately retrievable as well as a subject for subsequent journal prompts. Finally, the daily problems were being used as the legitimate source of learning material for which they had initially been designed.

The inclusion of the journal prompts and daily problems into the booklets turned what was initially just an organizational tool into a writing tool. I began to observe students making more thorough and complete connections between concepts and exhibiting a better anticipation of where concepts were leading. One particular instance of this was when a student in one of my geometry classes quickly recognized that the reason that we were developing the Laws of Sines and Cosines was because we wanted to be able to finally solve missing values in triangles that were not necessarily right triangles. Witnessing this kind of understanding of the curriculum helped to convince me that these booklets could be powerful teaching tools. This motivated me to make this kind of writing in mathematics the focus of this study, in hopes of formally investigating the concept booklets' potential learning effects. Specifically, I wanted to know the extent to which journaling in concept booklets affects student performance on traditional curriculum assessments, as well as more nontraditional open-ended assessments.

The effects that writing to learn mathematics have on learning are mixed but generally positive. The practice of reflective journaling has been shown to have a positive effect on students' abilities to understand new concepts and procedures, communicate mathematically (Jurdak & Rihad, 1998), and solve novel problems (Countryman, 1992). However, journaling alone has not necessarily had positive results on student performance on specific curricular assessments of the curriculum, probably because the nature of skill-specific mathematics instruction and open-ended problem-solving are vastly different (Jurdak & Rihab, 1998; Klein, 1999). Inspired by these conclusions, the objective of this study is to formally determine through experimentation whether a journaling process that is more closely aligned with specific course learning objectives, specifically the one involving the booklets described here, will have a positive effect on curriculum assessments, both of a traditional nature as well as open-ended problem-solving tasks.

Definitions of Key Terms

Concept booklet – A small, hand-made book, roughly five and a half inches wide by eight and a half inches tall, constructed from blank pieces of standard printer paper (see Appendix C). While the booklet serves as a location for taking notes, drawing diagrams, and writing solutions to daily problems, each also contains two other important features: responses to investigative journal prompts designed to motivate a lesson and responses to reflective journal prompts designed to bring attention to connections between past concepts or help students think through common incomplete conceptions. Booklets are made for each instructional unit, partly to provide a sense of closure to major concepts but also to provide the teacher time to examine individual booklets without depriving students of their entire record of course concepts.

Writing-to-learn mathematics – A style of writing "to help students understand, retain, analyze, and organize mathematical concepts" (Flores & Brittain, 2003, p. 112).

Conceptual understanding – Comprehension of overarching mathematical ideas. One instance of this is knowing that real roots to a quadratic polynomial found by using the quadratic formula correspond to the points where the graph of the polynomial intersects the x-axis of the coordinate plane. While the high-stakes testing that is prevalent in public education today is mainly focused on procedural skills, it is important to note that "(c)onceptual understanding is an important component of proficiency" (NCTM, 2000, p. 20).

Procedural understanding – Comprehension of mathematical algorithms and processes. The ability to set a quadratic equation to zero and properly substitute the coefficients and constant into the quadratic formula is one example of procedural understanding.

CHAPTER 2: LITERATURE REVIEW

Writing-To-Learn in the Mathematics Classroom

With the exception of occasional written proofs, writing has not historically been a major component of traditional mathematics classrooms. Various types of writing in mathematics have grown in prominence over the latter half of the last century, becoming even more strongly encouraged with the introduction of the National Council of Teachers of Mathematics' standards documents, first in 1989 and followed by the 2000 revision, *Principles and Standards for School Mathematics*. This shift in emphasis was primarily due to an identified need for our students to have more experiences that develop their abilities to communicate effectively about mathematics. In theory, students who communicate proficiently about mathematics are better equipped to develop the deeper conceptual understanding that is considered to be widely missing from our students' educations (Morgan, 1998).

According to Morgan (1998), while NCTM and similar organizations in the United Kingdom and New Zealand have urged an increase in the use of writing in mathematics classrooms, the extent to which it is currently being used is unclear. The writing that has been reported and examined, however, appears to be superficial compared to the types of writing discussed in the recommendation documents. Most often, the mathematics-classroom writing in which students engage consists of transcribing or copying lecture notes and example exercises done in class and does little to provide students with opportunities for creative mathematical writing. Some types of writing occurring in mathematics classrooms do not consist of simply copying or transcribing information but still do little to support learning. “To fulfill school or district writing requirements, many mathematics teachers report that they annually assign writing biographical reports to their students” (Bosse & Faulconer, 2008, p. 8). These types of biographical reports of mathematicians, while perhaps valuable, do little to provide students with opportunities to think critically about mathematical concepts. Much of the lack of creative writing opportunities can be attributed to students’ lack of familiarity with the language and symbols of mathematics. Most students are never formally taught to communicate about mathematics, only the way to perform mathematical computations and manipulations. Unfortunately, these types of writing do not achieve the goals set forth to accomplish a higher level of mathematical communication (Morgan, 1998). The type of writing intended to meet these goals, which has come to be known as writing-to-learn mathematics, is the focus of this study.

The writing utilized in the concept booklets falls under the writing-to-learn umbrella. Writing-to-learn discards the idea that mathematical knowledge is a set of facts and procedures to be recalled by the student, but rather a more dynamic body of knowledge to be applied in a variety of different situations to solve problems (Borasi & Rose, 1989; Countryman, 1992; Morgan, 1998). Examples of this type of writing include: reflective journaling, where students reflect on previously learned mathematical concepts, prior perceptions of concepts or mathematical ideas, the student’s academic performance on assignments and assessments, etc.; investigative journaling, where students respond to prompts designed to motivate their thinking about an upcoming concept or problem; and formal writing, where students write up formal solutions to often open-ended mathematical problems (Countryman, 1992; Morgan, 1998).

Journaling and Writing-to-Learn Mathematics

Journaling is, in the traditional sense, a place for students “to record their experience of learning mathematics” (Countryman, 1992, p. 27). Students may write about “reflections on material learned in class, reactions to readings or lectures, or even responses to open-ended assignments” (Borasi & Rose, 1989, p. 348). Journaling is not always this directed. Sometimes it serves as just a place where students can record anything they choose; however, some argue that journal prompts should not be open-ended, since some writers “find that putting into words something that they are doing or something that they know is easier than discussing something that they think or believe” (Mason & McFeetors, 2002, p. 533). For example, when teaching students about the purpose of the discriminant, $b^2 - 4ac$ in the quadratic formula, simply asking students what the discriminant tells us may elicit a wide range of correct responses, not all of which may be educationally valuable. On the other hand, asking students to use a graphing calculator to graph three sets of three unique quadratic functions after calculating the discriminant of each function, and then asking them to write about the connections that are seen between the number of real roots and the value of the discriminant, is much more likely to bring out responses that demonstrate a genuine understanding of what the discriminant really indicates about a function. With this in mind, students should be given prompts that provide them with a place to begin (Mason & McFeetors, 2002).

Borasi and Rose (1989) suggest a substantial list of potential benefits of journal writing, including individual benefits to the student, individual benefits to the teacher, and collective benefits to the student-teacher relationship. Specific individual benefits to the student that are cited include a positive effect on their emotional response to learning mathematics due to the opportunities to communicate their feelings about the course, as well as a better understanding of how the subject relates to other areas of study and their own reality, whether that reality pertains to future academic classes that they have to take, or perhaps to a potential career field that they have been exploring. More specific to this particular study, Borasi and Rose cite that students obtain an improved understanding of mathematical content through inquiry and reflection and improved problem-solving skills from the process of examining their experiences doing mathematics.

The first of the two types of journal prompts that are used in the concept booklets, those of the investigative nature, are used to motivate upcoming lessons or preview an upcoming topic. As an example of this type of prompt, for this study, students had to calculate resultant vectors. Before this process was formally introduced, I asked students to draw two sets of vectors, one where the resultant vector was equal to the sum of the vectors, and the other where the resultant vector was zero. No computations were done, but students had to visualize what the vectors would have to look like in order for each resultant to exist in that way. After this prompt, students were formally taught how to compute the resultant vector algebraically, but through the process of exploring the concept geometrically, it was my hope that the algebra would have more contextual meaning and subsequently help my students better retain the concept.

Morgan (1998) goes to great lengths to point out the controversial aspects of the kind of mathematical investigation used in journal prompts, specifically that investigative work can be divergent in nature, but also points out that they are important activities because they provide students the opportunity to engage in the kind of work that professional mathematicians do on a regular basis. The fact that this kind of work can be divergent, i.e. that different students can come to different conclusions, is not necessarily a drawback. It is just more important that journal prompts are carefully crafted to focus student learning toward a common objective. Additionally, through the process of sharing responses to journal prompts, students can see that alternative solution paths are sometimes equally as strong and frequently mathematically equivalent methods of completing a task (Countryman, 1992; Morgan, 1998).

Reflective prompts are the second type of journal prompts employed in the concept booklets. These are designed to help students reconsider the relationships between concepts, the ways in which they or other students may think about certain concepts, or simply the ways that study habits affected their performance on an assessment (Borasi & Rose, 1989; Countryman, 1992; Flores & Britain, 2003). One example of a reflective prompt that was used in this study was given the day after students used the Law of Sines to solve non-right triangles. They were asked to identify the circumstances under which the Law of Sines could be used and when it could not. Then, after the Law of Cosines was used, students had a concise set of rules to help them identify which law was appropriate for a given situation. This type of journaling is not only important in helping students monitor and adjust their understanding of course concepts (Borasi & Rose, 1989), but is also an essential component in a curriculum that develops conceptual understanding in students (Ben-Hur, 2006). In addition to these benefits, reflection is also noted to benefit students by allowing them to examine their emotional relationship to the subject and how it may affect performance on assessments (Flores & Brittain, 2003).

How Writing Affects Learning

Writing-to-learn mathematics exhibits four characteristics of successful learning; it forces the writer to integrate the hand, eye, and brain to express reality, it provides immediate feedback and reinforcement, it encourages semantic connections, and it "is self-rhythmed because it connects past, present and future through analysis and synthesis" (Morgan, 1998, p. 25). In this way, Morgan argues, writing has a positive effect on learning; however, the extent to which it affects learning and the specific ways in which it affects learning have not been adequately addressed, with most of the supporting literature lacking concrete evidence (Borasi & Rose, 1989; Morgan, 1998). Still, the reliable underlying premise here is that writing-to-learn mathematics, such as the type of writing found in the concept booklets, "forces construction of understanding, because we cannot write coherently about something we do not understand" (Talman, 1992, p. 107).

The type of mathematical writing that takes place through journaling can always be considered writing-to-learn mathematics since, although the subject of the writing often includes instructed curriculum concepts, it requires students to generate unique thoughts about these concepts, which is vital to any type of mathematics instruction considered to be conceptually based. This type of instruction, outlined by Ben-Hur (2006), relies on two principles. "One principle is that learning new concepts reflects a cognitive process. The other is that this process involves reflective thinking that is greatly facilitated through mediated learning" (p. 11). The mediated learning mentioned here simply means that the teacher provides numerous opportunities for the student to make individual connections on their own, but that these connections are made through experiences designed to guide students to specific outcomes and conclusions. When relating this to journaling, this means that journal prompts must be designed to help students reach a certain conclusion, rather than more open-ended prompts that are prone to elicit less specific responses not necessarily focused on the topic at hand.

In general, writing-to-learn mathematics through journaling has been shown to have positive effects on students' conceptual and procedural understandings and mathematical communications (Jurdak & Rihad, 1998). Oddly, though, the effects of journal writing on objective, curriculum assessments have been shown to be small to nonexistent (Jurdak & Rihad, 1998; Klein, 1999). The results of Klein's (1999) meta-analysis of the literature pertaining to journal writing found that while there are definite effects on learning, they are not always toward objective test performance. A variety of factors are cited for this phenomenon. For instance, the writer's prior language often prevents new learning, sometimes even perpetuating common incomplete conceptions. An example given in the review is of the phrase "warm sweater," which illustrates many students confusion of the scientific principles of heat-generation versus insulation. Additionally, when teachers ask students to engage in open-ended journal writing without prompts, usually called free writing (Countryman, 1992), the result is often the generation of many new ideas that may not be conceptually correct or in the direction of the intended curriculum objectives. On the other hand, when teachers provided journal prompts designed to elicit responses about specific ideas and components of major concepts, students frequently showed gains in conceptual understanding as a result of the process (Klein, 1999). This is consistent with Ben-Hur's (2006) outline of conceptually-based learning previously mentioned. Without teacher mediation, the learning is too open-ended to provide students with a focused path to the intended outcome. This notion, which is the chief motivation for this study, stimulates the question of whether the concept booklets can lead to improved performance on not only curriculum-based assessments, but also on more open-ended styles of assessments. The booklets should promote learning of curriculum concepts because they consist of journal prompts aimed at essential course concepts, rather than the traditional journaling approach involving open-ended journal prompts or no prompts at all.

Assessing Responses to Journal Prompts

Some more formal varieties of writing in mathematics, such as formal writing and written solutions to open-ended problems, require that students eventually submit a final copy free of errors, both mathematical and grammatical. In this way, it is appropriate to formally assess these texts (Countryman, 1992). In the case of journal writing, however, it is of the utmost importance that students feel free to communicate in a setting where they are unburdened with concerns about mathematical or grammatical errors. The journal is supposed to be part of the overall process of learning mathematics, not final evidence of successful learning. Instead, the process of journaling is designed to promote a deeper understanding of the subject matter that should be evident on formal objective assessments of the curriculum (Borasi & Rose, 1989; Countryman, 1992; Flores & Brittain, 2003; Mason & McFeetors, 2002). As a result, students are encouraged "to suspend judgment in the journal and to feel free to ask questions, to experiment, to make statements about what they did and did not understand" (Countryman, 1992, p. 27). This is not to say that they should be free of requirements. It just means that it is inappropriate to assign a formal grade to a journal or journal entry based on criteria other than the volume and frequency of their writing (Borasi & Rose, 1989).

Students received feedback about the content in their booklets from me, whether through conversations with them or written responses from me after having read their entries. Consistent with the recommendations about assessment, however, the journal writing that was done in the concept booklets was graded only for participation, with a completion grade assigned for their booklet at the end of the unit. All of the journaling occurred before the main lesson began each day, so it was usually very brief, only amounting to between three and five minutes on average. Students would participate not only through formulating their responses to each prompt, but also by sharing their responses with classmates. This sharing would sometimes take place in randomly assigned small groups, while at other times it was with the whole class. I would usually call on specific groups or individuals to present their responses to the rest of the class, based on the responses that I was seeing as they were writing. I felt that it was very important to try to get varied responses to each prompt, not only to show that there were many possible interpretations, but also to allow students to see the benefits of other points of view. In addition, I would sometimes paraphrase flawed responses, without identifying the student, so as to avoid any embarrassment, with the goal of pointing out that, frequently, flawed ways of thinking about something usually need only minor modification to be back on the right track.

CHAPTER 3: RESEARCH DESIGN

Design Overview

This study examined the effects of the use of concept booklets on students' academic performance on both objective curriculum-based assessments and nontraditional open-ended problem-solving tasks. The study was of an experimental design, with the independent variable or treatment being exposure to the concept booklets and dependent variable being performance on both types of assessments. The treatment group consisted of students in four sections of pre-calculus classes that I taught at a large suburban high school in Southwestern Idaho, which I will refer to as Lincoln High. Excluding a small number of freshmen and seniors who did not participate in the study, there were 84 students in the treatment group. The control group consisted of 41students, excluding freshmen and seniors, in two sections of pre-calculus classes taught by a colleague of mine, to whom I will refer as Ms. R, at another high school in the same school district, which I will call Washington High. Groups were examined for any differences in the dependent variable based on the null hypothesis, that any differences between the groups' performance could not be explained by receiving the treatment. The hypothesis, tested using two-tailed *t*-Tests, was designed to answer the research question, whether exposure to the concept booklets would have any effect on student performance on traditional and open-ended curriculum assessments.

In addition to analysis of each group's performance on the whole examination, I coded individual responses to each of the examination items in an effort to glean information about the possible ways in which the concept booklets influenced whether or not a student provided a correct solution to a problem. Coding included some algebraic and arithmetic issues such as whether a student's mistake was due to a computational error or an incorrectly applied formula. Out of this coding, I decided to focus on whether or not a student provided an accurate diagram of the problem. At that time, six examination questions were selected for analysis, based on whether a diagram was typically used to help in the discovery of a correct solution. The rationale for this analysis and the exercises that were examined for this purpose are described in detail in the next chapter.

Sampling Procedure

The sampling procedure used in this study was one of convenience. Prior to the year of the completion of the study, the author and Ms. R both taught pre-calculus classes at Lincoln High. The year of the study was the first year of existence for Washington High School. Students at Washington mostly came from the older high school, Lincoln. Because of this, both student bodies were strikingly similar in demographic composition and educational background. Moreover, almost all of the students entering the pre-calculus classes at both high schools had the same teacher, another colleague who is now at Washington High School, for Algebra 2, the previous course in the sequence. I taught all four of Lincoln High's Honors Pre-Calculus courses while Ms. R taught two of Washington's four Honors Pre-Calculus courses. In this way, randomness was already achieved through a convenience sampling. My students served as the treatment group, while students in Ms. R's classes at Washington High served as the control group.

Procedure and Instruments

At the beginning of the school year, I instructed students on the use of the concept booklets and began to use them as a regular component of their course assignments at Lincoln High. Concept booklets have been used by all of my students, including students in Geometry, Algebra 2, Pre-Calculus, and Introduction to Calculus, for the previous two years, so they were already existing components of each course. It is important to once again note that assessment of the concept booklets was only for volume and frequency of journal responses so as not to inhibit the journaling processes, both to investigative prompts, to introduce new concepts, and to reflective prompts, designed to motivate students to process previously learned topics on a much deeper level and identify important but often overlooked elements of those topics.

The content that served as the focus of the study was covered in an instructional unit over applications of trigonometry, including Laws of Sine and Cosine and vectors. This unit, the first one taught during the second semester, started on the 19^{th} of January and culminated with the unit examination on Tuesday and Wednesday, February 10 and 11, 2009. Treatment and control groups were compared using their Idaho Standards Achievement Test (ISAT) mathematics scores to establish no statistical difference between groups. Since the ISAT is a grade level examination, a sophomore's score cannot be compared to a junior's score, the data were stratified by high-school year, sophomores and juniors. As a result, the most recent ISAT scores for both the sophomore and junior groups were used to establish no statistical difference between Lincoln and Washington's sophomores and juniors. The numbers of freshmen and seniors that were enrolled in either school's pre-calculus classes were so small that students at these levels were not included in this study. The instructional unit was taught over the same number of days, using the same sets of practice exercises, as well as the same quizzes and tests for assessments of the course curriculum (see Appendix B). Both instructors had access to all assessment items prior to the beginning of the study. The treatment group responded to a number of journal prompts, both reflective and investigative, designed to deepen their understanding of daily concepts (see Appendix A).

Upon conclusion of the curriculum unit, test results were analyzed through the use of two-tailed t-Tests, with rejection of the null hypotheses occurring with statistical significance below the $p = .05$ level. All data analysis was performed with the Analysis Toolpak component of Microsoft Excel, Version XP.

Limitations

While the two high schools were predominantly formed from one high school, there were still a limited number of students that previously attended a third high school that has traditionally had students from a lower socioeconomic background. As a result, there were likely subtle differences between groups because of this factor.

Washington High School operated its opening year on a hybrid schedule that taught some classes on 60-minute periods every day and others over a 90-minute period every other day. However, all mathematics classes were taught in class periods that were the same length as those at Lincoln High. The hybrid nature of the teaching schedule caused a large number of student-scheduling conflicts and resulted in a large number of students from the third high school being placed in the wrong mathematics level at Washington High School. Shortly after the unit began, many, but not all, of these students dropped out of the pre-calculus classes; they had struggled significantly during the fall semester. It is logical to assume that some of these students' struggles were due, at least in part, to a different type or level of preparation in mathematics. As a result, the presence of students from the third high school in this study may support the belief that the study is limited due to this factor.

Another potential limitation of this study is the fact that the students that enroll in pre-calculus classes are generally considered "advanced" mathematics students and usually participate in their school's honors program. This may indicate that the groups studied here are not representative of the general student population. Consequently, the argument can be made that the generalizability of the study is in question.

Finally, due to the number of students that transferred to other classes at Washington High School after the start of this unit forced the overall sample size to be considerably smaller than expected. A larger number of students participating in the study would certainly have provided better evidence of the extent to which the results of this study generalize to the general population.

CHAPTER 4: RESULTS

Outcomes of Concept Booklets Influencing Exam Performance

Too often in classrooms I have seen instances where common teaching practices have undermined student understanding of curriculum concepts and, subsequently, the conceptual understanding that was deemed so important in Chapter 2. Certainly one of the most prevalent and arguably damaging practices is the presentation of formulas or algorithms without any opportunity for students to connect the underlying concepts to prior knowledge. Consequently, students learn skills without the ability to apply them in novel mathematical situations. This is especially true in the vector unit that was the focus of this study. Students often have great difficulty with certain vector concepts, such as vector projections and force diagrams. As a pre-calculus teacher who is bombarded by student requests to make the curriculum more accessible and perhaps “easier” to digest, I am often tempted to overlook the geometric relationships underlying these concepts and teach students to simply apply formulas without any context. Naturally, this provides little long-term benefit to students.

With this issue in mind, I spent a great deal of time after the beginning of this study thinking about the ways that concept booklets may have fostered student’s abilities to solve problems. Undoubtedly, more efficient organization of course concepts is a major factor, but as I coded the examination items, I began to think more and more about whether the concept booklets influenced whether or not students diagrammed the problems they encountered. I will further discuss aspects of students' solution processes in Chapters 4 and 5.

Necessary Changes to Initial Study Design

Originally, the study was intended to conclude on Thursday and Friday, February 5 and 6, but the unit assessments had to be given three school days later than originally planned due to Lincoln High School's participation in the 2009 Special Olympics. Many students participated in the games as volunteers at the opening ceremonies and assisted at several of the events. Lincoln's regular schedule was changed to compensate for this participation, resulting in one of the pre-calculus classes not actually meeting on one of the scheduled test days. Students at Washington High School took the exam on the originally scheduled dates. Although students took the exam on different dates, both groups of students received the same number of instructional days; students at Lincoln High began the next instructional unit rather than reviewed the content examined for this study for a longer time. As a result, this change of dates provided little to no advantage for students in the treatment group. One might even argue that time spent on a different instructional unit prior to being examined may be a disadvantage to the treatment group.

All tests given by Ms. R were photocopied before she graded them and sent to me so that I could grade both groups' exams using the same exam key. In addition to the regular evaluation, all test questions were coded for various types of error response categories as mentioned previously. The categorical coding of the exams was verified by a second independent evaluator who was aware of the research study and the literature and classroom experiences that led toward the development of the study. Inter-rater reliability was 100% between the two coders, confirming accuracy and consistency in the development of the categories and the evaluation of the student responses on the exam, the instrument.

Recall that upon examining the students' scores on the ISAT and the examination itself, it had become apparent that, for analysis purposes, it was necessary to stratify participants. I separated both the control and treatment groups into subgroups of sophomores and juniors, predominantly due to the fact that the ISAT is a grade level exam and cannot be used to make comparisons between students at different grade levels.

In addition to the formation of subgroups, it became apparent that the sample size of the control group was going to be limited significantly. Recall that scheduling issues at Washington High, which was using a hybrid schedule, necessitated a large number of schedule changes at the semester break immediately before this unit began. The decision was made to use a matched pair design for both the sophomore and junior subgroups; this matching was made based on students' base-line data, ISAT scores. This was a relatively easy task for the juniors, since the ISAT scores for the juniors aligned nicely. Unfortunately, though, for the sophomores, creating matched pairs was not as simple. Due to what appeared to be nothing but chance, eight sophomores and one junior from the control group had ISAT scores that were not shared by any student in the treatment group. As a consequence, the pairs were impossible to match perfectly, and any results, particularly from the sophomore subgroup, may be questionable. Further study may be necessary to examine the influence that concept booklets had on sophomores' conceptual understandings.

In addition to analysis of overall test results, analyses of individual parts of the test were also performed. The first part of the test consisted of very straight-forward items, such as exercise 7 (see Appendix B), which simply asks students to find the measure of the angle between two given vectors. There is a formula for this, and from my experience, students can memorize and apply the formula without really knowing what they are doing. On the other hand, the second part of the test was filled with much more open-ended items, such as exercise 5 (see Appendix B), which asks students to determine the possible locations that a boat traveling across a flowing river can reach the opposite side. The solution involves creatively assigning vectors to represent the situation and successfully applying the vector concepts and operations introduced in the instructional unit. The aim of implementing concept booklets in the first place was to help students improve their abilities to approach the types of problems that were encountered in the second part of the examination, so it was logical to analyze these two parts separately. Consequently, I performed statistical analyses on students' scores on the entire test as well as Parts I and II, which I will call Part I: Closed and Part II: Open for descriptive purposes.

It was discovered, after the exam was completed, that the results of two items from the second part of the examination would need to be omitted from the analyses. Unfortunately, Ms. R reported that she had given her students assistance on the intent of the word *resultant*, found in the first question on Part II, and she warned students about misreading the second question to find the height of the shed instead of the desired measurement, the length of the roof. Students at Lincoln High received neither of these advantages, so these two questions were omitted from the analyses.

Finally, the results of six questions were analyzed based on diagrams, one of the classification codes which emerged during the analysis of the exam. Some of the coding tracked computational and algebraic errors, but, as was mentioned previously, the main purpose was to analyze the performance of students who drew accurate diagrams of the problems presented on the exam against the performance of those students who did not. This analysis attempted to look at whether exposure to the concept booklets affected whether or not a student would provide a diagram, more importantly an accurate diagram, of the problem and a subsequent correct solution. Six questions, where the ability to diagram seemed beneficial to determining the solution, were identified and analyzed within the treatment and control groups, as well as for the pooled group. It is important to remember that these analyses were intended as a microcosmic evaluation of the connection between concept booklets and the use of diagrams and provided motivation for further examination of the benefits of concept booklets.

Sophomore Exam Results

As mentioned previously, the matched pairs of sophomores were not perfect, so a paired t-Test was performed based on their ISAT scores with a null hypothesis that there would be no difference in ISAT scores between control and treatment groups, significant at the .05 level. The test revealed mean ISAT scores for the treatment and control groups of 265.8 and 265.1, respectively, and a P-value of .182, so the null hypothesis was not rejected. In addition, the test revealed a strong Pearson Correlation of .984, meaning that there was a strong correlation between the two groups ISAT scores. In other words, a low test score for a student from Lincoln High tended to correspond to a low test score for a student from Washington High School, and a high test score in one group tended to correspond to a high test score in the other group.

Individual analyses of the whole exam and each of the two parts, shown in Table 1 revealed significantly better performance by the treatment group, with P-values of .006 for the whole examination and .017 and .038 for Part I: Closed and Part II: Open, respectively. These values indicate that, for the entire exam and both parts, we can reject the null hypothesis, that any differences between the groups could not be explained by receiving the treatment, with significance occurring at the $p = .05$ level. As a result, we can conclude that the implementation of concept booklets in these classrooms resulted in an improvement in students' performance on assessments. This appears to be the case, not only for traditional, straight-forward items designed to test procedural processes, but also for the types of open-ended items designed to test a student's ability to design and implement flexible solutions to more abstract problems.

Table 1.

Sophomore Performance on Whole Examination and Parts I and II.

Examination Part	Control Mean	Control Variance	Treatment Mean	Treatment Variance	*p*
Whole	149.1	468.7	165.0	363.8	.006*
Part I	107.9	194.8	118.0	151.4	.017*
Part II	41.3	121.7	47.0	107.9	.038*

*$p < .05$

Junior Exam Results

Analyses of juniors' performance on the whole examination, as well as Parts I and II, can be seen in Table 2. It is interesting to note that while the juniors in the treatment group did indeed have higher mean scores, their improved performance on the whole examination and Part I was not statistically significant. Performance on Part II of the examination, however, was significantly better at the .05 level, with a P-value of .008. My experience is that juniors enrolled in Lincoln High's Pre-Calculus are more akin to members of the general student population, as opposed to sophomores, who tend to be more naturally gifted mathematics students. The reason that juniors seem to have reached this level of mathematics is more often due to effort and to well-developed work ethics rather than intuitive understandings of mathematics. This may indicate one possible explanation for the reason that statistically significant results were found for the whole exam and both sub-parts for the sophomores, while results for the juniors were only significant on Part II: Open. This is purely speculation, however, and would require further investigation into these high-school level differences to answer with any sense of finality.

Table 2.

Junior Performance on Whole Examination and Parts I and II.

Examination Part	Control Mean	Control Variance	Treatment Mean	Treatment Variance	*p*
Whole	134.8	1142.4	152.3	256.7	.052
Part I	99.8	664.6	109.7	208.9	.193
Part II	35.0	171.4	43.1	58.0	.008*

*$p < .05$.

Concept Booklets and Diagram Analysis

To investigate the concept booklet's influence on students' likelihood to provide accurate diagrams of problems and subsequent correct solutions, six items, three from each part of the examination, were analyzed based on whether the student provided an accurate diagram for the problem or not. The three items from Part I: Closed were exercises 2, 9, and 10 (see Appendix B). The student samples that follow illustrate some of what was seen in the analysis.

Exercise 2 tested students' understanding of the ambiguous case of the Law of Sines. The given triangle measurements can result in two unique triangles. Generally, it is to the student's advantage to sketch the triangle in order to better understand the differences between triangles. Two examples of student responses to this question are shown in Figures 1 and 2.

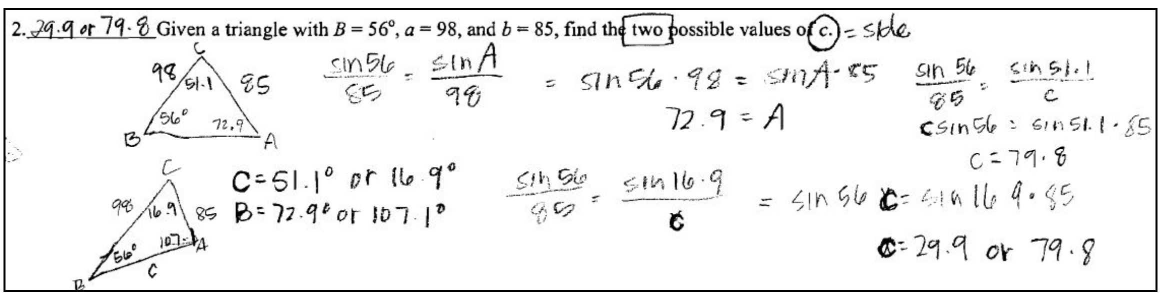

Figure 1. Accurate supporting sketches of the two correct solutions to the problem presented in exercise 2 of Part I.

Figure 1, which shows a solution by a junior from the treatment group, is an example of an accurate sketch of the scenario and a subsequent correct algebraic solution to the problem. Note that this student drew the two triangles with relatively accurate scale to represent the two different solutions to the problem. To contrast this, Figure 2, which was also done by a junior from the treatment group, although this student was not included in the junior strata, shows an inaccurate rendering of the scenario and a subsequent incorrect solution. In this situation, the larger triangle is an accurate depiction of one of the solutions, but the student attempted to change the angle located at vertex *B*.

This incomplete conception is evident in the sketch of the problem and led to an incorrect algebraic solution.

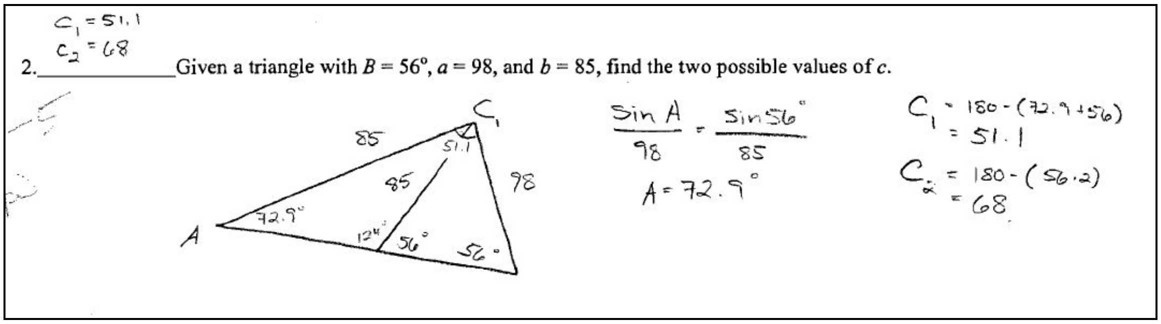

Figure 2. An inaccurate sketch of the problem presented in exercise 2 of Part I.

Exercise 9 involved finding the measure of the angle between two given vectors. While this can be a formula-driven skill, students usually benefit from sketching the vectors first to get a rough estimate of the size of the angle. Figure 3, done by a sophomore from the treatment group, shows the power of the ability to sketch accurately. This student elected not to use the traditional formula; instead, simple trigonometric ratios were used to find angle measures and subtract to find the angle between the vectors. This indicates a strong, yet flexible approach to problem solving supported by the ability to sketch a situation accurately.

In contrast, Figure 4 shows a miscalculation that might have been caught if the student had sketched the picture in addition to utilizing the formula. It is unclear how this student, who is a junior in the treatment group, came to 26 for the dot product of the two vectors, but it resulted in a smaller value for the cosine and a subsequently larger angle measure. A sketch could have helped the student recognize that the measure of the angle was actually smaller than the found value.

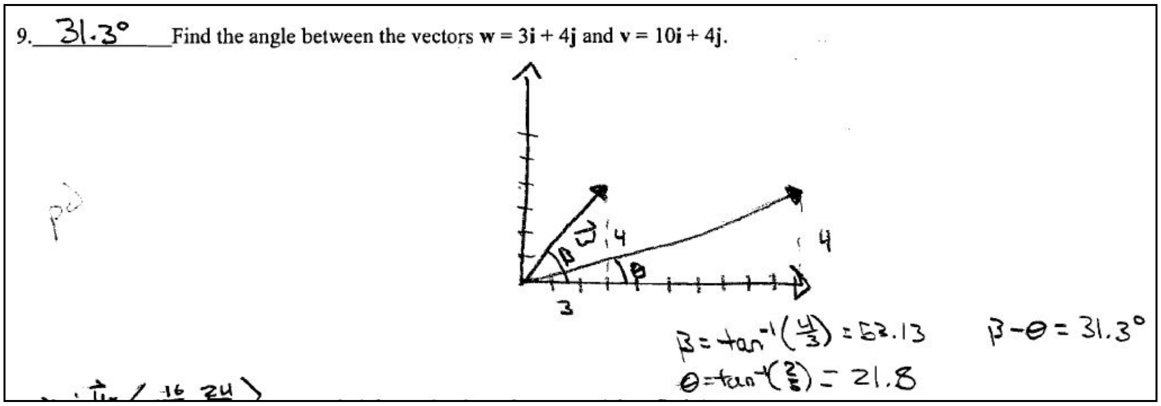

Figure 3. A novel approach to finding the angle measure in exercise 9.

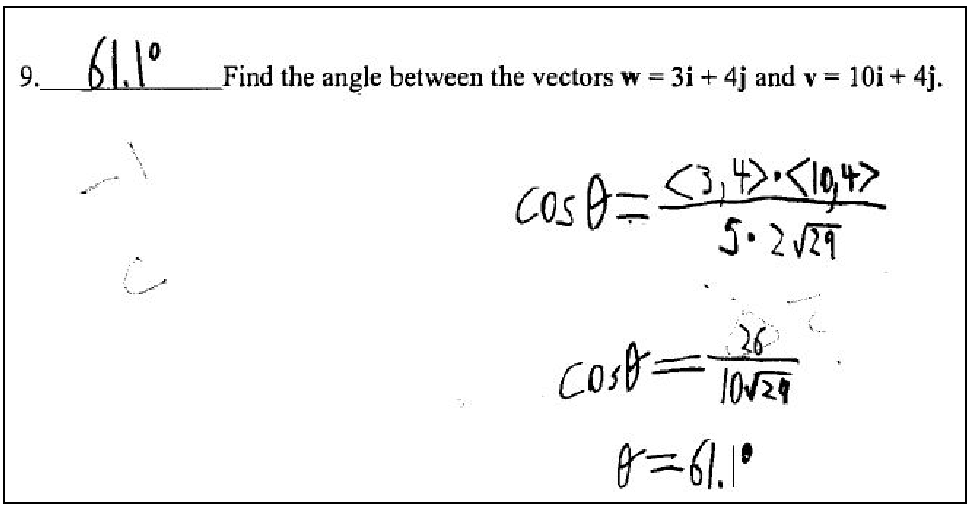

Figure 4. A miscalculation made without a sketch to indicate a smaller angle.

Figure 5 shows how one student can have great success with a sketch on one exercise and struggle on the very next problem. This student, who is a sophomore from the control group, did a wonderful job diagramming the vectors in exercise 9. Note that the solution in this case was the exact same novel solution shown in Figure 3, which does not rely on the typically-used formula. In contrast, the same student's response on exercise 10, which is discussed in the following paragraph, is completely wrong, despite having the formula apparently memorized. This is another illustration of how simply having a formula does not necessarily translate into an understanding of the mathematical relationships involved.

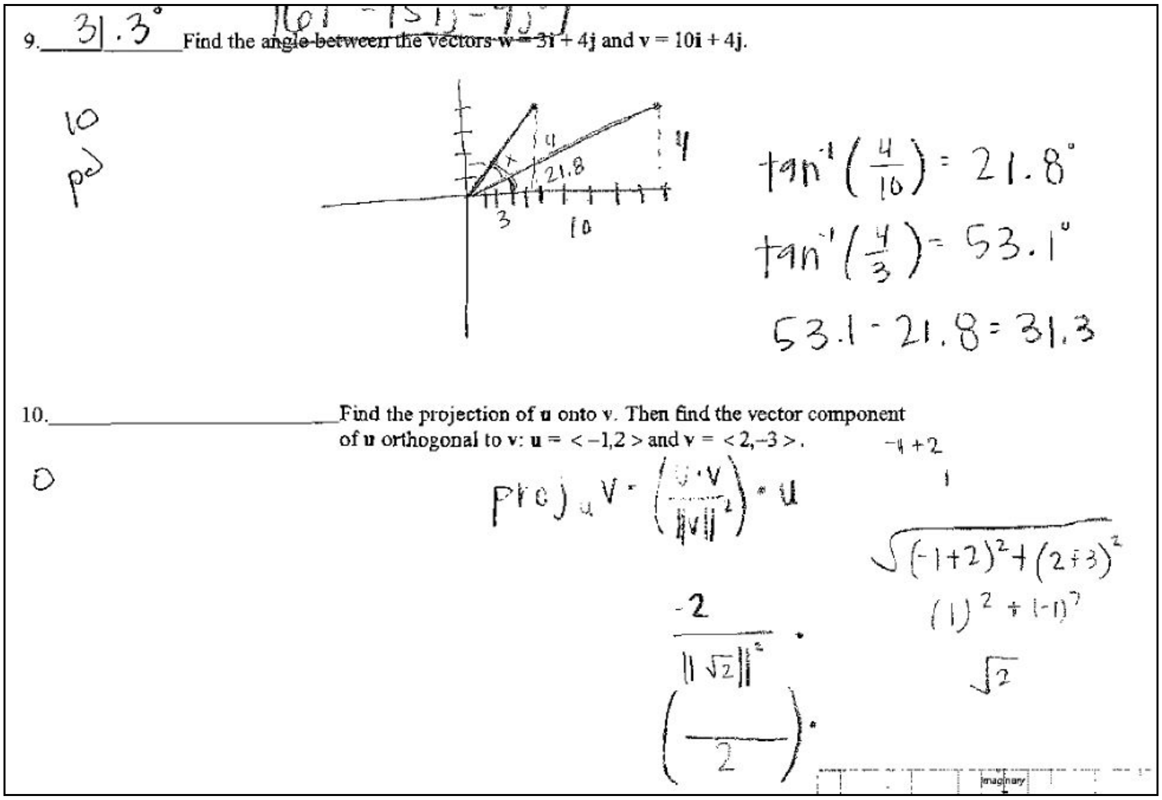

Figure 5. The same student uses a diagram effectively on exercise 9 and is unable to find the correct solution on the very next exercise without a diagram.

Exercise 10 involved students' ability to find the projection of one vector onto another, as well as a component of the vector that is orthogonal to the projection. In my and my colleagues' teaching experience, this is a skill that is notoriously difficult for students to master, but the ability to estimate the projection and orthogonal component from a sketch of the vectors seems to generally give students an advantage over those who lack this skill. This ability is evident in Figure 6. Note that the student, a junior from the treatment group, initially drew vector **u** incorrectly in the direction of the third quadrant but did not erase the incorrect vector. Vector **u** is shown without an arrow, and the projection and orthogonal components are shown with arrows. The orthogonal component is barely visible to the right of the origin since its magnitude is so small.

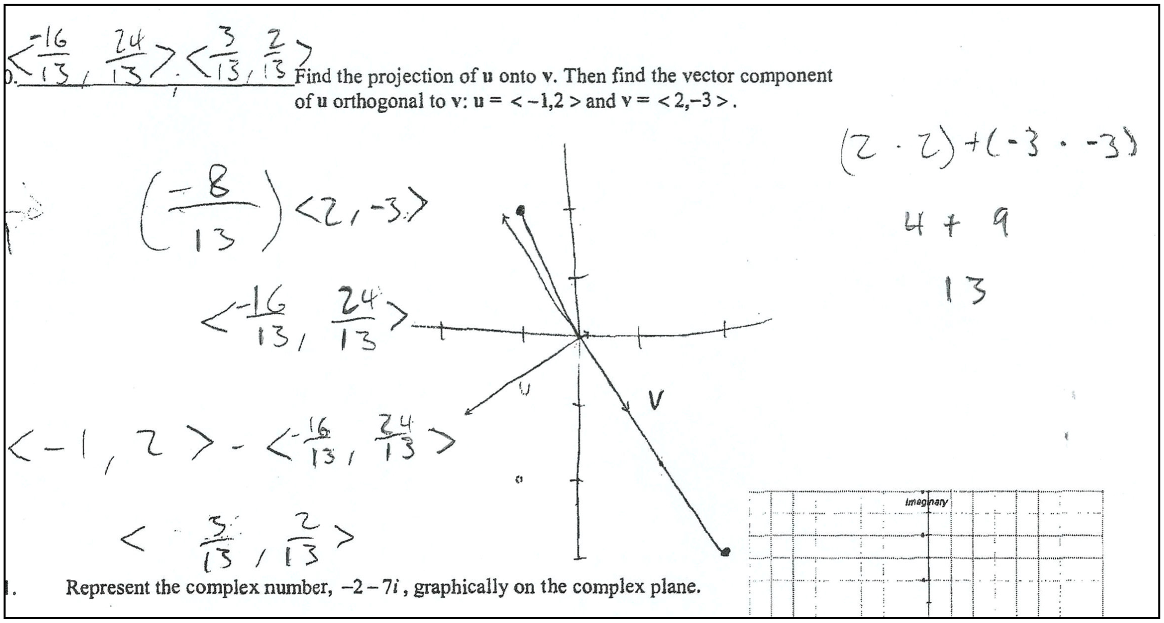

Figure 6. A sketch of the projection of vector **u** onto vector **v** and the orthogonal component of **u** aids the correct calculations of each vector.

In Figure 7, the student has calculated the projection correctly but mistakenly calculated the orthogonal component. It is reasonable to suggest that an accurate sketch of the vector would have helped this student, a junior from the treatment group, to recognize the implausibility of the answer.

Figure 8, which was done by a sophomore from the control group, illustrates the fact that an inaccurately drawn sketch is usually no better than the absence of a sketch. While the original vectors are drawn accurately, it appears that this student is unsure of how to sketch the projection and orthogonal vectors correctly. Again, an accurate sketch may have helped this student to better approximate the correct solution.

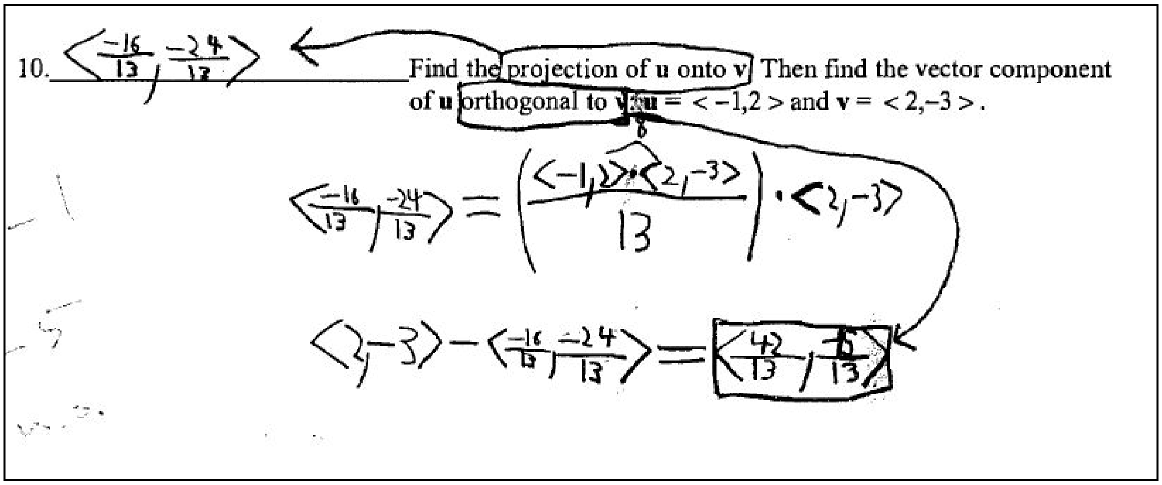

Figure 7. Without a sketch, the correct projection is found, but the orthogonal component is found incorrectly.

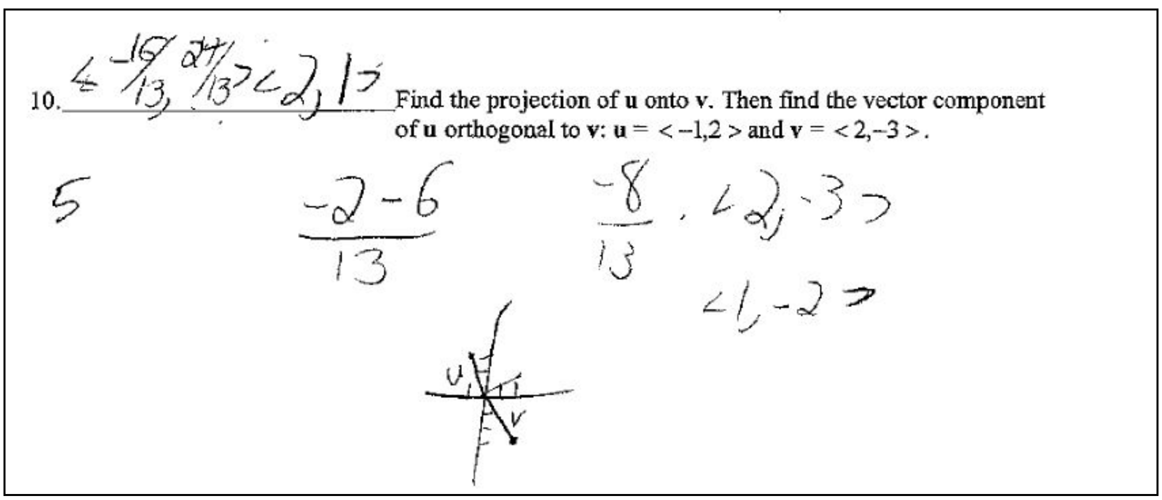

Figure 8. An incorrectly drawn sketch does little to help a student approximate the correct solution to exercise 10.

The three items chosen for analysis from Part II: Open were exercises 3, 4, and 5. Exercise 3 requires students to find the bearing that one would need to travel from one town to another given the relative locations of the two town and a third, additional town.

While the problem can be solved formulaically, sketching a map of the situation helps to find an easy path to the solution, as evidenced by the sample shown in Figure 9.

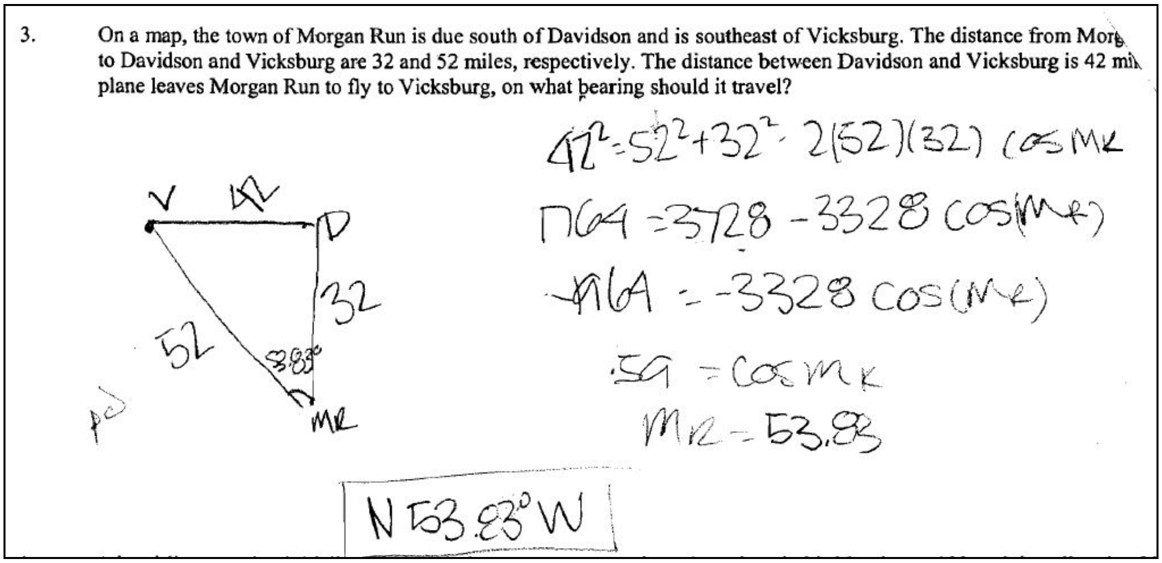

Figure 9. A junior from the treatment group correctly solves exercise 3 from Part II.

Exercise 4 involves finding the true velocity of an airplane in moving air. This is yet another item where the ability to sketch the scenario accurately provides an advantage in knowing roughly what the solution should be. Figure 10, which was done by a junior from the treatment group, illustrates this notion even though the relative lengths of the vectors are not very accurate. Despite this limitation, the student benefitted from seeing that, since both vectors are moving horizontally to the right, the resultant vector would have a horizontal component greater than the vectors' individual horizontal components.

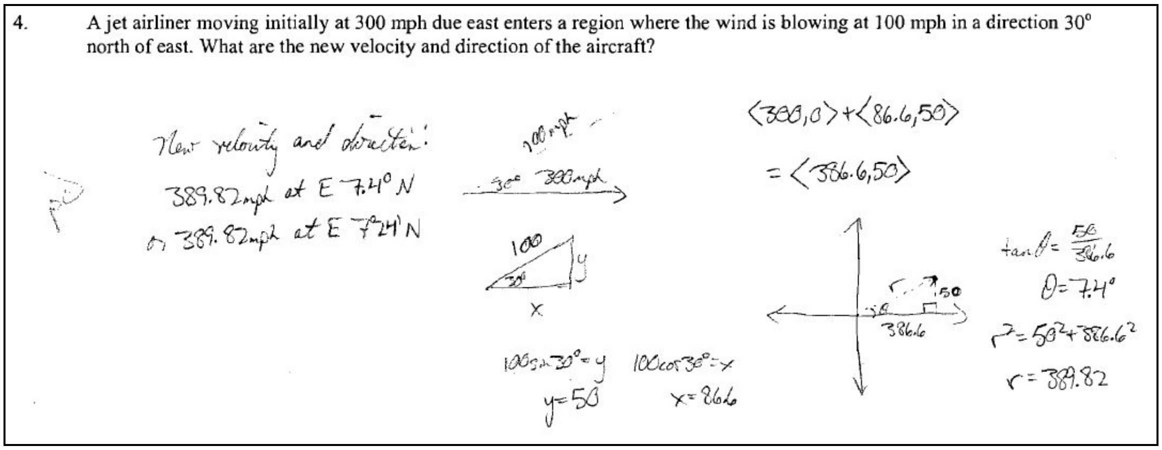

Figure 10. The sketch is beneficial by showing that both vectors are moving to the right.

For comparison to what is seen in Figure 10, note the improperly drawn vectors shown in Figure 11, which was done by a sophomore from the control group. The origin of this students' mistake is unclear, but the picture was directly responsible for the incorrect response, since the wind was drawn as having a northwesterly direction.

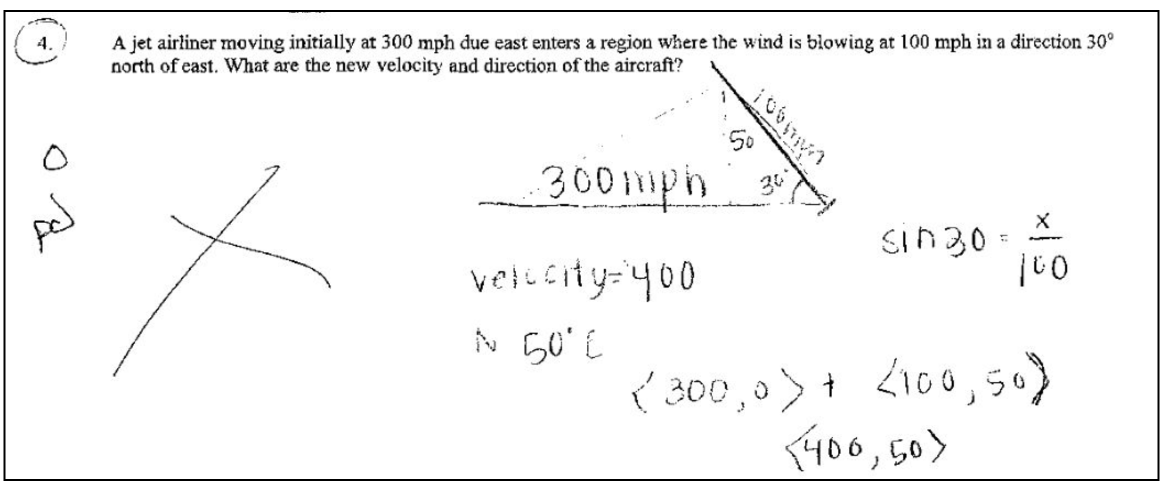

Figure 11. A response to exercise 4 that signifies an incomplete understanding of how to orient vectors.

Also of note in Figure 11 is the response on the right. This students' reference to the wind causing the plane to *rise* demonstrates confusion about direction and altitude.

This is an excellent case of a student who has well-developed procedural understanding, evidenced by their correct application of the Law of Cosines, but at the same time, a lack of conceptual understanding that prohibited a successful completion of the task. This is a clear instance of where concept booklets would allow me to communicate with students and attempt to remediate these types of incomplete conceptual understandings.

Finally, exercise 5 involves a boat crossing a flowing river in a fixed time. There are two potential landing sites, one upstream, and the other downstream, and again, the ability to sketch the scenario correctly is highly beneficial. Incidentally, this is the exercise that was reported most by students at both Lincoln and Washington as the hardest problem on the examination. The overall average score on this exercise was .984 out of 10 points, so it appears that students accurately gauged their performance on this problem.

5. A boat requires 2 minutes to cross a river that is 150 meters wide. The boat's speed relative to the water is 3 m/s, and the river current flows at a speed of 2 meters per second. At what upstream or downstream points could the boat reach the opposite shore in 2 minutes?

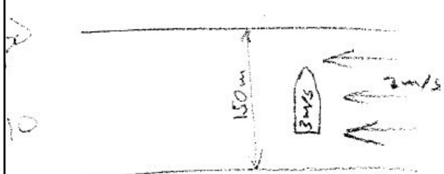

Figure 12. A response to exercise 5 that displays a lack of recognition that the boat cannot cross the river perpendicular to the banks.

Figure 12, shows one of the many incomplete interpretations of this problem. In this example, the student has not recognized that the boat cannot cross perpendicular to the bank since it requires two minutes to cross the river. When considered this way, it is easy to see how the study would be confused by the fact that the question asked them to find *two* points, one upstream and the other downstream, where the boat could land.

Most responses to this question were similar to the one in Figure 12, written by a junior from the treatment group, but there were some promising solutions, especially the one shown in Figure 13. The first interesting component of this solution, from a different junior in the treatment group, is the right triangle in the middle of the figure where the student was using the Pythagorean Theorem to solve for time. (This is a much more abstract application of the Pythagorean Theorem than is usually used.) Next, the solution to the triangle was used to find the angle at which the boat would have to travel. This angle was then used to break the boat's velocity vector into components. These components were then used to find how far upstream and downstream the boat would travel. It is difficult to imagine that this exercise could be solved in this manner without sketching an accurate representation of the figure. Developing this kind of sketching ability is, for this reason, one of the objectives of implementing the concept booklets.

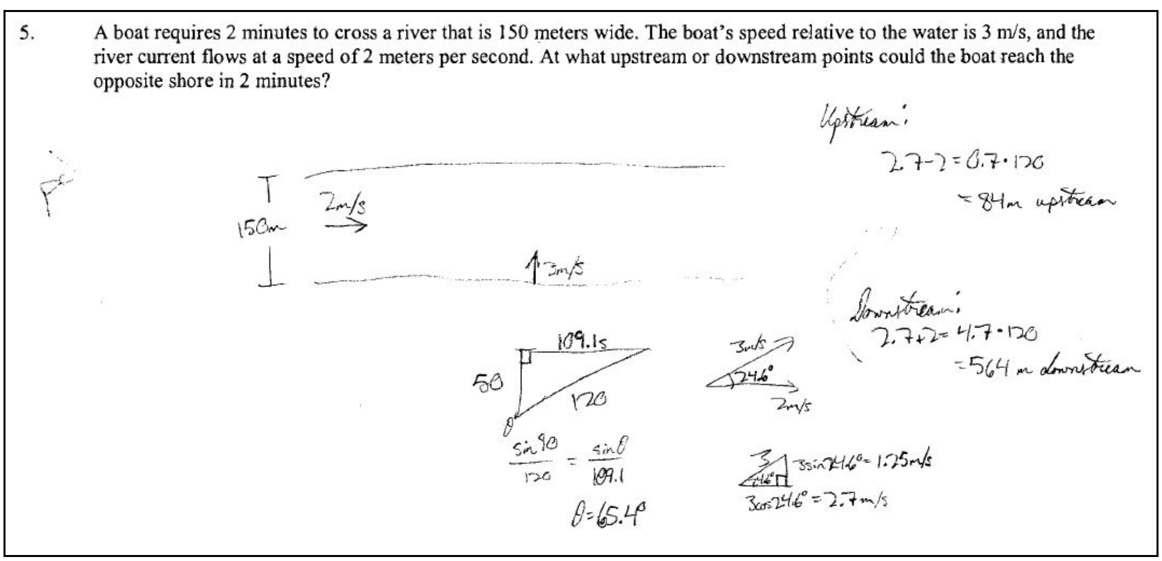

Figure 13. A response showing a deep understanding of how to apply vectors.

Six analyses were performed for each of the six items. The first three analyses were comparisons of subgroups of the sophomore control group, sophomore treatment group, and pooled group of sophomores. The second three analyses for each examination item were comparisons of the junior control group, treatment group, and pooled group of juniors. It is important to note that all of the previous t-Tests that were performed on the matched pair groups for overall examination performance assumed matched pairs. While these tests are examinations of the same groups of students, they are of subgroups of each group. As a result, they are not matched within each group, so an f-Test for variance was performed for all analyses of each of the six exam items. The null hypothesis of each f-Test was that there was no difference in the variances of the group. If the results of the f-Test returned a value less than .05, the null hypothesis was rejected, and the t-Test that was performed assumed unequal variances. Otherwise, all t-Tests that were performed assumed equal variances.

The results of the sophomore diagram analysis for the examination items from Part I: Closed are shown in Table 3. There are a few items of note from Table 3. First, note that *t*-Tests were not possible for exercises 2 and 9 for the control group and for exercise 2 for the treatment group, due to the incredibly low number of students in these subgroups. Also, while the results of the analysis of exercise 2 appear to show a level of statistical significance, it is important to pay attention to the fact that only two students did not provide an accurate diagram of the problem, and both of those students provided a correct solution. As a result, the P-value is not meaningful in this case.

Table 3.

Sophomore Diagram Analysis for Part I Examination Items

		Exercise 2	Exercise 9	Exercise 10
Control	DNP count	2	12	21
	DNP Mean	10.0	9.9	5.8
	DP count	20	10	1
	DP Mean	8.4	9.9	9.0
	p	-	.899	-
Treatment	DNP count	0	16	16
	DNP Mean	-	9.6	7.6
	DP count	22	6	6
	DP Mean	9.3	9.0	8.3
	p	-	.075	.510
Pooled	DNP count	2	28	37
	DNP Mean	10.0	9.8	6.6
	DP count	42	16	7
	DP Mean	8.9	9.6	8.4
	p	.000	.583	.162

Note. “DNP” denotes subgroups of students who did not provide an accurate diagram. “DP” denotes subgroups of students who did provide an accurate diagram. Also, all questions were out of ten points.

Finally, note that the performance of the subgroup of the treatment group that provided an accurate diagram actually performed worse than the subgroup that did not. This may indicate that either the diagram is not essential after all or that students understand this concept so well that the only individuals who drew the sketch were the ones that struggled with basic understanding of the problem. In general, the data in Table 3 suggests that, for the sophomores, diagramming was not necessarily beneficial to their performance on Part I of the examination.

Table 4 shows the results of diagram analysis of the juniors' performance on Part I of the examination. Again, three of the *t*-Tests could not be run, in this case because of the low numbers of students in one of the subgroups. However, two of the tests revealed significantly better performance, both on exercise 9. Recall that this exercise asked students to find the angle between vectors. It seemed that students who sketched a diagram were better equipped to know whether their answer was plausible based on the approximate size of the angle in their diagram. Similar to the sophomore subgroups, students who diagrammed exercise 10 actually had a lower mean score than those who did not. Again, this may be due to the reasons suggested for the sophomores.

Table 4.

Junior Diagram Analysis for Part I Examination Items

		Exercise 2	Exercise 9	Exercise 10
Control	DNP count	1	11	15
	DNP Mean	10.0	6.6	5.2
	DP count	18	8	4
	DP Mean	6.3	9.8	5.0
	p	-	.046*	.931
Treatment	DNP count	1	15	14
	DNP Mean	5.0	9.3	7.0
	DP count	18	4	5
	DP Mean	7.6	10.0	6.8
	p	-	-	.905
Pooled	DNP count	2	26	29
	DNP Mean	7.5	8.2	6.1
	DP count	36	12	9
	DP Mean	7.0	9.8	6.0
	p	.822	.022*	.961

* $p < .05$

Table 5 displays the sophomore results for the open-ended problems found on Part II of the examination. The only result here that shows a level of significance is the performance on exercise 3 by the sophomores from the treatment group. The reasons for this anomaly, however, are not clear.

Table 5.

Sophomore Diagram Analysis for Part II Examination Items

		Exercise 3	Exercise 4	Exercise 5
Control	DNP count	5	9	17
	DNP Mean	6.0	2.8	0.0
	DP count	17	13	5
	DP Mean	8.2	6.2	0.0
	p	.206	.102	-
Treatment	DNP count	6	13	2
	DNP Mean	6.8	5.7	2.5
	DP count	16	9	20
	DP Mean	9.7	7.6	1.3
	p	.000*	.296	.553
Pooled	DNP count	11	22	19
	DNP Mean	6.5	4.5	0.3
	DP count	33	22	25
	DP Mean	8.9	6.7	1.0
	p	.063	.096	.200

* $p < .05$

Finally, the junior performance on Part II is shown in Table 6. Note that several of these tests show significantly better performance regardless of whether it is the control, treatment, or pooled group. This indicates that providing an accurate diagram of the problem was beneficial to juniors in helping to find a correct solution to the problem. An interpretation of how this may be tied to exposure to concept booklets is offered in Chapter 5.

Table 6.

Junior Diagram Analysis for Part II Examination Items

		Exercise 3	Exercise 4	Exercise 5
Control	DNP count	8	11	10
	DNP Mean	5.6	.8	.0
	DP count	11	8	9
	DP Mean	9.0	9.1	.6
	p	.059	.000*	.347
Treatment	DNP count	4	8	9
	DNP Mean	8.8	2.5	.0
	DP count	15	11	10
	DP Mean	9.1	6.5	3.3
	p	.813	.038*	.007*
Pooled	DNP count	12	19	19
	DNP Mean	6.8	1.5	.0
	DP count	26	19	19
	DP Mean	9.0	7.6	2.0
	p	.058	.000*	.006*

* $p < .05$

CHAPTER 5: CONCLUSIONS

Potential Differences in Grade-Level

Certainly the most striking part of this study is how differently the sophomores and juniors performed in this study. The sophomore subgroup performed significantly better on the whole examination as well as the sub-components, Part I: Closed and Part II: Open. At the same time, the juniors only reached a level of significance on Part II: Open. Moreover, the diagram analysis suggested that the ability to generate an accurate diagram is more of a factor for juniors than sophomores. As mentioned previously, this may be an indication of the different types of students who wind up taking an honors mathematics class. It has been my experience over the past eleven years that it is often the sophomores in the room who rely heavily on intuition, while the juniors are more reliant on procedures and algorithms. As a result, sophomores may have benefited from the process of journaling in the concept booklets not only on the traditional closed questions from the first part of the test, but also on the open-ended questions in the second part of the exam. I hypothesize that this may be because, from my experience, they work more on intuition. In any case, adding a visualization component to their mathematical intuition helped them on both styles of questions.

In contrast, it has also been my experience that juniors may be more reliant than sophomores on rote memorization to master the skills that are tested on the examination, mainly because they may be at different stages in the development of their mathematical abilities. Juniors in this course are more often the students who make comments like, “I don't want to know *why* I'm doing it; just tell me *how* to do it." As a result, working with concept booklets did not seem to provide an advantage on the closed questions from Part I. This might be because juniors will practice a skill repeatedly until they are confident that they have mastered it, as opposed to trying to generate a sketch to interpret a problem. On the other hand, when the questions were more abstract and required the ability to look more flexibly at a not memorized situation, juniors began to benefit from the journaling.

Another potential motivation for investigating grade-level differences in this study may be to help explain some of the characteristics of the general student population. As was mentioned previously, my experience with juniors in pre-calculus classes is that they tend to exhibit qualities similar to general mathematics students who are not taking accelerated mathematics classes. I attribute this to their being closer to the average grade-level at which most students take a pre-calculus class. Accordingly, their response may be more indicative of how the general school population might respond to the concept booklets. Whatever the case, this is an area that deserves further examination, not only into how different grade-levels respond to this type of journaling, but also the extent to which it makes a difference in populations of honors students, mainstream students, and students who consistently struggle with mathematics.

Potential Differences in Self-Perception of Mathematical Ability

Another item of interest here is the performance on the entire examination by the juniors from the control group. What is unusual in this instance is the incredibly high variance shown in Table 2 by the junior subgroup of the Washington High School group. Note that the P-value returned for the whole examination was very close to the chosen level of significance. If not for this high variance, the separation between groups would certainly be enough to reach this level of significance. Of note, however, were potential causes for this. When evaluating and coding the examinations, it became immediately clear that many of the juniors either knew what they were doing, or they simply did not. To explain this, I return to the idea of older grade-level students in pre-calculus using lower level cognitive skills, such as rote memorization, to prepare for examinations. In other words, they are less likely to attempt to experiment until they find a workable solution. Generally, students in my classes who approach the curriculum in this manner will not attempt a problem that they do not immediately know how to do. One of the juniors who participated in this study is an excellent example of this difference. She spent an inordinate amount of time preparing for every examination she faced during the year. Despite all of the preparation, though, if she encountered a situation that wasn't exactly like the ones she had practiced, she would give up. I spent a great deal of effort trying to help her develop her abilities to interpret more abstract problems, but in the end she would always return to rote practice. More intuitive students, who seem to be more frequently encountered at the sophomore level, will play persistently with several ideas, hoping to find a path that leads to a working solution. This may explain the wider range of scores for the sophomores and suggest the importance of fostering confidence in our students; otherwise, they risk the possibility of giving up before they begin.

Coding of Examinations

During evaluation of the students' examinations or instrument, several themes began to surface in the responses. First, it was clear that many, if not most, students had not spent a great deal of time thinking about force and velocity interactions between objects, primarily those who had never taken a physics course, which is probably somewhere around 95% of my student population. Simple relationships like Newton's Third Law, the classic relationship between active and reactive forces, had simply never been considered by these pre-calculus students. This naturally makes understanding force diagrams trickier, a phenomenon I have seen through working with students who have been educated using a curriculum that emphasized physics before the other sciences. This may suggest the importance of the prerequisite courses for pre-calculus. So much of pre-calculus course-content depends on physical examples, students with a science background usually had an advantage. It was my experience that the students in the treatment group, who were usually better able to successfully diagram a relationship in their booklets, generally had better responses to the prompts. As a result, I decided that, if the examination was going to be thoroughly analyzed, these responses would have to be coded in order to better track some of the trends that were seen in the concept booklets.

Figures 14 and 15 illustrate the kinds of differences that were visible in the responses to the journal prompts. Figure 14 is an example of a student, a junior, who is computationally very strong but usually hesitates to answer concept-rich questions. This student's response seems to focus less on the question about the vectors and more about the part pertaining to *preventing arm and shoulder fatigue*. This response demonstrates a reluctance to discuss the concept in a technical manner, which may simply be a consequence of inexperience with physics.

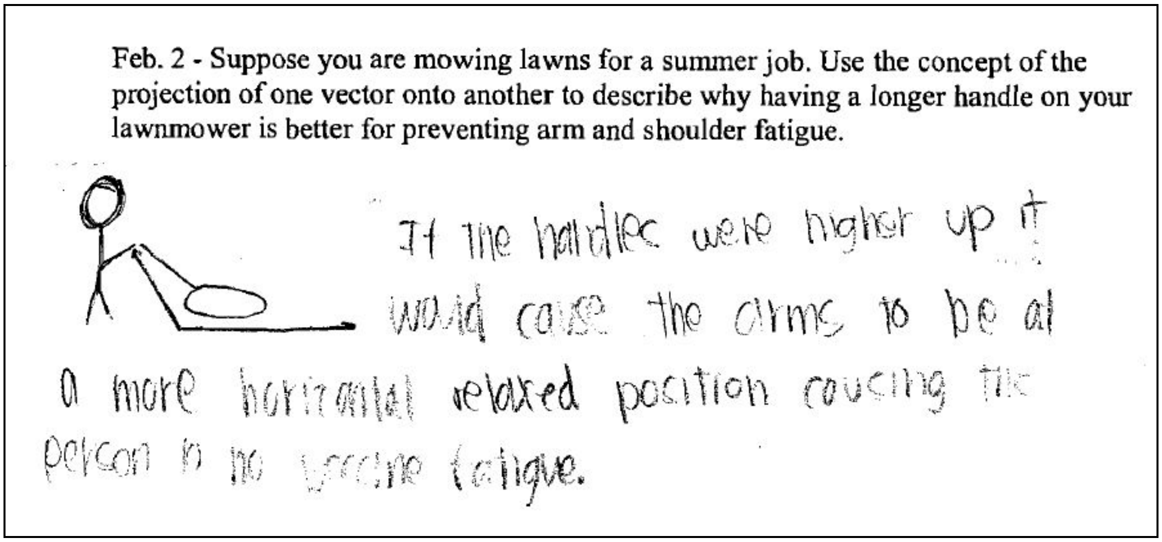

Figure 14. A response from a student with little physics experience.

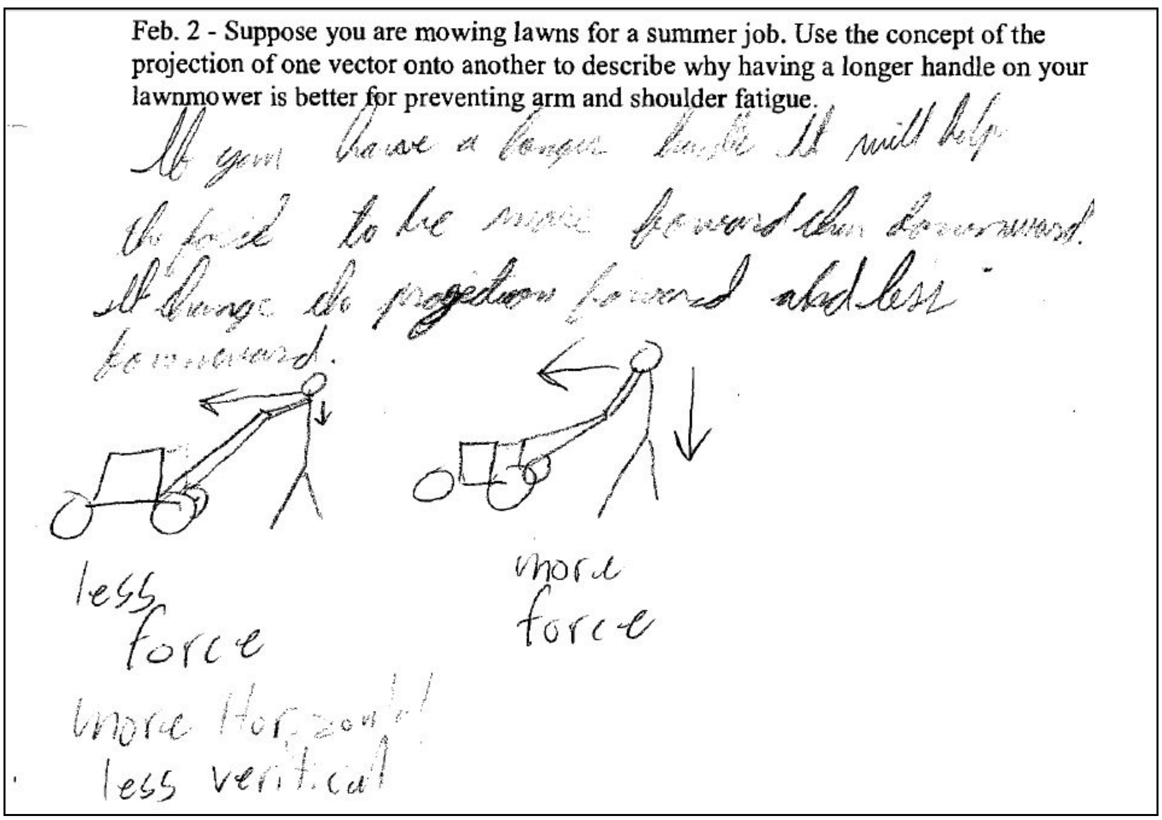

Figure 15. This response is by a student with physics experience.

Figure 15, on the other hand, shows a response by a student who had taken a conceptual physics course. This student, a sophomore, approached the question with much more technical precision, as can be seen in the vector components shown in both sketches.

How Concept Booklets Influenced Diagramming

In Chapter 4, I indicated some reasons for coding the examination items, primarily to investigate how concept booklets influenced whether or not a student provided an accurate sketch of the problem. The practice of diagramming the problem seemed most beneficial for the juniors on the second part of the examination, so my comments will pertain mainly to Table 6 in Chapter 4.

First, note that significantly better performances were seen by students who provided an accurate diagram of the problem versus those who did not. This is true regardless of whether they were a member of the control group or treatment group. This seems to indicate that an accurate diagram of the problem is beneficial to finding a correct solution to the problem, regardless of exposure to concept booklets. As a result, the question is not whether a sketch is beneficial to the discovery of a correct solution; it appears to be. Instead, the question becomes: were students who were exposed to the concept booklets more likely to provide an accurate sketch for the problem than students who were not exposed to the concept booklets? Although this study may not be able to fully answer all aspects of this question, there are some interesting results to be seen in Table 6. Note that, for all three of the open-ended exercises from Part II, the ratio of students who provided an accurate sketch of the problem to those who did not is higher in the treatment group. These ratios, summarized in Table 7, may explain some of why the juniors from the treatment group performed significantly better than the juniors from the control group on the open-ended items from Part II.

Table 7.

Ratio of Junior Students Who Provided an Accurate Diagram to Those Who Did Not on Open-Ended Questions From Part II

	Exercise 3	Exercise 4	Exercise 5
Control	11/8*	8/11*	9/10
Treatment	15/4	11/8*	10/9*

* Students from this group who provided an accurate diagram performed significantly better at the $p = .05$ level on this exercise.

This ratio may be higher in the treatment group because of the number of journal prompts that required students to sketch a diagram to answer the question. This was true for five of the ten journal prompts used during the unit, those on January 20, 27, and 30, as well as February 2 and 3 (see Appendix A). In each of these cases, a sketch was used by the vast majority of students in the treatment group. The prompt for January 20 required students to diagram a triangle, but the four other prompts involved vectors. In every one of these instances, the main focus in class while discussing their responses to the journal prompt was on whether or not students could accurately sketch the problem or scenario presented in the prompt. The concept booklets provided me with an opportunity to reinforce and encourage the use of drawings as an organizational tool for solving problems. Recall that students also received written journal feedback as well as participating in solution-discussions.

This study was not designed to answer this specific question using a statistical analysis; rather, it was to determine statistically if concept booklets influenced exam scores. During my qualitative examination of the students' work, it appeared that there was a relationship between the use of concept booklets and students' use of diagrams as part of their solution process. While qualitative analysis might suggest a relationship, statistically significant analysis pertaining to concept booklets and the likelihood of sketching an accurate diagram cannot be determined from this study. However, this is definitely worthy of further investigation. Qualitative or quantitative analysis could answer the questions of not only how the concept booklets affected whether or not a student provided an accurate diagram of the problem, but also why this seems to be more of a factor for juniors than sophomores.

Other Suggested Further Research and Final Comments

The statistical results of this study, which aimed to determine if concept booklets would have a positive effect on closed and open-ended curriculum assessments, signify significantly better examination results on the entire examination as well as Part I: Closed and Part II: Open for sophomores. While the juniors did not perform significantly better on the entire examination or Part I, their performance was significantly better on Part II. Concept booklets and the accompanying discussions are an instructional strategy that may improve mathematics performance on assessments of not only traditional skill-oriented problems, but also on open-ended problem-solving tasks. From a practical standpoint, concept booklets are easy to implement in class and add little to the workload for teachers in terms of assessing responses.

Due to limitations of the study, the extent to which concept booklets affect performance at different grade-levels and different levels of mathematical topics remains in question. As a result, it may be prudent to investigate further the differences between different grade-levels in a variety of mathematics classrooms, such as the differences between freshmen and sophomores in Algebra 2, or between sixth and seventh graders in Pre-Algebra.

It would also be beneficial to see this study implemented in other mathematics classes that are not generally considered to be honors classes. This study involved students who are typically college-bound and take their academic studies seriously. What would the study yield if it was implemented in a class aimed at the general student population, or even in a skills class for students who historically have struggled with mathematics?

Finally, what kinds of results would be seen if concept booklets were implemented earlier in the curriculum sequence, in classes such as geometry, pre-algebra, or even at the elementary level. Early experiences could potentially reduce some of the learning curve that comes with implementing this type of journaling in the math classroom, especially since so much of the time spent implementing this activity is dedicated to teaching students to learn how to respond to mathematical journal prompts. If this was a task built into the general mathematics curriculum, it could potentially become a more powerful tool than just the limited role it is shown to have in this study.

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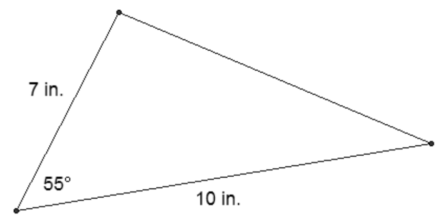
APPENDIX A

Journal Prompts

Date	Lesson Topic	Journal Prompt (Investigate = I, Reflective = R)
Mon., Jan. 19	N/A	No School – Martin Luther King, Jr. Day
Tue., Jan. 20	Law of Sines, including the ambiguous case (SSA)	I – Consider the figure below. Explain why the area of the triangle cannot be found with the traditional formula, $A = 0.5bh$, using any of the given measurements. Describe a method that could be used to find the area of this triangle based on the traditional formula. 
Wed., Jan. 21	Law of Cosines, Heron's Formula	R - Under what circumstances of given information can the Law of Sines be used to solve a triangle? Under what circumstances can it *not* be used to solve a triangle.
Thur., Jan. 22	Vectors in the Plane	I – Each of the following is a type of *vector quantity*: 30 miles to the north; 45 mph straight up; a force of 15 Newtons 20^o above the horizontal. Each of the following is *not* a vector quantity, but rather a *scalar quantity*: 35 inches; 17 mph; an acceleration of 30 ft. per second per second. Use the examples and counterexamples to define "vector quantity." Then describe other quantities that require description with a vector quantity.
Fri., Jan. 23	Vectors in the Plane	R – The mathematics of vectors is notoriously difficult. Describe some strategies that would be helpful when working with vector quantities.
Mon., Jan. 26	Vector Unit Quiz	None
Tue., Jan. 27	Vectors and Dot Products	R – In everyday conversation, the terms *velocity* and *speed* are often used a synonyms. In mathematics and science, however, these terms are considered to be different. Velocity is a magnitude and direction, whereas speed is a single-dimension quantity that can be represented by nonnegative real numbers. Consider an object that has a velocity v and a speed s. Discuss how these two quantities are related to each other. Which of the following statements is correct mathematically? Explain your reasoning. 1. While driving to work, I did not exceed a velocity of 55 miles per hour. 2. While driving to work, I did not exceed a speed of 55 miles per hour.

Wed., Jan. 28	Vectors and Dot Products	R – Consider two forces of equal magnitude acting on a point. 1. If the magnitude of the resultant is the sum of the magnitudes of the two forces, make a conjecture about the angle between the forces. 2. If the resultant of the forces is **0**, make a conjecture about the angle between the forces. 3. Can the magnitude of the resultant be greater than the sum of the magnitudes of the two forces? Explain.
Thur., Jan 29	Trigonometric Form of a Complex Number	I – Explain why finding powers of complex, imaginary numbers, such as $\left(-2+3i\right)^7$ is a considerably more demanding task than finding powers of real numbers, such as $(-2.3)^7$.
Fri., Jan. 30	Trigonometric Form of a Complex Number	R – What can be said about the vectors **u** and **v** if the following are true? Be sure to explain your answers. 1. The projection of **u** onto **v** equals **u**. 2. The projection of **u** onto **v** equals **0**.
Mon., Feb. 2	Review for Vector Unit Exam	R – Suppose you are mowing lawns for a summer job. Use the concept of the projection of one vector onto another to describe why having a longer handle on your lawnmower is better for preventing fatigue.
Tue., Feb. 3	Review for Vector Unit Exam	R – Consider the vectors, <-3, 5>, < 5, 2>, and <-1, 3>. Find the resultant vector in three different orders. What do your results say about the associativity of vector addition?
Wed., Feb. 4	Review for Vector Unit Exam	None
Thur., Feb. 5	**Vector Unit Exam, Part I**	None
Fri., Feb. 6	**Vector Unit Exam, Part II**	None

APPENDIX B

Assessment Instruments

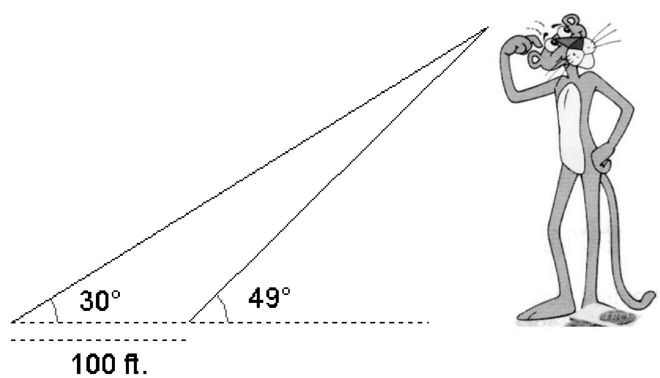
Honors Pre-Calculus – Vector Quiz – Spring 2009 **Name______________________________**

Be sure to show *all work* to receive credit for each solution! Do not leave any questions unanswered.

1.____________________Given a triangle with $A = 39°$, $B = 106°$, and c = 78, find a.

2.____________________Find the area of a triangle with $A = 71°$, b = 10, and c = 19.

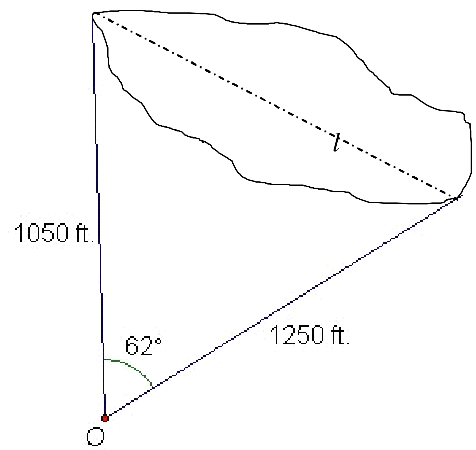
3.____________________Find the height of a giant helium balloon used in a Thanksgiving Day parade given that two guy wires are attached as shown in the figure below.



4.____________________________Given a triangle with a = 78, b = 15, and c = 91, find A, B, and C.

5.____________________A boat leaves a port and sails 16 miles at a bearing of S 20° E. Another boat leaves the same port and sails 12 miles at a bearing of S 60° W. How far apart are the two boats?

6.____________________A group of scientists wants to measure the length of a crater cause by a meteorite crashing into the earth. From a point, *O*, they measure the distance to each end of the crater and the angle between these two sides. What is the approximate length *l* of the crater? Round your answer to one decimal place.



7.____________________A vector has an initial point (3, 7) and a terminal point (3, -2). Find its component form.

8.____________________A vector has an initial point (2, 5) and terminal point (-1, 9). Find its magnitude and direction.

9.____________________Given $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{w} = 9\mathbf{i} + 5\mathbf{j}$, find $\mathbf{v} = \frac{1}{2}\mathbf{u} + 4\mathbf{w}$.

End of quiz. Check your answers.

Honors Pre-Calculus Vector Exam, Part I – Spring 2009 Name____________

Be sure to show *all* of your thought process and write your solution, with units where necessary, in the space provided.

$\mathbf{v}\cdot\mathbf{v} = \|\mathbf{v}\|^2$ $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$ $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\cdot\mathbf{v}$

$z = a+bi = r(\cos\theta + i\sin\theta)$, where $r = \sqrt{a^2+b^2}$ and $\tan\theta = \dfrac{b}{a}$

$z_1 z_2 = r_1 r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$ $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)]$

$z^n = [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$ $\sqrt[n]{r}\left(\cos\dfrac{\theta+2\pi k}{n} + i\sin\dfrac{\theta+2\pi k}{n}\right)$, where $k = 0,1,2,\ldots, n-1$

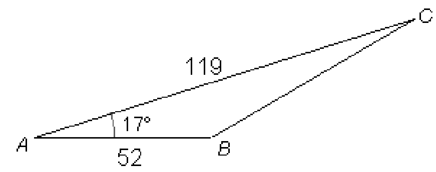
1.____________Given a triangle with $A = 102^\circ$, $B = 23^\circ$, and $c = 576.1$, find a.

2.____________Given a triangle with $B = 56^\circ$, $a = 98$, and $b = 85$, find the two possible values of c.

3.____________Find the area of the triangle with $A = 37^\circ$, $B = 78^\circ$, and $c = 250$.

4.____________Given a triangle with $a = 135$, $b = 71.6$, and $c = 69$, find B.

5.______________Given the triangle at right, find B.



6.______________A vector **v** has magnitude 27 and direction $\theta = 216°$. Find its component form.

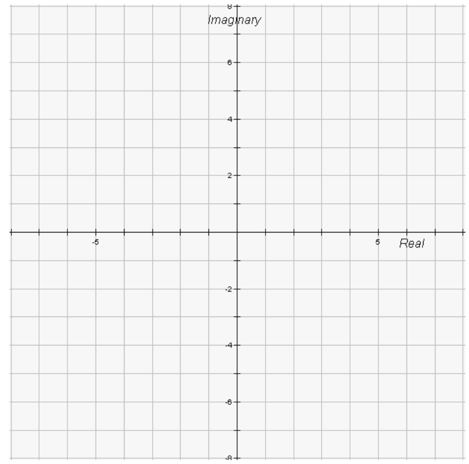
7.______________Given **v** = 3**i** + 2**j** and **w** = 6**i** + **j**, find the angle between **v** and **w**.

8.______________Given **v** = 3**i** - 9**j** and **w** = 2**i** + **j**, find **v** • **w**.

9.______________Find the angle between the vectors **w** = 3**i** + 4**j** and **v** = 10**i** + 4**j**.

10.____________________________Find the projection of **u** onto **v**. Then find the vector component of **u** orthogonal to **v**: **u** = $< -1, 2 >$ and **v** = $< 2, -3 >$.

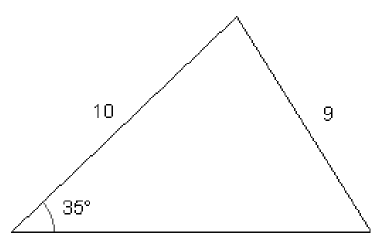
11. Represent the complex number, $-2-7i$, graphically on the complex plane.



12.__________________________________Multiply: $[16(\cos 33° + i\sin 33°)][8(\cos 17° + i\sin 17°)]$

13.__________________________________Evaluate: $(\sqrt{3}-i)^7$

Bonus:______________________________Use the *Law of Cosines* to find the *two* possible values for the missing side length in the triangle below.



End of Part I. Check your solutions!

Honors Pre-Calculus Vector Exam, Part II – Spring 2009 Name____________

Be sure to *clearly* show your entire solution and include units where necessary.

$\mathbf{v}\cdot\mathbf{v} = \|\mathbf{v}\|^2$ $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$ $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\cdot\mathbf{v}$

$z = a + bi = r(\cos\theta + i\sin\theta)$, where $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \dfrac{b}{a}$

$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$ $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

$z^n = [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$ $\sqrt[n]{r}\left(\cos\dfrac{\theta + 2\pi k}{n} + i\sin\dfrac{\theta + 2\pi k}{n}\right)$, where $k = 0,1,2,\ldots, n-1$

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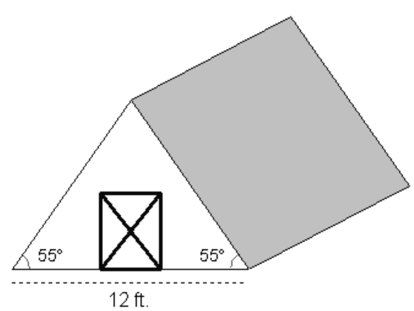
1. While traveling along a straight interstate highway you notice that the mile marker reads 260. You travel until you reach the 150-mile marker and then retrace your path to the 175 mile-marker. What is the magnitude of your resultant displacement from the 260-mile marker?

2. An A-frame tool shed is 12 feet wide. If the roof of the shed makes a 55° angle with the base of the shed, what is the length of the roof from ground level to the peak of the roof?



3. On a map, the town of Morgan Run is due south of Davidson and is southeast of Vicksburg. The distance from Morgan Run to Davidson and Vicksburg are 32 and 52 miles, respectively. The distance between Davidson and Vicksburg is 42 miles. If a plane leaves Morgan Run to fly to Vicksburg, on what bearing should it travel?

4. A jet airliner moving initially at 300 mph due east enters a region where the wind is blowing at 100 mph in a direction 30^o north of east. What are the new velocity and direction of the aircraft?

5. A boat requires 2 minutes to cross a river that is 150 meters wide. The boat's speed relative to the water is 3 m/s, and the river current flows at a speed of 2 meters per second. At what upstream or downstream points could the boat reach the opposite shore in 2 minutes?

6. A 120-foot tower is leaning. A 160-foot guy wire has been anchored 82 feet from the base of the tower. At what angle from vertical is the tower leaning?

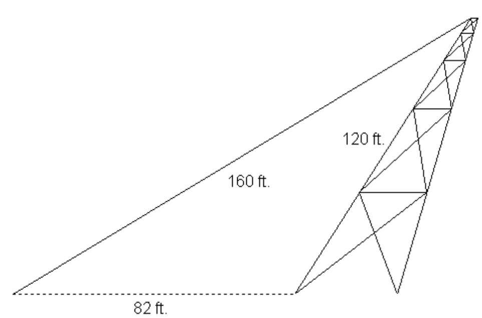
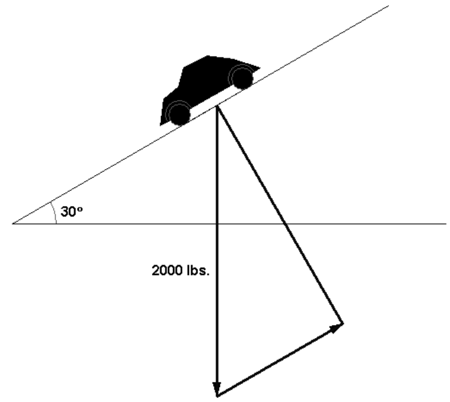


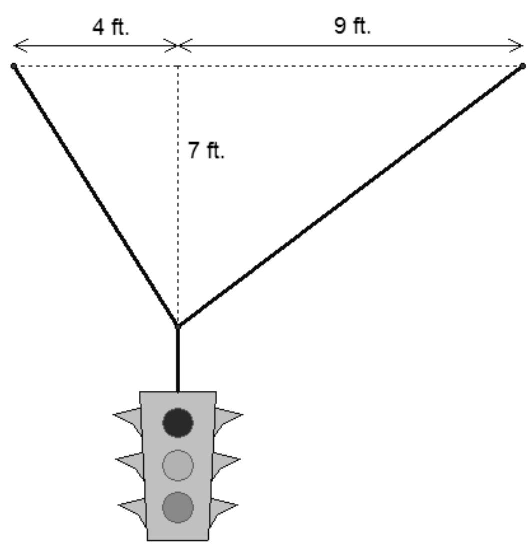
Figure not drawn to scale

7. What force is required to keep a 2000-pound vehicle from rolling down a ramp inclined at 30° from the horizontal? (See figure.)



8. A toy wagon is pulled by exerting a force of 20 pounds on a handle that makes a 25° angle with the horizontal. How much force is directed in the wagon's path of travel?

9. Use the figure to determine the tension in each cable supporting the stop light, which weighs 150 lbs.



End of Part II. Check your solutions.

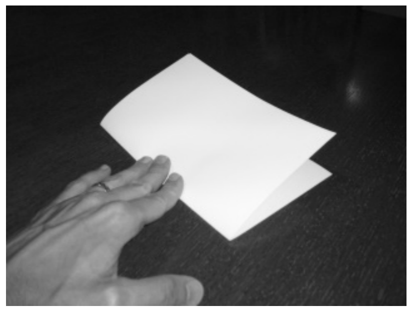
APPENDIX C

Instructions for Making Concept Booklets

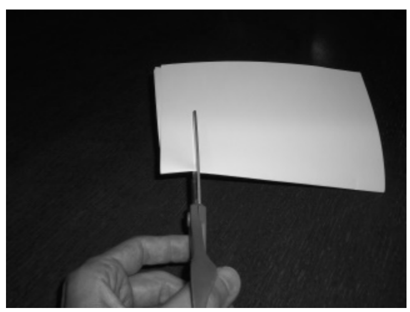
How to Make a Concept Booklet

The instructions found here are for making a booklet out of three sheets of paper. The number of sheets used to make a booklet, while always greater than one, can be modified as long as one set is cut along the spine from the edge to the incision and the other is cut along the spine from incision to incision, as shown in Steps 3 and 4.

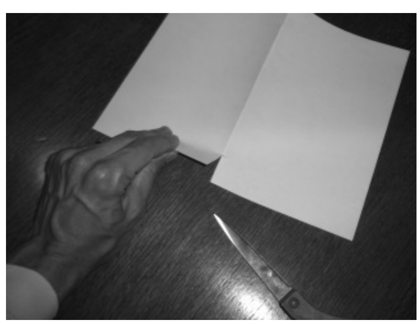
Step 1: Carefully fold all three pieces of paper "hamburger style" by folder the narrow way.



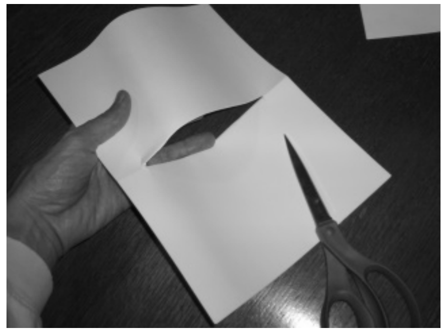
Step 2: Make two, roughly quarter-inch incisions on the spine of the folded pieces of paper approximately one inch from each side.



Step 3: Put aside two of the three sheets of paper. You will use these in Step 4. Take the remaining one sheet and cut along the edge of the spine from the edge of the paper to the incision. Repeat from the other edge of the paper.



Step 4: Cut the remaining pieces of paper along the spine from one incision to the other.



Step 5: Roll the paper from Step 3 "burrito style". Then insert the roll through the cut made in Step 4. Unroll the "burrito" and line up the incision. *Voila!* You have a booklet.