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POTENTIAL PITFALLS IN THE PRACTICAL APPLICATION OF THE RANDOM DECREMENT TECHNIQUE

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ABSTRACT

The Random Decrement Technique (RDT) has been widely used to extract as-built structural dynamic properties of civil engineering structures under ambient excitation, such as natural frequency, damping ratio, and their nonlinearity. This paper aims to clarify firstly that the RDT itself is not a damping evaluation technique (DET), but rather a data conditioning technique, akin to a filter. It results in what is called a "random decrement signature" (RDS) that is considered to represent the free decay response of the system, mode, or DOF being investigated. This paper also aims to show that a number of parameters influence the outcome of the RDT and the chosen DET. This was done by generating different sets of synthetic data, for which the actual damping and frequency values are known, which in turn are analyzed using RDT and appropriate DETs, and the results are then presented and discussed. Three important findings are: that damping may typically be overestimated especially the higher the noise is, any type of data filtering greatly affects the results, and some amplitude dependency may appear even if there was none. A discussion on how these potential pitfalls in the practical application of the RDT is then offered.

Keywords: Structural Health Monitoring, Random Decrement Technique, Full-Scale Civil Engineering Structures, Damping Evaluation Techniques, Logarithmic Decrement, Least Squares Approximation

1. BACKGROUND ON THE RANDOM DECREMENT TECHNIQUE AND MOTIVATION FOR STUDY

The Random Decrement Technique (RDT) or "Randomdec" as it is called by its inventor (Cole 1973), was developed initially for use in aerospace engineering applications. It has since become widely used in structural health monitoring, and specifically in the extraction of as-built structural dynamic properties such as natural frequency and damping, including their nonlinearity, from ambient vibration measurements of full-scale civil engineering structures that include buildings, bridges, floor systems, and long-span roofs. Some examples of the application of RDT in civil engineering were discussed by Jeary (1986), Tamura & Suganuma (1996), Tamura & Yoshida (2008), and Ku & Tamura (2009). Vandiver et al. (1981) provided a mathematical basis for the technique that is applicable to a linear system subjected to white noise excitation, which Cole (2014) commented as being simply assumptions that accompany the formulated theoretical basis, and clarified that in fact the RDT itself should apply even to nonlinear systems and those under colored noise excitation. Tamura & Suganuma (1996) also discusses the mathematics of the RDT in terms of conditional expectations.

The RDT results in a Random Decrement Signature (RDS) that resembles the autocorrelation function for sinusoidal waves obscured by noise, and is considered to be representative of the free decay response of the system (Fig. 1). Cole (2014) writes that the RDT provided an improvement in computational efficiency over the autocorrelation technique while yielding similar, sufficiently accurate results, which was important at that time when computers were far less powerful and far less accessible than they are today. While autocorrelation can detect amplitude dependency or nonlinearity, the RDT is able to preserve amplitude information, making it more appropriate for any system – linear or nonlinear (with amplitude-dependent properties) – whereas autocorrelation would be applicable only to a linear system, or otherwise a linear assumption is effectively (and sometimes unknowingly) made.

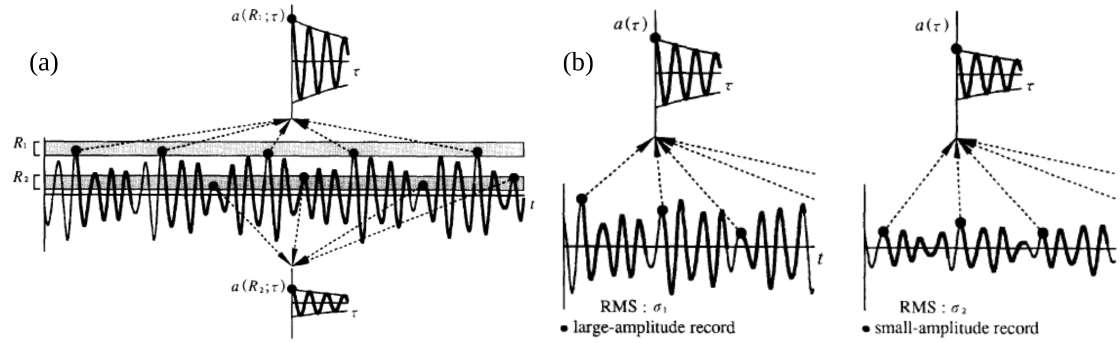

Figure 1: Illustration of RDT with two example triggering conditions from Tamura & Suganuma (1996).

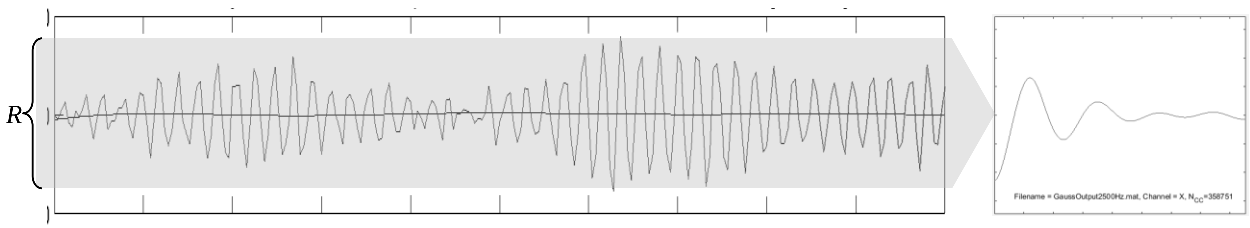

Figure 2: Autocorrelation as a special case of RDT.

The traditional RDT requires selecting a certain (positive) trigger amplitude and performing linear interpolation where the digital signal's resolution does not provide a data point at the selected trigger amplitude. Newer RDTs have suggested alternative triggering conditions and have specified certain requirements for length and minimum number of superimposed data segments. Cole (2014) provides an extensive discussion of these different triggering methods that have been developed over the years. In this sense, the RDT can be viewed as the general approach, with autocorrelation being a specific approach equivalent to an RDT having a trigger window spanning the full range of amplitudes (Figure 2).

It is important to emphasize that performing just the RDT itself does not yield the damping and natural frequency values right away. A separate Damping Evaluation Technique (DET) still needs to be applied on the RDS. In short, RDT is more of a data conditioning technique rather than a DET. The inventor of the RDT also mentioned this in his most recent publication (Cole 2014).

In many practical cases, in addition to performing RDT, some band-pass filtering needs to be applied, due to multiple modes in the recorded signal, or when there is very low signal-to-noise. However, this paper will show how filtering directly influences the resulting damping estimate. Likewise this paper will also show that, as for many other DETs, a low signal-to-noise would not yield acceptable results and that in some cases, amplitude dependency may appear even if there was none. The paper then offers a discussion on how these potential pitfalls in the application of RDT in practice may be addressed.

As the RDT results in an RDS which contains amplitude-dependent dynamic information, time-domain DETs are the primary point of discussion in this paper. However, frequency-domain DETs also have a purpose in performing time-domain DETs, which will be discussed later in this paper.

2. INDIVIDUAL EFFICACY OF THE RDT AND DETS

2.1 The Random Decrement Technique

To test how well the RDT really works, and specifically, the computer programs being used for this study to implement RDT, a random signal is first synthetically generated. For simplicity, only a zero-mean Gaussian white noise signal is used in this test but these results should indicate what to expect for other random signals. The expectation is that when the RDT is applied to this type of signal, the resulting RDS would be an all-zero flat line. The results are shown in Fig. 3(c) for a trigger amplitude equal to unity. It can be seen that it is not a perfectly flat line and that the mean is not exactly at zero. Fig. 3 also shows that the results are also very similar for different superimposed segment lengths. Meanwhile, Table 1 shows that as you use a longer data set, and effectively increase the number of superimposed segments, the mean and standard deviation values come closer and closer to the true values (both zero). This goes to show that the RDT algorithm (and triggering mechanism selected) is functioning as expected, upon the condition that a sufficiently long record is used.

2.2 The Logarithmic Decrement Technique

The simplest and yet most effective DET considered is herein called the logarithmic decrement technique (LDT). It is a time domain technique which relies on just two points (peaks) on a free decay plot to come up with a damping value (note that LDT has application even for nonlinear systems (Aquino & Tamura 2011)). To validate the technique, a linear 1DOF (one-degree-of-freedom) free decay with 2% viscous damping and 0.18 Hz frequency is synthetically generated, as LDT works only on 1DOF systems or single mode records. The results of LDT on the synthetic free decay data are shown in Fig. 4 for two different sampling rates. It can be seen that its precision is influenced by the sampling frequency, but otherwise, it comes up with sufficiently acceptable results.

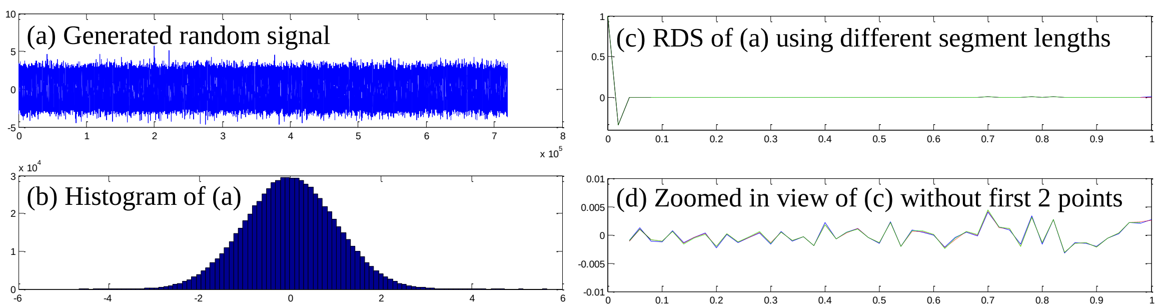

Figure 3: Generated random signal and resulting RDS.

Table 1: Results of RDT trials on zero-mean white noise

Data length (points)	Number of superimposed segments	|Mean|	Standard Deviation
10^3	4x10^2	0.002	0.05
10^4	>4x10^3	0.005	0.01
10^5	>4x10^4	0.001	0.005
10^6	>4x10^5	0.0001	0.002
10^7	>4x10^6	0.00002	0.0005

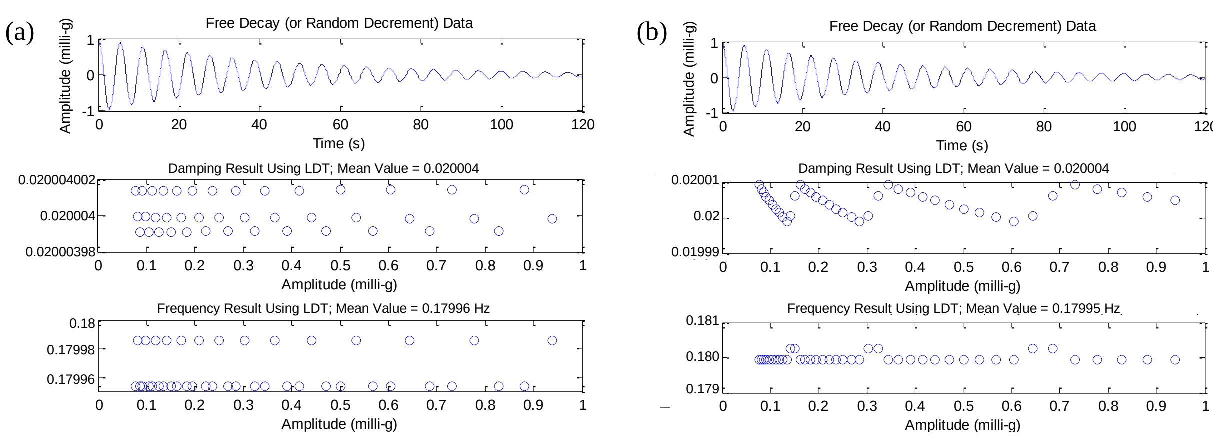

Figure 4: Generated free decay and results of LDT for sampling rates of (a) 1000 Hz and (b) 50 Hz.

Table 2: Results of ECFT

	1DOF system		2DOF system	
Initial Guesses	LSQerror	Error*	LSQerror	Error*
Exact Values	0.01%	Very low	Very low	Very low
10% off from exact	4.5%	4.8%	Very low	Very low
5% off from exact	0.05%	Low	5.4%	10%, 60%
50% off from exact	41%	132%	25%	Very high
10x exact values	Very high	Very high	97%	Very high
Damping 50% off	0.04%	Low	0.2%	Very low
Damping 10x exact	Very high	Very high	Very low	Very low
Frequency 50% off	0.25%	1.1%	1.3%	0%, 13%

*Direct comparison of damping results versus exact correct value. Note that frequencies are typically well estimated. There are two damping values for the 2DOF system, hence the two error values.

2.3 The Curve Fitting Technique

A technique proposed by Tamura & Suganuma (1996) that greatly reduces the influence of sampling rate and any other residual noise not removed by performing RDT, and only at the expense of longer computational time, is the curve fitting technique (CFT), or specifically, the use of nonlinear least squares approximation (NLSA). Simply put, it finds the best fitting 1DOF free decay curve to the obtained free decay data or RDS. NLSA together with the RDT have also been used to estimate damping and frequency for MDOF systems or multi-mode records, and is called the MRD technique (Tamura & Yoshida 2008). In addition, it provides for error quantification and evaluation as the fitted curve and the data itself can be compared such that results with unacceptably high error can be discarded. In this regard, CFT can be a far superior technique to the LDT. However, a small error tolerance and a large maximum number of iterations should be specified for improved accuracy, which increases the computational time further. Additionally, the initial guesses, especially of the natural frequency, need to be fairly accurate. However, this is where simpler frequency-domain or time-domain DETs that assume linear systems or provide average values (including, for 1DOF systems, taking the average of LDT results over a range of amplitudes) become useful.

There are 5 basic parameters for a 1DOF system (mean amplitude, damping, frequency, phase, and initial/peak amplitude) that need to be solved by the CFT. As an additional procedure to improve accuracy even further at the cost of even longer computational time, the authors propose that m×5 parameters instead are considered, where m-1 is the additional sets of parameters. For a 1DOF system and m set to 3, the CFT can therefore be used to solve for 15 instead of just 5 basic parameters. For a 2DOF system and m set to 3, 27 parameters are to be solved, instead of just 9. For example, the final damping result could be the result of three obtained parameters if m is set to 3:

[1] $$\zeta_{finalresult} = A_{\zeta} \times \zeta_{dummyresult} + B_{\zeta}$$

where A_{ζ} and B_{ζ} are 2 additional parameters to improve the damping estimate. They are like correction factors for which the initial guesses are 1 and 0, respectively. There are different ways of implementing this additional procedure, with the simplest being to consider m×5 parameters and let the curve fitting algorithm run as usual. Another approach is performing an initial curve fitting with 5 parameters, and then performing further curve fitting using the results from the initial curve fit as initial guesses, and this time solving for the correction factors (either at the same time or one at a time). Of course, multiple curve fittings using just 5 parameters at a time could also be done, with each step improving upon the one before. For now, the first and simplest approach of using say 3×5 parameters for 1DOF systems is considered. To differentiate from the standard CFT, the technique with this additional procedure is called Enhanced CFT or ECFT.

For the implementation of CFT or ECFT in this study, the simplex method is specifically used as it is readily available to use by the authors. It should be pointed out that it has limitations compared to NLSA. The CFT or ECFT results using the simplex method for the same linear 1DOF system discussed above, as well as for a synthetically generated linear 2DOF free decay, are presented in Table 2 for different initial guesses. Note that the "least squares error" as it is called herein is defined as:

[2] LSQerror = rms(**RDS**-**FC**)/rms(**FC**);

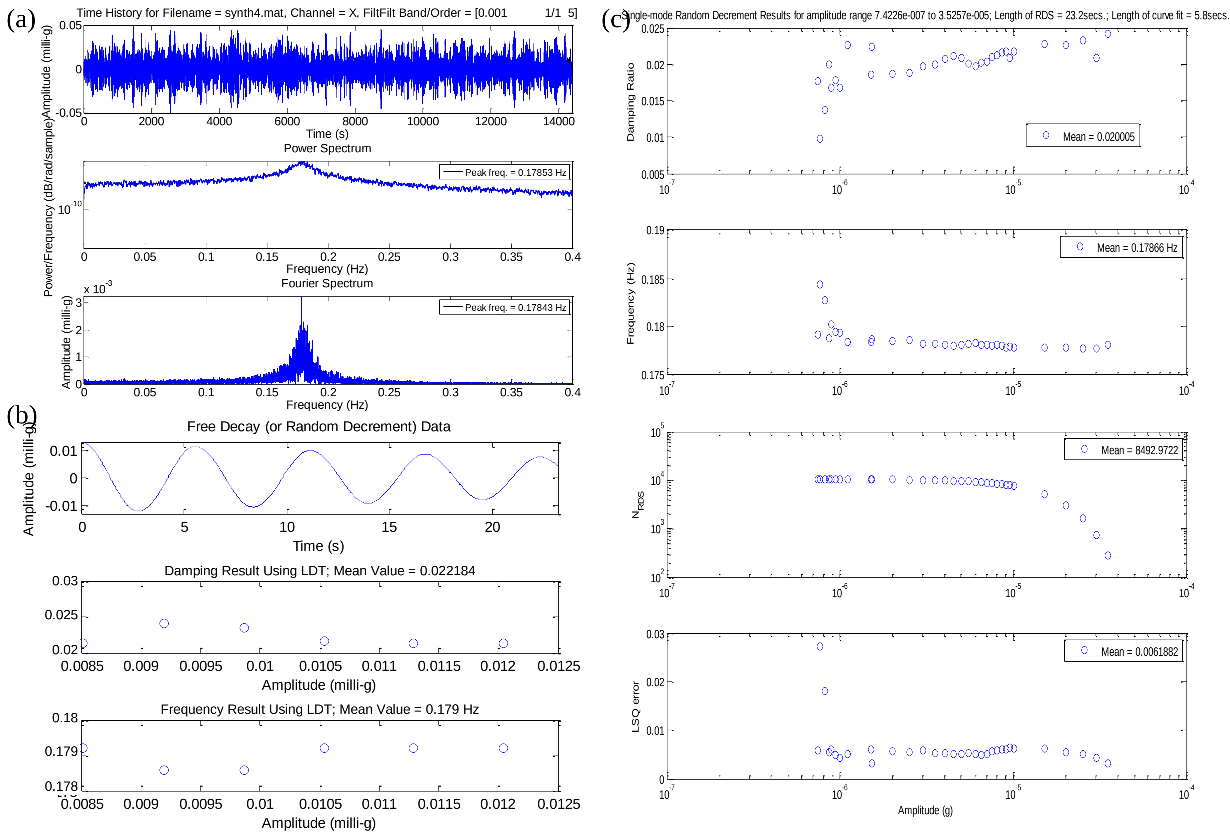

Figure 5: Linear 1DOF system under random excitation, example RDS, and LDT and RDMA+CFT results.

where LSQerror is the least squares error, **RDS** is the RDS or free decay data, **FC** is the fitted curve as a result of the CFT, **RDS-FC** is referred to as the residual, and rms is the root-mean-square function. **RDS** and **FC** are vectors of the same length corresponding to the same time vector. It is found here that LSQerror is ideally less than 0.5%, but in certain situations it could be very high and yet still yield very accurate results. In short, there is possibly some better way of calculating an error value that is applicable to all situations, but at this point, Eq. 2 is used to describe error quantities. Note that Table 2 shows that whenever LSQerror is high, the error in the estimate, particularly the damping, is also high. Also, it is noted in Table 2 that the frequency estimate is accurate in most cases, except perhaps when the initial guess is also very poor. It is important to have a good initial guess for frequency, which is not difficult to do, as discussed earlier. As for the damping initial guess, either operator skill or trial and error remains an important factor in achieving good fit results.

In summary, the RDT, LDT and CFT are capable of extracting the as-built structural dynamic properties of structures, with certain conditions that need to be met. In the case of RDT, it is important that a sufficiently long record is used. In the case of LDT, it requires a sufficiently high sampling rate, and is applicable only to a 1DOF system – unless a 1DOF equivalency is assumed. In the case of CFT, it is important that good initial guesses are obtained and used, results with high LSQerrors are discarded, and where possible, enhancements to improve the estimates are applied (iteratively) and additional computational time invested.

3. DAMPING EVALUATION CONSIDERING PRACTICAL CONDITIONS

3.1 Linear 1DOF System under Random Excitation

The free decay of the system is ideally directly obtained, but often it is easier to obtain ambient response measurements. The linear 1DOF system mentioned earlier is first excited using the same random signal also mentioned earlier, with a sampling rate of 50 Hz and a total length of 4 hours or 720,000 data points. The time history, power spectral density (PSD) plot, and Fourier spectrum are shown in Fig. 5a. The Peak-Picking Method (PPM), a very simple frequency-domain technique to obtain the frequency, results in frequency estimates of approximately 0.1785 Hz. This is used as an initial guess for the CFT. Note that while 0.18 Hz is the (actual, undamped) natural frequency of the system, the damped natural frequency with a viscous damping ratio of 2% is about 0.1782 Hz. If this were a real system, we would not know the damping ratio but for the purposes of this exercise, the exact value is used as an initial guess for the CFT. An example RDS with trigger amplitude of roughly 1/5 of the RMS of the whole signal is shown in Fig. 5b. The LDT is performed on this RDS, which results in the damping value being mostly overestimated, and seemingly exhibiting some nonlinearity. The calculated frequency appears to exhibit some nonlinearity as well, but a higher sampling rate would have improved these results. The average damping and frequency values are 2.2% and 0.179 Hz, respectively, which are higher than the exact values. Taking these average values, the error on damping is about 10%, but the error on frequency is under 1%.

The CFT results are shown in Fig. 5c for various RDS at different triggering amplitudes (herein referred to as RDMA, short for Random Decrement at Multiple Amplitudes). Note that the LSQerrors are under 1%, except at the lowest amplitudes, and the numbers of superimposed segments are about 10,000, except at the highest amplitudes. These are all very good indications. The mean damping and frequency values, if taken as estimates, are practically exact, although still a very slight overestimate. In any case, this example illustrates the superiority of CFT over LDT, but like LDT, the CFT results also appear to indicate some unexpected damping nonlinearity, even for this linear system.

The same data set was also analyzed after some filtering (Fig. 6). For some reason, the estimated (average) damping is now found to be higher using LDT (now 2.34% or having about 17% error), while CFT results in an average damping value of about 1.5% (or having about 25% error), despite LSQerrors under 0.6%. It can be argued that for this case, as there is no other mode and no signal noise, filtering was unnecessary. But this simply illustrates how filtering can distort the data, and particularly the resulting RDS, to significantly influence the final outcome and arrive at a lower damping value. Note that both LDT & CFT still seemingly exhibit nonlinear damping.

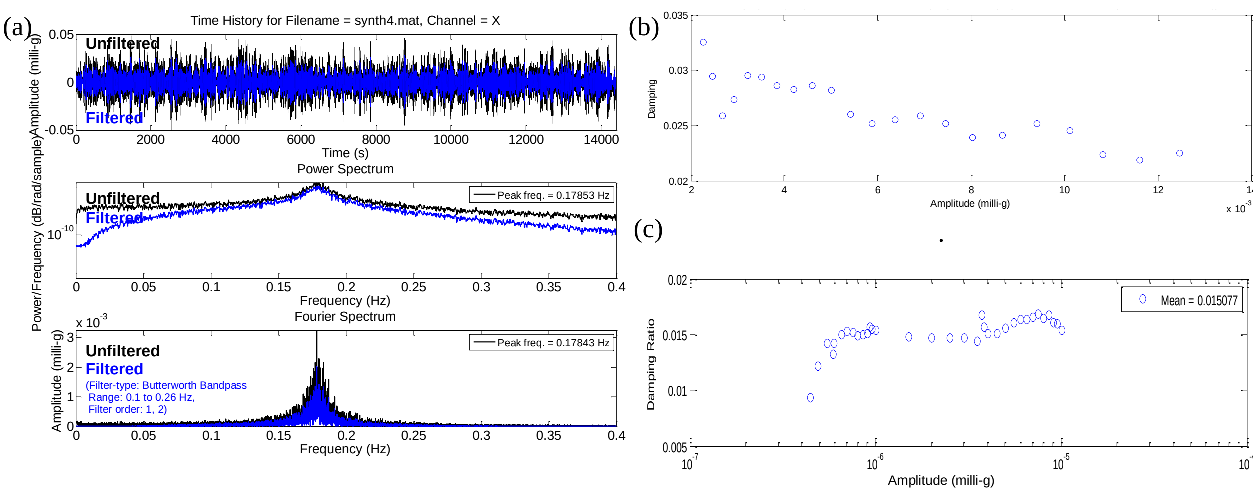

Figure 6: Filtered data from linear 1DOF system under random excitation, and LDT and RDMA+CFT results.

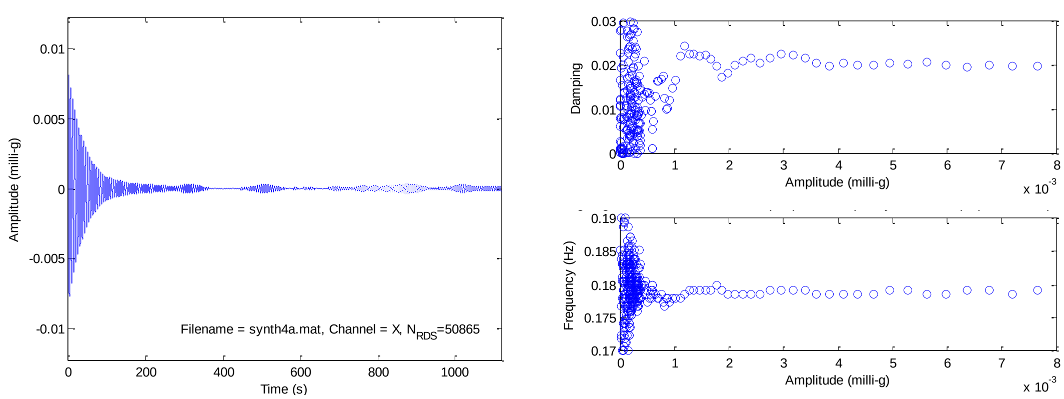

Figure 7: Very long RD signature from 24 hour record, and LDT results.

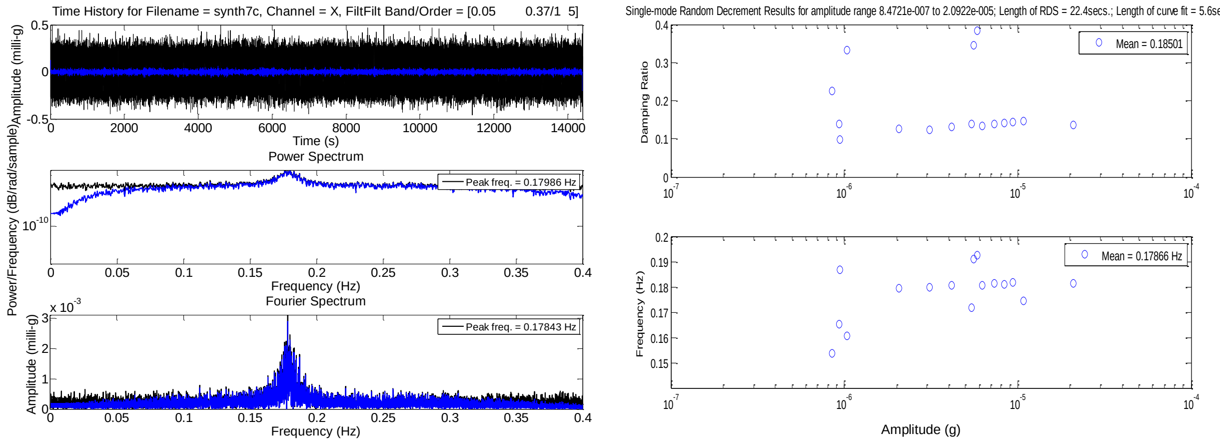

Figure 8: Linear 1DOF system under random excitation with added signal noise, and RDMA+CFT results.

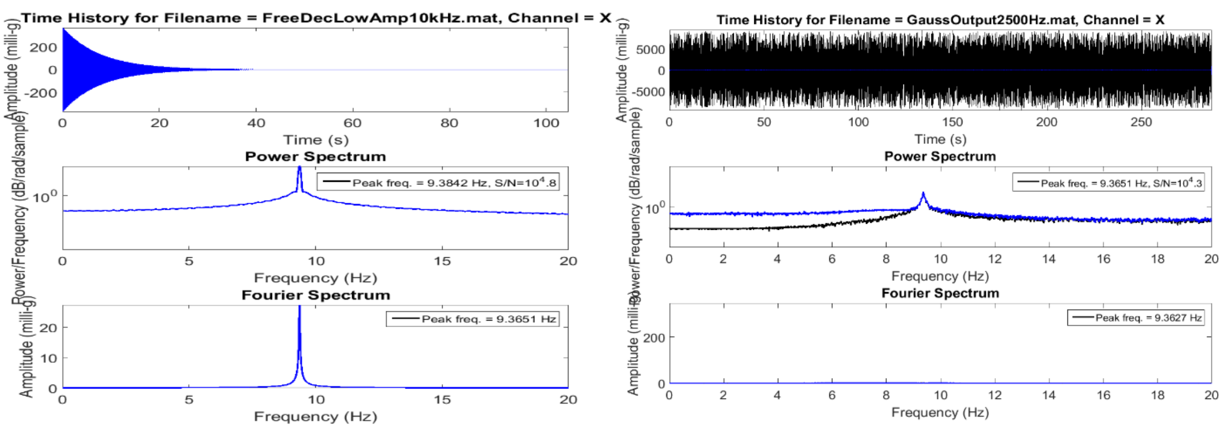

Figure 9: Nonlinear 1DOF system under free decay, and under random excitation.

A 24-hour record was next generated for the same system and left unfiltered, and the damping results are improved with under 1% error for LDT and under 3% for CFT. A longer RDS was also generated and while a sufficiently large number of superimposed segments could be obtained as shown in Fig. 7, note that the RDS fails at low trigger amplitudes – there appears to be enough noise left in after filtering, and so the damping and frequency results are quite scattered at the lower trigger amplitudes. Therefore the RDS length should not be too long, or that at relatively low amplitudes, it should be considered that the DET can still be influenced by other signals.

3.2 Linear 1DOF System under Random Excitation, with Added Signal Noise

Signal noise is now added to the data discussed in the previous section, prior to executing the same suite of analysis procedures, with some filtering also applied. The plots are shown in Fig. 8 and appear quite similar to the spectral plots in Figs. 5 and 6. LSQerrors are relatively high, ranging from 4% to 8%, as are the damping results (above 10% -- i.e. >400% error). This goes to show how signal noise might cause DETs to result in higher damping. At this point, a bandpass filter might be considered and a certain filter bandwidth might get us closer to the exact values, but we are artificially able to select the proper filter bandwidth in this case because we know the exact values. In practical cases, the bandwidth is almost arbitrarily selected but it could be adjusted to provide a desired damping outcome. That is, in certain cases, an engineer might have a preconceived notion that the damping should be around a certain value, and might select a filter bandwidth tailored to his or her expectation. For example, in earlier literature, a filter bandwidth of 1 Hz (±0.5 Hz around the natural frequency) has been recommended. This parameter should be investigated, similar to the effect of triggering condition, to aid in detecting user-induced bias or error.

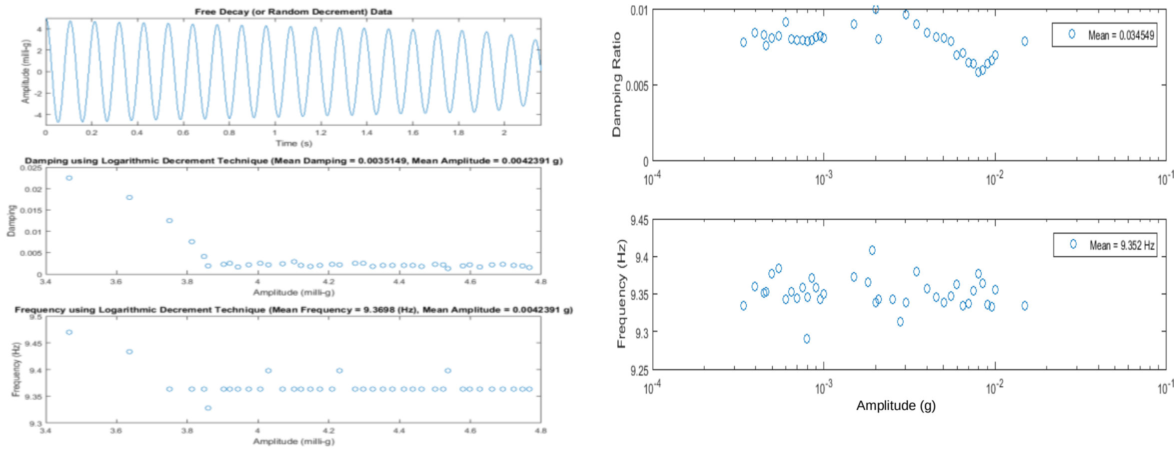

Figure 10: Nonlinear 1DOF system under random excitation: RDT+LDT and RDMA+CFT results.

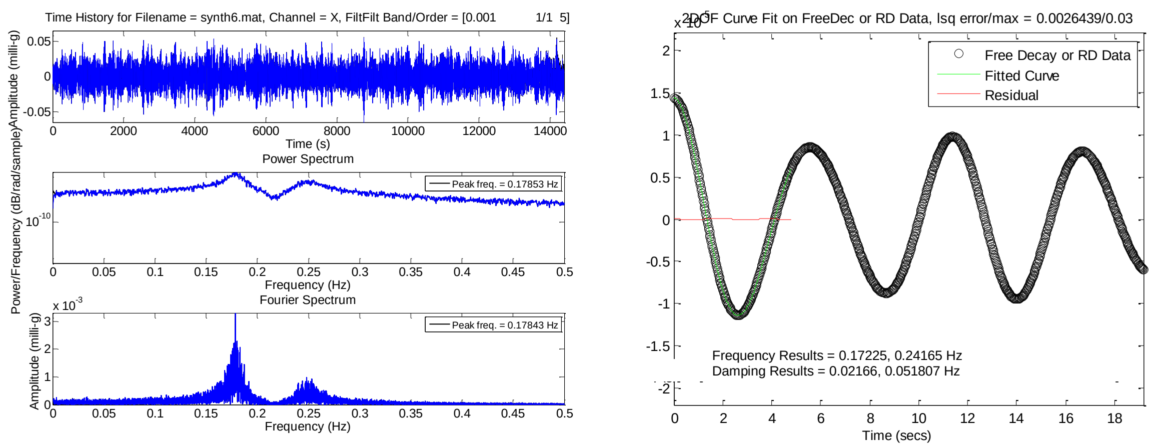

Figure 11: Linear 2DOF system under random excitation, and example RDT+CFT result.

3.3 Nonlinear 1DOF System under Random Excitation

The same nonlinear 1DOF system discussed by Aquino & Tamura (2011) is subjected to, first, an impulsive load to generate free decay, and next to random excitation. It has a baseline damping of 0.3% and rising to as high as around 7%, and a baseline natural frequency over 9 Hz and rising to over 11 Hz. The free decay response, and that under random excitation are shown in Fig. 9. In Fig. 10, RDT+LDT clearly shows some of the expected damping amplitude dependency but the CFT results look very different. Further, the maximum damping from LDT is over 2%, but it is only around 1% from the CFT results, although these are at different amplitude levels.

3.4 Linear 2DOF System under Random Excitation

A linear 2DOF system is next subjected to random excitation having 0.18 Hz and 2% as 1st mode frequency and damping, and 0.25 Hz and 3.5% for the 2nd mode (Fig. 11). These appear to be quite well separated modes so they should be quite distinct from each other. If RDT was performed only for a small segment length, enough to capture four periods of the 1st mode and therefore making an equivalent 1DOF assumption, the 2nd mode is almost obscured. After LDT (results not shown due to paper length restriction), a damping value as high as about 4% is obtained and an average frequency equal to the 1st mode frequency is obtained. But beating can be observed such that negative damping values are also obtained. The envelope technique could be used to ignore this beating, but ultimately, these DETs do not properly account for the multiple modes. It is for this reason that Tamura & Yoshida (2008) used the MRD or Multi-mode RD technique, together with the CFT or specifically NLSA that Tamura & Suganuma (1996) first proposed.

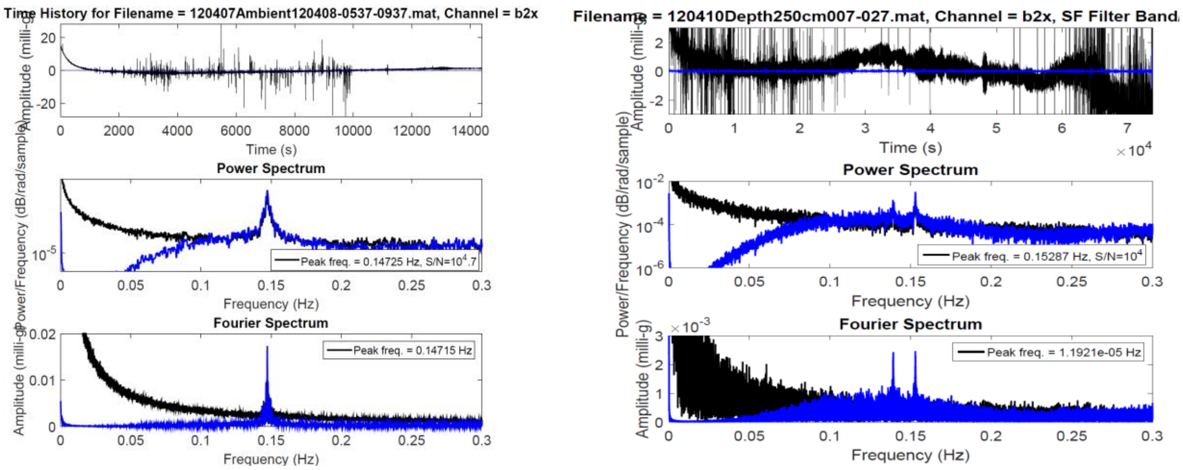

Figure 12: Full-scale tall building under random excitation: without and with TSD.

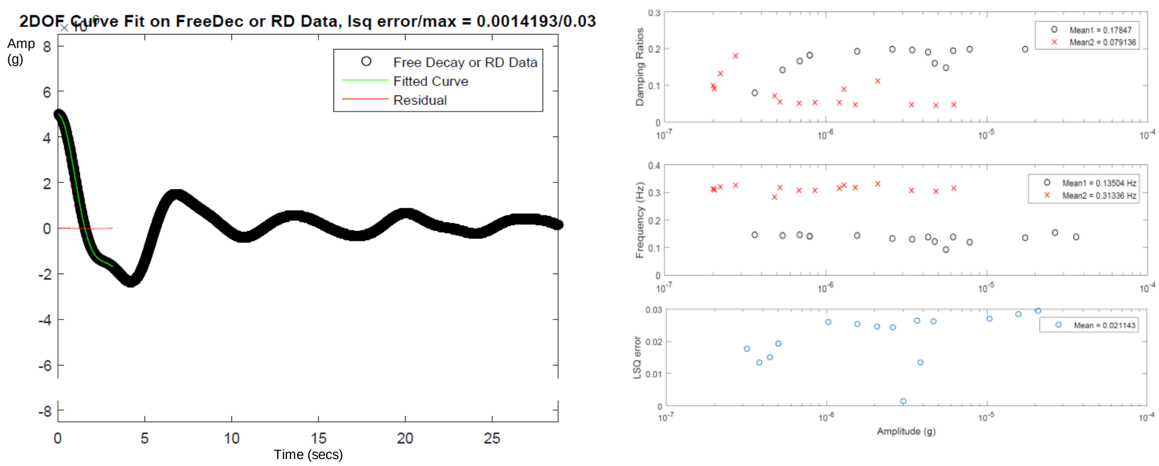

Figure 13: Full-scale tall building data with TSD: example RD+CFT and MRDma+CFT.

Carrying out this MRD technique for a range of amplitudes but using the simplex method, the results are far from accurate. The average values are 0.1686 Hz and 1.9% for Mode 1, and 0.2257 Hz and 6.9% for Mode 2 – errors of 6% and 46% on frequency, respectively, and 6% and 97% on damping, respectively, despite the LSQerrors being under 3%. Fig. 11 shows one RDS and one of the better RDT+CFT results, although the 2nd mode damping is still off by about 50%. A longer 72-hour record was also used but did not considerably improve the results.

3.5 Full-Scale Tall Building with TSD: 2DOF System Response to Random Excitation with Signal Noise

A specific example where RDT could be applied is on a full-scale building project equipped with a Tuned Sloshing Damper (TSD), which is essentially a secondary mass and splitting the building's primary mode into two modes, and effectively providing an equivalent additional damping to the system. A 4-hour recording without water in the TSD was obtained, followed by a 20-hour record with the TSD having the intended water depth. The data are shown in Figure 12. Firstly, it can be seen that without the TSD filled, the building has a natural frequency of about 0.1472 Hz based on the PPM in the frequency domain. With the TSD filled to the intended depth, it is clear from the frequency-domain at least that there are now 2 peaks, one at around 0.138 Hz and another at around 0.153 Hz. In both cases, there is a drift or very low frequency component in the data and so filtering is definitely necessary. The filter band is from 0.1 Hz to 0.4 Hz.

Performing LDT on an RDS from the without-TSD record yields damping generally under 1% and averaging around 0.6%, and average frequency of about 0.1471 Hz. Performing RDMA+CFT yields about 1% damping and 0.1461 Hz frequency, which are different from the LDT results. There also appears to be some amplitude dependency, but this is only illustrated here by a few data points.

Taking an RDS and performing LDT on the with-TSD record, and effectively assuming an equivalent 1DOF system, there appears to be an average damping and frequency of around 10% and 0.149 Hz (Fig. 13). Performing CFT on the same RDS results in a low LSQerror (0.1%), but the estimated frequencies are way off. RDMA+CFT appears to fare better at least only for the Mode 1 frequency (0.135 Hz), although LSQerrors are generally between 2% and 3%. But the Mode 1 damping (18%) appears very high. The extracted Mode 2 frequency and damping (0.313 Hz and 7.9%, respectively) do not make sense as well, and may have something to do with the filtering applied. This ordinary measurement data of a building-with-TSD is thus found to resist meaningful interpretation of the frequency and damping ratio, despite a solid understanding and application of sophisticated variations on the RDT plus DET.

4. SUMMARY AND CONCLUDING REMARKS

This paper has illustrated how certain aspects of actual data from full-scale measurements of civil engineering structures present potential pitfalls that affect the efficacy of RDT and any DET used in actual practice, even though it is shown that these should theoretically work. These aspects include the random excitation, the signal noise itself, and the required filtering to account for this noise, other modes, drift in the data, or general data imperfections. The presence of noise could generally increase the obtained damping ratio, even after performing RDT. Meanwhile, compensating by filtering and selecting a certain filter bandwidth can be highly subjective and is akin to selecting a desired outcome for damping. Different types of DETs can also be applied after performing RDT, using only filtering when absolutely necessary, and then engineering judgment may be employed to arrive at recommended values of damping and frequency for use in engineering and design based on the multiple results. But that would be highly subjective and less of a scientific approach. The best alternative with reliable results is forcibly exciting structures to cause them to exhibit free decay, which in turn is highly cost-prohibitive. Additionally, the work of Ku & Tamura (2009) shows promise in this regard, in that they have identified that the forcing function has a definite influence on the RDS as well, which should be considered.

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