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# The Security Prices and Default of a Firm under Anticipated Inflation

Muhammad Rashid

Baldev Raj

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Muhammad Rashid
and
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by

Muhammad Rashid
University of Western Ontario

and

Baldev Raj
University of Wilfrid Laurier
and
University of Western Ontario

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## UNDER ANTICIPATED INFLATION

## 1. Introduction

In the perfectly competitive taxless financial markets of classical economics, changes in the price level augment both the future cash flow of a financial security and its discount rate by comparable amounts, thus causing neutral effects on current values of financial securities. The introduction of taxation, however, causes non-neutral effects of inflation on prices of financial securities, as shown by Feldstein (1976) and Feldstein et al. (1978) among others. The purpose of this paper is to show that anticipated inflation (or deflation), in addition to non-neutral effects due to taxation, can cause non-neutral effects on the prices of corporate securities because the price of the product which the firm is selling may be changing at a different rate than the rate of change of the general price level. The rate of inflation is generally under-stood as the upward drift of the overall price index in the economy, therefore, it does not mean that the price of a given commodity will simultaneously be changing in the same direction and/or at the same rate. The price of a commodity is determined by the forces of its supply and demand and some of these forces are likely to be partially or totally un-affected by the rate of inflation (or deflation). Empirically, this fact can be easily confirmed by a cursory look at the time series of C.P.I. or GNP deflator and rates of changes of various product prices.

There is a general disagreement among economists about the effect of inflation on the value of the equity of a firm. The classic Fisher-Williams

conclusion is that in the absence of taxation and other imperfections, the value of the equity of a firm, like any other financial security, remains unaffected by inflation. Lintner (1974) has argued that inflation, in fact, decreases the value of the equity of a firm because the need for outside financing necessarily rises under inflationary situation. Still another belief is that the common stocks are a hedge against inflation, see Tobin (1963). This paper shows that even in a risk neutral world, the effect of changing rates of inflation on the value of the equity of a firm is, in fact, quite complex provided that the price of the product of the firm is changing at a different rate than the rate of change of the general price level. The complexity arises because the inflation affects the probability of default of the firm, the market value of its debt and its real cash flow generally not in the same direction and the value of equity depends on all these factors.

## 2. THE MODEL

As mentioned earlier, the object of this paper is to analyze the effect of anticipated rate of change of the general price level, $\pi$, (hereafter referred to as inflation) on prices of corporate securities of a firm and its probability of bankruptcy. For this purpose, we construct a single period model, where the portfolio investors are risk neutral and they have homogeneous expectations[1] about prices of the products and outputs of the firms at the end of the period, the financial markets are perfectly competitive except that there are costly bankruptcies and corporate income taxes, and the investment and financing decisions of the firms are

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[1] In this paper, the expectations about prices and outputs will be those of portfolio investors, which may or may not be the same as those of consumers and suppliers of inputs including managers.

given. It is further assumed that the exogenously determined real default

free rate of interest, r, is unaffected by changing rates of inflation.

Other assumptions of the model are stated below.

### 2.1 Relation Between the Rate of Change of Price of a Single Product and the Rate of Change of the Overall Price Index

Let $P_t$ and $P_{jt}$, respectively, be the general price level and the price

of product j that are anticipated by the portfolio investors to prevail at

the end of period t. Then, we may define:

$$(1) \quad \frac{P_{jt}}{P_t} = Q_{jt} \; .$$

The anticipated relative price $Q_{jt}$ is clearly always positive. However,

its magnitude, which depends on the demand and cost factors and the market

structure, is likely to vary from product to product. From definition (1), we

may obtain the following relationship:

$$(2) \quad \pi_{jt} = \pi_t + \sigma_{jt} \; ,$$

where $\pi_{jt} = \frac{1}{P_{jt}} \frac{dP_{jt}}{dt}$, $\pi_t = \frac{1}{P_t} \frac{dP_t}{dt}$, and $\sigma_{jt} = \frac{1}{Q_{jt}} \frac{dQ_{jt}}{dt}$.

For those firms or industries whose dynamic forces, which cause changes

in the structures of the supply and demand of their outputs and inputs, are

relatively different than those in aggregate, the $\sigma$'s will be non-zero. However,

whether the $\sigma$ for a single firm or industry is positive or negative depends on

the relative shifts of the supply and demand schedules of its product. Finally,

the $\sigma$ may depend on the anticipated rate of inflation. The explanation for this

may be found in the heterogeneity of expectations about inflation of the suppliers

of inputs to the firm and of the firm's management, their different bargaining

strengths and lags of adjustment in the input markets. However, this aspect of the

input market is not modelled in this paper. For the purpose of this paper, it is sufficient to assume that, for whatever reason both $\sigma$ and $\frac{d\sigma}{d\pi}$ may be positive, zero or negative.

## 2.2 Corporate Taxation, Costly Bankruptcy and Returns on Securities of a Firm

We assume that the firm has already acquired physical assets for investment equal to $A, and has issued securities[2] in order to finance these investment outlays. We further assume that the physical assets of the firm produce some stochastic quantity of a single product at the end of the period[3] $\tilde{X}$, and the production function is unaffected by changes in the prices. Finally, we assume that our firm dissolves at the end of the period.

Denoting the firm by subscript $j$, and the stochastic output and the anticipated price per unit at the end of the period respectively by $\tilde{X}_{j1}$ and $P_{j1}$; the net stochastic revenue, which is available to the security holders of the firm at the end of the period, is equal to $P_{j1}\tilde{X}_{j1}$. If the current price of the product of the firm is denoted by $P_{j0}$, then the net stochastic revenue of the firm can be written as

$$(3) \qquad P_{j1}\tilde{X}_{j1} = P_{j0}(1 + \pi_{j1})\tilde{X}_{j1} \ .$$

Under the assumptions of the model, this is also the nominal (stochastic) terminal value of the firm. Setting $P_{j0}$ equal to unity and substituting the value of $\pi_{j1}$ from equation (2), we have

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[2]Later on, we will specify the type of securities issued by the firm.

[3]Other inputs can be allowed in the production function of the firm. without affecting the analysis of the paper if it is assumed the non-capital real costs arising from the use of these inputs are not affected by changes in the rate of inflation. However, a more reasonable hypothesis may be that the prices and quantities of non-capital inputs are affected by inflation at least in the short run, and we think that it will be a useful extension of the model suggested above.

(4) $\quad P_{j1}\tilde{X}_{j1} = (1 + \pi_1 + \sigma_{j1})\tilde{X}_{j1}$ .

Since we will be dealing with a single firm in a one period model, we will henceforth drop all subscripts in order to simplify the notation.

As usual it will be assumed that both investment outlays at historic cost and the nominal contractual interest payments are tax deductible and the corporate earnings are subject to taxation at a constant rate $\tau$. Further, the associated possibility of bankruptcy with corporate debt is assumed. It is assumed that if the firm goes bankrupt, it incurs some non-random real bankruptcy costs, c, which are also assumed to be unaffected by inflation. Denoting the real contractual interest rate, which the firm promises to pay on its bonds by h, and the current market value of the bonds of the firm by B, we may write the nominal stochastic terminal value of the debt of the firm at the end of the period as

$$\tilde{W}_b = \begin{cases} (1 + h)(1 + \pi)B & \text{, whenever bankruptcy does not occur} \\ \\ \tilde{X}(1 + \pi + \sigma) - c(1 + \pi) & \text{, otherwise.} \end{cases}$$

Assuming a limited liability for the bondholders of the firm, the real stochastic terminal value of the firm must be equal or greater than real costs of bankruptcy, i.e., $\tilde{X}(1 + \frac{\sigma}{1 + \pi}) \geq c$; and we assume this condition holds.

The nominal stochastic value of the equity of the firm at the end of the period is somewhat more difficult to specify. Consider the tax obligation of the firm: we have already assumed that both the nominal contractual interest payments and the investment outlays at historic costs are tax deductible, which means that the firm has the nominal tax obligation equal to:

$\tau[\tilde{X}(1 + \pi + \sigma) - A - \{(1 + h)(1 + \pi) - 1\}B]$, where $\{1 + h)(1 + \pi) - 1\}$ is the nominal contractual rate of interest. Whenever the firm remains solvent, the equity holders of the firm will receive the sales revenue net of the nominal tax, the nominal contractual interest and the principal payments. However, if the firm goes bankrupt, the equity holders will receive zero payment under the limited liability provision. Formally the nominal stochastic returns of the equity holders of the firm can be written as

$$(6) \quad \tilde{W}_e = \begin{cases} [\tilde{X}(1 + \frac{\sigma}{1 + \pi}) - (1 + h)B](1 - \tau)(1 + \pi) + \tau(A - B), & \text{whenever } \tilde{X} \geq q \\ \\ 0 & , \text{otherwise} \end{cases}$$

where $q = \frac{(1+h)(1+\pi)}{1+\pi+\sigma} B$ .

The probability that the firm will not be able to meet its real contractual interest obligation and principal debt payments at the end of the period is

$$(7) \quad F = \int_c^q f(\tilde{X})dX \ ,$$

where $f(\tilde{X})$ is the density function of $\tilde{X}$ and c are the real costs of bankruptcy which enters as the lower limit of the integral due to the assumption that the bondholders of the firm are also protected by the limited liability provision.

To this point, we have expressed the tax payment and terminal values of stocks and bonds of the firm in nominal terms. Given the anticipated

rate of inflation, $\pi$, we can convert them into real terms by deflating each of them by $1+\pi$. Accordingly, the tax payment in real terms is equal to

$\tau[\tilde{X}(1+\frac{\sigma}{1+\pi}) - \frac{A}{1+\pi} - \{(1+h) - \frac{1}{1+\pi}\}B]$, while the real (stochastic) returns on stocks and bonds of the firm, respectively, are

$$(8) \quad \tilde{\omega}_e = \frac{\tilde{W}_e}{1+\pi} = \begin{cases} [\tilde{X}(1+\frac{\sigma}{1+\pi}) - (1+h)B](1-\tau) + \frac{\tau}{1+\pi}[A-B] & , \text{ whenever } \tilde{X} \geq q \\ \\ 0 & , \text{ otherwise} \end{cases}$$

and

$$(9) \quad \tilde{\omega}_b = \frac{\tilde{W}_b}{1+\pi} = \begin{cases} (1+h)B & , \text{ whenever } \tilde{X} \geq q \\ \\ \tilde{X}(1+\frac{\sigma}{1+\pi}) - c & , \text{ otherwise} . \end{cases}$$

From (8) and (9), we may define the stochastic real rates of return on the equity and the debt of the firm as follows:

$$(10) \quad \tilde{r}_e = \frac{\tilde{\omega}_e}{Z} - 1 \quad , \text{ and}$$

$$(11) \quad \tilde{r}_b = \frac{\tilde{\omega}_b}{B} - 1 \quad ,$$

where $Z$ in equation (10) is the market value of the equity of the firm.

Denoting the mathematical expectation of a random variable by a overhead bar, we can write the current values of the equity and debt of the firm, from equations (8) to (11) as

$$(12) \quad Z = \frac{\bar{\omega}_e}{1+\bar{r}_e} = \frac{1}{1+\bar{r}_e} \int_q^\infty [\{\tilde{X}(1+\frac{\sigma}{1+\pi}) - (1+h)B\}\{1-\tau\} + \frac{\tau(A-B)}{1+\pi}]f(\tilde{X})d\tilde{X}$$

and

$$(13) \quad B = \frac{\bar{\omega}_b}{1+\bar{r}_b} = \frac{1}{1+\bar{r}_b} [\int_q^\infty (1+h)B f(\tilde{X})d\tilde{X} + \int_c^q \{\tilde{X}(1+\frac{\sigma}{1+\pi} - c\} f(\tilde{X})d\tilde{X}] .$$

In the risk neutral world of perfectly competitive financial markets, the equilibrium condition requires that the expected real rate of return on every financial security be equal to the default free real rate of interest, $r$. Therefore, we write

(14)    $\bar{r}_e = r$ , and

(15)    $\bar{r}_b = r$ .

Substituting these equilibrium equations into (12) and (13) and using the definition of probability of bankruptcy as given in equation (7), the equilibrium current values of stocks and bonds of the firm may be written as

(16)    $Z = \dfrac{1}{1+r} \left[ (1-\tau)\{(1+\dfrac{\sigma}{1+\pi})E_{+q}(\tilde{X}) - (1+h)\, B(1-F)\} + \dfrac{\tau(1-F)(A-B)}{1+\pi} \right]$ ,

and

(17)    $B = \dfrac{1}{1+r} \left[ (1+h)B\,(1-F) + (1+\dfrac{\sigma}{1+\pi})E_{-q}(\tilde{X}) - c(1-F) \right]$ ,

where $E_{+q}(\tilde{X}) = \displaystyle\int_{q}^{\infty} \tilde{X} F(\tilde{X})d\tilde{X}$ , and $E_{-q}(\tilde{X}) = \displaystyle\int_{c}^{q} \tilde{X} F(\tilde{X})d\tilde{X}$ are the partial moments of the random variable $\tilde{X}$.

## 3.    THE COMPARATIVE STATIC RESULTS

In this section, we present the effect of change in the anticipated rate of inflation on the current values of the equity and bonds and the probability of bankruptcy of the firm. From equations (7), (16) and (17), it may be noted that the value of equity, Z, depends, among other things, on the market value of debt, B, and probability of bankruptcy, F. On the other hand, the B and F, while depending on each other, do not depend on Z. Therefore, first we analyze the effect of $\pi$ on B and F.

### 3.1 The Effect of Change in the Anticipated Rate of Inflation, on the Market Value of the Debt, B, and the Probability of Bankruptcy, F

Differentiating F in (7) with respect to $\pi$ and noting that r, h and c are independent of $\pi$ by assumption, we obtain

$$(18) \quad \frac{dF}{d\pi} = f(q)[\{\frac{1+h}{1+\pi+\sigma} - \frac{(1+h)(1+\pi)}{(1+\pi+\sigma)^2} - \frac{(1+h)(1+\pi)}{(1+\pi+\sigma)^2} \frac{d\sigma}{d\pi}\}B + \frac{(1+h)(1+\pi)}{(1+\pi+\sigma)} \frac{dB}{d\pi}]$$

$$= f(q)[\frac{q\sigma(1-\eta)}{(1+\pi+\sigma)(1+\pi)} + \frac{q}{B} \frac{dB}{d\pi}],$$

where f(q) is the density function of $\tilde{X}$ at q and $\eta = \frac{d\sigma}{d\pi} \frac{1+\pi}{\sigma} = \frac{d\sigma}{d(1+\pi)} \frac{(1+\pi)}{\sigma}$, is the elasticity of the rate of change of the product price with respect to $1+\pi$. Similarly, differentiating equation (17) with respect to $\pi$ we obtain

$$(19) \quad \frac{dB}{d\pi} = [(1+h)(1-F)\frac{dB}{d\pi} - (1+h)B\frac{dF}{d\pi} + (1+h)B\frac{dF}{d\pi} - \frac{\sigma}{(1+\pi)^2}E_{-q}(\tilde{X})$$

$$+ \frac{1}{(1+\pi)}E_{-q}(\tilde{X})\frac{d\sigma}{d\pi} - c\frac{dF}{d\pi}]/(1+r).$$

Substituting the value of $\frac{dF}{d\pi}$ from equation (18) into equation (19), multiplying throughout by $(1+r)$ and explicitly solving for $\frac{dB}{d\pi}$, we obtain

$$(20) \quad \frac{dB}{d\pi} = \frac{\frac{(\eta-1)\sigma}{1+\pi}[\frac{cf(q)q}{(1+\pi+\sigma)} + \frac{E_{-q}(\tilde{X})}{1+\pi}]}{[(1+r) - (1+h)(1-F) - \frac{cf(q)q}{B}]}.$$

From this equation, we can explain the effect of a change in $\pi$ on B under different assumptions about the market parameters $\sigma$ and $\frac{d\sigma}{d\pi}$ (or $\eta$). These results are stated in the form of two propositions below.:

Proposition 1. Given the assumptions of the model and $\frac{d\sigma}{d\pi} = 0$, then

$$\frac{dB}{d\pi} \gtrless 0 \text{ if } \sigma \lessgtr 0.$$

Proof: Under the condition: $\frac{d\sigma}{d\pi} = 0$, equation (20) reduces to

$$(21) \quad \frac{dB}{d\pi} = \frac{\frac{-\sigma}{1+\pi}(\frac{cf(q)q}{(1+\pi+\sigma)} + \frac{E_{-q}(\tilde{X})}{1+\pi})}{[(1+r) - (1+h)(1-F) + \frac{cf(q)q}{B}]}.$$

Substituting the value of $(1+r)B$ from equation (17), we can write the denominator in (21) as

$$(22) \quad \frac{1}{B}[cf(q)q + (1+\frac{\sigma}{1+\pi})E_{-q}(\tilde{X}) - c(1-F)].$$

Under the limited liability provision of the bondholders $(1+\frac{\sigma}{1+\pi})E_{-q}(\tilde{X}) - c(1-F)$ is positive. Further, $cf(q)q$ is also positive.[4] Therefore, the denominator of equation (21) is positive. Consequently, the sign of the numerator is opposite to the sign of $\sigma$ since $cf(q)q$, $(1+\pi+\sigma)$, $E_{-q}(\tilde{X})$ and $(1+\pi)$ all are positive. Thus the sign of $\frac{dB}{d\pi}$ is opposite to the sign of $\sigma$.

This proposition may be explained in simple words as follows. If $\sigma = 0$, then from equation (2) $\pi_j = \pi$. This means that the stochastic real terminal value of the debt of the firm becomes independent of $\pi$ (see expression (9)), because $h$, $\tilde{X}$, $r$ and $c$ are assumed to be independent of inflation. Therefore, the current value of the debt of the firm remains unaffected by variation in $\pi$.

Consider now $\sigma > 0$: Given some value of $\pi = \pi^o$ there will be correspondingly some mean value of the debt of the firm, $\bar{\omega}_b^o$ and its current market value, $B^o$. Starting from an equilibrium situation, we must have $r_b^o = (\bar{\omega}_b^o - B^o)/B^o = r$. Now let $\pi$ rise to a new value $\pi^1$ corresponding to which there is now a lower mean real value of the debt of the firm at the end of the period, which we may denote by $\bar{\omega}_b^1 < \bar{\omega}_b^o$. Therefore, at the market value of the debt of the firm equal to $B^o$, the required real rate of return on bonds of the firm becomes $(\bar{\omega}_b^1 - B^o)/B^o$, which is clearly less than the default free real rate of interest $r$. This causes portfolio investors to shift their demand away from the bonds

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[4] We are ignoring here some implausible rates of deflation such as $\pi < -1$ and $\pi < -1 - \sigma$.

of the firm which in turn results in a decrease in the selling price of these
bonds. This process continues until the price of these bonds has fallen to a
level where, once again, the bonds of the firm yield the same mean real rate
of return as promised by default free financial assets in the market. Follow-
ing the similar argument we can show that if $\sigma < 0$, an increase in the inflation
rate would require that the bond price of the firm must rise in order to
equilibrate the market of the debt of the firm.

**Proposition 2.** Given the assumptions of the model, $\sigma \neq 0$ and $\frac{d\sigma}{d\pi} \neq 0$,

then (a) $\frac{dB}{d\pi} \gtreqless 0$ if $\sigma > 0$ and $\eta \gtreqless 1$, and (b) $\frac{dB}{d\pi} \gtreqless 0$ if

$\sigma < 0$ and $\eta \gtreqless 1$.

**Proof:** The proof of this proposition follows immediately from
equation (20). For $\sigma > 0$, we see that $\frac{dB}{d\pi} \gtreqless 0$ as $\eta \gtreqless 1$ but for $\sigma < 0$,
$\frac{dB}{d\pi} \gtreqless 0$ as $\eta \lesseqgtr 1$. For both $\sigma \gtrless 0$, $\eta = 1$ implies that $\frac{dB}{d\pi} = 0$.

This proposition has a natural interpretation. If $\sigma > 0$, then an increase
in the rate of inflation reduces the mean real returns to the bondholders
since it is deflated at a higher rate. However, the total change in the real
returns to the bondholders in addition, depends on whether $\eta \gtreqless 1$. Thus, if
$\eta > 1$, then the reduction in the mean real returns caused by $\sigma > 0$ is more
than offset by the inflation elasticity of the rate of change of the relative
price of the product sold by the firm. Therefore, as a whole, the mean real
returns of the bondholders rise requiring a higher current market value of the
bonds so that the equilibrium can be obtained where $\bar{r}_b = r$. For $\sigma > 0$ in
combination with $\eta < 1$, however, the former factor dominates causing the market
value of the debt to fall as a result of changes in $\pi$. Finally for $\sigma > 0$ in
combination with $\eta = 1$, the current value of the debt remains unchanged in
response to a change in $\pi$ because in this case the two opposing effects on the

mean real returns cancel each other exactly.

For $\sigma < 0$, however, the signs are reversed. Consider $\sigma < 0$ in combination with $\eta < 0$. With an increase in the anticipated rate of inflation, $\bar{\omega}_b$ rises for two reasons: First the anticipated real price of the product rises because $\sigma < 0$ is discounted at a high rate of inflation. Secondly, $\eta < 0$ implies that $\frac{d\sigma}{d\pi} > 0$ which means that the anticipated real product price rises even further. Consequently, in this situation the equilibrium market value of the bond of the firm rises at a faster rate. For $\sigma < 0$, but $0 < \eta < 1$, $\frac{dB}{d\pi}$ is still positive but less so as compared to previous cases because now $\frac{d\sigma}{d\pi} < 0$. When $\eta > 1$, along with $\sigma < 0$, then an increase in $\bar{\omega}_b$ due to $\sigma < 0$ is more than offset since the proportionate change in the anticipated rate of inflation causes a proportionate decrease in the rate of change of the relative price of the output. Consequently, as a whole, $\bar{\omega}_b$ falls which requires a lower equilibrium value of the debt of the firm.

From the above two propositions, it is apparent that under the assumptions of our model, there are three sets of sufficient conditions for the market value of the debt of the firm to be neutral with respect to the rate of inflation. They are (i) $F = 0$, (ii) if $F \neq 0$, then both $\sigma$ and $\frac{d\sigma}{d\pi}$ are equal to zero, and (iii) if $F$, $\sigma$ and $\frac{d\sigma}{d\pi}$ are non-zero, then $\eta$ is equal to unity. Conditions (ii) and (iii) indicate the importance of values of $\sigma$ and $\frac{d\sigma}{d\pi}$ (or $\eta$) for the non-neutral effect of inflation on the market value of the debt of the firm. Since in general $\sigma$ and $\frac{d\sigma}{d\pi}$ are non-zero, the effect of change in the anticipated rate of inflation on the market value of the debt of the firm is in general non-neutral.

Having analyzed the effect of a change in the anticipated rate of inflation on the market value of debt of the firm, we will refer to this effect in the subsequent analysis in terms of the elasticity defined as: $\eta_B = \frac{1+\pi}{B} \frac{dB}{d\pi} = \frac{(1+\pi)}{B} \frac{dB}{d(1+\pi)}$.

We now turn our attention towards the effect of change in the anticipated rate of inflation on the probability of default of the firm. This effect is summarized in the propositions (3) and (4) below.

__Proposition__ 3.   Given the assumptions of the model and $\frac{d\sigma}{d\pi} = 0$, then

(a) $\frac{dF}{d\pi} \gtreqless 0$, if (i) $\sigma > 0$ and (ii) $|\eta_B| \lesseqgtr \frac{\sigma}{(1+\pi+\sigma)}$ , and

(b) $\frac{dF}{d\pi} \lesseqgtr 0$, if (i) $\sigma < 0$ and (ii) $\eta_B \lesseqgtr |\frac{\sigma}{1+\pi+\sigma}|$.

__Proof:__          Substituting $\frac{d\sigma}{d\pi} = 0$ in equation (18), we obtain:

(23)    $\frac{dF}{d\pi} = \frac{f(q)q}{1+\pi} [\frac{\sigma}{(1+\pi+\sigma)} + \eta_B]$

From proposition 1, we note that if $\sigma \gtrless 0$, then $\frac{dB}{d\pi} \lessgtr 0$ and hence $\eta_B \lessgtr 0$. Further, since $\frac{f(q)q}{1+\pi}$ is positive, it follows from equation (23) that if $\sigma > 0$ and $\frac{\sigma}{1+\pi+\sigma} \gtreqless |\eta_B|$, then $\frac{dF}{d\pi} \gtreqless 0$, which proves Proposition 3a. Similarly if $\sigma < 0$ and $|\frac{\sigma}{1+\pi+\sigma}| \gtreqless \eta_B$, then it follows from equation (23) that then $\frac{dF}{d\pi} \lesseqgtr 0$, which proves Proposition 3b.

The rationale for the results in Proposition 3 follows: The conditions $\sigma > 0$ and $\frac{d\sigma}{d\pi} = 0$ imply that as the anticipated rate of inflation increases, the total real cash flow of the firm falls which in turn implies that the probability that the firm will not be able to meet its contractual interest payments rises. However, in this situation, the market value of bonds of the firm also falls reducing the contractual interest obligations of the firm and lowering the probability of bankruptcy. The former factor is represented by the term $\frac{\sigma}{1+\pi+\sigma}$ in equation (23) and the latter by $\eta_B$ and both of these are weighted by the same positive term, $\frac{f(q)q}{1+\pi}$ . The probability of bankruptcy rises if the former factor dominates but remains constant or falls otherwise.

For $\sigma < 0$ and $\frac{d\sigma}{d\pi} = 0$, the effect of change in $\pi$ on the real cash flow of the firm and its contractual real interest obligations are of opposite signs.

That is, the term, $\frac{\sigma}{1+\pi+\sigma}$ , exerts a dampening effect on F while the term $\eta_B$ exerts a booster effect on F.

Under the conditions that both $\sigma$ and $\frac{d\sigma}{d\pi}$ are non-zero, an effect of the variation in the anticipated rate of inflation is given in the following proposition.

**Proposition 4.** Given the assumption of the model, $\sigma \neq 0$ and $\frac{d\sigma}{d\pi} \neq 0$, then

(a) $\frac{dF}{d\pi} = 0$ if $\eta = 1$ and (b) if $\eta \neq 1$, then the following are true:

(1) Given that $\sigma > 0$ and $\eta > 1$, then $\frac{dF}{d\pi} \gtreqless 0$ if

$$\frac{\sigma(1-\eta)}{1+\pi+\sigma} \lesseqgtr |\eta_B|$$

(2) Given that $\sigma > 0$ and $\eta < 1$, then $\frac{dF}{d\pi} \gtreqless 0$ if

$$\left|\frac{\sigma(1-\eta)}{1+\pi+\sigma}\right| \lesseqgtr \eta_B$$

(3) Given that $\sigma < 0$ and $\eta < 1$, then $\frac{dF}{d\pi} \gtreqless 0$ if

$$\left|\frac{\sigma(1-\eta)}{1+\pi+\sigma}\right| \lesseqgtr \eta_B$$

(4) Given that $\sigma < 0$ and $\eta > 1$, then $\frac{dF}{d\pi} \gtreqless 0$ if

$$\frac{\sigma(1-\eta)}{1+\pi+\sigma} \gtreqless |\eta_B| .$$

**Proof:**    We may write equation (19) as

$$(24) \quad \frac{dF}{d\pi} = \frac{f(q)q}{1+\pi} \left\{ \frac{\sigma(1-\eta)}{1+\pi+\sigma} + \eta_B \right\}$$

Since $\frac{f(q)q}{1+\pi}$ is positive, the sign of $\frac{dF}{d\pi}$ depends on the sign of the terms in curly brackets. Now if $\eta = 1$, then the first term in the curly bracket is equal to zero, and also simultaneously $\eta_B$ is equal to zero from Proposition 2. Therefore, $\frac{dF}{d\pi} = 0$. However, if $\eta \neq 1$, then the sign of the expression:

$\frac{\sigma(1-\eta)}{1+\pi+\sigma} + \eta_B$ is ambiguous without further prior restrictions on the sign

and/or magnitude of each of $\sigma$, $\eta$ and $\eta_B$. For example, if $\eta < 1$ and $\sigma > 0$,

then the term $\frac{\sigma(1-\eta)}{1+\pi+\sigma}$ is positive and the term $\eta_B$ is negative from

Proposition 2, and hence the sign of $\frac{dF}{d\pi}$ is ambiguous. Consider now the case

where $\eta > 1$ and $\sigma > 0$. The term $\frac{\sigma(1-\eta)}{1+\pi+\sigma}$ is negative but the term

$\eta_B$ is positive. Thus once again, the sign of $\frac{dF}{d\pi}$ is ambiguous. Similarly, it

is easily verified that under various other values of $\sigma$ and $\eta$, $\frac{\sigma(1-\eta)}{1+\pi+\sigma}$

and $\eta_B$ will have opposite signs, therefore, the sign of $\frac{dF}{d\pi}$ is ambiguous. However,

if the values of $\sigma$, $\eta$ and $\eta_B$ are restricted as in conditions (1), (2), (3)

and (4) of Proposition 4, the results stated therein can be verified from

equation (24) and Proposition 2.

The interpretation of this proposition is analogous to Proposition 3

above. The term $\frac{\sigma(1-\eta)}{1+\pi+\sigma}$ represents the effect of change in $\pi$ on the

real net cash flow of the firm. If it is negative (or positive), the probability

of bankruptcy falls (or rises). On the other hand, the term, $\eta_B$, is a

manifestation of changes in the contractual interest payments of the firm

resulting from the change in $\pi$. If this term is negative, the interest

obligation of the firm falls resulting in a fall in the probability of

bankruptcy as well. It is clear from Proposition 4 that the terms $\frac{\sigma(1-\eta)}{1+\pi+\sigma}$

and $\eta_B$ are of opposite signs. Thus the final affect of a change in $\pi$ of F

depends on their relative strengths.

In conclusion, the sets of sufficient conditions for $\frac{dF}{d\pi}$ to be equal

to zero are: (i) both $\sigma$ and $\frac{d\sigma}{d\pi}$ are equal to zero, and (ii) $\eta = 1$. Thus

the effect of $\pi$ on F is non-neutral unless conditions (i) and (ii) are met.

### 3.2 The Effect of Change in the Anticipated Rate of Inflation on the Market Value of the Equity of the Firm

Differentiating the equilibrium market value of the equity of the firm

in equation (16) with respect to $\pi$ and noting, once again, that r, h, c and

$\tilde{X}$ are independent of $\pi$, by assumption, we get

$$(25) \quad \frac{dZ}{d\pi} = \frac{1}{1+r} \left[ \left\{ E_{+q}(\tilde{X}) \frac{\sigma}{(1+\pi)^2} (\eta - 1) \right\} (1-\tau) - \left\{ (1-\tau)(1+h) + \frac{\tau}{1+\pi} \right\} (1-F) \frac{dB}{d\pi} \right. $$

$$ \left. - \left\{ \frac{\tau(A-B)}{1+\pi} \frac{dF}{d\pi} \right\} - \left\{ \frac{\tau(A-B)}{(1+\pi)^2} (1-F) \right\} \right]$$

Each term in the rectangular brackets on the right-hand side of (25) represents a different type of effect of variation in $\pi$ on Z. The term, $E_{+q}(\tilde{X}) \frac{\sigma}{1+\pi} (\eta - 1) = K$, say, represents the effect of $\pi$ on Z which results due to changes in the real sales of the firm. If $\sigma > 0$, then $K \gtreqless 0$ whenever $\eta \gtreqless 1$. Similarly, if $\sigma < 0$, then $K \gtreqless 0$ whenever $\eta \lesseqgtr 1$. The term

$-\left\{(1+h)(1-\tau) + \frac{\tau}{1+\pi}\right\}(1-F) \frac{dB}{d\pi} = L$, say, arises because of the effect of

$\pi$ on B. Since $\left\{(1+h)(1-\tau) + \frac{\tau}{1+\pi}\right\}(1-F)$ is positive, therefore, the sign of L is opposite to the sign of $\frac{dB}{d\pi}$. The effects of the change in $\pi$ on B under different assumptions about $\sigma$ and $\frac{d\sigma}{d\pi}$ are given in Propositions 1 and

2 above. The term, $-\tau \left\{\frac{A-B}{(1+\pi)^2}\right\}$ represents two types of effects of a change

in $\pi$ on Z. One, the capital consumption allowance effect, $-\frac{\tau A}{(1+\pi)^2}$, arises because of the deductibility of investment outlays of the firm at historic costs rather than replacement costs. This clearly reduces the value of the equity of the firm under rising inflation. Second, the effect from $\frac{\tau B}{(1+\pi)^2}$ arises because we have allowed for the provision that only the interest payment is tax deductible and not both interest payment and principal payment. Now assuming that $A \geq B$, that is, the market value of the debt never exceeds the total investment outlays of the firm, then $\theta \leq 0$. Finally, the term, $-\frac{\tau(A-B)}{(1+\pi)} \frac{dF}{d\pi}$ among other things, represents the effect of $\pi$ on F. To sum up: If $\sigma > 0$, then $K \gtreqless 0$ whenever $\eta \gtreqless 1$. Correspondingly, from Proposition 2, $\frac{dB}{d\pi} \gtreqless 0$, which implies that $L \lesseqgtr 0$. Thus whatever may be the sign of $\sigma$ and $\lambda$, the sign of $\frac{dZ}{d\pi}$ is ambiguous from the first two terms on

the right-hand side of equation (25). A similar result will hold if

$\sigma < 0$. The effect of change in $\pi$ on $Z$ can however be unambiguously

stated under some restrictive conditions. This has been done in the

following two propositions. The proofs and explanation straight-

forwardly follow and hence are omitted.

**Proposition** 5. Given that $F = 0$ and $\tau = 0$, then (a) $\frac{dZ}{d\pi} \gtreqless 0$ if $\sigma > 0$ and

$\eta \gtreqless 1$, and (b) $\frac{dZ}{d\pi} \gtreqless 0$ if $\sigma < 0$ and $\eta \lesseqgtr 1$.

**Proposition** 6. Given the more general situation where $F$ and $\tau$ are not

equal to zero, then $\frac{dZ}{d\pi} = \frac{-\tau (A - B)(1 - F)}{(1 + \pi)^2}$

if the following conditions hold: (i) $\sigma = \eta = 0$ or

(ii) $\eta = 1$.

## 4. CONCLUSION

Given that the technology is unaffected by variations in inflation,

we have shown that the real sales of the firm will still be affected provided

that the rate of change of anticipated product price changes differently

than the rate of change of anticipated inflation. In a risk neutral single

period model, it is shown that when the real sales revenues of the firm rises

(or falls), then the value of its debt also rises (or falls) but the effect

of a change in the anticipated rate of inflation on the probability of default

and the value of the equity of the firm, is in general, ambiguous. For the

probability of default, this ambiguity arises because while it varies negatively

with real sales it varies positively with the value of the debt of the firm.

On the other hand, the value of equity depends, among other things, on the

real sales revenue, value of the debt and probability of default. Further,

the effect of the anticipated inflation on these three factors happen not to

move in the same direction under changing inflationary situations. Thus, the

effect of anticipated inflation on the price of the equity of the firm is ambiguous.

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