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CALIBRATION OF THE NONLOCAL DYNAMIC DEFORMATION MODEL OF A FLEXURAL BEAM BASED ON NUMERICAL EXPERIMENT RESULTS

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Abstract: In this paper, the problem of numerical modeling of the dynamic behavior of bending beams made of the materials with a developed internal structure is considered. The simulation is performed taking into account the non-local elastic properties of the material in time. According to the principals of nonlocal mechanics, it is assumed that elastic forces in a structure depend on the entire history of its deformation, and not only on the instantaneous deformed state. The proposed dynamic deformation model is proposed as an alternative to detailed three-dimensional models. The nonlocal dynamic deformation model is integrated into the algorithm of the FEA method to make it applicable to solving applied engineering problems. The numerical implementation of the model is performed in Python. A technique for selecting the scale parameter of a non-local model based on experimental data using the least squares method and the dichotomy method has also been developed and implemented. To identify the possibility of extrapolation of the scale parameter values determined according to the developed methodology, a series of numerical experiments were conducted, on the basis of which scale parameters for beams of different lengths were obtained and the stability of the nonlocal model was evaluated.

Keywords: Nonlocal mechanics, nonlocal damping, numerical simulation, finite element method

КАЛИБРОВКА НЕЛОКАЛЬНОЙ МОДЕЛИ ДИНАМИЧЕСКОГО ДЕФОРМИРОВАНИЯ ИЗГИБАЕМОЙ БАЛКИ ПО РЕЗУЛЬТАТАМ ЧИСЛЕННОГО ЭКСПЕРИМЕНТА

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Аннотация: в настоящей работе рассматривается задача численного моделирования динамического поведения изгибаемых балок, выполненных из материалов с развитой внутренней структурой. Моделирование выполнено с учетом нелокальных во времени упругих свойств материала. Согласно положениям нелокальной механики предполагается, что упругие силы в конструкции зависят от всей истории её деформирования, а не только от мгновенного деформированного состояния. Представленная модель динамического деформирования предлагается в качестве альтернативы подробным трёхмерным моделям. Нелокальная модель динамического деформирования интегрирована в алгоритм метода конечных элементов, чтобы сделать её применимой для решения прикладных инженерных задач. Численная реализация модели выполнена на языке Python. Также разработана и реализована методика подбора масштабного параметра нелокальной модели на основании экспериментальных данных с применением метода наименьших квадратов и метода дихотомии. Для выявления возможности экстраполяции определённых по разработанной методике значений масштабного параметра, была проведена серия численных экспериментов, на основе которых были получены масштабные параметры для балок разной длины и проведена оценка устойчивости нелокальной модели.

Ключевые слова: Нелокальная механика, нелокальное демпфирование, численное моделирование, метод конечных элементов

INTRODUCTION

The active use of composite materials to address various production challenges requires new approaches to describe their behavior under dynamic loads. To achieve sufficient accuracy in numerical calculations, detailed three-dimensional finite element models are often employed, allowing for a reasonably accurate representation of materials with complex internal structures. However, such models are resource-intensive and laborious to analyze.

To optimize the calculation and design process, there arises a need to employ one-dimensional rod element models which are constructed on the base of specific mathematical hypotheses. Such hypotheses become necessary as classical viscoelastic models, proposed, for instance, in the works of W. Kelvin [1], J. Maxwell [2], J. Rayleigh [3], and Voigt [4], do not always accurately describe the behavior of materials with intricate internal structures.

As an alternative to classical models ones based on the principals of nonlocal mechanics can be used. Among such models the nonlocal elasticity model proposed in article [5], the model of nonlocal damping in space [6], and model [7] that combines the nonlocality of elastic and damping properties can be mentioned. The continuous spatial nonlocal damping model is described in article [8], while the nonlocal temporal internal friction model, based on the finite element method, is presented in article [9]. Article [10] discussed various damping mechanisms for a quasi-isotropic pultruded composite beam, including: viscous damping; internal damping dependent on strain rate; spatial hysteresis; and temporal hysteresis. Additionally, the article examines various combinations of these mechanisms [11].

In this work a proposed model is based on the assumption of the material's nonlocality in its elastic properties over time.

DYNAMIC MODEL OF BENDING BEAM DEFORMATION WITH CONSIDERING NON-LOCAL IN TIME ELASTIC PROPERTIES OF THE MATERIAL

The non-local model for dynamic deformation has been integrated into the finite element method algorithm. This approach allows using of the model in applied analyses of structures with the relatively complicated geometry. In the finite element method algorithm, the equation of motion is formulated in matrix form and is expressed in displacements [12]:

$$M \cdot \ddot{V}(t) + D \cdot \dot{V}(\tau) + K \cdot V(t) = F(t) \quad (1)$$

Here: K is the stiffness matrix, M is the mass matrix, D is the damping matrix, $F(t)$ is the vector of external force influences, $V(t)$ is the displacement vector.

When accounting for non-local material properties, it is assumed that the elastic forces within the system at the considered time moment t depend not only upon the instantaneous value of displacement $V(t)$, but also upon the displacement values at prior time instances τ. Moreover, the impact of time points on each other weakens proportionally with increasing time gap [13]. To formulate a model for the deformation of a beam element considering material memory, let us express equation (1) in the subsequent manner:

$$\begin{gathered} M \cdot \ddot{V}(t) + D \cdot \dot{V}(\tau) + \\ +K \cdot \int_0^t R(t-\tau) \cdot V(t) d\tau = F(t) \end{gathered} \quad (2)$$

Here, the stiffness matrix K is superimposed with the kernel of the non-local elasticity operator $R(t\text{-}\tau)$ or influence function. This function characterizes the attenuation of the influence of displacements at previous time instances τ on the elastic forces at the current moment t, while ensuring adherence to the normalization condition.

$$\int_0^t R(t-\tau)\,d\tau = 1 \qquad (3)$$

For the memory function modeling, kernels of various types can be used [6]. In the present study, the non-local operator kernels employ the error function, which takes the form when condition (3) is satisfied:

$$R(t-\tau) = \frac{2\eta}{\sqrt{\pi}} \cdot e^{-\eta^2 (t-\tau)^2} \qquad (4)$$

Like for all non-local models, in this case, a defining characteristic is the scale parameter η, which determines the nonlocality in the elastic properties of the material over time. Figure 1 illustrates how this parameter influences the shape of the kernel.

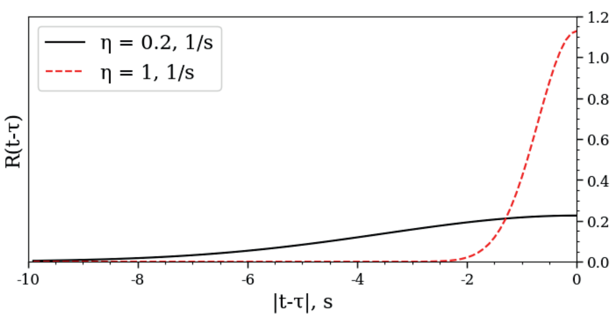


Figure 1. Graphs showing the effect of the parameter η on the nonlocality of the model

To solve the equation of motion, an implicit scheme (Newmark's method) [14] was employed. In this case, the velocities and accelerations of the finite element model nodes are represented as:

$$\dot{V}_{i+1} = \frac{V_{i+1} - V_i}{\Delta t};$$
$$\ddot{V}_{i+1} = \frac{2}{\Delta t^2}\left(V_{i+1} - V_i - \dot{V}_i \cdot \Delta t\right) - \ddot{V}_i \qquad (5)$$

Here: V_i is the vertical displacement of a node at a time t_i, $\dot{V}_i$ is the speed of the node at a given time t_i, $\ddot{V}_i$ is the node acceleration at a point in time t_i, Δt is the time step

Since, in the case of employing an implicit scheme, the memory function is imposed on the entire deformation process starting from the moment t_{i+1}, the discrete analog of the integral kernel has been divided into two parts [15]: α - the weighting coefficient associated with V_{i+1}, β - the sum of all other weights:

$$\alpha = \frac{2\eta}{\sqrt{\pi}} \cdot e^{-\eta^2 (t_{i+1}-t_i)^2} \cdot \Delta t;$$
$$\beta = \Delta t \sum_{j=1}^{i} \frac{2\eta}{\sqrt{\pi}} \cdot e^{-\eta^2 (t_i - t_{j-1})^2} \qquad (6)$$

Then, the expression (1) can be presented as:

$$M \cdot \left[\frac{2}{\Delta t^2} \cdot \left(V_{i+1} - V_i - \dot{V}_i \cdot \Delta t\right) - \ddot{V}_i\right] + D \cdot \frac{1}{\Delta t} \cdot (V_{i+1} - V_i) + K \cdot \beta + K \cdot \alpha = \bar{F}_{i+1} \qquad (7)$$

After all transformations, the computational scheme takes the form:

$$Q \cdot V_{i+1} = F_{i+1} + \mathrm{M} \cdot \ddot{V}_i + Q_1 \cdot \dot{V}_i + Q_2 \cdot V_i - K \cdot \beta, \qquad (8)$$

here:

$$Q = \frac{2}{\Delta t^2} M + \frac{1}{\Delta t} D + K \cdot \alpha, \quad Q_1 = \frac{2}{\Delta t} M, \ Q_2 = \frac{2}{\Delta t^2} M + \frac{1}{\Delta t} D \qquad (9)$$

ASSESSMENT OF THE INFLUENCE OF THE SCALE PARAMETER ON THE NATURE OF THE OSCILLATORY PROCESS.

For further improvement of the model, it is necessary to determine how the variation of the scale parameter affects the nature of the vibrations. In the implementation of this model using the MATLAB software, a 12-meter beam clamped at both ends was considered. The material chosen for the beam is a thermoactive vinyl ester glass-reinforced plastic of Class I, with a Young's modulus in the longitudinal direction equal to 17.2 GPa. The beam, in its cross-sectional profile, is a rectangle with a height of 0.3 meters and a width of 0.2 meters. The material's relative damping coefficient is set at 0.042. The beam is subjected to an instantly applied and uniformly distributed load with an intensity of -10 kN/m. At the first step, with i = 1, the initial conditions are set as $\bar{V}_0 = 0$.

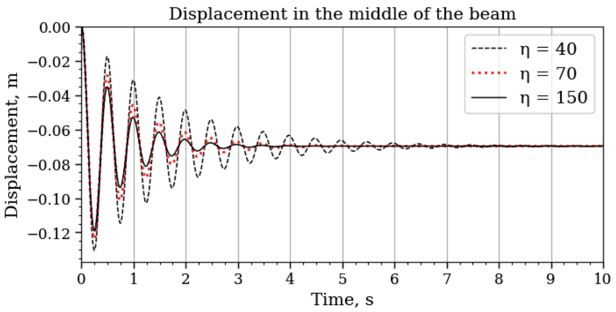


Figure 2. Graphs of the vertical displacement of the central node of the beam obtained for three calculation scenarios with different scale parameters

Analyzing the presented graphs, it can be observed that decreasing the parameter η (increasing the degree of non-locality) leads to an increase in the amplitude of oscillations.

DETERMINATION OF THE NON-LOCAL PARAMETER BASED ON THE RESULTS OF THE NUMERICAL EXPERIMENT.

In order to apply the non-local material model in practical calculations, it is necessary to determine the value of the non-local parameter. For this purpose, in the current study a calibration methodology of the non-local model using the least squares method was used based on the results of the numerical experiment [16]. The numerical experiment data consisted of the outcomes of three-dimensional finite element modeling of the beam. The geometrical and physical characteristics are the same the ones discussed in the previous section. The model was constructed using the MIDAS-Civil computational software, taking into account the orthotropic material properties. The material characteristics used in the modeling are presented in Table 1.

Table 1. Characteristics of thermoactive vinyl ester glass-reinforced plastic Class I:

Young's Modulus (Longitudinal), E_{lw}	17.2 GPa
Young's Modulus (Transverse), E_{cw}	12.2 GPa
Poisson's Ratio (Longitudinal), μ_{lw}	0.32
Poisson's Ratio (Transverse), μ_{cw}	0.15
Density, ρ	1900 kg/m^3
Damping Coefficient, γ	0.042

As a result of the calculation, a displacement diagram of the mid-section of the analyzed beam was obtained. Subsequently, the ordinate values of the oscillation graph obtained in MIDAS-Civil were exported and used to determine the scale parameter of the non-local model with an algorithm that was implemented with Python.

The numerical method employed to search for the minimum value of the sum of squared deviations between experimental data and numerical results was the bisection method.

The resulting diagrams are presented in Fig. 3 and Fig. 4. In comparison with the classical local deformation model (fig. 3), the calibrated non-local bending beam model (fig. 4) gives better alignment with the results of the numerical experiment.

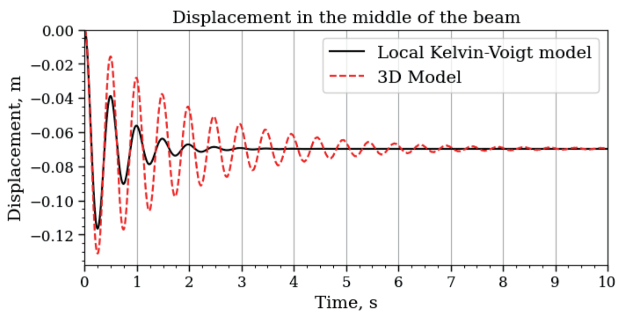


Figure 3. Comparison of numerical experiment results with one-dimensional modeling results using a model approximating the classical local Kelvin-Voigt model

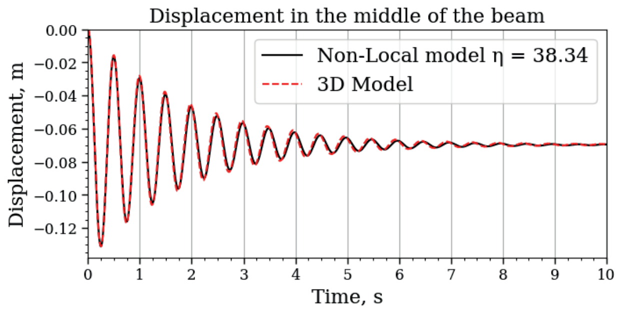


Figure 4. Comparison of numerical experiment results with one-dimensional modeling results using the calibrated time-nonlocal deformation model with the utilization of the error function kernel

As a result of calibration, the value of the scale parameter has been obtained η = 38.34 1/s. The root mean square error is 0.0034 m.

EVALUATION OF THE SCALE PARAMETER EXTRAPOLATION POSSIBILITY

The application of models requiring constant determination of the scale parameter by the experimental data may be impractical and can be extremely labor-intensive in application. Therefore, the question arises about the method for determining the scale parameter of a non-local model as a material constant or identifying dependencies that enable the determination of this parameter for elements of various geometry.

The damping model employed in conducting numerical experiments within the computational suite Midas is frequency dependent, hence the viscosity coefficient depends on the geometric attributes of the beam. Therefore, to demonstrate the correlation between the viscosity coefficient and the scale parameter, a series of numerical experiments were conducted involving beams of varied lengths. The outcomes tabulated in Table 2.

It can be seen from Table 2 that there is a correlation between the ratios of the viscosity coefficients and the ratios of the scale parameter. Consequently, given numerical or empirical data obtained for a beam of specific length and a non-local model calibrated by this data, it becomes possible to determine the scale parameter by values of frequency ratios or viscosity coefficients applicable to beams of different geometries but composed of the same material. Should damping in the material prove frequency-independence, the scale parameter for said material will inherently remain constant.

Table 2. The results of a series of experiments

L, m	χ, Pa·s	*η*, 1/s	The ratios of viscosity coefficients	The ratios of scale parameters
10	0,014	52.9	1,00	1.00
11	0,017	44.8	0,83	0.85
12	0,021	38.3	0,69	0.73
13	0,024	34	0,59	0.64
14	0,028	29.6	0,51	0.56
15	0,032	26.3	0,44	0.50
16	0,036	22.9	0,39	0.43
17	0,041	20.6	0,35	0.39
18	0,046	18.3	0,31	0.35

Here **L** is the Beam length, **χ** is the Viscosity coefficient, ***η*** is the Scale parameter

For clarity the proportion between viscosity coefficients and the proportion between scale parameters is visually presented in Figure 5.

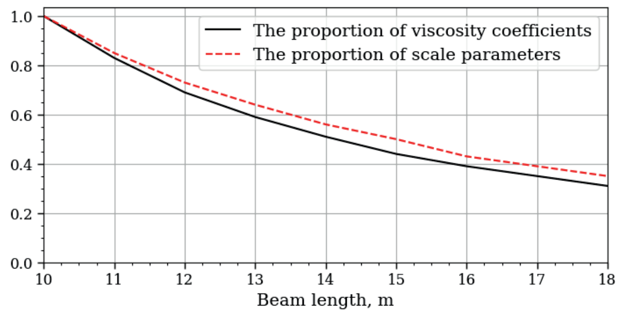
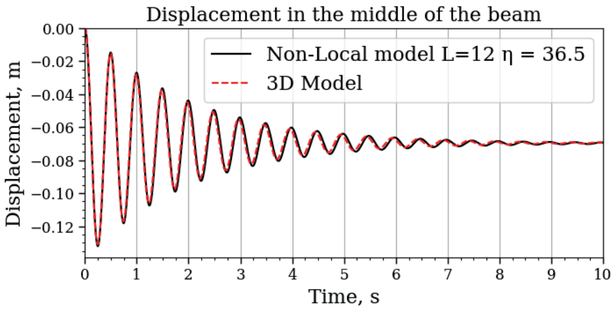


Figure 5. The correlation between the proportion of viscosity coefficients and the proportion of scale parameters

Thus, knowing the scale parameter for a 10-meter beam, it is possible to determine the scaling parameter for a 12-meter beam with sufficient accuracy. According to Table 2, the scaling parameter for a 10-meter beam is η = 52.9 1/s, and the viscosity coefficient ratio for a 12-meter beam relative to a 10-meter beam is 0.69. Therefore, by multiplying these values, the approximate scaling parameter for a 12-meter beam can be determined, which in this example will be η = 36.50 1/s. The obtained root mean square error for the results obtained using the calculated scaling parameter will be 0.0051 м (Fig. 6).



<u>Figure 6.</u> Comparison of numerical experiment results with one-dimensional modeling results using the time-nonlocal deformation model at the calculated scaling parameter η = 36.50 1/s

CONCLUSION

During the present study, a non-local deformation model of a bending beam was constructed using the finite element method. The Newmark method was applied for the numerical solution of the motion equation. The relationship between the scale parameter η and the amplitude of the bending beam oscillations has been demonstrated. It was shown that an increase in the material's nonlocality leads to an increase in the amplitude of oscillations. The model has been supplemented with a calibration algorithm for the scale parameter based on experimental results. The developed calibration algorithm has been tested using numerical experiment data, taking into account the orthotropic properties of the material. In comparison with the classical local model, the calibrated non-local deformation model of the bending beam allows for better convergence with the results of the numerical experiment. Additionally, a correlation between the viscosity coefficient of the model and the scale parameter has been identified, that allows to determine of the scale parameter through frequencies or viscosity coefficients ratio. Thus, the scale parameters can be determined for the beams of different geometry using this correlation, and there is no need to determine it from experimental data all over again. The correlation was obtained for the numerical experiment results obtained with the frequency dependent damping model. For the materials characterized by frequency independent damping properties, the scale parameter will be the constant of material.

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