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THE SEMILOGARITHMIC PORTFOLIO BALANCE  
SCHEDULE IS TENUOUS\*

by

Geoffrey H. Kingston

July 1980

Abstract

A well-known procedure in the context of the semilogarithmic portfolio balance schedule is to impose an arbitrary terminal condition that rules out the occurrence of runaway inflations in the absence of runaway growth in the money supply. Notwithstanding the fact that a formal justification for this procedure is sometimes available in the context of equations that emerge from optimum problems, this paper finds that such a justification is not available in the context of several leading optimizing models of money that either conceivably or actually deliver the Cagan schedule.

\*I wish to thank R. Boyer, W. Brock and A. Robson for helpful conversations on this topic. Responsibility for any errors or shortcomings is mine.

## I. INTRODUCTION

This paper shows that the monetary equilibrium characterized by the semilogarithmic portfolio balance schedule is tenuous. That is, if any one of several leading optimizing models of the demand for fiat money is used to obtain the semilogarithmic portfolio balance schedule, and its associated perfect foresight price paths (see Sargent and Wallace, 1973, and Sargent, 1979), from the "economically more satisfactory case when the demand for money is derived from utility maximization" (Calvo, 1978, p. 143), then the resulting "fundamentals-based" equilibrium does not dominate a continuum of anomalous equilibria characterized by exploding price levels and imploding real balances.

Thus this paper is on uniqueness. Its point can be made another way. A well-known procedure of, for example, Sargent and Wallace (1973) and Sargent (1979), in the context of the semilogarithmic portfolio balance schedule, is to impose "an arbitrary terminal condition that rules out the occurrence of runaway inflations in the absence of runaway growth in the money supply" (Sargent, 1979, p. 194). Notwithstanding the fact that "a formal justification for this procedure is sometimes available in the context of difference [or differential] equations that emerge from optimum problems" (Sargent, 1979, p. 177), such a justification is not available in the context of several leading optimizing models of money that either conceivably or actually deliver the Cagan schedule.

That schedule has long played an important role in the analysis of money and inflation, beginning with the well-known paper by Cagan (1956). Recently it has been prominent in the literature on rational expectations, and on the monetary approach to exchange-rate determination. Its attractive properties include tractability, a money demand function whose domain is the whole real line, and an associated revenue function such that government revenue from

money creation attains an interior maximum. Thus the microeconomic features of the schedule are worth investigating. That those foundations are shaky is disturbing.

Tenuousness can be defined more precisely as individual preferences such that if an explosive price path were to arise then the representative household would have no incentive to act in a way that, in aggregate, crushes the anomalous path (i.e., "pricks the bubble"), even if monetary growth were moderate. This idea goes back a long way. But the first demonstration relevant to this paper that tenuousness can afflict an otherwise well-behaved optimizing model of a monetary economy is Brock (1978), which refers to his (1974, 1975) model based on the notion of real balances in the utility function. With standard assumptions on tastes and technology, and for cases when explosive price paths cannot be ruled out by simple feasibility considerations, it turns out that the necessary condition for non-tenuousness (or "essentiality") is that if the unique fundamentals-based price path were to prevail, and if monetary growth were to go to infinity, then inflation tax proceeds would be bounded away from zero. See Brock (1978) and also Scheinkman (1980).

Tenuousness is also investigated by Wallace (1980) and by Brock and Scheinkman (1980), in the context of the overlapping generations model of money. A comment on the Wallace paper by Scheinkman (1980) is especially noteworthy because it shows that the above condition applies also to significant versions of the overlapping generations model and the Lucas (1980) transactions model. Scheinkman gives the following insight into the economics of the Brock condition. "The condition  $\lim_{\mu \rightarrow \infty} \frac{x_{\mu}}{\mu} > 0$  [ $\mu$  = growth rate of money, and  $x_{\mu}$  = real balances associated with the equilibrium which is stationary in real balances] also has the interpretation that no matter how expensive it becomes to hold money people still hold a large quantity of it; that is, money is very necessary to the system" (p. 96).

The condition in question enables the following "one-line" proof that the semilogarithmic portfolio balance schedule is tenuous: a simple version of the relevant money demand function is  $x_{\mu} \equiv e^{\alpha\mu}$ ,  $\alpha < 0$ , whence

$$\lim_{\mu \rightarrow \infty} \mu x_{\mu} = \lim_{\mu \rightarrow \infty} \mu e^{\alpha\mu} = 0. \quad (\text{So if the semilogarithmic portfolio balance schedule were the true model, then money would not be "very necessary to the system".})$$

This proof is not very informative, however, for two reasons. First, it does not demonstrate that the semilogarithmic schedule can be viewed as the aggregate outcome of optimization by a representative household. If, somehow, that schedule were to be rescued from tenuousness, then such a demonstration would be a necessary first step. Second, most of the interesting recent applications of the schedule, beginning with Sargent and Wallace (1973), explain the price level and the demand for money by a weighted average of all future (known or projected) money supplies, which might change over time, rather than by a single money growth rate that is known or expected to remain constant.

The remainder of this paper shows tenuousness in a setting that takes care of the two foregoing limitations. The setup is Brock (1974,1975) with more specific utility functions and with more general money supply processes. Thus agents are assumed to have perfect foresight, and real balances appear in their utility functions. Both discrete and continuous time are considered. Surprisingly, a different instantaneous utility function is needed for each case. For example, the relevant continuous-time instantaneous utility function satisfies an Inada-type condition at the origin, whereas its discrete-time counterpart does not. Nevertheless it turns out that tenuousness afflicts both cases.

Section 2 considers the discrete-time case. Section 3 deals with continuous time. Section 4 offers some concluding comments.

## 2. DISCRETE TIME

The semilogarithmic portfolio balance schedule, in the case of perfect foresight and discrete time, asserts that at any date  $t$ , the nominal money supply,  $M$ , and the general price level,  $p$ , will be related by

$$\ln(M_t/p_t) = \alpha \ln(p_{t+1}/p_t), \quad \alpha < 0 \quad (1)$$

where the money supply (exogenous but variable) always equals money demand by virtue of the assumption that markets clear. (For further discussion see, e.g., Sargent, 1979, Ch. IX, Section 6, eqns. (40) and (42).) This section first derives equation (1) from utility maximization. Thus we shall consider the representative household's problem, and invoke market-clearing assumptions. We then show that the equilibrium characterized by (1) is tenuous.

The representative household (cf. Brock, 1975) at date  $t$  seeks to maximize:

$$U_t \equiv \sum_{j=0}^{\infty} \beta^j [u(c_{t+j}) + a \frac{M_{t+j}}{P_{t+j}} - \frac{b}{\gamma} (\frac{M_{t+j}}{P_{t+j}})^{\gamma}] ; \quad a, b > 0; \quad 0 < \beta, 1/\gamma < 1 \quad (2)$$

subject to

$$P_{t+j} c_{t+j} + M_{t+j} - M_{t+j-1} = P_{t+j} y + \prod_{i=0}^j \sigma_{t+i} - \prod_{i=0}^{j-1} \sigma_{t+i} \quad j=0,1,2,\dots \quad (3)$$

$M_{t-1}$  given.

The new variables here are  $U$ , lifetime utility;  $c$ , real consumption;  $y$  real income, assumed constant; and  $(\prod_{i=0}^j \sigma_{t+i} - \prod_{i=0}^{j-1} \sigma_{t+i})$ , which describes nominal cash transfers from the government at date  $t+j$ , and will also be denoted by  $H_{t+j}$ .

The money supply is seen to be unity at date  $t-1$  (a value and date chosen merely for convenience), and is then driven by the exogenous but variable one-period growth factors  $\sigma_{t+i}$ . By contrast, Brock (1974, 1975) allows for at most

a once-over change. However, we do assume the following bounds on monetary growth; i.e., on  $\ln \sigma_{t+j}$ :

$$|\ln \sigma_{t+j}| \leq \ln \bar{\sigma}, \quad j=0,1,2,\dots; \quad 1 \leq \bar{\sigma} < \infty \quad (4)$$

These bounds are less general than those assumed by, for example, Sargent (1979) in related contexts (pp. 193 and 197 of that text), but are not restrictive.

The functional form describing instantaneous utility gained from consumption, i.e.,  $u(c)$ , will be left unspecified except for the standard restrictions  $u'(c) > 0$ ,  $u''(c) < 0$ ,  $u'(0) = +\infty$ . The form of the instantaneous utility gained from real balances is seen to be a modified quadratic specification. It obeys diminishing marginal utility, and embodies the fairly standard notion of a "satiety" level of real balances  $m$ . (See, e.g., Brock, 1974, 1975.) On the other hand it does not satisfy an Inada-type condition at the origin (even though its continuous-time counterpart does satisfy such a condition). Moreover, to get from the representative household's problem to an equation as simple as (1) we need more conditions on the parameters  $a$ ,  $b$ , and  $\gamma$ :

$$a = u'(y), \quad b = \beta a, \quad \gamma = 1 - 1/\alpha \quad (5)$$

These conditions are legitimate, albeit highly restrictive, since they amount merely to very specific assumptions on preferences (recall that  $y$  is fixed).

Given the assumptions (5), the first-order necessary conditions for an interior solution are

$$\frac{u'(c_{t+j})}{P_{t+j}} = \frac{u'(y)}{P_{t+j}} \left[ 1 - \beta \left( \frac{M_{t+j}}{P_{t+j}} \right)^{\gamma-1} \right] + \frac{\beta u'(c_{t+j+1})}{P_{t+j+1}} \quad j=0,1,2,\dots \quad (6)$$

Brock (1975, eqn. (3) et seq.) gives an economic interpretation of such conditions. Also following Brock, we assume market clearing, that is,

$$c_{t+j} = y \quad (7)$$

$$M_{t+j} = \prod_{i=0}^j \sigma_{t+i} \quad j=0,1,2,\dots \quad (8)$$

for the entire economy. Upon substituting (7) into (6), and simplifying, we get (1), as required. In the following discussion it should be clear from the context whether the variables in (7) and (8) are being referred to in an individual or in a market context.

Following, e.g., Sargent (1979, p. 193), (1) can be solved forward to get perfect foresight price paths  $\{p_t\}$  in terms of the forcing function (8):

$$\ln p_t^{(k)} = -\frac{1}{\alpha\gamma} \sum_{j=0}^{\infty} \gamma^{-j} \ln M_{t+j} + k\gamma^t \quad (9)$$

where  $\gamma \equiv 1 - 1/\alpha > 1$ , and  $k$  is a constant, possibly zero. The equality  $k=0$  defines the unique "fundamentals"-based price path, the relationship  $k \neq 0$  defines "bubbles", and the absence of incentives that crush the  $k > 0$  paths defines "tenuousness". Since Sargent and Wallace (1973), equation (9) or variants of it have played an important part in the analysis of inflation and exchange rates. Those analyses impose the "arbitrary" terminal condition  $k = 0$ .

One can manipulate (9) directly, or else get forward solutions to (1) expressed "in real balance form" (see Brock, 1974, 1975; see also the discussion preceding (13) below), to get sequences of nominal balances  $\{M_t\}$  in terms of the forcing function (8):

$$\ln M_t = \ln p_t^{(k)} - \sum_{j=0}^{\infty} \gamma^{-(j+1)} \ln \sigma_{t+j+1} - k\gamma^t \quad (10)$$

At the level of the representative household, by construction such a plan is feasible, and satisfies the household's budget constraint and the first-order necessary conditions. It therefore describes candidate equilibria.



The remainder of this section determines values of  $k$  for which the representative household has no incentives to choose an alternative feasible plan. In other words, the objects of interest are  $k$  values that correspond to equilibrium price levels and money demands.

Consider first the case  $k < 0$ , i.e., imploding prices and exploding real balances. To show that this case does not yield equilibria one can proceed along the lines of the estimation argument given by Brock (1974, 1975).

Suppose the representative household adopts an alternative plan such that at date  $t+T$  ( $T$  is specified below) an amount  $\mathcal{E}/p_{t+T}$  units of consumption is purchased ( $\mathcal{E}$  small), whereas at each subsequent date the original plan  $c_{t+j} = y$ ,  $j=0,1,2,\dots$ , is implemented. Thus from date  $t+T$  on, the representative household's nominal balances are  $\mathcal{E}$  dollars less than those entailed by the original plan.

The present value seen from date  $t$  of the consumption utils gained by virtue of adopting the alternative plan is just  $\beta^T u'(y) \mathcal{E}/p(k)_{t+T}$ . Now the instantaneous utility derived from real balances,  $v(m)$ , is  $u'(y)m(1-\beta m)^{\gamma-1/\gamma}$ . Hence the present value of the real-balances utils foregone by virtue of adopting the alternative plan, noting that  $v'(m) = u'(y)(1-\beta m)^{\gamma-1}$ , is

$$\beta^T u'(y) \mathcal{E} \cdot \sum_{j=0}^{\infty} \beta^j (1-\beta m_{t+T+j}^{\gamma-1}) / p(k)_{t+T+j}.$$

The value of this infinite sum depends, inter alia, on the sign of  $k$  and the value of  $T$ . With  $k < 0$ , (4) and (10) together imply that every element of the sequence  $\{m_{t+T+j}\}_{j=0}^{\infty}$  is not less than  $(\bar{\sigma})^{\alpha} \exp(|k|\gamma^{t+T})$ . It follows that a sufficiently large  $T$  ensures that every element of the sequence  $\{1-\beta m_{t+T+j}^{\gamma-1}\}_{j=0}^{\infty}$ , and thus the above infinite sum, will be non-positive. Hence the alternative plan has been shown to yield a net gain over the original plan, as required.

Consider now the case  $k \geq 0$ , which encompasses both the fundamentals-based price path ( $k=0$ ), and a continuum of anomalous explosive price paths ( $k > 0$ ) that entail imploding real balances. To show that this case yields equilibria we give an argument similar to that used by Brock (1974, 1975), although he confines attention, in effect, to the sub-case  $k = 0$ . Brock's argument is, in turn, similar to the concavity argument that is commonly used to get transversality conditions in optimal control problems.

All candidate price paths are generated, in aggregate, when the representative household's plan is given by the sequence  $\{c_{t+j} = y, M_{t+j} = \prod_{i=0}^j \sigma_{t+i}\}_{j=0}^{\infty}$ . Consider any alternative plan  $\{c_{t+j} = c'_{t+j}, M_{t+j} = M'_{t+j}\}$  that is feasible and consistent with the budget constraint when the ruling price sequence is

$\{p_{t+j}^{(k)}\}_{j=0}^{\infty} \Big|_{k \geq 0}$ . To simplify the notation, define  $\{\tilde{p}_{t+j}\}_{j=0}^{\infty} \equiv \{p_{t+j}^{(k)}\}_{j=0}^{\infty} \Big|_{k \geq 0}$ ,

$\{\tilde{M}_{t+j}\}_{j=0}^{\infty} \equiv \{\prod_{i=0}^j \sigma_{t+i}\}_{j=0}^{\infty}$ ,  $\{\tilde{m}_{t+j}\}_{j=0}^{\infty} \equiv \{M_{t+j}/\tilde{p}_{t+j}\}_{j=0}^{\infty}$ , and  $\{m'_{t+j}\}_{j=0}^{\infty} \equiv$

$\{M'_{t+j}/\tilde{p}_{t+j}\}_{j=0}^{\infty}$ .

We shall prove that if  $k \geq 0$ , then

$$\begin{aligned} \sum_{j=0}^T \beta^j \{u[\tilde{p}_{t+j} y + M'_{t+j-1} - M'_{t+j} + H_{t+j}]/\tilde{p}_{t+j}\} + v(m'_{t+j}) - u(y) - v(\tilde{m}_{t+j}) \\ \cong u'(y) \beta^{T+1} (\bar{\sigma}^{-\alpha} - M'_{t+T}/\tilde{p}_{t+T}) \bar{\sigma}. \end{aligned} \quad (11)$$

Notice that the right-hand side is bounded above by zero as  $T \rightarrow \infty$ .

The concavity of  $u(c)$  and  $v(m)$  implies that the left-hand side of (11) is less than or equal to

$$\sum_{j=0}^T \beta^j \{u'(y)(M'_{t+j-1} - M'_{t+j} + H_{t+j})/\tilde{p}_{t+j} + v'(\tilde{m}_{t+j})(m'_{t+j} - \tilde{m}_{t+j})\} \quad (12)$$

Use  $v'(m) = u'(y)(1 - \beta m^{\gamma-1})$ , and then use the fact that (1) may be expressed in what Brock 1974, 1975 terms "real balance form", i.e., as

$\tilde{m}_{t+j}^{\gamma} = \tilde{m}_{t+j+1} / \sigma_{t+j+1}$ ,  $j=0,1,2,\dots$ , to restate (12) as

$$u'(y) \sum_{j=0}^T \beta^j [(M'_{t+j-1} - \tilde{M}_{t+j-1}) / \tilde{p}_{t+j} - \beta (M'_{t+j} - \tilde{M}_{t+j}) / \tilde{p}_{t+j+1}] \quad (13)$$

which, given the initial condition  $\tilde{M}_{t-1} = M'_{t-1}$ , reduces to

$$u'(y) \beta^{T+1} (\tilde{m}_{t+T} - m'_{t+T}) (\tilde{p}_{t+T} / \tilde{p}_{t+T+1}) \quad (14)$$

Finally, the conditions (4) and equations (9) and (10) together imply that (14) is less than or equal to the right-hand side of (11) as required.

In summary, for any  $k \geq 0$ , the original plan is at least as good as the class of alternative plans under consideration, so that the representative household would have no incentive to change his original plan. In particular, there is no incentive that would, in aggregate, serve to crush an explosive price path ( $k > 0$ ), were such a path to arise.

### 3. CONTINUOUS TIME

The standard semilogarithmic portfolio balance schedule, in the case of perfect foresight and continuous time, is given by

$$\ln[M(t)/p(t)] = \alpha D \ln p(t), \quad \alpha < 0 \quad (15)$$

where  $D$  denotes the right-hand derivative operator with respect to time.

(See, e.g., Sargent, 1979, p. 35.) Paralleling Section 2, this section derives (15) from utility maximization, and proceeds to show that the equilibrium characterized by (15) is tenuous.

The representative household is assumed to solve

$$\max_t \int_t^{\infty} e^{-\beta(s-t)} \{u[c(s)] + \left[\frac{u'(y)}{-\alpha}\right] \cdot [1 - \ln\left(\frac{M(s)}{p(s)}\right) - \alpha\beta] \} ds \quad (16)$$

s. t.

$$DM(s) = p(s) \cdot y - p(s) \cdot c(s) + \theta(s) \cdot \exp\left(\int_t^s \theta(x) dx\right), \quad t \leq s \leq \infty, \quad (17)$$

$M(t)$  given.

The new variable here is  $\theta(s)$ , the continuous-time growth rate of money at date  $s$ .

Hence  $\theta(s) \exp\left(\int_t^s \theta(x) dx\right) \equiv H(s)$  say, is just the continuous-time counterpart of  $H_{t+j}$ .

It is assumed that  $p$  and  $\theta$  are continuous and right-differentiable on  $[t, \infty)$ .

Analogous to the conditions (4) it is assumed that

$$|\theta(s)| < \bar{\theta}, \quad t \leq s \leq \infty, \quad 0 < \bar{\theta} < \infty \quad (18)$$

The instantaneous utility gained from real balances is now given by

$v(m) = [-u'(y)/\alpha]m(1 - \ln m - \alpha\beta)$  so that  $v'(m) = u'(y)(\ln m + \alpha\beta)/\alpha$ , whence  $v'(0) = +\infty$  and  $v''(m) < 0$ .

To solve the representative household's problem, consider the Hamiltonian

$$\mathcal{H}\left(\frac{M}{p}, c, \lambda, s\right) \equiv e^{-\beta(s-t)} \left\{ u[c(s)] + v\left[\frac{M(s)}{p(s)}\right] + \lambda(s) [p(s)y - p(s)c(s) + H(s)] \right\}$$

where  $\lambda$  is the costate variable to  $M$ . The necessary conditions for an interior solution are

$$u'(c) = \lambda p \quad (19)$$

$$D\lambda = \beta\lambda - v'(M/p)p^{-1} \quad (20)$$

Eliminate  $\lambda$  from these equations to get

$$D[-p^{-1}u'(c)] = v'(M/p)p^{-1} + \beta[-p^{-1}u'(c)],$$

which is identical to eqn (11) of Brock, 1974, Section 4, notwithstanding his different specification of the household's optimizing problem. Upon using our particular functional form for  $v(M/p)$ , and the first of the two market-clearing conditions

$$c(s) = y \quad (21)$$

$$M(s) = \exp \left[ \int_t^s \theta(x) dx \right] \quad t \leq s \leq \infty \quad (22)$$

we get (15), as required.

Following Section 2, the next step is to solve (15) in the forward direction to get general (fundamentals cum bubbles) price paths. Again it suffices to reproduce the relevant result in Sargent (1979)--in the present case we need p. 37 of that text:

$$\ln p^{(k)}(t) = -\frac{1}{\alpha} \int_t^{\infty} e^{(s-t)/\alpha} \ln M(s) ds + ke^{-t/\alpha} \quad (23)$$

Paralleling (10), we get a closed-form solution for the log of nominal balances in terms of future money growth rates:

$$\ln M(t) = \ln p^{(k)}(t) - \int_t^{\infty} e^{(s-t)/\alpha} \theta(s) ds - ke^{-t/\alpha} \quad (24)$$

The case  $k < 0$  does not yield equilibria. This can be shown by an estimation argument entirely similar to that given in Section 2. Accordingly, the details are not reported here.

Turning to the case  $k \geq 0$ , define  $\tilde{p}(s) \equiv p^{(k)}(s) \Big|_{\substack{t \leq s \leq \infty \\ k \geq 0}}$ ,  $\tilde{M}(s) \equiv \exp \left[ \int_t^s \theta(x) dx \right] \Big|_{\substack{t \leq s \leq \infty \\ \theta \geq 0}}$ , and  $\tilde{\lambda}(s) \equiv [u'(y)/\tilde{p}(s)] \Big|_{t \leq s \leq \infty}$ . The price level is always nonnegative (recall (23)), so that

$$e^{-\beta T} \cdot \tilde{\lambda}(t+T) \geq 0, \quad T \geq t.$$

Moreover, (23) and (24) together imply

$$0 \leq e^{-\beta T} \cdot \tilde{\lambda}(t+T) \cdot \tilde{M}(t+T) \leq u'(y) e^{-(\beta T + \alpha \bar{\theta})}, \quad T \geq t.$$

Passing to the limit as  $T \rightarrow \infty$ , it follows that the  $k \geq 0$  case yields equilibria. (See, e.g., Proposition 8 of Arrow and Kurz, 1970, Ch. II.) A corollary is that

tenuousness holds also for the standard continuous-time semilogarithmic portfolio balance schedule.

#### 4. CONCLUDING REMARKS

Two aspects of the micro foundations of portfolio balance schedules warrant further investigation.

First, this paper by no means exhausts the many ways that one might go about deriving the Cagan schedule from an optimizing problem. Hence there might be such a problem that generates a non-tenuous Cagan schedule. However, it was observed in Section 1 that Scheinkman's (1980) result rules out several natural approaches.

Second, there might be some other portfolio balance schedule that satisfies the Brock-Scheinkman condition for non-tenuousness, yet shares the attractive properties of the Cagan schedule. It was noted in Section 1 that these attractions add up to a tall order. On the other hand, the Cagan schedule does fail the Brock-Scheinkman condition. "Since inconvertible fiat money seems only to appear in economic systems in which the division of labor had led to tremendous costs to pure barter, it may well be that assumptions such as [the condition in question] are not unnatural in a highly aggregated model." (Scheinkman, 1980, p. 96.) Hence the tenuousness of the Cagan schedule could be more serious than a mere technical difficulty that can be safely bypassed via a terminal condition of the Sargent-Wallace type.

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