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CATEGORIZING RISKS IN THE INSURANCE INDUSTRY: THE CASE OF SYMMETRIC INFORMATION*

by

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Abstract

Models of insurance markets are often characterized by the assumption of asymmetric information. In this paper the alternative assumption of symmetric (but improving) information is considered. Since both firms and consumers are initially unaware of risk class membership, there is no role for improved information to play in the reduction of the extent of adverse selection that persists in the asymmetric information models. However, it is shown in this paper that if firms are risk averse or face a bankruptcy constraint which is costly to maintain, information which leads to the categorization of risks will provide a "production efficiency" which leads to a fall in the average price of insurance. Policies which avoid possible adverse distributional effects of categorization are investigated.

I. Introduction

The fact that insurance firms are frequently unable to identify the risk class to which their customers belong can create serious consequences which affect the "normal" functioning of insurance markets. For example, Rothschild and Stiglitz (1976) have shown that a Nash equilibrium may not exist when insureds know the risk class to which they belong but insurers (firms) do not. Under this assumption of asymmetric information high risk types are unwilling to reveal their risk class membership since, quite naturally, they prefer to be treated as low risk types.

It is not necessarily the case, however, that information concerning risk class membership will be asymmetric. For example, suppose two types of automobiles, H and L, are assumed by both insurers and insureds to have identical accident probabilities, *ceteris paribus*. Initially, one expects no differentiation of insurance premiums[1] on the basis of an automobile being of type H or L. Suppose that, after some time has passed, information which distinguishes H as a "high risk" automobile and L as a "low risk" automobile becomes available to both consumers and firms. Provided firms can (costlessly) identify the type of automobile being insured, this situation can be described as one of symmetric information with "new" information becoming available simultaneously to all agents. Such information may quite naturally lead to a categorization scheme whereby high and low risk types are charged different premiums. The investigation of the welfare implications for the symmetric information case constitutes the purpose of this paper.

If, for the symmetric information case, firms are assumed to be risk neutral, then the welfare implications of categorizing risks are trivial. According to the assumption of symmetric information, consumers believe themselves to be (and are treated by firms as) "average" risks. Under the assumptions of perfect competition, firm risk neutrality, and zero administrative costs,[2] firms will offer insurance at (pooled) actuarially fair rates and consumers will purchase full coverage insurance. After categorization, high and low risk types are costlessly identified and are offered insurance at their (specific) risk class actuarially fair rates. Although high risk types must pay a higher price than before categorization, they will continue to purchase full coverage insurance since they will hold the belief that they are indeed high risk types and that the new (higher) price is in fact the actuarially fair one. Similarly, low risk types will also purchase full insurance after categorization. Therefore, there are no "consumption" or "production" efficiencies associated with categorization. The "average" price of insurance and the level of coverage chosen are identical before and after categorization.

However, if firms are concerned with the amount of risk present in their portfolios (e.g., because they are risk averse or face a bankruptcy constraint) then information relating to the risk class membership of consumers may be desirable since it reduces the "structure risk" associated with providing insurance and, therefore, will reduce the "average risk charge" levied on consumers. That is, information which provides firms with the ability to choose, with certainty, the proportions of high and low risk types that they insure reduces the riskiness of their portfolios and, therefore, induces competitive firms to reduce the average price of insurance. As in the case with risk neutral firms, high and low risk types will be charged different premiums after categorization. Given that consumers do not initially (i.e., before categorization) know their risk class, they will view categorization as a process which randomizes their premiums. Individuals may, therefore, receive a higher level of expected utility from the possibility of no categorization than from the possibility of categorization.

It is interesting to compare the welfare implications of imperfectly categorizing risks for the asymmetric information case with those derived in this paper for the symmetric information case. If one employs the Wilson E2 equilibrium concept to the asymmetric information model, the nonexistence outcome associated with a Nash equilibrium (see Wilson, 1976) vanishes.[3] If firms behave according to Wilson's equilibrium concept, they will use information, even if imperfect, to categorize risks. The welfare implications of this activity are analyzed in Hoy (1981a, 1981b) where it is shown that, in some circumstances, imperfect categorization leads to a Pareto-type welfare improvement. However, it may also be the case that, after categorization, some individuals are better off (i.e., those assigned to the low risk category) while others are worse off (i.e., those assigned to the high risk category).

Perhaps more interesting is the analysis of how consumers treat the possibility of a prospective categorization scheme before they know the risk category to which they will be assigned (see Hoy, 1981b). Although individuals know their risk type, they will not know the risk category to which they will be assigned if it is assumed that the information on which the categorization scheme is based is imperfect (i.e., if some individuals are misclassified). The result is that both high <u>and</u> low risk types may prefer, ex ante to knowing the risk category to which they will be assigned, that a particular categorization scheme not be implemented. That is, everyone may receive a higher level of expected utility from the possibility of no categorization than from the possibility of categorization. This result is similar to the one derived in this paper (and mentioned above) when information is symmetric between insureds and insurers. That is, it is possible that individuals will all be made worse off by the use of information which leads to categorization. Therefore, there is an incentive for governments to suppress the use of information which leads to the categorization of risks.

Although the welfare implications of categorization are similar for the symmetric and the asymmetric information models, the sources of the results are different. In the asymmetric information model, the "purpose" of categorization is for firms to obtain information concerning risk class membership so that the extent of adverse selection can be reduced. In the symmetric information case, categorization provides a "production efficiency" in the sense that structure risk, which results from the uncertainty involved in selecting different risk types from a heterogeneous population, can be reduced by categorization. For this reason, the government may not wish to suppress the use of information which leads to categorization. They may prefer to introduce more sophisticated policies which allow risk averse firms to use information concerning risk class membership in order to reduce the riskiness of their portfolios but, at the same time, do not allow firms to charge a different premium for different risk types. The construction and viability of such programs are discussed later in this paper.

In Section II the statistical background to the result that categorization reduces the riskiness of a firm's portfolio (by eliminating structure risk) is introduced. A model with risk averse firms (and consumers) who are initially unaware of the existence of different risk classes is presented in Section III. The welfare implications of categorizing risks for this model are discussed in Section IV. In Section V another model with risk averse firms is presented.

In this case (unlike the model in Section III) firms are aware, even before categorization, of the existence of the two risk classes in the population. However, they are unable to identify which individuals belong to which class. Consumers are also unable to determine risk class membership. Therefore, the model is called a "partial information" one. In Section VI a model with risk neutral firms who face a bankruptcy constraint (imposed by the government) is considered. In each of the models in Sections III, IV, and VI, categorization leads to a lower average premium being charged by firms. In Section VII the results of the paper are summarized.

II. Simple Random vs. Stratified Random Sampling

If firms cannot identify the risk class to which individuals belong then two kinds of risk associated with the provision of insurance arise. One type is associated with the random nature of losses while the other, called structure risk, arises from the random nature of selecting clients from a population containing two different risk classes. If risk class membership is not known, then the firm's selection of clients represents simple random sampling from a population which is composed of subpopulations. On the other hand, if the risk class membership of consumers is known by firms then the selection of clients represents stratified random sampling. Since the proportions of high and low risk types are random in the former case but certain in the latter, it follows that stratified sampling is more efficient than random sampling. This result, which can be found in several statistics texts,[4] is demonstrated by Proposition 1 below.

If a variable or parameter refers to the situation before categorization (or rather, if there is no categorization) then the index 0 will be used. If a variable or parameter refers to the situation after categorization then the index 1 will be used to denote information for high risk types and 2 will be used to denote information for low risk types. Therefore, let $f_1(x)$ be the probability density function (p.d.f.) of losses for high risk types and $f_2(x)$ be the p.d.f. for low risk types, with means $\mu_1 = \int_{-\infty}^{\infty} xf_1(x)dx > \mu_2 = \int_{-\infty}^{\infty} xf_2(x)dx$ and variances $\sigma_1^2 = \int_{-\infty}^{\infty} (x-\mu_1)^2 f_1(x)dx$ and $\sigma_2^2 = \int_{-\infty}^{\infty} (x-\mu_2)^2 f_2(x)dx$. Assume that all means and variances are finite. Let q_1 and q_2 be the proportions of high and low risk types in the population. If there is no information concerning risk class membership, the distribution of losses for an individual selected at random will have pooled p.d.f. $f_o(x) = q_1 f_1(x) + q_2 f_2(x)$, with mean $\mu_o = q_1\mu_1 + q_2\mu_2$ and variance $\sigma_o^2 = \int_{-\infty}^{\infty} (x-\mu_o)^2 f_o(x)dx$.

Proposition 1. Let σ_o^2, σ_1^2, and σ_2^2 represent the variances and μ_o, μ_1, and μ_2 the means for the pooled population, high risk types, and low risk types, respectively. If the pooled population is composed of proportions q_1 and q_2 of high and low risk types, then

$$\sigma_o^2 = q_1\sigma_1^2 + q_2\sigma_2^2 + q_1(\mu_1-\mu_o)^2 + q_2(\mu_2-\mu_o)^2 \tag{2.1}$$

For proof, see footnote 4.

Therefore, the riskiness of a firm's portfolio can be reduced by the knowledge of individuals' risk class membership. The following discussion illustrates this claim.

Suppose firms sample N clients and know that N_1 of them are high risk types and N_2 are low risk types, with $\frac{N_1}{N} = q_1$ and $\frac{N_2}{N} = q_2$. The variance of losses associated with this collection of risks is $N_1\sigma_1^2 + N_2\sigma_2^2 = N(q_1\sigma_1^2 + q_2\sigma_2^2)$. On the other hand, it follows from Proposition 1 that, if firms select N clients at random from a population composed of proportions q_1 and q_2 of high and low risk types, the variance associated with this latter collection of risks is $N\sigma_o^2 = N(q_1\sigma_1^2 + q_2\sigma_2^2 + q_1(\mu_1-\mu_o)^2 + q_2(\mu_2-\mu_o)^2)$. The variance is greater in the second case because the firm is not certain of the proportions of high and low risk types selected--even though the "expected value" of the composition of risks in the second situation is equal to the "known" composition of risks in the first situation. Furthermore, as the number of firms increases in either "experiment" the population of risks will be exhausted. It is in this sense that a "production efficiency" arises from categorization if firms are risk averse.

Consider the special case with $\sigma_1^2 = \sigma_2^2$. Suppose firms levy a "risk charge" on each consumer, which is a monotonically increasing function of the variance which results from providing him with insurance. Using the results of Proposition 1 it follows that $\sigma_o^2 > \sigma_1^2, \sigma_2^2$. Therefore, individuals are assessed a lower risk charge in the situation where the risk class membership of individuals is known. This result does not, however, imply that information which is used to categorize risks will unambiguously lead to a Pareto-type improvement in welfare. High risk types[5] expose the firm to a higher expected loss. This larger expected cost may outweigh the reduction in the "risk charge" which is associated with categorization. Therefore, after categorization, the high risk individuals may face a higher price of insurance and be made worse off.

In this section the implications of categorization are considered in a very superficial manner. In the remainder of the paper more explicit rules determining the behavior of firms and consumers are considered. Also, the welfare implications of categorization are investigated in more depth.

III. Risk Averse Firms

In this section it is assumed that firms are identical, risk averse, and display constant absolute risk aversion. Administrative costs are assumed to be zero. The insurance contract offered by firms is such that a fraction (r) of any loss incurred by a client is paid to him by the firm. Before categorization, all agents believe that losses $\tilde{X}_o$ are normally distributed with mean $\mu_o > 0$ and variance $\sigma_o^2 < \infty$ (i.e., $\tilde{X}_o \sim N(\mu_o, \sigma_o^2)$). The mean and variance are substantiated by pooled claims experience.

Let $\Psi_o(r)$ represent the price for coverage level r. Therefore, a firm selling N policies at coverage level r and price $\Psi_o(r)$ will be faced with the following mean and variance of return:[6]

$$\mu = N(\Psi_o(r) - r\mu_o) \tag{3.1}$$

$$\sigma^2 = Nr^2\sigma_o^2 \tag{3.2}$$

Let $\bar{V}$ represent the firm's expected utility when not selling insurance (i.e., mean and variance of return equal zero) and let V_f be the expected utility received by the firm if it does sell insurance. $\Psi_o(r)$ is said to be an equilibrium price schedule if firms are indifferent to entry or exit from the industry. Therefore, to satisfy the no entry-exit requirement $\Psi_o(r)$ must satisfy the following condition:[7]

$$V_f(\mu, \sigma^2) = - e^{-\gamma\mu + \gamma^2\sigma^2/2} = \bar{V} \tag{3.3}$$

The indifference curves of any utility function, V, with constant absolute risk aversion are straight lines in μ-σ^2 space with

$$\left.\frac{d\mu}{d\sigma^2}\right|_{dV=0} = - \frac{\partial V/\partial\sigma^2}{\partial V/\partial\mu} = \frac{\gamma}{2} \tag{3.4}$$

Now, if equation (3.3) holds for all r then $\frac{\partial V_f}{\partial r} = \frac{\partial V_f}{\partial \mu} N(\Psi_o' - \mu_o) + \frac{\partial V}{\partial \sigma^2} N2r\sigma_o^2 = 0,$

which gives

$$\Psi_o'(r) = \mu_o + \gamma r \sigma_o^2 \qquad (3.5)$$

The no entry-exit condition implies that $\Psi(0) = 0$ so that:[8]

$$\Psi_o(r) = r\mu_o + \frac{\gamma}{2} r^2 \sigma_o^2 \qquad (3.6)$$

It is a well-known consequence when using the mean-variance approach that firms with constant absolute risk aversion will charge a price equal to expected marginal cost plus a constant multiplied by the variance of return (see Leland, 1974, pp. 141-2). Equation (3.6) corresponds to this result.

Setting $\frac{\partial V_f}{\partial r} = 0$ above coincides with the firm's first order condition for the maximization of utility. However, the firm can also choose N, the number of contracts sold. Upon setting $\frac{\partial V_f}{\partial N} = 0$ we get equation (3.6); that is, no "additional information" is obtained so that firm size is indeterminate. This result follows because of the assumption of constant absolute risk aversion. In this context, if $V_f > \overline{V}$ (i.e., actual price is greater than the equilibrium price) then firms wish to sell an infinite number of policies while if $V_f < \overline{V}$, they wish to sell none. This result is analogous to the case of constant returns to scale with perfect competition and certainty. Since it is implicitly assumed that the market price adjusts to equate V_f to $\overline{V}$, the issue of firm size need not be considered here. We can assume that firm size is determined by some criterion other than profit maximization.

Let us now consider the consumer. Suppose each consumer has certain wealth $\overline{W}$ as well as random losses denoted by $\tilde{X}_o$ (i.e., before categorization) If he purchases coverage level r his mean and variance of wealth will be:

$$\mu = \overline{W} - \Psi_o(r) - (1-r)\mu_o \qquad (3.7)$$

$$\sigma^2 = (1-r)^2\sigma_o^2 \qquad (3.8)$$

Let $V_c(\mu,\sigma^2) = -e^{-\beta\mu+\beta^2\sigma^2/2}$ represent the consumer's utility function (recall, consumers are identical). Although it is not necessary that consumers' utility functions display constant absolute risk aversion, it does simplify the analysis. For example, it is easy to show that utility maximizing consumers will purchase coverage level:

$$r = \frac{\beta}{\gamma+\beta} \qquad (3.9)$$

As one might expect, $\frac{\partial r}{\partial \beta} > 0$ while $\frac{\partial r}{\partial \gamma} < 0$. That is, as the level of the consumer's risk aversion increases so does the quantity of insurance he purchases, while if the level of risk aversion of firms increases (i.e., if the "risk charge" increases), the consumer purchases less insurance. It is also easy to check that the consumer's second-order condition $(\frac{d^2V_c}{dr^2} < 0)$ holds.

The analysis for the situation after categorization is similar to that above. The high risk types' losses are represented by the random variable $\tilde{X}_1 \sim N(\mu_1,\sigma_1^2)$ while losses for low risk types are represented by $\tilde{X}_2 \sim N(\mu_2,\sigma_2^2)$ with $\mu_1 > \mu_2 > 0$. Therefore, according to the previous analysis we find:

$$\Psi_1(r) = r\mu_1 + \frac{\gamma}{2} r^2\sigma_1^2 \qquad (3.10)$$

$$\Psi_2(r) = r\mu_2 + \frac{\gamma}{2} r^2\sigma_2^2 \qquad (3.11)$$

These price schedules clearly satisfy the no entry-exit condition if each firm specializes by selling contracts to only one of the two risk classes. Furthermore, it can be shown that by selling a proportion k $(0 < k < 1)$ of N contracts to high risk types and 1-k to low risk types, a firm can neither increase nor decrease its level of utility. Therefore, equations (3.10) and (3.11) represent equilibrium price schedules even if firms do not specialize.[9] The remainder of this paper is concerned with the effects of categorization on the price of insurance to high and low risk types.

The conditions under which an individual of risk type i faces a higher, identical, or lower price of insurance after categorization (i.e., $\Psi_i(r) \gtreqless \Psi_o(r)$) are summarized below. Equation (3.12) corresponds to the appropriate condition for high risk types while equation (3.13) corresponds to the condition for low risk types.

$$\Psi_1(r) \gtreqless \Psi_o(r) \quad \text{as} \quad (\mu_1-\mu_o) + \frac{\gamma}{2}r(\sigma_1^2-\sigma_o^2) \gtreqless 0 \tag{3.12}$$

$$\Psi_2(r) \gtreqless \Psi_o(r) \quad \text{as} \quad (\mu_2-\mu_o) + \frac{\gamma}{2}r(\sigma_2^2-\sigma_o^2) \gtreqless 0 \tag{3.13}$$

The first result to be derived states that at least one of the two risk classes must face a price of insurance after categorization which is less than the price before categorization (see Proposition 2 below).

<u>Proposition 2</u>. It is not possible that categorization will lead to higher prices for both high and low risk types; that is, at least one of $\Psi_1(r)$ and $\Psi_2(r)$ must be less than $\Psi_o(r)$.

<u>Proof</u>: The proof is by contradiction.

Suppose $\Psi_1(r) \geq \Psi_o(r)$ and $\Psi_2(r) \geq \Psi_o(r)$

then $q_1\Psi_1(r) \geq q_1\Psi_o(r)$, $q_2\Psi_2(r) \geq q_2\Psi_o(r)$

$$\Rightarrow q_1\Psi_1(r) + q_2\Psi_2(r) \geq \Psi_o(r) \text{ since } q_1 + q_2 = 1$$

$$\Rightarrow q_1 r\mu_1 + q_1 \frac{\gamma}{2} r^2\sigma_1^2 + q_2 r\mu_2 + q_2 \frac{\gamma}{2} r^2\sigma_2^2 \geq r\mu_o + \frac{\gamma}{2} r^2\sigma_o^2$$

$$\Rightarrow q_1\sigma_1^2 + q_2\sigma_2^2 \geq \sigma_o^2 \text{ since } q_1\mu_1 + q_2\mu_2 = \mu_o$$

The last equation contradicts the results of Proposition 1.

q.e.d.

Since it must be the case that categorization leads to the reduction in the price of insurance for the members of at least one of the two risk classes it is natural to inquire if it is possible for both risk classes to be faced with a lower price of insurance after categorization. As Example 1 below illustrates, it is indeed possible that the reduction in structure risk associated with categorization may be sufficiently large that the corresponding fall in the "risk charge" levied by firms will lead to an overall decrease in the price of insurance for both risk classes.

Example 1

Suppose: $q_1 = q_2 = 0.5$

$\mu_1 = 100,\ \mu_2 = 60 \Rightarrow \mu_o = 80$

$\sigma_1^2 = \sigma_2^2 = 100 \Rightarrow \sigma_o^2 = 500$ (by Proposition 1)

$\gamma = 1$ (recall, $r = \frac{\beta}{\gamma+\beta}$).

Then, using equations (3.11), (3.10), (3.9), and (3.6) it follows that $\Psi_1(r) < \Psi_o(r)$ provided $\beta > \frac{1}{9}$ and that $\Psi_2(r) < \Psi_o(r)\ \forall\ \beta$.

Therefore, in Example 1, both high and low risk types face a price of insurance which is less after categorization than before categorization. This result is summarized in Proposition 3 below (Example 1 serves as its proof).

Proposition 3. It is possible that members of both risk classes will incur a lower price of insurance as a result of categorization.

Although it is required, by Proposition 1, that $q_1\sigma_1^2 + q_2\sigma_2^2 < \sigma_o^2$, it is not necessarily the case that both σ_1^2 and σ_2^2 be less than σ_o^2. In particular, if $\sigma_2^2 > \sigma_o^2$ it is possible that categorization will lead to a price of insurance for low risk types which is higher than the price before categorization. This result is not as strange as it may initially seem. Low risk types, as defined by the mean of the loss distribution (i.e., $\mu_2 < \mu_1$), will face higher probabilities for certain "large" losses than do high risk types if $\sigma_2^2 > \sigma_1^2$. Therefore, the possibility of "seemingly perverse" results, such as high risk types being made better off as a result of categorization while low risk types are made worse off,[10] is not really counterintuitive. Nevertheless, it can be shown that if $\sigma_2^2 > \sigma_o^2$ and $\sigma_1^2 < \sigma_o^2$, the "seemingly perverse" results mentioned above are more likely to occur the greater is the degree of risk aversion of firms and/or consumers. This result is somewhat intuitive since it is the introduction of risk aversion on the part of firms which creates the possibility of these results.

It was suggested in the Introduction that if firms are risk neutral (γ=0), then the implications of categorization are trivial; namely, that no "consumption" or "production" efficiencies are incurred. That this is the case is easily seen in the model presented in this section. As $\gamma \rightarrow 0$ we find that $\Psi_o(r) \rightarrow r\mu_o$, $\Psi_1(r) \rightarrow r\mu_1$, $\Psi_2(r) \rightarrow r\mu_2$, and $r \rightarrow 1$. Therefore, consumers purchase 100% coverage both before and after categorization. Also, the average price of insurance after categorization is equal to the (average) price of insurance before categorization (since $\mu_o = q_1\mu_1 + q_2\mu_2$). This result is in contrast to the outcome when firms are risk averse, in which case the average price after categorization is less than the (average) price before categorization.[11]

IV. Welfare Implications and Policy Suggestions

In Section III the implications of categorization with respect to the price of insurance were considered. In this section some welfare implications of categorization are derived for that model. The first part of this section employs the usual ex ante and ex post welfare analyses.

This is followed by a consideration of how individuals might assess the process of categorization on the basis of their selfish interests before they know the particular category to which they will be assigned. Since the individual's perspective in this latter analysis is considered before he knows either the categorization scheme to which he will be assigned or the particular loss that will be realized, the approach is referred to as "ex ante-ex ante analysis".

Information which leads to the categorization of risks affects an individual's ex ante utility in two ways since, after categorization, the individual is faced with a different price of insurance and a different set of expectations. Since individuals purchase less than full coverage insurance this latter aspect of the information cannot be ignored. Nevertheless, as a result of the assumptions made concerning firm and consumer behavior, it is shown below (see Theorem 1) that the welfare implications of categorization are, for both ex ante and ex post utilities, analogous to the price effects.[12] That is, if individuals discovered to be in risk class i are assessed a <u>lower</u> price after categorization than before categorization ($\Psi_i(r) < \Psi_o(r)$) then they will also experience <u>higher</u> levels of both ex ante and ex post utility after categorization. Similarly, if they are faced with a <u>higher</u> price ($\Psi_i(r) > \Psi_o(r)$), they will experience <u>lower</u> levels of ex ante and ex post utility.

In order to derive the equivalence result between price movements and ex ante welfare implications we need to derive expressions for a consumer's ex ante utility before and after categorization. Ex ante utility refers to the expected utility of an individual before he knows the particular loss which he ultimately incurs. His expected utility before categorization (i.e., if no categorization scheme is or is expected to be implemented) incorporates the assumption that he does not know the risk class to which he belongs.[13]

Consumers receive certain wealth $\bar{W}$ as well as some possible loss. Presented with one of the above price schedules for insurance, each individual maximizes his utility by choosing the optimal level of coverage, which is $r = \frac{\beta}{\gamma+\beta}$. The result is that individuals are faced with one of the following pairs of mean (μ) and variance (σ^2) of wealth

before categorization:
$$\mu = \bar{W} - \Psi_o(r) - (1-r)\mu_o \qquad \sigma^2 = (1-r)^2\sigma_o^2 \tag{4.1}$$

after categorization,

high risk types:
$$\mu = \bar{W} - \Psi_1(r) - (1-r)\mu_1 \qquad \sigma^2 = (1-r)^2\sigma_1^2 \tag{4.2}$$

low risk types:
$$\mu = \bar{W} - \Psi_2(r) - (1-r)\mu_2 \qquad \sigma^2 = (1-r)^2\sigma_2^2 \tag{4.3}$$

Let V_c^o denote an individual's ex ante utility before categorization while V_c^1 and V_c^2 denote high and low risk types' ex ante utility (respectively) after categorization. To determine an individual's utility level for each of the possible situations one must substitute the appropriate pair of expressions from equations (4.1), (4.2), and (4.3) into the utility function $V_c(\mu,\sigma^2) = -e^{-\beta\mu+\beta^2\sigma^2/2}$. The following expressions can then be derived.

$$V_c^o = -e^{k_o}, k_o = -\beta(\bar{W}-\Psi_o(r) - (1-r)\mu_o) + \frac{\beta^2(1-r)^2\sigma_o^2}{2} \tag{4.4}$$

$$V_c^1 = -e^{k_1}, k_1 = -\beta(\bar{W}-\Psi_1(r) - (1-r)\mu_1) + \frac{\beta^2(1-r)^2\sigma_1^2}{2} \tag{4.5}$$

$$V_c^2 = -e^{k_2}, k_2 = -\beta(\bar{W}-\Psi_2(r) - (1-r)\mu_2) + \frac{\beta^2(1-r)^2\sigma_2^2}{2} \tag{4.6}$$

Therefore, according to his ex ante utility, an individual of risk type i is better off with categorization if $k_i < k_o$ and worse off if $k_i > k_o$. Also, according to his ex post utility, he is better off with categorization if he ultimately incurs a higher level of wealth W^i after categorization than before categorization (W^o). Using these relationships and some previous results, Theorem 1 is stated and proved below.

Theorem 1. The conditions (i) $\Psi_i(r) \lesseqgtr \Psi_o(r)$, (ii) $V_c^i \gtreqless V_c^o$, and (iii) $W^i \gtreqless W^o$ are equivalent.

Proof: The theorem is proved in two parts. In Part A it is shown that (i) and (ii) are equivalent while in Part B it is shown that (i) and (iii) are equivalent.

Part A

From equations (3.6), (3.9), (3.10), and (3.11) it follows that

$$\text{(i)}\quad \Psi_i(r) \lesseqgtr \Psi_o(r)$$

$$\Leftrightarrow r\mu_i + \frac{\gamma}{2} r^2\sigma_i^2 \lesseqgtr r\,\mu_o + \frac{\gamma}{2} r^2\sigma_o^2$$

$$\Leftrightarrow (\mu_i - \mu_o) + \frac{\gamma}{2}\left(\frac{\beta}{\gamma+\beta}\right)(\sigma_i^2 - \sigma_o^2) \lesseqgtr 0$$

If, in addition, one uses the results from equations (4.4), (4.5), and (4.6) it follows that

$$\text{(ii)}\quad V_c^i \gtreqless V_c^o \Leftrightarrow k_i \lesseqgtr k_o \Leftrightarrow k_i - k_o \lesseqgtr 0$$

$$\Leftrightarrow -\beta(\bar{W} - \Psi_i(r) - (1-r)\mu_i) + \frac{\beta^2(1-r)^2}{2}\sigma_i^2$$
$$- (-\beta(\bar{W} - \Psi_o(r) - (1-r)\mu_o) + \frac{\beta^2(1-r)^2}{2}\sigma_o^2) \lesseqgtr 0$$

$$\Leftrightarrow \Psi_i(r) + (1-r)\mu_i - \Psi_o(r) - (1-r)\mu_o + \frac{\beta(1-r)^2}{2}(\sigma_i^2 - \sigma_o^2) \lesseqgtr 0$$

$$\Leftrightarrow (\mu_i-\mu_o) + (\frac{\gamma}{2} r^2 + \frac{\beta(1-r)^2}{2})(\sigma_i^2-\sigma_o^2) \lesseqgtr 0$$

$$\Leftrightarrow (\mu_i-\mu_o) + \frac{\gamma}{2}(\frac{\beta}{\gamma+\beta})(\sigma_i^2-\sigma_o^2) \lesseqgtr 0$$ upon expansion of $\frac{\gamma}{2} r^2 + \frac{\beta(1-r)^2}{2}$ and substitution for $r = \frac{\beta}{\gamma+\beta}$.

Therefore, conditions (i) and (ii) are identical.

Part B

(iii) $W^i \gtreqless W^o$

$$\Leftrightarrow \overline{W} - \Psi_i - (1-r)L \gtreqless \overline{W} - \Psi_o - (1-r)L,$$ where L is the particular loss realized

$$\Leftrightarrow \Psi_i \lesseqgtr \Psi_o$$

Therefore, conditions (i) and (iii) are identical, so that, in conjunction with Part A it follows that conditions (i), (ii), and (iii) are identical.

q.e.d.

In summary, the answer to the query of whether an individual is made better off as a result of categorization (w.r.t. the model presented in Section III) will be the same for the following criteria:

(i) An individual is better off if, after categorization, he is required to pay a lower price for the same level of insurance coverage as was purchased before categorization.

(ii) An individual is better off provided he receives a higher level of ex ante utility (i.e., expected utility before the particular loss is realized).

(iii) An individual is better off provided he receives a higher level of ex post utility (i.e., utility after the particular loss is realized).

These results allow the following reinterpretation of Propositions 2 and 3 (respectively).

Proposition 2'. It is not possible that categorization will lead to lower levels of ex ante or ex post utility for both high and low risk types.

Proposition 3'. It is possible that members of both risk classes will incur higher levels of ex ante and ex post utility as a result of categorization.

The implications of categorization from the point of view of the consumer before he knows the risk class to which he will be assigned will now be considered. This perspective differs from the above analysis since, before categorization, the individual is contemplating the consequences of some prospective categorization scheme which he believes will be implemented. In the previous analysis the consumer, before categorization, assumed that the status quo (i.e., no categorization) would prevail.

In the spirit of symmetric information it is assumed that all individuals hold the same probabilities of being assigned to the high or low risk category. It is, furthermore, assumed that individuals' subjective probabilities (concerning risk class assignments) are in accord with the actual population proportions. Therefore, the probability that individuals hold for being assigned to the high risk class is q_1, the actual proportion of high risk types, and the probability of being assigned to the low risk class is $q_2 = 1-q_1$, the actual proportion of low risk types. Each individual who is assigned to risk class i is assumed by everyone, including himself, to be of that risk type.

Let the index "a" identify variables or parameters which refer to the situation (before categorization) in which individuals anticipate that a particular categorization scheme will be implemented. The individual knows the parameters of the categorization scheme, but not the actual assignment of individuals to risk classes. Let V_c^a refer to the utility that an individual receives in anticipation of this particular categorization scheme. Therefore

$$V_c^a = q_1 V_c^1 + q_2 V_c^2 \tag{4.7}$$

That is, the expected utility that an individual receives in anticipation of a categorization scheme is equal to the probability that he will be assigned to the high risk class (q_1) multiplied by the expected utility he would receive if this were the case (V_c^1) plus the probability of being assigned to the low risk class (q_2) multiplied by his expected utility if he were (V_c^2).

If V_c^a, the expected utility received upon anticipation of a particular categorization scheme, is less than V_c^o, the expected utility received under the status quo (i.e., no categorization), then individuals prefer, before knowing the risk class to which they will be assigned, that the categorization scheme not be implemented.[14] This result is possible even though the average price of insurance after categorization is lower than the average price of insurance before categorization. The reason for this combination of results is that, while information which leads to the categorization of risks reduces the structure risk faced by firms and, hence, the average risk charge levied on consumers, it also introduces premium risk. Therefore, even though categorization represents a "better than actuarially fair gamble" to consumers, they may prefer the status quo (because of their risk aversion). This possibility is stated and proved in Proposition 4 below.

Proposition 4. The anticipation of a categorization scheme may lead to a Pareto-type worsening of welfare in terms of individuals' (V_c^a -) ex ante utilities.

Proof: An example will suffice to illustrate this proposition

Suppose $q_1 = q_2 = 0.5$; $\mu_1 = 85$, $\mu_2 = 75$; $\sigma_1^2 = \sigma_2^2 = 10$; $\overline{W} = 100$ and $\gamma = 0.1$ are the parameters for a specific situation.

If $\beta = 1$ then $r = .9091$ is the level of coverage chosen before and after categorization.

Therefore, $\Psi_o = 74.17$, $\Psi_1 = 77.69$, $\Psi_2 = 68.60$

and $V_o = -0.1011657 \times 10^{-7}$

$V_1 = -0.481936 \times 10^{-6}$

$V_2 = -0.2187986 \times 10^{-10}$

It follows that $q_1V_1 + q_2V_2 = V_c^a < V_o$ (Note that $q_1\Psi_1 + q_2\Psi_2 < \Psi_o$.)

The use of information which leads to the categorization of risks provides a "production efficiency" to risk averse firms. This efficiency results from the reduction of structure risk which is associated with the provision of insurance to members of a heterogeneous population of risks. The consequence of this efficiency is that the average price of insurance after categorization is less than that before categorization. It may even be the case that both high and low risk types are better off after categorization (see Proposition 3').

However, it is also possible that individuals belonging to one of the two risk classes may face a higher premium level as a result of categorization.[15] This type of "premium risk" may be viewed as undesirable if one takes the perspective of the individual before he knows the risk class to which he belongs (see Proposition 4). The possibility that a prospective categorization scheme could be unanimously rejected by consumers, before they know the particular risk class to which they will be assigned, presents a dilemma to an individual who wishes to determine the desirability of the process of categorization. If information which leads to the categorization of risks is suppressed then the "production efficiencies" associated with the reduction of structure risk would be lost. The possibility of constructing policies which resolve this conflict is discussed briefly below.[16]

Suppose the government imposes a regulation which, after categorization, requires that firms (i) hold portfolios of clients with proportions q_1 and q_2 of high and low risk types (respectively) and (ii) charge a single price for all individuals. The consequences of adopting such a policy would be that the price charged to each individual would be equal to $q_1\Psi_1(r) + q_2\Psi_2(r)$ (i.e., this price satisfies the no entry-exit condition). The reason that this price is less than the price before categorization (recall $\Psi_o > q_1\Psi_1 + q_2\Psi_2$) is that, after categorization, firms know the proportion of high and low risk types in their portfolios to be q_1 and q_2 while, before categorization, these proportions are random variables (with expected values q_1 and q_2). Therefore, structure risk is eliminated without the introduction of premium risk.

Another method which eliminates premium risk is the introduction of a system of taxes and subsidies. For example, let $\overline{\Psi}(r) = q_1\Psi_1(r) + q_2\Psi_2(r)$ denote the "average price" of insurance after categorization if there were no government intervention. Assume further, without loss of generality, that $\Psi_1 > \overline{\Psi} > \Psi_2$. If, after categorization, the government subsidizes the sale of insurance to high risk types by the amount $\Psi_1 - \overline{\Psi}$ and taxes the sale of insurance to low risk types by amount $\overline{\Psi} - \Psi_2$ then firms will charge all individuals the price $\overline{\Psi}$. Once again, premium risk is avoided, but not at the cost of losing the production efficiency associated with categorization schemes.

In both of the above policies it was implicitly assumed that individuals purchase the same quantity of insurance after categorization as they do before categorization.[17] Since consumers also know the risk class to which they belong (after the information is revealed), it seems unlikely that both risk types will purchase the same level of insurance when offered the same price. The government could, of course, introduce a type of compulsory insurance regulation whereby consumers are required to purchase the same level of coverage after categorization as they did before categorization. Although this type of regulation is attractive for a model such as that presented in Section III, the informational requirements for the successful implementation of such a policy are very demanding for a world in which consumers have different tastes (i.e., varying degrees of risk aversion) and, therefore, purchase different levels of insurance.

In the remainder of this paper different models in which firms have some concern for the level of risk in their portfolios are investigated. In each case it is shown that categorization leads to a reduction in the average price of insurance. Therefore, Propositions 2 and 3 continue to hold in each case.

V. Risk Averse Firms: A Partial Information Model

For the model presented in Section III, agents (i.e., firms and consumers) were assumed not to be aware, before categorization, of the existence of the underlying risk structure associated with the losses being insured. In particular, it was assumed that firms (and consumers) treated all individuals identically and (mistakenly) assumed that losses were normally distributed with mean μ_o and variance σ_o^2 (i.e., before categorization). Although this mean and variance would in fact be substantiated by pooled claims experience, the pooled distribution would not be a normal one. It is, instead, a distribution of normal distributions (e.g., see Figure I below where the pooled distribution is bimodal).

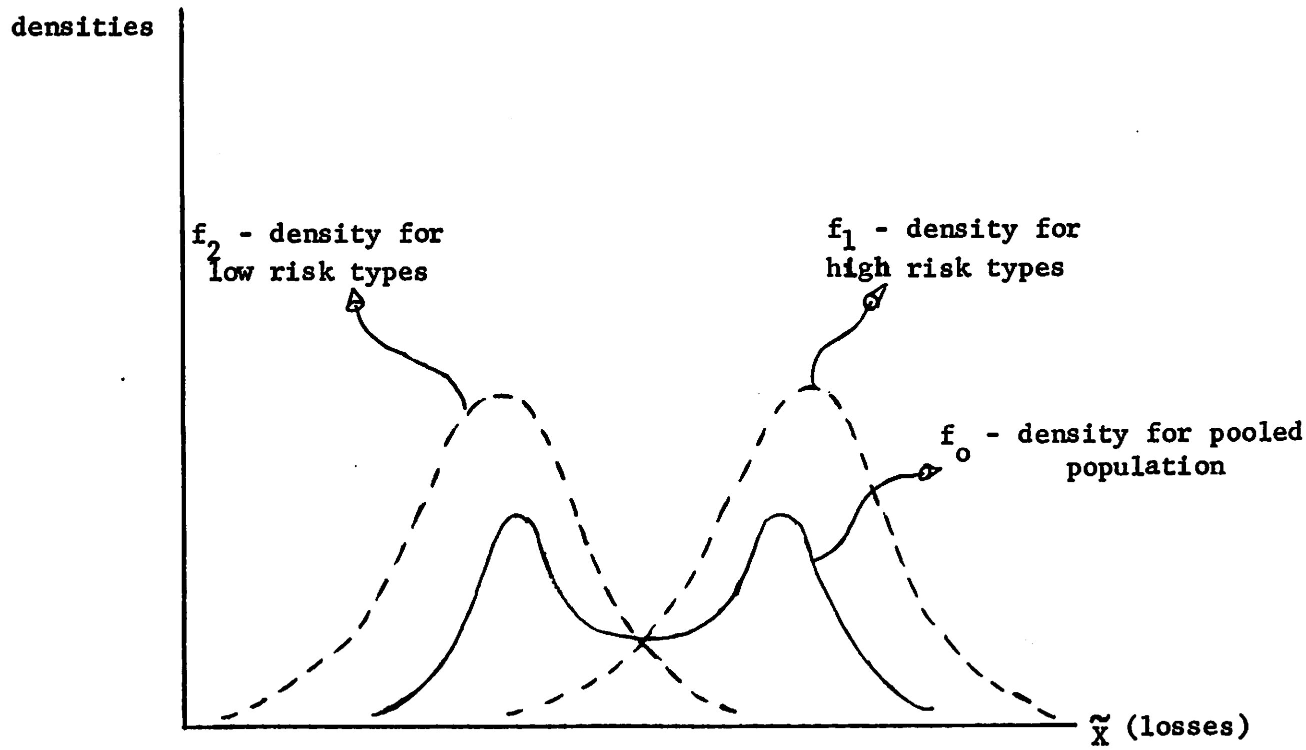

Figure I

In this section a "partial information model" is developed. It is assumed (in opposition to Section III) that firms and consumers are aware of the existence of the two risk classes and know the relevant parameters associated with these risk classes even before categorization. That is, it is known that the population is composed of a proportion q_1 of high risk types, who face the distribution of losses $\tilde{X}_1 \sim N(\mu_1,\sigma_1^2)$, and a proportion q_2 of low risk types, who face the distribution of losses $\tilde{X}_2 \sim N(\mu_2,\sigma_2^2)$. However, information which enables individuals (both firms and consumers) to identify the risk class to which they belong is not initially available. Since, before categorization, the existence of risk classes and associated parameters are known but the actual assignment of individuals to risk classes is not, this model is called a "partial equilibrium model".

Therefore, before categorization each consumer's subjective probability distribution of losses ($\tilde{X}_o$) is a compound distribution which can be represented by the density function $f_o(x) = q_2 f_2(x) + q_1 f_1(x)$, where q_2 is the probability of being a low risk type, f_2 is the probability density for low risk types, q_1 is the probability of being a high risk type, and f_1 is the probability density for high risk types. Firms treat the selection of risks (clients) in an identical fashion. After information concerning risk class membership becomes available, both firms and consumers can readily identify the risk class to which each individual belongs. Therefore, the situation after categorization is the same as that for the model in Section III.

In order to facilitate the analysis of this model the expected utility of a consumer, when no insurance market exists, is derived. The consumer's wealth, $\tilde{W}$, is distributed according to the following distribution.

$$\tilde{W} \sim N(\bar{W} - \mu_o, \sigma_2^2) \text{ with probability } q_2$$
$$\sim N(\bar{W} - \mu_1, \sigma_1^2) \text{ with probability } q_1 .$$

The density function, then, is

$$f_o(w) = q_2 f_2(w) + q_1 f_1(w), \text{ where}$$
$$f_2(w) = \frac{1}{\sigma_2\sqrt{2\pi}} e^{-(w - (\bar{W}-\mu_2))^2/2\sigma_2^2}$$
$$f_1(w) = \frac{1}{\sigma_1\sqrt{2\pi}} e^{-(w - (\bar{W}-\mu_1))^2/2\sigma_1^2}$$

If $f_o(w)$ is thought of as the probability that an individual receives wealth w then this probability is equal to q_2, the probability of being a low risk type, multiplied by $f_2(w)$, the probability that a low risk type will receive wealth w, plus q_1, the probability of being a high risk type, multiplied by $f_1(w)$, the probability that a high risk type will receive wealth w.

Given that $U(w) = -e^{-\beta w}$ is the consumer's elementary utility function, the following represents his expected utility.

$$V_c^o = E(-e^{-\beta w}) = \int_W -e^{-\beta w}(q_2 f_2(w) + q_1 f_1(w))dw \ .$$

Using the above expressions for f_2 and f_1 and the results from the derivation of the moment generating function for the normal distribution (e.g., see Taylor, 1974, pp. 52-54 and 82-84) we get

$$\begin{aligned} V_c^o &= q_2 \int -e^{-\beta w} f_2(w)dw + q_1 \int -e^{-\beta w} f_1(w)dw \\ &= q_2 \ (-e^{-\beta(\bar{W}-\mu_2)+\sigma_2^2\beta^2/2}) \\ &+ q_1 \ (-e^{-\beta(\bar{W}-\mu_1)+\sigma_1^2\beta^2/2}) \end{aligned} \qquad (5.1)$$

where V_c^o in this case represents utility for an individual who purchases no insurance.

Now, consider the case of a firm who sells insurance at price $\Psi_o(r)$ for coverage level r (recall that r is the coinsurance rate). If the firm sells the contract to a low risk type, his returns will be distributed according to $N(\Psi_o - r\mu_2, r^2\sigma_2^2)$--represented by density g_2--while if he sells it to a high risk type, his returns will be distributed according to $N(\Psi_o - r\mu_1, r^2\sigma_1^2)$--represented by density g_1. Since the firm cannot determine the risk class membership of its clients, it is assumed that returns are selected randomly from the population. That is, firms are faced with density g_2 with probability q_2, and density g_1 with probability q_1. Therefore, $g_o(w) = q_1 g_1(w) + q_2 g_2(w)$ represents the probability density function for the firms' returns resulting from the sale of an insurance contract (before categorization). The elementary utility function for firms is $-e^{-\gamma w}$ so that expected utility is $V_f = E(-e^{-\gamma w}) = \int_W -e^{-\gamma w} g_o(w)dw = \int_W -e^{-\gamma w}(q_2 g_2(w) + q_1 g_1(w))dw$. Therefore,

$$V_f = q_2(-e^{-\gamma(\Psi_o - r\mu_2)+\gamma^2 r^2 \sigma_2^2/2}) + q_1(-e^{-\gamma(\Psi_o - r\mu_1)+\gamma^2 r^2 \sigma_1^2/2}) = \bar{V} \qquad (5.2)$$

represents the no entry-exit condition which is used to define the equilibrium price schedule $\Psi_o(r)$. It can be shown, as for the model in Section III, that firm size (i.e., the number of contracts sold per firm) is indeterminate.

After categorization, the equilibrium price schedules for high and low risk types are, as in Section III,

$$\Psi_1(r) = r\mu_1 + \frac{\gamma}{2} r^2 \sigma_1^2$$

$$\Psi_2(r) = r\mu_2 + \frac{\gamma}{2} r^2 \sigma_2^2$$

These price schedules give rise to expected utility $\bar{V}$ for firms; that is, they satisfy the no entry-exit condition for the situation after categorization (see equation (3.3)).

It is shown in Theorem 2 (below) that, in order for $\Psi_o(r)$ to satisfy the no entry-exit condition for the partial information model before categorization (see equation (5.2)), it must be the case that $\Psi_o(r) > q_1\Psi_1(r) + q_2\Psi_2(r)$. Given this result it follows that Proposition 2 (see Section III) also holds for the "partial information model" of this section.

<u>Theorem 2</u>. If, for the "partial information model", $\Psi_o(r) = q_1\Psi_1(r) + q_2\Psi_2(r)$ then $V_f < \bar{V}$. That is, this price is not sufficiently large for the no entry-exit condition to be satisfied. Therefore, in equilibrium, $\Psi_o(r) > q_1\Psi_1(r) + q_2\Psi_2(r)$.

<u>Proof</u>:

Suppose $\Psi_o = q_2\Psi_2 + q_1\Psi_1$

$$= r\mu_o + q_2 \frac{\gamma}{2} r^2\sigma_2^2 + q_1 \frac{\gamma}{2} r^2 \sigma_1^2$$

Now, $V_f = -q_2 e^{-\gamma(\Psi_o - r\mu_2)+\gamma^2 r^2 \sigma_2^2/2} - q_1 e^{-\gamma(\Psi_o - r\mu_1)+\gamma^2 r^2 \sigma_1^2/2}$

Upon substitution for Ψ_o above, it follows that

$$V_f = -q_2 e^{-\gamma u} - q_1 e^{-\gamma v}$$

where $u = r(\mu_o - \mu_2) + q_2 \frac{\gamma}{2} r^2 \sigma_2^2 + q_1 \frac{\gamma}{2} r^2 \sigma_1^2 - \frac{\gamma}{2} r^2 \sigma_2^2$

$v = r(\mu_o - \mu_1) + q_2 \frac{\gamma}{2} r^2 \sigma_2^2 + q_1 \frac{\gamma}{2} r^2 \sigma_1^2 - \frac{\gamma}{2} r^2 \sigma_1^2$

Now, let $F(x) = -e^{-\gamma x}$

Then $\bar{V} = F(0)$

and $V_f = \lambda F(u) + (1-\lambda)F(v)$, where $\lambda = q_2$.

It is easy to show that $\lambda u + (1-\lambda)v = 0$.

$\therefore \quad \bar{V} = F(0) = F(\lambda u + (1-\lambda)v) > \lambda F(u) + (1-\lambda)F(v) = V_f$,

since $F(\cdot)$ is strictly concave.

q.e.d.

Therefore, if the no entry-exit condition is to be satisfied, before categorization, it must be the case that the price $\Psi_o(r)$ is greater than the average price of insurance after categorization $(q_1\Psi_1(r) + q_2\Psi_2(r))$. From this result it follows that Proposition 2 also holds for the partial information model. That is, since $\Psi_o > q_1\Psi_1 + q_2\Psi_2$ (see the proof of Proposition 2 in the appendix), it is not possible for both Ψ_1 and Ψ_2 to be greater than Ψ_o. Therefore, individuals from at least one of the two risk classes will be faced with a lower price of insurance after categorization. Also, since the average price after categorization is less than the price before categorization, it may be possible to construct policies which guarantee that equal and lower prices will be charged after categorization. However, as was argued in Section IV, such policies may not be feasible in a "real world" context.

VI. The Problem of Bankruptcy

In this section it is assumed that firms are risk neutral but that the government imposes a cash reserve requirement on insurance firms to ensure that the probability of bankruptcy is less than or equal to some specified level α. The result of these assumptions is that firms will be concerned with the amount of risk associated with the contracts in their portfolio.

It is assumed that firms satisfy the government's reserve requirement by borrowing funds (in the form of bonds). In the event that accumulated claims exceed premiums collected the insurer must use whatever part of these reserves necessary to cover "excess" claims. If the amount of reserves is at least sufficient to cover any "excess" claims then the firm refinances by (a) obtaining funds from the owners to repay previous loans and (b) offering bonds in order to replenish reserves. However, if the firm's reserves are (more than) exhausted in paying claims then the firm is liquidated and not allowed to continue underwriting insurance. In this case the firm defaults on its loans and, it is assumed, any additional amount of claims is paid by the government. On the other hand, if premiums are in excess of losses, the owners receive the difference as dividends and replenish reserves in the usual fashion (see Borch (1974, pp. 313-24) for a more complete discussion on the saving of a firm from ruin).

To justify the government's reserve requirement is not one of the purposes of this paper, although some discussion is necessary. One might argue that firms should be allowed to choose their financial policies without the interference of government and that consumers may then choose to purchase their contract from whichever firm they so desire. However, this argument seems to require the very demanding assumption that consumers can costlessly (or nearly so) assess probabilities of ruin by studying the financial statements of insurance firms. The alternative argument (adopted in this paper) for government regulation in the form of the imposition of reserve requirements is neatly summarized by Borch[18] (p. 345, 1974):

'the public complains only too often, about difficulties in understanding and interpreting the fine print in the insurance contract. If in addition the public should be asked to read the company's balance sheet and evaluate the company's ability to fulfill the promises made in big print, the public may well revolt and ask for government protection...(as a result)...in most countries the government has stepped in to protect the insurance-buying public. Often the government supervision has been established at the explicit request of the insurance companies, simply because they found it difficult to do business without some official stamp of approval.'

Now, suppose a firm obtains reserve funds R by selling bonds which offer a gross return $(1+q)R$ when the firm remains solvent but nothing if the firm becomes bankrupt. That is,

solvent (probability = $1-\alpha$), return = $(1+q)R$

not solvent (probability = α) , return = 0

Reserves are held in cash (at zero interest) and are made available to cover losses which are in excess of premiums. The expected return for the lenders of these funds is $(1+q)R\ (1-\alpha) + 0 \cdot \alpha = (1-\alpha)(1+q)R$. Suppose lenders are risk neutral and that the general equilibrium, riskless, gross return for an investment of size R is $(1+p)R$. Then competition among lenders requires that

$$(1-\alpha)(1+q)R = (1+p)R \tag{6.1}$$

$$\text{i.e.,}\ (1+q) = \frac{(1+p)}{(1-\alpha)}$$

Equation (6.1) represents the cost for maintaining reserves R. We can now go on to consider the remaining variables which are relevant to the firm's decisions as well as the size of R.

Suppose a firm insures risks $\tilde{X}_i$, $i=1,2,\ldots,N$ which are i.i.d. with mean μ and variance σ^2. Let r be the coverage level for these policies. Assume that all consumers have identical utility functions so that r is unique. Let P(r) be the equilibrium market price for coverage level r and assume that perfect competition prevails (i.e., firms are price takers). We will later use the assumption that in equilibrium expected profits must be zero. Let C(N) be direct underwriting costs (including a return to equity holders). Note that C does not depend on r but does depend on N, the number of contracts sold. Later we will use the assumption that $C' > 0 \ \forall N$ and $C'' > 0$ for $N \geq N_o$ where $N_o > 0$. Let TR denote total revenue and $\tilde{TC}$ denote total cost (which is random). For convenience, reserves (R) are included in the revenue function as well as in the cost function. This is because the loan is repaid in amount (1+q)R only if the firm remains solvent. Therefore,

$$TR = NP(r) + R \tag{6.2}$$

if solvent (probability = $1-\alpha$)

$$\tilde{TC} = r \sum_{i=1}^{N} \tilde{X}_i + C(N) + (1+q)R \tag{6.3}$$

not solvent (probability = α)

$$\tilde{TC}' = r \sum_{i=1}^{N} \tilde{X}_i + C(N) \tag{6.4}$$

Letting E denote the expectations operator we get

$$E(\tilde{TC}) = rN\mu + C(N) + (1-\alpha)(1+q)R$$

which, using equation (6.1) gives

$$E(\tilde{TC}) = rN\mu + C(N) + (1+\rho)R \tag{6.5}$$

Let $\tilde{\pi}$ denote profits. Then using equations (6.2) and (6.5) we get

$$E(\tilde{\pi}) = NP(r) - rN\mu - C(N) - \rho R \tag{6.6}$$

For P(r) to be an equilibrium price it must satisfy the condition $E(\tilde{\pi}) = 0$. From equation (6.6) this gives

$$P(r) = r\mu + \frac{C(N)}{N} + \frac{\rho R}{N} \tag{6.7}$$

The optimal firm size (N^*) will be discussed later. First we will consider the size of the reserve requirement R.

The firm has funds $Nr\mu + R$ available to cover the cost of claims (when solvent) with all other funds used to cover underwriting costs. The government wishes to ensure that R is large enough so that

$$Pr[r \sum_{i=1}^{N} \tilde{X}_i \geq Nr\mu + R] \leq \alpha$$

where Pr denotes probability. Therefore, the minimum reserve requirement satisfies

$$Pr[r \sum_{i=1}^{N} \tilde{X}_i \geq Nr\mu + R] = \alpha \tag{6.8}$$

It is assumed that the minimum reserve requirement is binding. That is, without government intervention firms would hold reserves less than R. Suppose N is sufficiently large to justify using the central limit theorem with $\frac{\sum \tilde{X}_i - N\mu}{\sqrt{N}\,\sigma} \sim N(0,1)$. Then

$$Pr\left[\frac{\sum_{i=1}^{N} \tilde{X}_i - N\mu}{\sqrt{N}\,\sigma} \geq Z_\alpha\right] = \alpha$$

where Z_α is the appropriate critical point for the standardized normal. Rewriting this expression we get

$$Pr\left[\frac{r(\sum_{i=1}^{N} \tilde{X}_i - N\mu)}{r\sqrt{N}\,\sigma} \geq Z_\alpha\right] = \alpha$$

or

$$Pr[r \sum_{i=1}^{N} \tilde{X}_i \geq Nr\mu + Z_\alpha r \sqrt{N}\,\sigma] = \alpha \tag{6.9}$$

Therefore, $R = Z_\alpha r \sqrt{N}\,\sigma$ is the minimum (binding) reserve requirement. Upon substitution for R into equation (6.7) we get

$$P(r) = r\mu + \frac{C(N)}{N} + \frac{\rho Z_\alpha r\sigma}{\sqrt{N}} \tag{6.10}$$

As is seen above the equilibrium price is a function of N (firm size). Since it is assumed that firms maximize expected profits, the appropriate value for N is N*, the optimal firm size. To find N* we must investigate the first order conditions for firms maximizing expected profits. The firm can attempt to vary r to alter expected profits but the equilibrium price P(r) is such that firms earn zero expected profits at each possible value of r. Furthermore, since consumers' utility functions are identical, only one coverage level r is relevant to the firm's decision.[19] As a result firm size N is the only relevant choice variable. Therefore, firms maximize expected profits by choice of N, taking r as given. This gives rise to the following first order condition (see equation (6.6) for $E(\tilde{\pi})$)

$$\frac{dE(\tilde{\pi})}{dN} = P(r) - r\mu - C'(N) - \rho\frac{dR}{dN} = 0$$

where $\frac{dR}{dN} = \frac{1}{2} Z_\alpha r\sigma N^{-\frac{1}{2}}$ which gives

$$P(r) - r\mu - C'(N^*) - \frac{1}{2}\rho Z_\alpha r\sigma N^{*^{-\frac{1}{2}}} = 0.$$

Upon substitution from equation (6.7), with N = N*, we get

$$r\mu + \frac{C(N^*)}{N^*} + \rho Z_\alpha r\sigma N^{*^{-\frac{1}{2}}} = r\mu + C'(N^*) + \frac{1}{2}\rho Z_\alpha r\sigma N^{*^{-\frac{1}{2}}} \tag{6.11}$$

Equation (6.11) is the usual (long run) condition for expected profit maximizing firms; that is, expected average cost equals expected marginal cost equals price. It is easy to see that the optimal value for N occurs where average cost attains a minimum. The second order condition requires that $C''(N^*) > \frac{1}{4}\rho Z_\alpha \sigma N^{*^{-3/2}}$. This condition must hold if there is to be a finite optimal firm size.[20]

Using equation (6.10) we can write the price of insurance coverage r as

$$P_o(r) = r\mu_o + \frac{C(N^*)}{N^*} + \frac{\rho Z_\alpha r\sigma_o}{\sqrt{N^*}} \tag{6.12}$$

where N* refers to optimal firm size. Before categorization, $P_o(r)$ is the price faced by all insureds, with aggregate mean claim μ_o and pooled variance σ_Q^2 (this notation is consistent with section III).

After categorization, firms can identify the risk class to which individuals belong. Let μ_1 be the mean loss for individuals belonging to the high risk class and μ_2 for the low risk class ($\mu_1 > \mu_2 > 0$). Let σ_1^2 represent the variance of the loss distribution for members of the high risk class and σ_2^2 the variance of the loss distribution for the low risk class. No restrictions on the relative sizes of σ_1^2 and σ_2^2 need to be assumed (i.e., $\sigma_1^2 \gtreqless \sigma_2^2$). This notation is consistent with section III.

If firms specialize so that one group of firms sells insurance to one risk class while the remaining firms sell insurance to the other risk class, then we can use the analysis already developed in this section to derive the following price schedules:

$$P_1(r_1) = r_1\mu_1 + \frac{C(N_1^*)}{N_1^*} + \frac{\rho Z_\alpha r_1 \sigma_1}{\sqrt{N_1^*}} \tag{6.13}$$

$$P_2(r_2) = r_2\mu_2 + \frac{C(N_2^*)}{N_2^*} + \frac{\rho Z_\alpha r_2 \sigma_2}{\sqrt{N_2^*}} \tag{6.14}$$

P_1 is the price charged to high risk types for coverage level r_1 while P_2 is the price charged to low risk types for coverage level r_2. N_j^* is the optimal firm size for companies who sell insurance to risk type j. It is shown in the appendix that firms will specialize rather than sell to both risk types. Therefore, equations (6.13) and (6.14) do in fact represent the price schedules faced by high and low risk types, respectively, after categorization. The following example should help to illustrate the above analysis.

Suppose a firm is faced with an administrative cost function $C(N) = aN^{3/2}$ where N represents the number of contracts sold. Using this cost function we get the following first order condition for expected profit maximizing firms (see equation (6.6))

$$\frac{dE(\tilde{\pi})}{dN} = P(r) - r\mu - \frac{3}{2} aN^{*\frac{1}{2}} - \frac{1}{2} \rho Z_\alpha r\sigma N^{*-\frac{1}{2}} = 0$$

Upon substitution for the equilibrium price (see equation (6.12) we get the optimal number of contracts to be

$$N^* = \frac{1}{a} \rho Z_\alpha r\sigma \tag{6.15}$$

The equilibrium price must be evaluated at optimal firm size since the condition of no entry or exit is assumed to hold for profit maximizing firms. Therefore,

$$P(r) = r\mu + 2(a\rho Z_\alpha r\sigma)^{1/2} \tag{6.16}$$

It is easy to show that the second order condition is satisfied (i.e., $\frac{d^2E(\pi)}{dN} < 0$). For this example the pre- and post-categorization price schedules are

$$P_o(r) = r\mu_o + 2(a\rho Z_\alpha r\sigma_o)^{1/2} \tag{6.17}$$

$$P_1(r_1) = r_1\mu_1 + 2(a\rho Z_\alpha r_1\sigma_1)^{1/2} \tag{6.18}$$

$$P_2(r_2) = r_2\mu_2 + 2(a\rho Z_\alpha r_2\sigma_2)^{1/2} \tag{6.19}$$

Note that after categorization the optimal firm size, N_j^*, will be different from that before categorization. This may lead to a change in the number of firms in the industry.

Although firms are risk neutral in this model, they are concerned with the level of risk in their portfolios because of the solvency constraint which is imposed on them. One might expect, therefore, that the use of information which leads to the categorization of risks will reduce structure risk and provide firms with a production efficiency (i.e., reduce the costs of maintaining the solvency constraint). This result is shown to be the case in Theorem 3 below which presents a conclusion similar to the one in Proposition 2; that is, the average price of insurance is lower after categorization than before categorization.

Theorem 3. The average price of insurance after categorization is lower than before categorization for the "bankruptcy model". A corollary of this result is that the price must fall after categorization for at least one of the two risk classes.

Proof: The proof is by contradiction. Define $P_j(r,N^*)$ as the equilibrium price of coverage r for group j (after categorization) if firms operate at size N^*. Since N_j^* is the optimal size for firms providing coverage of risk type j then $P_j(r,N^*) \geq P_j(r)$. Now, assume that the statement of the proposition is false and that

$$P_1(r) > P_o(r)$$

and $$P_2(r) > P_o(r)$$

Then, a fortiori,

$$P_1(r,N^*) > P_o(r)$$

$$P_2(r,N^*) > P_o(r)$$

therefore, $$q_1P_1(r,N^*) > q_1P_o(r)$$

$$q_2P_2(r,N^*) > q_2P_o(r)$$

Upon adding both sides we get

$$q_1P_1(r,N^*) + q_2P_2(r,N^*) > P_o(r) \quad \text{since } q_1 + q_2 = 1$$

Upon substitution for $P_j(r,N^*) = r\mu_j + \frac{C(N^*)}{N^*} + \frac{\rho Z_\alpha r\sigma_j}{\sqrt{N^*}}$ and $P_o(r)$ and using the fact that $\mu_o = q_1\mu_1 + q_2\mu_2$ we get

$$q_1\sigma_1 + q_2\sigma_2 > \sigma_o$$

or $$\sigma_o^2 < (q_1\sigma_1 + q_2\sigma_2)^2$$

But, from proposition 1 we know that $\sigma_o^2 > q_1\sigma_1^2 + q_2\sigma_2^2$ and since $(q_1\sigma_1 + q_2\sigma_2)^2 \le q_1\sigma_1^2 + q_2\sigma_2^2$ (substitute $q_2 = 1-q_1$) we have a contradiction.

q.e.d.

In closing, a few comments concerning the model presented in this section are made.

The average cost of the reserve requirement decreases as the number of contracts sold increases[21] (recall, $R = z_\alpha r\sigma \bar{N}$). Government regulation sometimes takes account of this fact in one fashion or another. For example, Britain's Companies Act of 1967 required insurance companies to hold minimum free reserves according to the following schedule (Revell, p. 411):

General Premium Income	Free Assets
Up to £250,000	£50,000
£250,000 up to £2.5 million	20% of general premium income
Over £2.5 million	20% on first £2.5 million of general premium income and 10% of excess.

One consequence of such regulatory action is to provide an incentive for firms to become large. In the model presented in this section, the optimal size of firms is finite because it is assumed that the portion of costs associated with the administration (or management) of policies is increasing. It may be this factor which is responsible for the following observation made by Borch (1974, p. 267):

> 'The foundation of insurance is the law of large numbers. It turns out, however, that the number of insurance contracts in the portfolio of a company is not usually "large enough", i.e., one cannot apply the law and ignore deviations from expected values.'

VII. Conclusion

Two "pricing principles" often encountered in the insurance literature are the variance principle (with price equal to the expected claim plus a constant multiplied by the variance of the claim cost) and the standard deviation principle (with price equal to the expected claim plus a constant multiplied by the standard deviation of the claim cost).[22] The variance principle is consistent with the model presented in Section III while the standard deviation principle is consistent with the "bankruptcy model" of Section VI. However, the constant which is multiplied by the standard deviation (in the latter case) is, for the model in Section VI, dependent on firm size, which in turn depends on the particular cost function and other parameters of the model. Therefore, the models discussed in this paper can be related to some of the pricing models used in the insurance literature.[23]

Furthermore, the analysis of pricing behavior when firms (and consumers) encounter information relating to risk class membership is similar when using either the approach adopted in this paper or the application of the pricing principles (mentioned above) in conjunction with credibility theory. Bühlman (1970, p. 93) defines the credibility premium as 'the approximation of the risk premium by a function dependent upon the actual claims.' For the model presented in Section III the premium charged is $\Psi = \mu + \frac{\gamma}{2}\sigma^2$, where μ is the expected claim and σ^2 is the variance of the claim cost. Before categorization, $\mu = r\mu_o$ and $\sigma^2 = r^2\sigma_o^2$ while after categorization $\mu = r\mu_i$ and $\sigma^2 = r^2\sigma_i^2$ (i=1,2) for risk class i. Bühlman (1970, pp. 93-100) shows that this would be the pricing behavior resulting from the application of the variance principle and the use of actual claims experience (via credibility rating). The price is based on $\mu = r\mu_o$, $\sigma^2 = r^2\sigma_o^2$ when no claims experience is available, and

$\mu = r\mu_i$, $\sigma^2 = r^2\sigma_i^2$ in the limit as claims experience approaches infinity. These results intuitively correspond to the "no information" case before categorization and the "full information" case after categorization (respectively) for the model presented in Section III of this paper. Therefore, although the main purpose of this paper is to analyze the welfare implications of categorization rather than to investigate the use of various "pricing principles" employed in the insurance industry, it is interesting to note the similarities. The use of credibility theory in the ratemaking literature leads to pricing behavior which is very similar to that which results from the analysis in this paper.[24] Therefore, the welfare implications of using credibility formulae are likely to be very similar to the welfare implications of categorization that are derived in this paper.

In each of the models presented in this paper (i.e., Sections III, V, and VI) it is assumed that firms are concerned about the level of risk associated with the provision of insurance. This is because firms are either risk averse or face a solvency constraint. In each case, information which leads to the categorization of risks also leads to a "production efficiency" by eliminating structure risk, a phenomenon which is associated with the selection of different risks from a heterogeneous population. Therefore, one of the results of categorizing risks is a reduction in the average price of insurance. It may even lead to a reduction in the price of insurance for members of both the high and low risk classes.[25]

However, the use of categorization schemes also introduces "premium risk" in the sense that the members of one of the two risk classes may experience an increase in the price of insurance after categorization. It was shown in Section IV that if the ex ante utility of consumers is considered <u>before</u> they know the risk class to which they will be assigned, it is possible that the use of a categorization scheme may make everyone worse off. Although policies which avert the "undesirable" effects of premium risk but do not exclude the "benefits" of eliminating structure risk (associated with categorization) are considered, the "informational requirements" of such policies are likely to be too great if they are to be implemented in any "realistic" situations.

Appendix

In this appendix it is shown that firms in section VI will not sell contracts to both high and low risk types. That is, after categorization, firms will specialize by selling to one risk type only.

Suppose firms are faced with market price schedules $P_1(r_1)$ and $P_2(r_2)$ after categorization. If firms specialize by selling to only one category then expressions for expected profit are analogous to equation (6.12). Equation (A.1) represents expected profit for firms that specialize by selling to high risk types while (A.2) represents expected profit for firms selling only to low risk types.

$$E(\tilde{\pi}_1) = N_1 P_1(r_1) - N_1 r_1 \mu_1 - C(N_1) - pZ_\alpha r_1 \sigma_1 N_1^{1/2} \tag{A.1}$$

$$E(\tilde{\pi}_2) = N_2 P_2(r_2) - N_2 r_2 \mu_2 - C(N_2) - pZ_\alpha r_2 \sigma_2 N_2^{1/2} \tag{A.2}$$

Firms choose N_i to maximize profits and $P_i(r_i)$ is an equilibrium price schedule which yields zero expected profits for firms operating at optimal size.

Suppose firms do not specialize. Instead they sell n_1 contracts to high risk types and n_2 contracts to low risk types (with $N = n_1 + n_2$). By following the same analysis as in section VI we get the following expressions for total revenue and expected total cost:

$$TR = n_1 P_1(r_1) + n_2 P_2(r_2) + R \tag{A.3}$$

$$E(TC) = n_1 r_1 \mu_1 + n_2 r_2 \mu_2 + C(N) + (1+p)R \tag{A.4}$$

The minimum reserve requirement R must be found for nonspecializing firms. Let $\tilde{X}_1^i$, $i=1,2,\ldots,n_1$ represent losses for the set of high risk contracts and $\tilde{X}_2^j$, $j=1,2,\ldots,n_2$ the set of low risk contracts. Then R must be large enough to ensure that

$$\Pr[r_1 \sum_{i=1}^{n_1} \tilde{X}_1^i + r_2 \sum_{j=1}^{n_2} \tilde{X}_2^j \geq n_1 r_1 \mu_1 + n_2 r_2 \mu_2 + R] = \alpha \qquad (A.5)$$

Applying the central limit theorem and the fact that $\mathrm{Var}[r_1 \Sigma \tilde{X}_1^i + r_2 \Sigma \tilde{X}_2^j] = n_1 r_1^2 \sigma_1^2 + n_2 r_2^2 \sigma_2^2$ we get

$$\Pr\left[\frac{r_1 \Sigma \tilde{X}_1^i + r_2 \Sigma \tilde{X}_2^j - n_1 r_1 \mu_1 - n_2 r_2 \mu_2}{(n_1 r_1^2 \sigma_1^2 + n_2 r_2^2 \sigma_2^2)^{1/2}} \geq Z_\alpha\right] = \alpha \qquad (A.6)$$

Upon comparison of (A.6) with (A.5) we get

$$R = Z_\alpha (n_1 r_1^2 \sigma_1^2 + n_2 r_2^2 \sigma_2^2)^{1/2} \qquad (A.7)$$

Now, by using equations (A.3), (A.4) and (A.7) we can derive expression (A.8) which represents expected profit for a nonspecializing firm.

$$E(\tilde{\pi}) = n_1 P_1(r_1) + n_2 P_2(r_2) - [r_1 n_1 \mu_1 + r_2 n_2 \mu_2 + C(N) + \rho Z_\alpha (n_1 r_1^2 \sigma_1^2 + n_2 r_2^2 \sigma_2^2)^{1/2}] \qquad (A.8)$$

Since nonspecializing firms sell contracts to both risk types we can write $n_1 = kN$ and $n_2 = (1-k)N$ with $0 < k < 1$. If firms do not specialize and $P_1(r_1)$ and $P_2(r_2)$ are equilibrium price schedules then these prices must satisfy $E(\tilde{\pi}) = 0$ in equation (A.8). Furthermore, it must be the case that $E(\tilde{\pi}_i) \leq 0$ in both equations (A.1) and (A.2)--otherwise firms could earn positive expected profits by specialization. Now, equations (A.1) and (A.2) give expected profits for firms selling N_1 and N_2, the optimal number of contracts. It follows, a fortiori, that if $E(\tilde{\pi}_i) \leq 0$ then the following also holds:

$$E(\tilde{\pi}_1, N) = NP_1(r_1) - Nr_1\mu_1 - C(N) - \rho Z_\alpha r_1 \sigma_1 N^{1/2} \leq 0$$

$$E(\tilde{\pi}_2, N) = NP_2(r_2) - Nr_2\mu_2 - C(N) - \rho Z_\alpha r_2 \sigma_2 N^{1/2} \leq 0$$

Therefore, if firms do not specialize it must be the case that

$$kE(\tilde{\pi}_1,N) + (1-k)E(\tilde{\pi}_2,N) \leq 0 = E(\tilde{\pi}) \qquad \text{(A.9)}$$

Upon substitution we get

$$kr_1\sigma_1 + (1-k)r_2\sigma_2 \geq (kr_1^2\sigma_1^2 + (1-k)\sigma_2^2\sigma_2^2)^{1/2}$$

After squaring both sides and collecting terms we get

$$k(k-1)(r_1\sigma_1 - r_2\sigma_2)^2 \geq 0 \qquad \text{(A.10)}$$

Consider first the case where $r_1\sigma_1 \neq r_2\sigma_2$. Then (A.10) cannot hold when $0 < k < 1$ and we have a contradiction. If $r_1\sigma_1 = r_2\sigma_2$ then the equality of (A.9) holds. However, $E(\tilde{\pi}_i,N)$ is evaluated at nonoptimal firm size so that firms may still make positive expected profits by specialization. If $E(\tilde{\pi}) = E(\tilde{\pi}_1) = E(\tilde{\pi}_2) = 0$ then firms will be indifferent to specialization but in this case we can still derive equilibrium prices from equations (A.1) and (A.2) so that this possibility does not change the analysis.

The results of this appendix can be summarized as follows. If there is a set of prices $P_1(r_1)$ and $P_2(r_2)$ which provide an equilibrium for nonspecializing firms (i.e., $E(\tilde{\pi}) = 0$) then either firms can earn positive profits through specialization or will be indifferent between the two strategies. In the former case competitive market forces will drive down at least one of the prices and nonspecializing firms will incur losses. In the latter case we can still derive equilibrium prices according to the firm strategy of specialization.

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Footnotes

[*]The author is indebted to Richard Arnott, Lucien Foldes, and Mike Peters for comments on an earlier (much altered) version of this paper. Any remaining errors are the responsibility of the author.

[1]Before information relating to risk class membership becomes available it is assumed that individuals (i.e., both insureds and firms) base their subjective beliefs concerning possible losses on the pooled claims experience of the population. After categorization, risk class specific claims experience is relevant to the members of each risk category.

[2]The inclusion of a per client administrative charge which is independent of the level of coverage chosen does not significantly alter the analysis.

[3]By departing from the usual Nash assumption that firms ignore the consequences that their actions will have on the behavior of other firms, Wilson (1976) derived an equilibrium concept which escapes the nonexistence difficulty which sometimes arises under the assumption of asymmetric information.

[4]For a general proof see Bühlman (1970, pp. 88-89). A similar theorem, for finite populations, is given in Cochran (1977, p. 99).

[5]The criterion of high risk types being faced with a greater expected loss than for low risk types ($\mu_1 > \mu_2$) is intuitive when the variances are equal ($\sigma_1^2 = \sigma_2^2$). Using the results of the following section it can be argued that this distinction is irrelevant when the variances are no longer assumed to be equal (i.e., especially when $\sigma_2^2 > \sigma_1^2$).

[6]It is assumed (later) that consumers also have identical risk averse utility functions. Therefore, r will be unique.

[7]Condition (3.3) is analogous to the zero profit condition for competitive firms under certainty. The expression for $V_f(\mu,\sigma^2)$ is derived from the moment generating function for the normal distribution.

[8]Note that by substituting equation (3.6) into equations (3.1) and (3.2) and the resulting expressions into equation (3.3) one can see that $V_f(\mu,\sigma^2) = \overline{V}$; that is, $\Psi_o(r)$ is indeed an equilibrium price schedule.

[9]Suppose a firm sells a proportion k of N contracts to high risk types and 1-k to low risk types so that $\mu = kN(\Psi_1(r)-r_1\mu_1) + (1-k)N(\Psi_2(r)-r_2\mu_2)$ and $\sigma^2 = kNr_1^2\sigma_1^2 + (1-k)Nr_2^2\sigma_2^2$. By using equations (3.10) and (3.11) we can see that the no entry-exit condition, equation (3.3), is satisfied for any $0 < k < 1$. If either of the price schedules Ψ_i is less or greater than that specified in equations (3.10) and (3.11), the no entry-exit condition will be violated.

[10]The following example illustrates such a possibility. Suppose $q_1 = q_2 = 0.5$, $\mu_1 = 100$, $\mu_2 = 90$, $\sigma_1^2 = 10$, $\sigma_2^2 = 100$, and $\gamma = 1$. Then $\mu_o = 95$ and $\sigma_o^2 = 80$. It follows that $\Psi_1(r) < \Psi_o(r)$ for $\beta > \frac{1}{6}$ and $\Psi_2(r) > \Psi_o(r)$ for $\beta > 1$.

[11]See the proof to Proposition 2 from which it follows that $q_1\Psi_1(r) + q_2\Psi_2(r) < \Psi_o(r)$.

[12]The result that the quantity of insurance purchased ($r = \frac{\beta}{\gamma+\beta}$) is unchanged by categorization is clearly essential to the correspondence between the price effects and ex post welfare implications of categorization.

[13]This result, of course, follows from the assumption of symmetric information.

[14]A long-term insurance contract which does not permit the insurance firm to alter the individual's premium may be a consequence of this possible phenomenon. It is implicitly assumed in this paper that, for whatever economic or institutional reasons, such long-term contracts are not viable alternatives.

[15]It is not possible, however, that members of both risk classes can be made worse off by categorization (see Proposition 2').

[16]One could consider this problem in a piecemeal fashion by analyzing the consequences of each particular "increase in information" which occurs. However, the irreversibility of information accumulation makes this possibility unattractive.

[17]In the first of these policies the result that $\overline{\Psi}(r) = q_1\Psi_1(r) + q_2\Psi_2(r)$ clearly requires that both high and low risk types purchase the same level of coverage, r, after categorization. In the second case the government's tax-subsidy scheme requires that r be constant if the scheme is to "break-even" (i.e., for $q_1(\Psi_1(r) - \overline{\Psi}(r)) = q_2(\overline{\Psi}(r) - \Psi_2(r))$.

[18]There may be other reasons for the government to implement solvency requirements. For instance, when default gives rise to a transfer of ownership there may be some deadweight costs imposed on the economy (e.g., legal costs, temporary shut-down costs, agency costs and so forth). For a discussion of these issues see Harris [1976].

[19]It is assumed that the result of consumers maximizing utility by choice of r and subject to P(r) gives rise to a unique choice for r and that $0 < r < 1$.

[20]The average cost of holding reserves falls with N (the number of contracts sold). Therefore, if C(N) were linear ($C'' = 0$), the average total cost would be downward sloping (everywhere).

[21]It is implicitly assumed in this paper that no use is made of reinsurance agreements, or other related methods, which allow the firm to reduce the probability of ruin.

[22]See Bühlman (1970, pp. 85-87) for a discussion of these pricing principles.

[23]It is interesting to note that the "partial information model" does not lead to a simple application of the variance principle.

[24]It appears that ratemaking procedures implicitly assume that symmetric information (rather than asymmetric information) prevails.

[25]Therefore, it is possible that categorization may lead to a Pareto type welfare improvement in terms of ex ante and ex post utilities (see Section IV).