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Linear Precoding for MIMO Multiple Access Channels with Discrete-Constellation Inputs

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***Abstract*—In this paper, we study linear precoding for multiple-input multiple-output (MIMO) multiple access channels (MAC) with discrete-constellation inputs. We derive the constellation-constrained capacity region for the MIMO MAC with an arbitrary number of users. Due to the non-concavity of the objective function, we obtain the necessary conditions for the weighted sum rate (WSR) maximization problem through Karush-Kuhn-Tucker (KKT) analysis. To find the optimal precoding matrices, we propose an iterative algorithm utilizing alternating optimization strategy and gradient descent update. Numerical results show that when inputs are digital modulated signals and the signal-to-noise ratio (SNR) is in the medium range, our proposed algorithm offers significantly higher sum rate than non-precoding and the traditional method which maximizes Gaussian-input sum capacity. Furthermore, the bit error rate (BER) results of a low-density parity-check (LDPC) coded system also indicate that the system with the proposed linear precoder achieves significant gains over other methods.**

I. Introduction

The problem of linear precoding for multiple-input multiple-output (MIMO) multiple access channels (MAC) has been investigated in the literature in the last few years. Most of existing methods employed information theoretical analysis which shows that the optimal input signals are Gaussian distributed, and the optimal input covariance matrices can be found by maximizing the weighted sum rate (WSR) [1]–[4]. Other criteria in linear procoding design of MAC are also utilized, such as mean square error (MSE) minimization [5], and signal-to-interference and noise ratio (SINR) maximization [6].

However, there are some drawbacks of the aforementioned methods. Precoders obtained based on Gaussian input assumption may lead to non-optimal information rate when the inputs are actually replaced by discrete constellations such as phase-shift keying (PSK), pulse-amplitude modulation (PAM), or quadrature amplitude modulation (QAM). On the other hand, MSE minimization and SINR maximization approaches may not necessarily provide minimum bit error rate (BER) or maximum date rate.

To overcome the shortcomings of optimization through Gaussian-input capacity, MSE, SINR, etc., the mutual information with discrete-constellation inputs has recently been employed for precoding design in point-to-point communication scenarios [7]–[10], relay networks [11] and broadcast channels [12], [13]. For 2-user single-input single-output (SISO) MAC, [14] found the optimal angle of rotation and designed the code pairs based on trellis coded modulation.

To the best of our knowledge, little research has been done on MIMO MAC precoding based on mutual information with discrete-constellation inputs. In this paper, the maximum mutual information with discrete-constellation inputs with uniform distribution is referred to as constellation-constrained capacity [14], while the maximized mutual information with Gaussian inputs is called Gaussian-input capacity. We derive the constellation-constrained capacity region for the MIMO MAC with an arbitrary number of users and find that the boundary can be achieved by solving the problem of weighted sum rate maximization with constellation and individual power constraints. Since the weighted sum rate is no longer a concave function of precoding matrices as opposed to the case of Gaussian inputs, we obtain the necessary conditions through Karush-Kuhn-Tucker (KKT) analysis [15]. To find optimal linear precoders for all users, we propose an iterative algorithm utilizing alternating optimization strategy with gradient descent update method. Numerical results show that the proposed algorithm converges fast under various signal-to-noise ratios (SNRs). In addition, when inputs are digital modulated signals, and the SNR is in the medium regime, our proposed algorithm offers much higher sum rate than non-precoding and the traditional power allocation method which maximizes Gaussian-input sum capacity. The BER results of a low-density parity-check (LDPC) coded multiuser system also show that the system with the proposed linear precoder achieves significant gains over other methods.

The rest of the paper is organized as follows. Section II describes the model of MIMO MAC and a brief overview of the existing results on capacity region with Gaussian input signals. The constellation-constrained capacity region of MIMO MAC with discrete-constellation inputs is derived in Section III. Section IV discusses necessary condition of the weighed sum rate maximization problem, the details of the iterative algorithm, and the MIMO system over MAC with iterative detection and decoding. Numerical results are provided in Section V, and Section VI draws the conclusions.

II. System Model and Existing Results of MIMO MAC with Gaussian Inputs

Consider a K-user communication system with multiple antennas at transmitters and the receiver over multiple access

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channels. The signal model is given by

$$\begin{aligned}\mathbf{y} &= \mathbf{H}_1\mathbf{G}_1\mathbf{x}_1 + \mathbf{H}_2\mathbf{G}_2\mathbf{x}_2 + \cdots + \mathbf{H}_K\mathbf{G}_K\mathbf{x}_K + \mathbf{v} \\ &= \mathbf{H}\mathbf{G}\mathbf{x} + \mathbf{v}\end{aligned} \quad (1)$$

where $\mathbf{H} = [\mathbf{H}_1, \mathbf{H}_2, \cdots, \mathbf{H}_K]$, $\mathbf{G} = \textit{block-diag}\{\mathbf{G}_1, \cdots, \mathbf{G}_K\}$, and $\mathbf{x} = [\mathbf{x}_1^T, \cdots, \mathbf{x}_K^T]^T$. Suppose there are N_r antennas at the receiver, and each user has N_t transmit antennas. $\mathbf{H}_i \in \mathbb{C}^{N_r \times N_t}$ represents the complex channel matrix between the i-th transmitter and the receiver. We assume that the receiver knows the channels of all users, and each transmitter knows its own channel state information. Throughout this paper, we constrain each user's precoding matrix to be a square matrix, which is denoted as $\mathbf{G}_i \in \mathbb{C}^{N_t \times N_t}$. The vector $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$ is the received signal, and the receiver noise $\mathbf{v} \in \mathbb{C}^{Nr \times 1}$ is a zero mean circularly symmetric complex Gaussian vector with covariance matrix $\sigma^2\mathbf{I}$, i.e. $\mathbf{v} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$.

Assume all signal vectors $\mathbf{x}_i$ of different users are independent from one another, and elements of each $\mathbf{x}_i$ are independent and identically distributed (i.i.d) with unit energy, i.e, $\mathbb{E}[\mathbf{x}_i\mathbf{x}_i^h] = \mathbf{I}_{N_t}$. The covariance matrix of transmitted signal of user i is $\mathbf{Q}_i = \mathbb{E}[\mathbf{G}_i\mathbf{x}_i\mathbf{x}_i^h\mathbf{G}_i^h] = \mathbf{G}_i\mathbf{G}_i^h$.

We now briefly review existing results of MIMO MAC based on Gaussian inputs. For K-user MAC, it is well known that the capacity region is a convex hull of polyhedrons, and the boundary of the capacity region can be fully characterized by maximizing the weighted sum rate $\sum_{i=1}^K \mu_i R_i$ for all nonnegative μ_i. Assuming that $\mu_1 \geq \cdots \geq \mu_K$, and $\sum_{i=1}^K \mu_i = K$, then the optimal covariance matrices which maximize the capacity region can be found through solving the following optimization problem [2], [3]:

$$\max_{\mathbf{Q}_1,\cdots,\mathbf{Q}_K} \mu_K \log\left|\mathbf{I} + \sum_{i=1}^K \mathbf{H}_i\mathbf{Q}_i\mathbf{H}_i^h\right| + \sum_{i=1}^{K-1}(\mu_i - \mu_{i+1})\log\left|\mathbf{I} + \sum_{l=i}^{l}\mathbf{H}_l\mathbf{Q}_l\mathbf{H}_l^h\right| \quad (2)$$

$$\text{subject to} \quad \mathrm{Tr}(\mathbf{Q}_i) \leq P_i, \mathbf{Q}_i \succeq \mathbf{0}, i = 1, \cdots, K. \quad (3)$$

The above problem is a convex optimization problem, which can be solved by efficient numerical methods [15], [16].

III. Constellation-Constrained Capacity Region of MAC with Discrete-Constellation Inputs

We derive the constellation-constrained capacity region with finite discrete inputs for MIMO MAC in this section. Let the set A and its complement A^c partition all users into two groups, where $A = \{i_1, i_2, \cdots, i_{K_1}\} \subseteq \{1, 2, \cdots, K\}$, and $A^c = \{j_1, j_2, \cdots, j_{K_2}\}$, $K_1 + K_2 = K$. With the assumptions of $\mathbf{x}_A = [\mathbf{x}_{i_1}^T, \mathbf{x}_{i_2}^T, \cdots, \mathbf{x}_{i_{K_1}}^T]^T$, and $\mathbf{x}_{A^c} = [\mathbf{x}_{j_1}^T, \mathbf{x}_{j_2}^T, \cdots, \mathbf{x}_{j_{K_2}}^T]^T$, it is known that the achievable rate region of K-user MAC is the closure of the convex hull of the rate vectors $(R_1, R_2 \cdots, R_K)$, which satisfies the constrains $\sum_{i\in A} R_i \leq I(\mathbf{x}_A; \mathbf{y}|\mathbf{x}_{A^c})$, $\forall A \subseteq \{1, 2, \cdots, K\}$ for some independent input distributions $p(\mathbf{x}_1), p(\mathbf{x}_2), \cdots, p(\mathbf{x}_K)$ [1].

In practical digital communication systems, transmitted signals $\mathbf{x}_i$ are often equiprobably drawn from certain discrete constellations such as PSK, PAM, or QAM. Assuming that M_i is the number of constellation points in each component of $\mathbf{x}_i$, then the number of all possible vectors of $\mathbf{x}_i$ is $N_i = M_i^{N_t}$. Assuming that $\mathbf{H}_A = [\mathbf{H}_{i_1}, \mathbf{H}_{i_2}, \cdots, \mathbf{H}_{i_{K_1}}]$, and $\mathbf{G}_A = \textit{block-diag}\{\mathbf{G}_{i_1}, \mathbf{G}_{i_2}, \cdots, \mathbf{G}_{i_{K_1}}\}$, we have the following proposition which generalizes the results of 2-user MAC achievable rates in [14].

Proposition 1: When the discrete signals $\mathbf{x}_i$ of all users are i.i.d, $I(\mathbf{x}_A; \mathbf{y}|\mathbf{x}_{A^c})$ is given as follows:

$$I(\mathbf{x}_A; \mathbf{y}|\mathbf{x}_{A^c}) = \log N_A - \frac{1}{N_A}\sum_{i=1}^{N_A}\mathbb{E}_{\mathbf{v}} \left[\log\sum_{k=1}^{N_A}\exp\left(\frac{-\|\mathbf{H}_A\mathbf{G}_A(\mathbf{x}_A^i - \mathbf{x}_A^k) + \mathbf{v}\|^2 + \|\mathbf{v}\|^2}{\sigma^2}\right)\right] \quad (4)$$

where $\mathbf{v} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$, and $N_A = \prod_{i\in A} N_i$.

Proof: From the definitions $I(\mathbf{x}_A; \mathbf{y}|\mathbf{x}_{A^c}) = H(\mathbf{y}|\mathbf{x}_{A^c}) - H(\mathbf{y}|\mathbf{x}_A, \mathbf{x}_{A^c})$, we can prove (4). The details are omitted for brevity. ■

According to [17], the boundary of the constellation-constrained capacity region can be characterized by the solution of the weighted sum rate optimization problem. Without loss of generality, we assume the weights $\mu_1 \geq \cdots \geq \mu_K \geq \mu_{K+1} = 0$, i.e. decoding user K first and user 1 last. Then, the WSR maximization is equivalent to solving the following optimization problem:

$$\max_{\mathbf{G}_1,\cdots,\mathbf{G}_K} \quad g(\mathbf{G}_1, \cdots, \mathbf{G}_K) = \sum_{i=1}^K \Delta_i f(\mathbf{G}_1, \cdots, \mathbf{G}_i). \quad (5)$$

$$\text{subject to} \quad \mathrm{Tr}(\mathbf{G}_i\mathbf{G}_i^h) \leq P_i, \quad i = 1, 2, \cdots, K \quad (6)$$

where $\Delta_i = \mu_i - \mu_{i+1}, i = 1, \cdots, K$, and $f(\mathbf{G}_1, \cdots, \mathbf{G}_i)$ is equal to $I(\mathbf{x}_1, \cdots, \mathbf{x}_i; \mathbf{y}|\mathbf{x}_{i+1}, \cdots, \mathbf{x}_K)$, which can be obtained by (4). When $\mu_1 = \cdots = \mu_K = 1$, the weighted sum rate maximization problem reduces to sum rate maximization.

IV. General Results of Weighted Sum Rate Maximization

In this section, we solve the problem of WSR maximization with discrete-constellation inputs described in (5) and (6).

A. Necessary Conditions

In general, the objective function $g(\mathbf{G}_1, \cdots, \mathbf{G}_K)$ is nonconcave on precoding matrices $\{\mathbf{G}_1, \cdots, \mathbf{G}_K\}$. Thus, the WSR maximization with discrete-constellation inputs is not convex, and we can only find necessary conditions for this optimization problem, as given in the following proposition.

Proposition 2: The necessary condition of weighted sum rate maximization described in (5) and (6) satisfies:

$$\lambda_i\mathbf{G}_i = \frac{\log e}{\sigma^2}\sum_{j=i}^K \Delta_j\mathbf{H}_i^h\mathbf{H}_{A_j}\mathbf{G}_{A_j}\mathbf{E}_{A_j}^i \quad (7)$$

$$\lambda_i\left[\mathrm{Tr}\left(\mathbf{G}_i\mathbf{G}_i^h\right) - P_i\right] = 0, i = 1, 2, \cdots, K. \quad (8)$$

where $\lambda_i \geq 0$, and $\text{Tr}(\mathbf{G}_i\mathbf{G}_i^h) - P_i \leq 0$. $\mathbf{E}_{A_j}$ is the minimum mean square error (MMSE) matrix [18] of $\mathbf{x}_{A_j}$ given $\mathbf{y}$ and $\mathbf{x}_{A_j^c}$, and $\mathbf{E}_{A_j}^i \in \mathbb{C}^{N_t j \times N_t}$ is the i-th column block of $\mathbf{E}_{A_j}$,

Proof: The proposition can be proved through standard KKT analysis. The Lagrangian for (5) and (6) is given by

$$\mathcal{L}(\mathbf{G}, \lambda) = -g(\mathbf{G}_1, \cdots, \mathbf{G}_K) + \sum_{i=1}^{K} \lambda_i \left[\text{Tr}\left(\mathbf{G}_i\mathbf{G}_i^h\right) - P_i\right] \tag{9}$$

in which $\lambda_i \geq 0, i = 1, \cdots, K$. Then,

$$\nabla_{\mathbf{G}_i}\mathcal{L} = -\nabla_{\mathbf{G}_i} g(\mathbf{G}_1, \cdots, \mathbf{G}_K) + \lambda_i \mathbf{G}_i = 0 \tag{10}$$

By the gradient of mutual information [19], we find

$$\nabla_{\mathbf{G}_i} f(\mathbf{G}_1, \cdots, \mathbf{G}_j) = \frac{\log e}{\sigma^2}\mathbf{H}_i^h \mathbf{H}_{A_j}\mathbf{G}_{A_j}\mathbf{E}_{A_j}^i \tag{11}$$

Substituting (11) to (10), we can prove (7). ■

B. Iterative Algorithm For Weighted Sum Rate Maximization

From Proposition 2, it can be seen that the optimal precoders of different users depend on one another. We adopt the alternating optimization method [5], [20] to maximize the weighed sum rate with finite discrete inputs. During each iteration of the algorithm, only one user's precoding matrix $\mathbf{G}_i$ is updated while others are fixed. For i-th user at n-th iteration, we first generate $\widetilde{\mathbf{G}}_i^{(n)}$ based on the gradient of $g(\mathbf{G}_1, \cdots, \mathbf{G}_K)$ with respect to $\mathbf{G}_i$ as follows

$$\widetilde{\mathbf{G}}_i^{(n)} = \mathbf{G}_i^{(n)} + t\nabla_{\mathbf{G}_i} g(\mathbf{G}_1^{(n)}, \cdots, \mathbf{G}_K^{(n)}) \tag{12}$$

where t is the step size. If $\|\widetilde{\mathbf{G}}_i^{(n)}\|_F^2 > P_i$, we project $\widetilde{\mathbf{G}}_i^{(n)}$ to the feasible set $\text{Tr}(\mathbf{G}\mathbf{G}^h) \leq P_i$ to obtain the update [19]:

$$\mathbf{G}_i^{(n+1)} = \left[\widetilde{\mathbf{G}}_i^{(n)}\right]^+_{\text{Tr}(\mathbf{G}\mathbf{G}^h) \leq P_i} = \sqrt{P_i}\widetilde{\mathbf{G}}_i^{(n)} / \|\widetilde{\mathbf{G}}_i^{(n)}\|_F. \tag{13}$$

For fast convergence, we use backtracking line search [15] to determine the step size t in gradient update. Detailed steps of the proposed algorithm are shown in Table I.

The proposed algorithm can only find local optimum. To overcome this drawback, we run the iterative algorithm with random initialization multiple times and choose the one with maximal weighted sum rate to be the final solution [9]. We use Monte Carlo simulation method to estimate $g(\mathbf{G}_1, \cdots, \mathbf{G}_K)$ via (4), and calculate $\mathbf{E}_{A_j}$ as follows:

$$\mathbf{E}_{A_j} = \mathbf{I}_{N_t j} - \frac{1}{N_A}\sum_{m=1}^{N_A} \mathbb{E}_{\mathbf{v}}\left\{\frac{\left[\sum_{l=1}^{N_A}\mathbf{x}_{A_j}^l q_{m,l}(\mathbf{v})\right]\left[\sum_{k=1}^{N_A}(\mathbf{x}_{A_j}^k)^h q_{m,k}(\mathbf{v})\right]}{\left[\sum_{i=1}^{N_A} q_{m,i}(\mathbf{v})\right]^2}\right\} \tag{14}$$

where $A_j = 1, 2, \cdots, j$, and the function $q_{m,l}(\mathbf{v})$ is defined as

$$q_{m,l}(\mathbf{v}) = \exp\left(-\frac{\|\mathbf{H}_{A_j}\mathbf{G}_{A_j}\left(\mathbf{x}_{A_j}^m - \mathbf{x}_{A_j}^l\right) + \mathbf{v}\|^2}{\sigma^2}\right). \tag{15}$$

TABLE I
WSR MAXIMIZATION ALGORITHM WITH DISCRETE-CONSTELLATION INPUTS

initialize $\mathbf{G}_i^{(0)}$ with $\text{Tr}\left(\mathbf{G}_i\mathbf{G}_i^h\right) = P_i$, $i = 1, 2, \cdots, K$.
repeat
compute $g^{(n)} = g(\mathbf{G}_1^{(n)}, \cdots, \mathbf{G}_K^{(n)})$, and $\mathbf{E}_{A_j}^{(n)}$ for $j = 1, \cdots, K$.
for $i = 1 : K$
$\nabla_{\mathbf{G}_i} g(\mathbf{G}_1^{(n)}, \cdots, \mathbf{G}_K^{(n)})$
$= \frac{\log e}{\sigma^2}\sum_{j=i}^{K}\Delta_j \mathbf{H}_i^h \mathbf{H}_{A_j}\mathbf{G}_{A_j}^{(n)}(\mathbf{E}_{A_j}^i)^{(n)}$.
set step size $t := 1$.
do
$\widetilde{\mathbf{G}}_i^{(n)} = \mathbf{G}_i^{(n)} + t\nabla_{\mathbf{G}_i} g(\mathbf{G}_1^{(n)}, \cdots, \mathbf{G}_K^{(n)})$.
$\mathbf{G}_i^{(n+1)} = \sqrt{P_i}\widetilde{\mathbf{G}}_i^{(n)} / \|\widetilde{\mathbf{G}}_i^{(n)}\|_F$, if $\|\widetilde{\mathbf{G}}_i^{(n)}\|_F^2 > P_i$.
compute $g^{(n+1)}$ based on $\mathbf{G}^{(n+1)} =$ *block-diag* $\{\mathbf{G}_1^{(n)}, \cdots, \mathbf{G}_{i-1}^{(n)}, \mathbf{G}_i^{(n+1)}, \mathbf{G}_{i+1}^{(n)}, \cdots, \mathbf{G}_K^{(n)}\}$.
$t := \beta t$.
while $g^{(n+1)} < g^{(n)} + \alpha t\|\nabla_{\mathbf{G}_i} g(\mathbf{G}_1^{(n)}, \cdots, \mathbf{G}_K^{(n)})\|_F^2$.
end
until the $g(\mathbf{G}_1^{(n)}, \cdots, \mathbf{G}_K^{(n)})$ converges or n reaches maximum iteration number.

C. Iterative Detection and Decoding for MAC

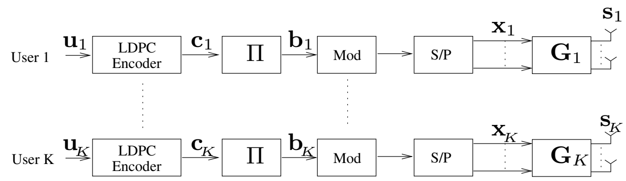

Fig. 1. MIMO uplink transmitters with precoding.

Fig. 1. shows a bank of parallel transmitters of K users. Each data block $\mathbf{u}_i$ is encoded by the LDPC encoder and then interleaved. After modulation, the symbol $\mathbf{x}_i$ is multiplied by the individual precoding matrix $\mathbf{G}_i$, and transmitted to the space through N_t antennas. We note that all users can use identical LDPC encoders and interleavers, while the linear precoders may differ from each other according to Section IV.

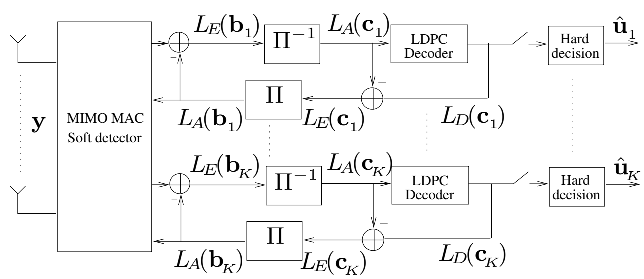

Fig. 2. Iterative receiver of MIMO multiple access channel.

The iterative receiver is given in Fig. 2. We choose to implement the MAP method [21] for the MIMO MAC soft detector to obtain optimal BER results. Within each iteration, the information intended for all users is iteratively exchanged between a MIMO MAC soft detector and a bank of LDPC soft channel decoders. At the final iteration, hard decisions are made upon $L_D(\mathbf{c}_i), i = 1, \cdots, K$.

V. Numerical Results

In this section, we provide numerical results of the constellation-constrained capacity region, sum rate with finite discrete inputs and BER performance for 2-user MAC. Assume that $N_r = 2$ and $N_t = 2$ for each user. Suppose the maximum individual power $P_1 = P_2 = P$, and all users adopt the same modulation scheme. The signal to noise ratio can be defined as SNR $= P/\sigma^2$, when the channels are normalized. In our simulations, we choose the noise power $\sigma^2 = 1$.

For illustrative purpose, we consider an example of two fixed channel matrices for two users, which are given by

$$\mathbf{H}_1 = \begin{bmatrix} 1.3898 & 0.1069j \\ -0.1069j & 0.2138 \end{bmatrix}, \mathbf{H}_2 = \begin{bmatrix} 1.2247 & 0 \\ 0 & 0.707 \end{bmatrix}.$$

Each channel matrix has normalized power with Tr $\left(\mathbf{H}_i \mathbf{H}_i^h\right) = N_r = 2$, as in [8].

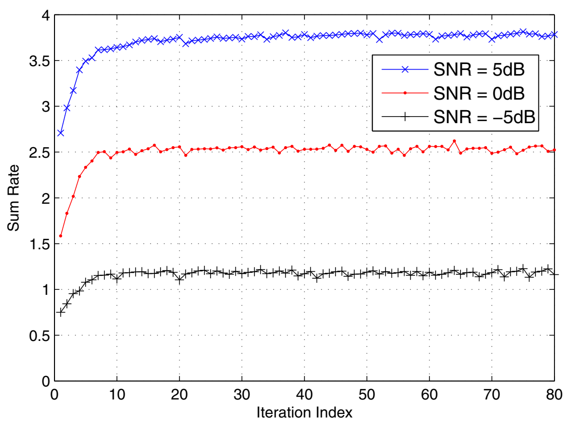

Fig. 3. Convergence of sum rate maximization algorithm with BPSK inputs.

Fig. 3 plots the convergence behavior of the sum rate maximization algorithm with BPSK inputs. we can see that the proposed algorithm usually converges after 10 iterations under different SNRs. We choose $\alpha = 0.1$ and $\beta = 0.5$, for the algorithm in Section IV. The Monte Carlo simulation number for both the sum rate and MMSE matrix is set to 500.

Fig. 4 shows the sum rate of various precoding schemes with QPSK modulation. In the method of "QPSK, Gaussian sum capacity max", the optimal input covariance matrices $\{\mathbf{Q}_1, \cdots, \mathbf{Q}_K\}$ are obtained by maximizing Gaussian-input sum capacity described in (2) and (3). After using standard convex optimization tool [16] to solve this problem, we decompose each covariance matrix as $\mathbf{Q}_i = \mathbf{V}_i \mathbf{\Sigma}_i \mathbf{V}_i^h$, and choose the precoder to be $\mathbf{G}_i = \mathbf{V}_i \mathbf{\Sigma}_i^{\frac{1}{2}}$. Then, we replace Gaussian inputs to QPSK signals and calculate the sum rate of this precoding scheme using (4). When SNR is 20 dB, we find that the covariance matrices obtained by the Gaussian-input sum capacity maximization method are

$$\mathbf{Q}_1 = \begin{bmatrix} 98.63 & 11.61j \\ -11.61j & 1.37 \end{bmatrix}, \mathbf{Q}_2 = \begin{bmatrix} 2.64 & -16.04j \\ 16.04j & 97.36 \end{bmatrix}.$$

With eigenvalue decomposition $\mathbf{Q}_i = \mathbf{V}_i \mathbf{\Sigma}_i \mathbf{V}_i^h$, we find that $\mathbf{\Sigma}_1 = \mathbf{\Sigma}_2 = diag\{100, 0\}$. In this case the precoder acts as beamforming by allowing each user to transmit only one modulated symbol in vector $\mathbf{x}_i$. Therefore, the Gaussian-input sum capacity maximization method fails to serve as the optimal strategy for practical finite alphabet signals.

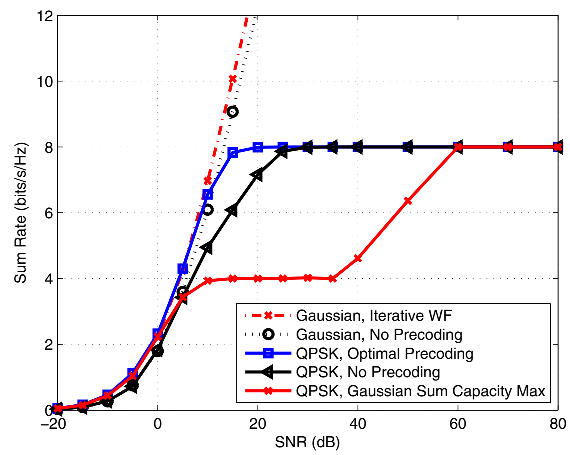

Fig. 4. Sum rate of 2-user MAC with QPSK inputs.

From the numerical results in Fig. 4, we observe that our proposed algorithm, denoted as the "optimal precoding", outperforms the non-precoding (i.e., $\mathbf{G}_i = \sqrt{\frac{P_i}{N_t}}\mathbf{I}_{N_t}$) with finite discrete inputs for a wide SNR range from -15 dB to 20 dB. In addition, we find that when the SNR is less than 10 dB, the optimal precoding with QPSK inputs obtains the same sum rate as the iterative WF [3] with Gaussian inputs.

The precoding matrices obtained via optimal precoding method when SNR $= 10$ dB are

$$\mathbf{G}_1^{opt} = \begin{bmatrix} 1.90 + 0.59i & 0.48 + 2.06i \\ 0.50 - 0.21i & 1.00 - 0.54i \end{bmatrix}, \text{and}$$

$$\mathbf{G}_2^{opt} = \begin{bmatrix} -0.35 - 1.16i & 0.62 - 0.12i \\ 0.71 + 1.31i & 1.18 + 2.12i \end{bmatrix}.$$

The constellation-constrained capacity region of optimal precoding with QPSK inputs when SNR = 8 dB is illustrated in Fig. 5. We also plot the rate regions achieved by non-precoding and Gaussian-input sum capacity maximization schemes, which can be obtained through (4). The curve of optimal precoding is obtained by varying μ_1 and μ_2 using weighted sum rate maximization algorithm with discrete inputs in Section IV. We can see that the constellation-constrained capacity region of optimal precoding is much larger than the rate regions of non-precoding and Gaussian-input sum capacity maximization method.

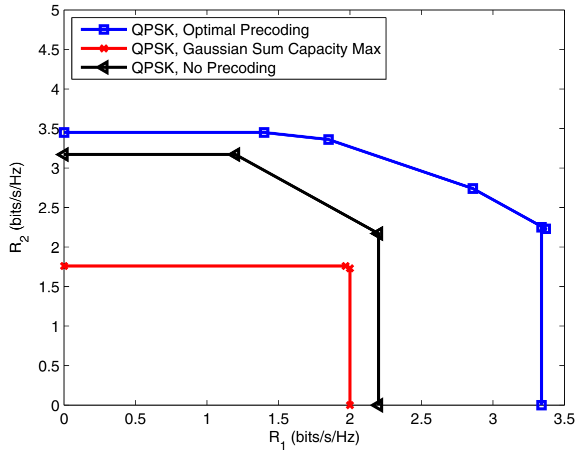

Fig. 5. Capacity region of 2-user MAC with QPSK inputs, when SNR = 8dB.

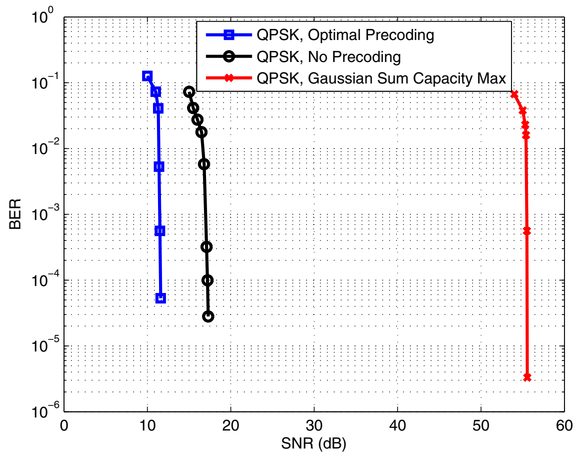

Fig. 6. BER of 2-user MAC with QPSK inputs.

Fig. 6 plots the BER curves of QPSK inputs. We use the LDPC encoder and decoder simulation package [22] with codeword length of 9600 bits and $3/4$ coding rate. The iteration number between MAP detector and LDPC decoder is 5. At the BER of 10^{-4}, the SNR of optimal precoding is about 12 dB, which is close to the limit predicted by the sum rate vs. SNR curve. In addition, the performance gains of optimal precoding over non-precoding and Gaussian-input sum capacity maximization well match the results in Fig. 4.

VI. Conclusion

In this paper, we studied the linear precoder for MIMO MAC with finite discrete inputs. From the information theoretical perspective, we derived the constellation-constrained capacity region and proposed an iterative algorithm to obtain the optimal precoding matrices for all users. The convergence behavior of our algorithm was verified by simulations. It showed that for the the medium SNR range, our precoding method offers significantly higher sum rate than the traditional power allocation method which maximizes Gaussian-input sum capacity but with input signals replaced by finite discrete inputs. BER simulation results of an LDPC coded system also indicated significant SNR advantage achieved by the proposed precoding scheme.

Acknowledgments

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