A DYNAMIC PROGRAMMING APPROACH FOR ARTERIAL SIGNAL OPTIMIZATION IN A CONNECTED VEHICLE ENVIRONMENT

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ABSTRACT

Within the Connected Vehicle (CV) environment, vehicles are able to communicate with each other and with infrastructure via wireless communication technology. The collected data from CVs provide a much more complete picture of the arterial traffic states and can be utilized for signal control. Based on the real-time traffic information from CVs, this paper enhances an arterial traffic flow model for arterial signal optimization. Then a dynamic programming optimization model is created to solve the signal optimization application. A real-world arterial corridor is modeled in VISSIM to validate the algorithms. This approach is shown to generate good results and may be superior to well-tuned fixed-time control.

Keywords: Arterial signal optimization, connected vehicle, link traffic dynamic, dynamic programming

1. INTRODUCTION

Traffic lights at intersections are the major control measure in urban road networks; however, when traffic demand approaches or exceeds the network capacity, operations may become less efficient. For example, one limited congestion incident triggered by temporarily and locally excessive demand may lead to an unstable escalation and the creation of secondary congestion when no suitable control actions are employed. Subsequently, it may lead to restricted mobility of the entire system and result in degraded operational efficiency (Chaudhary et al. 2010, Liu et al. 2013). If traffic becomes congested, traffic intersections are not isolated and the traffic states of roads will interact with each other. Hence, it is necessary to understand the behavior of arterial traffic and to investigate corridor-wide coordinated signal control strategies. Providing an efficient signal control system has become increasingly important due to the effects of high congestion levels on the urban environment and the quality of life. Therefore, an optimal control decision must be used to keep the arterial traffic well organized.

Adaptive control strategy adjusts, in real time, signal timing plans in response to real-time traffic flow fluctuations. With advances in computation and sensing, it has become an increasingly attractive option and been researched for the last three decades. Some adaptive control strategies proactively adjust signal timing plans to meet predicted traffic states before vehicles arrive at intersections. Proactive control strategy uses macroscopic, mesoscopic or microscopic traffic flow models to predict the future traffic states, and develop optimization tools to search for the best future control decisions based on the predicted traffic states (He et al. 2012, Liu et al. 2014). Therefore, this strategy, also called as the model-based adaptive control strategy, can make the best control decisions from a long-term point of view. However, the efficiency of corridor-wide coordination strategies is still needs to be further improved. It is very important to find a trade-off between the accuracy and the computational complexity, so that the model-based control strategies can make better control decisions and also continue to be applicable in practice.

The latest development in Intelligent Transportation Systems (ITS) is Connected Vehicle (CV) technology, the core of which is wireless communication within a high-mobility traffic environment. Vehicles connected via various wireless communication technologies can provide high-resolution vehicle trajectory data for traffic analysis. There

are several key advantages of CV technology: i) CV technologies can capture the unique signatures of individual vehicles and match those signatures at different locations; and ii) CV technologies can cost-effectively capture high-resolution trajectories and maneuvers of individual vehicles, facilitating large-scale vehicle trajectory data collection. Applying CV technologies to arterial performance monitoring resolves two long-standing issues: i) CV technologies can directly and accurately measure, rather than estimate, segment-based speed, which significantly reduces bias in arterial traffic state estimation, especially travel time estimations; and ii) CV technologies can directly measure high-resolution vehicle trajectories, providing improved traffic dynamic details and more opportunities to enhance traffic dynamic analysis at intersections as well as on corridors or over the network (Feng et al. 2015).

This study proposes a decentralized traffic signal control strategy. Each intersection is equipped with a local controller, where an enhanced SFM-based signal optimization model is used to address dynamic queue interactions among different lanes and adjacent intersections. Then the projected traffic flow from the adjacent intersection is the coordination parameter confining the possible control space. Then dynamic programming (DP) is used to find the assignment of green time to each phase of a cycle, which is a multi-stage control problem. The remainder of this paper is organized into sections: (a) a literature review regarding proactive control systems; (b) the enhanced SFM describing the traffic flow dynamic along arterial links and nodes; (c) the dynamic programming model; (d) evaluation of the new control model in a simulation study; and (e) some concluding remarks and recommendations for further research.

2. LITERATURE REVIEW

Macroscopic urban traffic models representing the traffic state evolutions in signalized arterial networks can be classified into the following three generalized categories: (a) the Cell Transmission Model; (b) the Store-and-Forward Model; and (c) the Dispersion-and-Store Model. Some studies have used different terms, such as the Input-and-Output Model, the Queue Equilibrium Model, and the Platoon-Dispersion-Model (Pavlis and Recker 2004). The Store-and-Forward model (SFM) was first proposed by Gazis (1974) to represent traffic conditions at oversaturated intersections and has since been used in various works, most notably for road traffic control (Papageorgiou et al. 2003). In this modeling approach, it is first assumed that vehicles entering a link are traveling at a fixed travel time. Then, the vehicles are either stored at the end of this link (during a red signal), or further forwarded to downstream links at a saturation flow rate (during a green signal). The concept of network queue management was formalized in the work by Lieberman et al. (1992 and 2000) where the researchers first identified internal traffic metering to maintain stable queues in the congested network. Diakaki (2000, 2002, and 2003) developed the traffic-responsive urban control system (TUC) by using SFM as the underlying traffic flow model. After initial development, TUC was expanded to perform real-time cycle and offset control, and to allow for public transport priority. Compared with TUC, Aboudolas et al. (2009) presented two other novel control methodologies based on the SFM, an open-loop quadratic-programming control (QPC) and an open-loop nonlinear optimal control (NOC). Later, Aboudolas et al. (2010) investigated the efficiency of the QPC, which aimed to balance the link queues and minimize the risk of queue spillback.

Research on the use of CV data in the development of signal control strategies is emerging. Head (2008) described the potential and limitations of traffic control in a V2I environment. Venkatanarayana (2011) proposed the use of probe data for better queue management and clustering of vehicle platoons. The University of Virginia developed their Cooperative Vehicle Intersection Control (CVIC) system, which enables cooperation between vehicles and infrastructure for effective intersection operations and management when all vehicles are fully connected and no traffic signal control is needed (Lee et al. 2012; 2013). There are other studies that have used CV technology for adaptive traffic control (Goodall et al. 2013) and transit priority (Liao and Davis, 2007; Koonce, 2012). Smith et al. studied how to better manage queue and cluster-approaching platoons with CV probe vehicles (Smith et al. 2012). Liao et al. developed a Mobile Accessible Pedestrian Signal System for blind pedestrians to cross signalized intersections (Liao et al. 2011).

Dynamic programming (DP) is an exact solution for optimization over time. It decomposes a control problem into a series of sub-problems (i.e., steps), which correspond to discrete segments of time in a real-time control problem. At each step, a set of state variables provide information on the controller and the traffic states at that time (Yin et al. 2015). The Bellman's equation is recursively calculated backwards step-by-step to find the optimal action, which transfers the system from the current state to a new state. In summary, DP is a global optimization strategy for multistage decision processes and it provides a standard against which all other strategies can be compared. Application of DP to the signal control problem can be found in Cai et al. (2009). Unfortunately, the implication of

DP for real-time traffic signal control is limited. Firstly, the computational demand is exponential to the size of the state space, the information space and the action space. Furthermore, in practice it is difficult to obtain complete information on the time period in which the controller seeks optimization. For example, traffic detectors may supply only 5-10 seconds (s) of data for future arriving vehicles. Finally, most of the outputs from the program are never implemented because optimized policies are generated for all possible combinations of initial conditions at each stage of the control period. In practice, only one optimum policy would be implemented. By being able to produce the theoretically optimal control strategy for each input state, DP usually serves as a standard for evaluation of the relative effectiveness of other strategies that can be implemented in practice.

3. SYSTEM THROUGHPUT OPTIMIZATION

This section describes the mathematical equations that represent dynamic traffic states for the arterial traffic network. The equations have the following key features: 1) modeling of traffic flow evolution along arterial links and nodes; 2) modeling of the merging and diverging of vehicle movements at intersections; 3) capturing the physical queue formation and dissipation process; and 4) representing the interaction between control parameters and dynamic traffic states. The traffic dynamic includes a process: upstream arrivals, propagation to the end of the queue, merging into lane groups, and departing, as shown in Figure 1. In order to describe the model, we define J as the set of nodes (intersections) and L as the set of links (streets) in the urban traffic network. Link j_W is marked by its downstream node j and the direction of west. The sets of links of input flow and output flows for link j_W are defined as I_{j_W} and O_{j_W}.

3.1 Upstream Arrivals

The upstream arrival equation describes the flow evolution, which arrives at the upstream of one link over time. Similar to most other research, SFM formulates the inflow to the link j_W as the sum of departure flows from I_{j_W}, as shown in Equation (1).

[1] $$q_{j_W}^{in}(k) = \sum_{l \in I_{j_W}} q_{l,j_W}^{out}(k)$$

Where, $q_{j_W}^{in}(k)$ is the upstream arrival flow of link j_W during time step k; $q_{i,j_E}^{out}(k)$ is the departing flows from link l that merge into j_W, and l belongs to I_{j_W}.
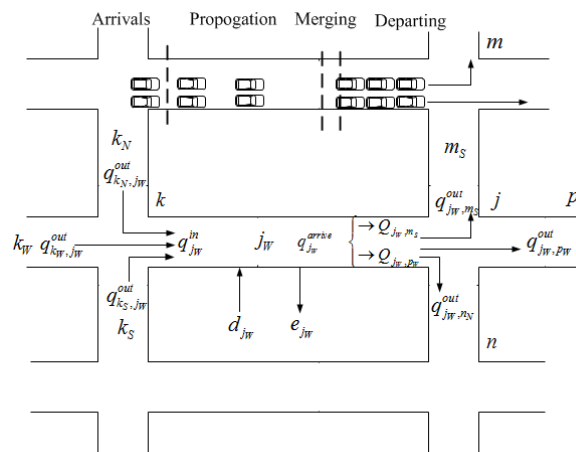

Figure 1: Dynamic traffic flow evolutions along arterial streets.

3.2 Propagation to End of Queue

Then upstream arrival flow propagates to the end of the queue. In the SFM, the discrete-time step T is equal to cycle length. Vehicles entering a link are either stored at the end of this link (during a red signal), or further forwarded to downstream links at the saturation flow rate (during a green signal). Therefore, SFM does not consider the propagation process. Traffic probe data is available in a CV environment, which can serve as an input to the traffic dynamic model. The average speed of moving equipped vehicles is used to represent the propagation process, and the arriving flow at the end of the queue at link j_W is stated as follows:
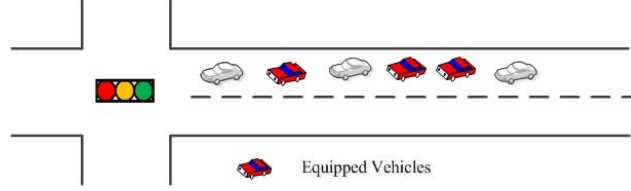

Figure 2: Illustration of approaching speed of moving equipped vehicles.

[2] $\qquad q_{j_W}^{arrive}(k) = (1-\alpha(k)) \cdot q_{j_W}^{in}(k-\beta(k)) + \alpha(k) \cdot q_{j_W}^{in}(k-\beta(k)-1)$

[3] $\qquad \alpha(k) = rem \left\{ \dfrac{(C_{j_W} - Q_{j_W}(k)) \cdot l_{veh}}{N_{j_W} \cdot v_{j_W}(k) \cdot c(k)} \right\} \quad \beta(k) = floor \left\{ \dfrac{(C_{j_W} - Q_{j_W}(k)) \cdot l_{veh}}{N_{j_W} \cdot v_{j_W}(k) \cdot c(k)} \right\}$

Where, $floor\{x\}$ is the largest integer that is smaller than or equal to $\{x\}$; $rem\{x\}$ is the remainder; $c(k)$ is the cycle length at time step k; l_{veh} is the average vehicle spacing; $v_{j_W}(k)$ is the average approaching speed; $q_{j_W}^{arrive}(k)$ is the flows arrive at the tail of the queue during time step k; C_{j_W} is the capacity of link j_W, number of vehicles; $Q_{j_W}(k)$ is the queue length, number of vehicles; N_{j_W} is the number of lanes.

3.3 Merging into Lane Groups

Upon arriving at the end of a queue at a link, vehicles may change lanes and merge into different lane groups, according to the driver's destination. The merging flow into lane group o at time step k can be approximated:

[4] $\qquad q_{j_W,o}^{arrive}(k) = \beta_{j_W,o}(k) \cdot q_{j_W}^{arrive}$

Where, $\beta_{j_W,o}(k)$ is the turning ratio for different turning movements. This study considers $\beta_{j_W,o}(k)$ as predefined, and there is a large body of research on real-time O-D estimation.

3.4 Departing Process

The next step is the queue discharge for different lane groups o. The departing flow $q_{j_W,d}^{out}(k)$ from different lane groups at time step k is given by the following equation:

[5] $\qquad q_{j_W,d}^{out}(k) = \min \left\{ S_{j_W,o}(k) \cdot g_{j_W,o}(k)/T, Q_{j_W,o}(k)/T + q_{j_W,o}^{arrive}(k), (C_{j_W,d} - n_{j_W,d}(k))/T \right\}$

Where, $S_{j_W,o}(k)$ is the saturation flow rate of lane groups o; $g_{j_W,o}(k)$ is the green phase duration of lane groups o; $Q_{j_W,o}$ is the queue length of lane groups o; d is belongs to the set of downstream nodes of output links of link j_E; $n_{j_W,d}(k)$ is the number of vehicles in a link.

The first term of Equation (5) considers congested conditions; the second term considers uncongested conditions; and the third term considers the available storage space of the destination link. Saturation flow rate is calculated using the method from HCM 2010, which estimates the saturation flow rate of any lane group based on known prevailing traffic parameters. The algorithm takes this form:

[6] $\qquad S_i = S_o \cdot N \cdot \prod_i f_i$

Where, S_o is the saturation flow rate per lane under base conditions; f_i is the multiplicative adjustment factor for each prevailing condition I; and N is the number of lanes in the lane group.

3.5 Queue Evolution

Queues at lane groups are updated at every time step k.

[7] $\qquad Q_{j_W,o}(k+1) = Q_{j_W,o}(k) + T \cdot (q_{j_W,o}^{arrive}(k) - q_{j_W,d}^{out}(k))$

3.6 Flow Conservation

The evolution of the total number of vehicles present at link j_W can be stated as follows:

[8] $\qquad n_{j_W}(k+1) = n_{j_W}(k) + (q_{j_W}^{in} - \sum_{d \in D_{j_W}} q_{j_W,d}^{out}) \cdot T + (d_{j_W} - e_{j_W}) \cdot T$

d_{j_W} and e_{j_W} are the demand flow and exit flow of links during time step k, respectively.

In congested conditions, the control objectives need to be decidedly different, as mobility is restricted. For example, the delay minimization strategy provides user-optimal delay minimization in uncongested conditions, but can sometimes work not in favor of minimizing total delay when systems become congested. Instead, the signal plans should be timed such that every green second should be serving traffic at its maximum flow rate. In this research, the following represents the objective for maximizing the throughput in the controlled sub-network.

[9] $\qquad max \ \sum_{k=1}^{M} \sum_{l \in L} q_l^{in}(k)$

One type of principal constraint is the dynamic traffic state evolution along the arterial network. Another is the queue length constraint for left-run and through queues, as shown in Equation (10). The queue length cannot be larger than the capacity of the corresponding lane groups.

[10] $\qquad Q_{l,o}(k) \le C_{l,o}$

As the enhanced SFM considers different movements of one link, the two-ring, eight-phase structure from the National Electrical Manufacturers Association (NEMA) is formulated as another type of constraint.

[11] $\qquad g_{j_N}^{left}(k) + g_{j_S}^{through}(k) = g_{j_W}^{left}(k) + g_{j_E}^{through}(k)$

[12] $\qquad g_{j_S}^{left}(k) + g_{j_N}^{through}(k) = g_{j_E}^{left}(k) + g_{j_W}^{through}(k)$

[13] $\qquad g_{j_N}^{left}(k) + g_{j_S}^{through}(k) + g_{j_W}^{left}(k) + g_{j_E}^{through}(k) = c(k)$

Where, $g_{j_N}^{left}(k)$ and $g_{j_N}^{through}(k)$ represent the green split for left turn and through movement of approach j_N, respectively; $g_{j_S}^{left}(k)$ and $g_{j_S}^{through}(k)$ represent the green split for left turn and through movement of approach j_S, respectively; $g_{j_W}^{left}(k)$ and $g_{j_W}^{through}(k)$ represent the green split for left turn and through movement of approach j_W, respectively; $g_{j_E}^{left}(k)$ and $g_{j_E}^{through}(k)$ represent the green split for left turn and through movement of approach j_E, respectively.

The following is the common minimum and maximum green constraint.

[14] $\qquad g_{l,o}^{min} \leq g_{l,o}(k) \leq g_{l,o}^{max} \quad l \in L$

4. CORRIDOR COORDINATION CONTROL

As the intersections in a corridor are interconnected, the decisions made at one intersection inevitably affect those made at the adjacent intersection. Proper determination of intersection offsets provides for the efficient movement of platoons through multiple intersections during the green signal phase, resulting in significantly reduced delays and improved driver satisfaction. Incorporating coordination into the local controllers is equivalent to a multistage sequential decision-making problem. DP is a long-established technique for solving multistage decision problems, which converts a multistage decision process into a series of single-stage problems. A cycle length is considered as a number of consecutive stages. At each stage, the traffic condition is described collectively by a state. As there are four phases in one cycle, the decision process is divided into four stages. Decisions must be made to move on from one stage to another and the possible sets of decisions depend on the stage and state the junction is in. The system throughput optimization part described in the previous section is adopted as a local controller and it determines an effective green time g for each stage. Coordination control introduces an adjustment parameter t_a, which can be either positive or negative. The resulting sum $t_s = t_g + t_a$ is the green-time allocation (the stage decision) at one stage. The flowchart of this DP coordination system is illustrated in Fig. 3. The objective of coordination is to minimize the average delay on the vehicles.

The state variables that describe the current traffic condition consist of $n_{j_W}(s)$, $Q_{j_W,o}(s)$ and $g(s)$. t_s is one factor enabling the intersection to proceed from one stage to the next. In addition, it determines the state the intersection should be in at the next stage. In order to compare the performance at each stage transformation, another important consideration is that, at a stage, the state space should be finite. Therefore, the possible values of t_a need to be finite, which means a finite state space in the next stage. Equation (15) shows the assumed size of t_a:

[15] $\qquad t_a \in \{-15, -10, -5, 0, 5, 10, 15\}$

However, the the total number of possible states is still enormous. This study used state merging to reduce the number of possible states, which is conducted when two or more states are equivalent: (1) queue lengths at the approach are similar; and (2) $g(s)$ is the same.
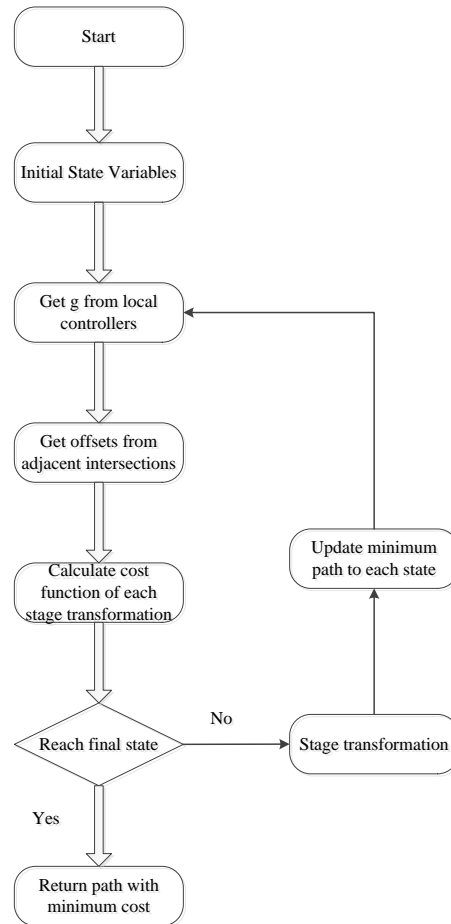
Figure 3: Flow chart of the DP coordination

5. SIMULATION EXPERIMENT

5.1 Simulation Platform

All the performance analyses and evaluations were conducted in a fine-grained simulation engine. VISSIM with the ASC/3 module, a full-scale signal emulator, works as a traffic simulator. A Microsoft Visual C++ application is created to control the simulation process and continuously read traffic state data from VISSIM using the Component Object Model (COM) interface. At the same time, the signal timing data are collected via the NTCIP standards supported by the ASC/3 module. Once such information is collected, it is sent to the "Adaptive Algorithms" module in MATLAB to obtain the optimal signal timing. After optimization, the new signal timing plan goes back into the C++ application via the .NET framework. Finally, the optimal timing plans are sent back to the simulator through a series of NTCIP messages. Specifically, the current timing plan is first saved in a different split plan in ASC/3 and then replaced with the new optimal signal timing.

5.2 Simulation Results

The testing site is about 7.4 km long and consists of eight signalized intersections. The signal controllers were configured as actuated with virtual loop detectors in VISSIM. For detailed information about the lane assignment and signal timing plan of each intersection, please refer to the previous study (Liu and Qiu 2016). The PM peak period, from 15:30 to 17:30, was selected for simulation. The warm-up time was 10 minutes and cool-down time was also 10 minutes. A reference case was needed for comparison with the proposed model. Therefore, one optimized fixed-time control plan was generated with SYNCHRO for both the current and 15% increase traffic demand. For the reference

case, the offsets were constant during the VISSIM simulation. Each of the scenarios was simulated multiple times and results were tested for statistical significance.

The scenario with 15% increase in traffic demand exhibited extensive queues, which propagated to block the upstream intersection. The average delay for the whole corridor was used as the Measures of Effectiveness (MOE). As shown in Table 1, the total average delay is improved by 16% and 23% under current and 15% increase demand scenarios, respectively. Figure 4 compares the average delay of different intersections. The proposed model decreases the delays of each individual intersection.

Table 1: Delay Comparison of VISSIM Simulation Results

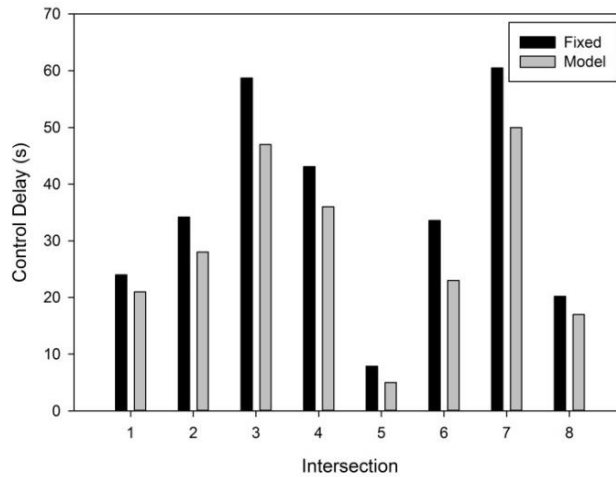| Scenarios	MOE	Simulation Results from VISSIM		
		Proposed Model	Fixed-time	Improvement
Current	Average Delay(s)	31.6	37.7	16%
15%		37.3	48.7	23%

Figure 4: Average delay at intersections.

6. CONCLUSIONS

This paper first enhances an arterial traffic flow model for arterial signal optimization using the average approaching speed of all equipped vehicles in a CV environment. Then a dynamic programming optimization model is proposed to solve the signal optimization application. Simulation results show that, in terms of average delay, the proposed model provides better performance than fixed-time control plan does.

ACKNOWLEDGEMENTS

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