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IMPERFECT CAPITAL MARKETS AND GAMBLING WITH RISK AVERSION

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and Gambling with Risk Aversion

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Imperfect Capital Markets and Gambling with Risk Aversion

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1. Introduction

Since the classic paper by Friedman and Savage [1948] it has generally been accepted that the observed fact that individuals or firms[1] participate in unfair lotteries and other forms of unfair risk taking[2] must be explained by a section in the individual's utility function in which the individual shows risk preference rather than risk aversion.

However, in the last two decades there has emerged a considerable literature examining the behaviour of individuals under uncertainty in which the assumption has almost invariably been made that economic units are risk averse over the relevant ranges of income or wealth. Thus, whilst the Friedman-Savage view that an individual may show both risk aversion and risk preference is accepted as necessary to explain phenomena such as gambling, it is in general ignored in the literature which makes the tidier assumption of individuals who show only risk aversion.

It is the purpose of the paper to suggest that it is possible to explain participation in unfair lotteries without relaxing the risk aversion assumption, nor imputing utility to the act of taking a risk. The explanation of unfair gambling by individuals provided focuses on the market constraints facing the individual rather than on his preferences. In particular, we show that when certain capital market imperfections exist, a risk averse individual may participate in unfair lotteries. Furthermore, we show that in general risk aversion is neither necessary nor sufficient for an individual to reject fair lotteries.

In section 2 we discuss certain capital markets imperfections and their implications about the nature of rates of return on investment. In section 3 we provide a rationale for gambling with risk aversion and in section 4 we consider the joint problem of portfolio choice and gambling.

2. Rates of Return and Capital Market Imperfections

A number of recent studies discuss the effects of uncertainty, imperfect information and various transactions costs, on capital markets and show that the result may be that capital markets are characterized by certain imperfections. These imperfections are usually in terms of the non-existence of certain markets and the fact that there may not be free and equal access to other markets. Uncertainty and the possibility of costly default may lead lenders to introduce collateral requirements, or take default costs into account in their loan rates. Consequently, it can be said that one has to have certain assets (providing collateral services) in order to have easier or cheaper access to capital markets.

Barro [1976] and Benjamin [1978] derive these conclusions explicitly and show that market imperfections will lead to loan rates being functions of loan sizes and available collateral. In particular, they derive a loan supply function which is constant for some initial range and then becomes an increasing and convex function of loan sizes.

In the same vein Jaffee and Modigliani [1968] show that as a result of uncertainty and imperfect and costly information, capital markets may be characterized by credit rationing. They show that beyond a certain point load rates will generally depend on loan sizes, with a possible upper bound on loan sizes. Furthermore, they provide empirical evidence supporting these types of imperfections.

Empirical evidence indicating capital markets imperfections is also given by Eckstein [1961] and Nerlove [1968] who find capital markets to be charcterized by differential rates of return on given investments.

The important implication of the capital market imperfections is that individuals face a variety of capital market constraints and, therefore, do not have free and equal access to the market. Consequently, capital provides additional services by either weakening the accessibility constraints or by reducing the cost of the acquisition of capital. By providing collateral services capital, therefore, increases the set of "feasible activities" or reduces their cost, so that its "full rate of return" should take this additional role into account.

In some sense, assets which provide an insurance from the lender's point of view, are used to resolve an "adverse selection" like problem,[3] by establishing the relative credit worthiness of individuals. A similar role is played by education and human capital in the signaling literature.[4] An analogy may also be drawn between the capital market access role of capital and the liquidity services of money in the theory of the demand for money.[5]

A consequence of these capital market imperfections is that for some levels of asset holdings the rates of return on assets may depend on the levels of the asset holdings, even from an individual's (rather than an aggregate) point of view. The range within which this is likely to happen is when wealth levels are low, but not below some minimum level. When an individual's wealth is below some minimum level the capital market constraints may be so effective that except for the possibility of obtaining small loans the market is in effect inaccessible. Since small loans can usually be obtained at constant loan rates (as is shown in Jaffee and Modigliani [1968], Barro [1976] and Benjamin [1978]), the additional role of asset holding is ineffective within this initial range and consequently the rates of return will not depend on asset holdings. However, as an individual's asset holdings increase above some minimum level, his higher level of wealth will provide him with the additional "collateral services" and rates of return within this range will depend on the levels of wealth.

Since rates of return are usually not known with certainty, the dependence of rates of return on asset holdings is in a probabilistic sense. In other words, the rates of return probability distribution function changes with asset holdings, i.e., the distribution is conditional on asset holdings.

The form of the dependence of the distribution on asset holdings may be of a very general nature and could be reflected in the various characteristics of the distribution. More specifically, however, it follows from the above discussion that the dependence is such that, at least over some range, the rates of return distribution becomes more "favourable", as asset holdings increase. A natural way to represent the change in the distribution is in terms of its stochastic dominance.[6] Thus, over some range, an increase in asset holdings will make the distribution stochastically dominant over the ones corresponding to lower levels of asset holdings.

The simplest example of such an effect is an increase in the expected rates of return; as asset holdings increase, we may over some range, get higher expected rates of return. Beyond a certain point as asset holdings increase, access to the capital markets becomes easier and eventually accessibility constraints may become ineffective, so that the expected rate of return may eventually converge to a constant value. Of course, as the individual accumulates more and more of an asset, he will eventually become "big", i.e., a monopolist, so that his expected rate of return will again become a (decreasing) function of the asset holdings.

In addition to the above considerations, rates of return may be increasing functions of wealth (over some initial range) for other reasons as well. Firstly, a large number of fixed costs may be incurred in effecting profitable investment. Information costs incurred in locating high return investments may be considerable and a large element of these costs may be relatively fixed. Transaction costs, where again the large part is fixed are also likely to comprise a high percentage of small investments. In the presence of these costs the rate of return will be an increasing function of the investment. Secondly, there may exist significant indivisibilities which (especially in view of the capital market constraints) will imply initial increasing returns to scale.

In view of these considerations we conclude that individuals, especially those with small or moderate levels of wealth, may very often face various capital market constraints which lead to rates of return being functions of wealth. Furthermore, the relationship between wealth and the rates of return is such that rates of return are (at least over some range) increasing (in a probabilistic sense) functions of wealth.

3. Gambling with Risk Aversion

Having discussed some of the capital market imperfections and the constraints they impose on individuals, we now consider their effects on the individual's attitudes towards gambling.

In order to focus on the gambling problem and to separate it from the investment problem, we initially make the assumption that the rate of return is non-random. This assumption will be relaxed later, and as it turns out, it does not change the results. For the sake of simplicity, we also assume that there is only one asset, an assumption that is later dropped.

In line with the discussion in the previous section, it will be assumed that the individual faces a rate of return R on his investment, A, such that

$$R = R(A) \qquad \begin{array}{ll} R' = 0 & 0 \leq A \leq A^* \\ R' > 0 & \text{for } A^* \leq A \leq A^{**} \end{array} \tag{1}$$

where R' is the partial derivative of R. In other words, the rate of return is constant over some initial range and increasing over some subsequent range.

Regarding the curvature of R(A), it is not clear whether it is concave or convex. As it turns out, however, the curvature of R(A), whilst having an effect, is not crucial since what ultimately matters is the curvature of final wealth as a function of A.[8]

An individual investing an amount A will end up with final wealth W, such that

$$W = A[1+R(A)]. \tag{2}$$

Clearly, convexity of R(A) over some range is sufficient, but not necessary for the convexity of W(A) over the same range; both convex and concave rate of return functions may lead to a convex W(A).

Let us now consider the individual's decision problem. Following the literature,[7] we assume the individual has a utility function defined on final wealth, U(W), and that

$$U' > 0, \qquad U'' < 0 \tag{3}$$

i.e., the individual is risk averse.

The individual is assumed to wish to maximize his expected final utility. In the absence of uncertainty his utility is

$$U(A(1+R(A))) \tag{4}$$

where his initial wealth is A. Consider, however, his position if he participates in a lottery with a single net prize, S, entrance ticket price, P, and a probability of winning, q. The lottery is instantaneous.[10] The expected utility from the gamble is

$$qU((A+S)\ (1+R(A+S))) + (1-q)U((A-P)\ (1+R(A-P)) \tag{5}$$

so that the individual will prefer the gamble to no-gamble if expression (5) is greater than expression (4).

Using the mean value theorem to express $U[(A+S)(1 + R(A+S))]$ and $U[(A-P)(1 + R(A-P))]$ we get that the condition for the individual to prefer gambling is equivalent to[11]

$$q > \frac{P}{S+P} - \frac{qS^2K_1 + (1-q)P^2K_2}{U'(A)\ [1+R(A)+AR'(A)](S+P)} \tag{6}$$

where

$$\begin{aligned} K_1 &= \tfrac{1}{2}[\bar{U}''.(1+\bar{R}+A\bar{R}') + \bar{U}'.(2\bar{R}'+A\bar{R}'')] \\ K_2 &= \tfrac{1}{2}[\bar{\bar{U}}''.(1+\bar{\bar{R}}+A\bar{\bar{R}}') + \bar{\bar{U}}(2\bar{\bar{R}}'+A\bar{\bar{R}}'')] \end{aligned} \tag{7}$$

and where a single bar denotes a value at $A + \alpha S$ $0 \leq \alpha \leq 1$ and a double bar denotes a value at $A - \beta P$ $0 \leq \beta \leq 1$.

Let us examine condition (6). The probability required to make the lottery fair is $q = \frac{P}{S+P}$. Hence, if the second term on the right hand side of (6) is (ignoring the minus sign) negative, the individual requires better than fair odds. If, however, this term is positive, he is prepared to participate in an unfair lottery. Whether or not this term is in fact negative, depends on the curvature of the utility function, the properties of the return function R and the initial level of wealth A. What is clear is that risk aversion (i.e., $U'' < 0$) is necessary and sufficient for the individual to reject a fair gamble, only within the initial range where the rate of return is constant. In the range where the rate of return is increasing with wealth it is, however, neither necessary nor sufficient. The capital market imperfections introduce an additional effect which may explain why some risk averse individuals participate in unfair gambles.

The condition for the term to be positive is clearly seen to require that the increasing rate of return effect outweighs the risk aversion tendencies of the individual and, if necessary, the declining rate of increase of the rate of return (in the event of a concave R(A)).

$$2q.S^2\bar{U}\bar{R}' + 2(1-q)P^2\bar{\bar{U}}\bar{\bar{R}}' < -\{qS^2\bar{U}''.(1+A\bar{R}'+\bar{R}) + (1-q)\ P^2\bar{\bar{U}}.\ (1+\bar{\bar{R}}+A\bar{\bar{R}}')\}$$
$$-\{qS^2\bar{R}''\bar{U}'.A+(1-q)P^2\bar{\bar{R}}''\bar{\bar{U}}'A\}. \tag{8}$$

In order to see this very simply it may be of interest to consider the case of P being small and equal to S. In that case the condition simply becomes

$$R' > -\{\frac{U''.(1+R+AR') + U'AR''}{2U}\} \tag{9}$$

Diagrammatically, the result is demonstrated for this last case in Diagram 1.

The individual starts with wealth $A_1 > A^*$. In the absence of a gamble he ends up with a final wealth W_1 and utility level U_1. If he participates in a fair lottery as described above, he has a 0.5 probability of ending up with final wealth W_2 and utility level U_2 ($A_2 = A_1 - P$) and a 0.5 probability of ending up with final wealth W_3 and utility level U_3 ($A_3 = A_1 + S$ and $S = P$), clearly, in the diagram the expected utility of the gamble, given by HJ (being the level of U at the middle point of the chord XY) is greater than the utility of no gambling, given by KL. Here the risk averse individual is seen to (strictly) prefer participation in a fair lottery to no gamble. Hence, he is prepared to pay a price for participating in the lottery. The lottery need not be fair.

It will be noted that, whilst we have assumed that $R(\cdot)$ is twice differentiable everywhere, this is not necessary for our results. The rate of return may, in fact, show jumps as, say, enough wealth is obtained to carry out a given project which was not feasible previously. Thus, we can consider the case where the rate of return is always given by some constant, but this constant value is different for different wealth levels, with higher (constant) rates of return being associated with higher wealth levels. As a result of the existence of jumps in the rate of return, the wealth function has jumps as is shown in Diagram 2. That our results are still valid in this case is shown in Diagram 2 which is self-explanatory since it uses the notation of Diagram 1.

Our analysis also indicates that an individual may simultaneously participate in gambling and purchase insurance. This conclusion can be obtained using either Diagram 1 or Diagram 2. In Diagram 1 the rate of return is constant for wealth levels below A^*, thus implying that over this initial range the individual's utility function is concave with respect to initial wealth. Over the subsequent range, however, R is an increasing function of A so that U may be convex in A. Finally, as A increases even further, the capital market constraints become ineffective and again R is a constant (and may eventually decrease with A) so that over this range U is concave in A. This pattern of the utility function is similar to the one suggested by Friedman and Savage and clearly allows for simultaneous insurance and gambling. Here, however the individual is globally risk averse.

The same conclusions are obtained using Diagram 2. The existence of the jumps in the rates of return implies the existence of jumps in the

Diagram 1

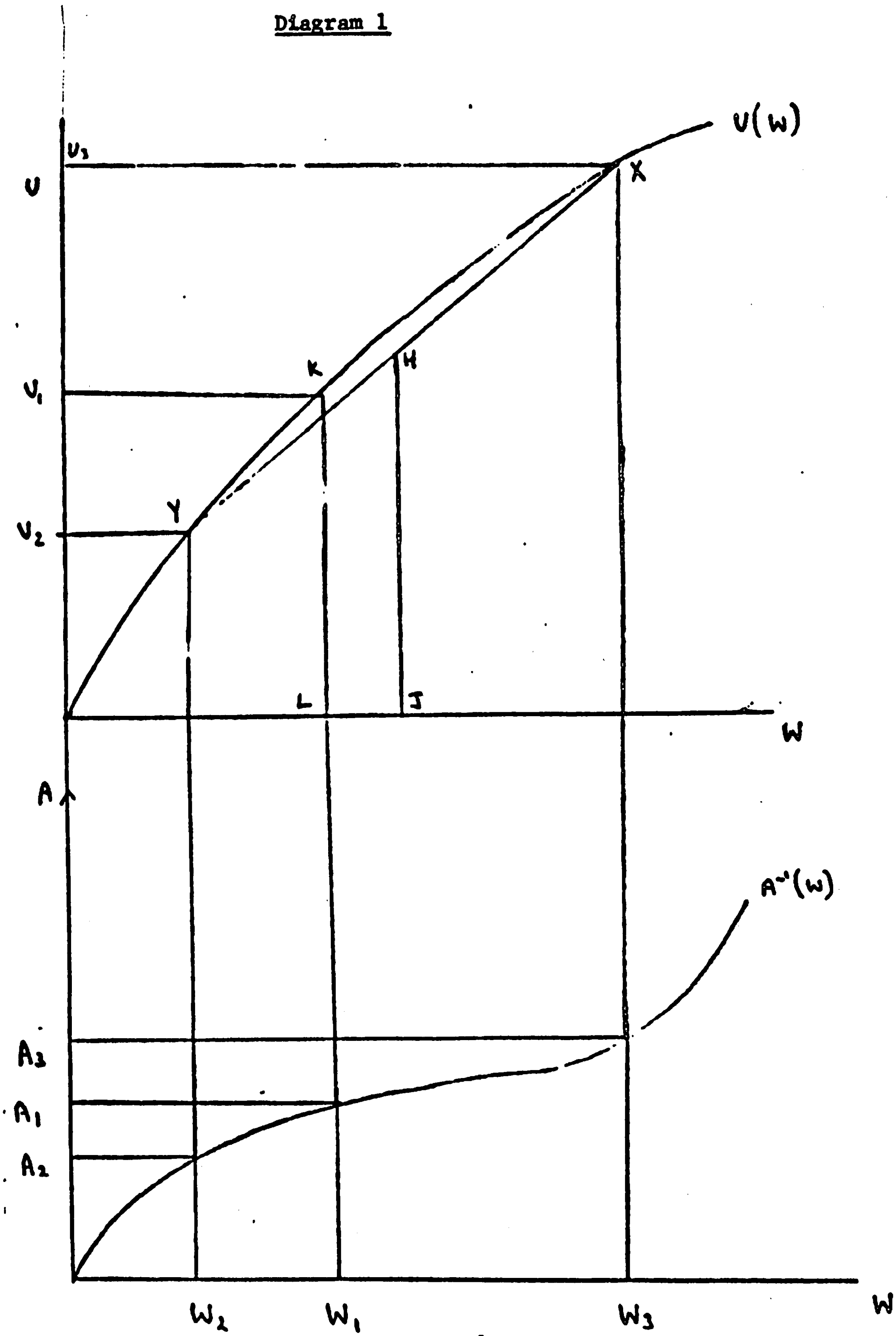


Diagram 2

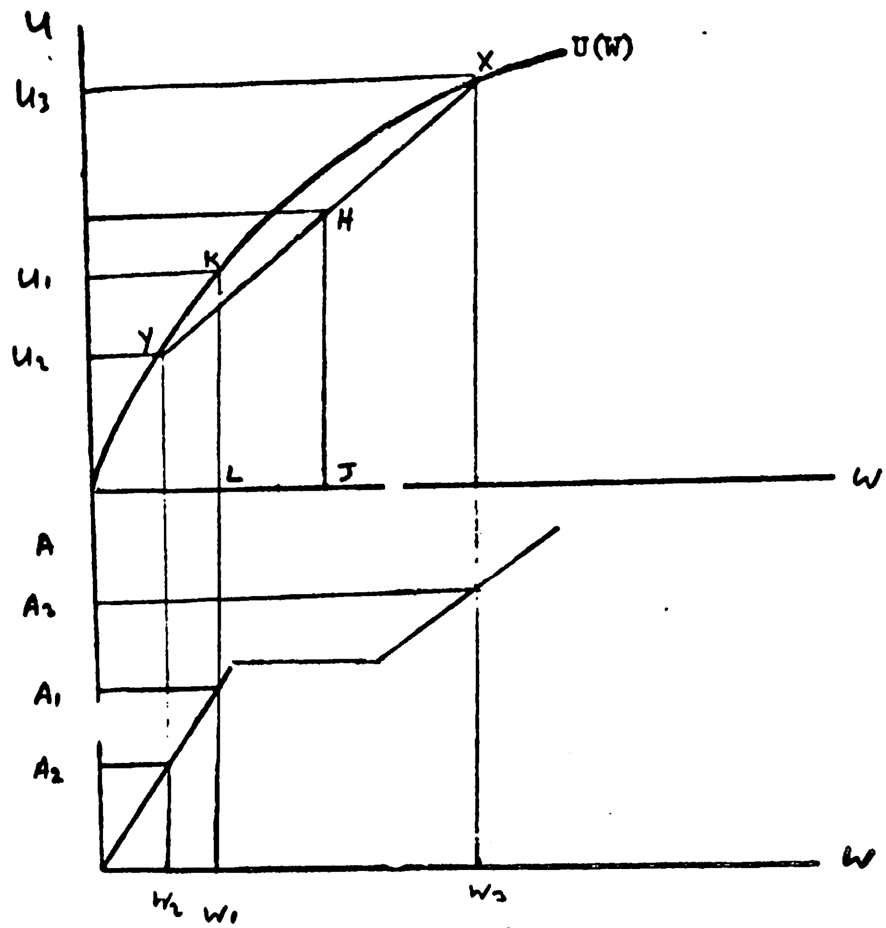
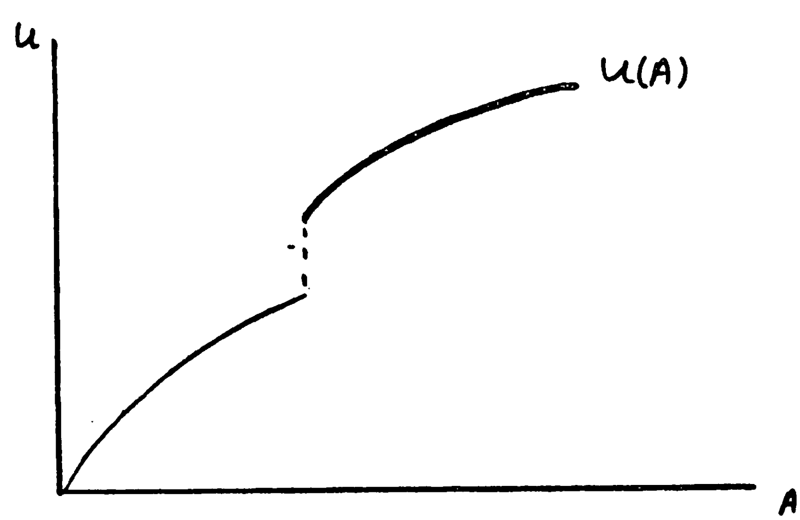


Diagram 3



individual's utility function as a function of initial wealth. This is seen in Diagram 3. While the individual's utility function here, is composed of two (or more) concave segments, it is nevertheless not concave in A globally, thus again allowing for simultaneous gambling and insurance.

Hence, the theory put forward is capable of explaining the phenomenon explained by Friedman and Savage of simultaneous gambling and insurance without, however, relaxing in any way the assumption of a diminishing marginal utility of wealth.

An alternative way of illustrating these results is to employ the state preference approach and consider the effects of the capital market imperfections on the set of acceptable gambles.

Given the lottery described above we define the set of acceptable gambles as

$$T(A) \equiv \{P,S : qU[(A+S)(1+R(A+S))] + (1-q)\ U[(A-P)(1+R(A-P))] \geq U[A(1+R(A))]\} \tag{10}$$

i.e., it is the set of (P,S) that yield at least as much expected utility as the utility obtained from the initial wealth (i.e., (P,S) such that expression (5) is greater than (4)). The boundary of this set is the set of indifferent gambles, obtained by solving for those values of P and S that satisfy the constraint in (10) with strict equality. Define this iso-expected utility curve as

$$P = G(S;A) \tag{11}$$

As is well known[12] a necessary and sufficient condition for an individual with initial wealth A, not to engage in a fair bet is that the set of acceptable gambles is convex, i.e., the function G is concave. Let us consider the shape of the function G. It is easy to show that locally, at $S = P = 0$,

$$\frac{dP(0,A)}{dS} = \frac{q}{1-q} > 0 \tag{12}$$

$$\frac{dP^2(0,A)}{dS^2} = \frac{q}{(1-q)^2}\left[\frac{U''(\cdot)}{U'(\cdot)} + \frac{2R'(A)+AR''(A)}{[1+R(A)+AR'(A)]^2}\right](1+R(A)+AR'(A)) \tag{13}$$

where $U''(\cdot)$, $U'(\cdot)$ are evaluated at $A(1+R(A))$. Thus, locally at $S = P = 0$ the iso-expected utility curve is increasing but its curvature is unknown. The expression in the square brackets in (13) may be positive or negative depending on A and the exact characteristics of the rate of return function R(A). The set of acceptable gambles defined by (10) may or may not, therefore, be convex. Thus, the concavity of the utility function is neither necessary nor sufficient for the convexity of the acceptable gambles set. The conclusion, then, is that individuals may engage in unfair gambling even if their utility function is a concave function of wealth. An examination of (13) shows that there are two effects to consider: (i) the effect of an increase in wealth on marginal utility, i.e., the curvature of the utility function (ii) the effect of an increase in wealth on the marginal wealth, i.e., the curvature of the wealth function. The first effect $U''(\cdot)/U'(\cdot)$ is nothing but the (minus) Arrow-Pratt measure of (local) absolute risk aversion.[13] In our case, however, we cannot look at this risk aversion measure only; it is both effects that determine whether an individual, with initial wealth A, takes a fair gamble or not.

This suggests, that, it is more appropriate to look at a measure of the curvature of the iso-expected utility curve, rather than usual risk aversion measures. The two measures will be the same if the rate of return effect vanishes. Otherwise the use of the risk aversion measure may yield wrong results.

This result in effect says that two pieces of information are important in explaining individuals' behavior; their preferences and their constraints. The traditional measure of risk aversion provides information about preferences only and, therefore, is not sufficient to explain individuals' behavior. When we observe an individual's behavior, it may look as if he is risk averse or risk loving, where in effect his actual behavior reflects both his constraints and preferences. Thus we may get risk averse individuals engaging in unfair gambling.

It is clear that as the effect of the increase in returns becomes stronger (as a result of market imperfections) the iso-expected utility curve becomes locally, less concave, (its slope however remains the same at $P=S=0$). Thus we can conclude that the set of acceptable gambles will be enlarged and will, locally, contain the set that corresponds to no market imperfections. Therefore, the individual will be willing to accept additional (previously undesirable) gambles, that is, take higher risks.

Finally, it is important to note that most of the results are local, in the sense that they correspond to a given level of initial wealth. As the level of wealth changes the individual may change his behavior even if his preferences remain the same. In general we would expect the capital market constraints to become less important as wealth increases, thus increasing the importance of the risk aversion effect. Beyond a certain point the individual may become a monopolist, and again there will be two effects to consider. In this latter range, however, market imperfections will clearly reinforce risk aversion. Indeed, risk aversion is, at these levels of wealth, not necessary for fair gambles to be rejected.

4. Uncertain Returns

So far it was assumed that the rate of return was non-random. This was an appealing framework, since it enabled us to concentrate on the effects of market constraints without having to worry about two types of uncertainty; the outcome of the lottery and the outcome of the investment.

In this section we allow for uncertain rates of return. We now have two gambles; the lottery and the investment, and an individual, even if he is risk averse, may want to buy a lottery in order to, what we may loosely speaking call, improve the odds in his investment gamble. The crucial point here is that due to the capital market imperfections, the outcome of the second gamble is not independent of the outcome of the first. To explain individuals' behavior it is, therefore, necessary to examine both gambles, otherwise, rational behavior may seem as if it is irrational.

In view of the above analysis it is clear that the probability distribution function of the rate of return depends on the level of wealth. In other words, it is the conditional distribution that we have to consider, rather than the marginal.

Let R be a random variable whose conditional density function is given by $f(R/A)$. This of course implies that in general, A affects the whole distribution of R. The simplest way to capture this effect is, as was mentioned in section 2, to assume that A increases the mean of the distribution.

In general we can assume that as A increases (within some range) the conditional distribution of R becomes more stochastically dominant. In other words: if $A_2 > A_1$ then $f(R/A_2)$ stochastically dominates $f(R/A_1)$, where by stochastic dominance[14] we mean that $\int_a^r f(R/A_1)\, dR \geq \int_a^r f(R/A_2)\, dR$ for all $a \leq r \leq b$ and $[a,b]$ is the interval over which R is distributed. This general effect of A, does include the effect on the mean as a special case, but also allows for effects on other characteristics of the distribution.

The individual, who is again assumed to be risk averse, has an initial wealth A, all or part of which he can invest in an uncertain project whose conditional distribution is given by $f(R/A)$. In general there will be more than just one possible investment project, but for the sake of simplicity and in order to separate the gambling problem from the many asset portfolio selection problem, we consider one possible investment prospect only.

With no lottery the individual allocates his initial wealth between investment in the risky asset, X, and cash $A - X$, where $0 \le X \le A$. His final wealth is given by $W = (A-X) + (1+R(A))X = A + R(A)X$, and his problem is

$$\max_{X} \{E[U(A+R(A)X)] : 0 \le X \le A,\ R\sim f(R/A)\} \tag{14}$$

This is the well-known (simple) portfolio choice problem (See Arrow [1965]), which has three possible solutions

$$\begin{array}{rlll} \text{(i)} & X^* = 0 & \text{if} & E[U'(A)R(A)] \le 0 \\ \text{(ii)} & X^* = A & \text{if} & E[U'(A+R(A)A)R(A)] \ge 0 \\ \text{(iii)} & 0 < X^* < A & \text{if} & E[U'(A+R(A)X^*)R(A)] = 0 \end{array} \tag{15}$$

where X^* is the optimal solution. Cases (i) and (ii) give boundary solutions and case (iii) gives an interior solution where the individual diversifies. In general we can expect to get the interior solution.

Now suppose the individual has the choice of participating in the lottery described above. Clearly, he will participate in the lottery if and only if his expected utility given by the solution to (14) is smaller than the maximum expected utility from the joint investment-lottery gamble. The final wealth of such a double gamble is given by

$$W' = (A+Y) + R(A+Y)X \tag{16}$$

where Y is a random variable describing the outcome of the lottery, i.e., its distribution is given by

$$Y = [{}^{S}_{-P} \quad {}^{q}_{1-q} \tag{17}$$

The corresponding expected utility is, therefore,

$$\begin{aligned} E[U(W')] = q \int_a^b U[(A+S) + R(A+S)X]\ f(R/A+S)\ dR \\ + (1-q) \int_a^b U[(A-P) + R(A-P)X]\ f(R/A-P)\ dR \end{aligned} \tag{18}$$

Let the maximum expected utility in problem (14) (no lottery) be given by E^o. Then an individual will buy a lottery ticket iff

$$E' \equiv \max \{E[U(W')] : 0 \leq X \leq A,\ R \sim f(R/A)\} > E^o \tag{19}$$

Suppose the solution to (14) is given by X* as in (15). Now, consider all those (P,S) that yield at least as high an expected utility as E^o, holding X fixed at X*. Since by assumption the market imperfections make stochastic dominance an increasing function of wealth, we can apply our previous analysis and conclude that the iso-expected utility curve in (P,S) space need not be concave at P = S = 0.[15] Thus, given X*, gambling is possible. Furthermore, if gambling is a possibility when X is fixed at X*, it must clearly be a possibility when we allow X to vary optimally. Therefore, we conclude that even if the individual is risk averse, he may still participate in unfair gambling, since by doing so he increases the stochastic dominance of the rate of return distribution function, i.e., improves the odds on his investment gamble. Whether or not this occurs depends on the degree of risk aversion, the properties of the conditional distribution and the initial wealth. As before, we have two effects; the increase in stochastic dominance and the risk aversion, and the question is which is the dominant effect. Neither single effect is necessary nor sufficient for explaining gambling.

It should be noted that the possibility of participation in a lottery changes the individuals optimal behavior. In other words his lottery and investment choices are interdependent. For example, even if in the absence of a lottery the individual chooses not to invest, he may change his behavior if a lottery is available. The reason being, of course, that the additional gamble may improve the odds on his investment gamble. If on the other hand, capital markets are perfect in the sense that the rates of return do not depend on asset holdings, or alternatively, A is large enough so that capital market constraints are ineffective, then the individual will not engage in a fair gamble if he is risk averse.[16]

5. Conclusion

This paper provides possible explanations for participation in unfair gambling by risk averse individuals. It suggests that the existence of market imperfections, in particular in the capital markets, may impose various constraints on individuals and thus effect their behavior. The existence of these constraints could lead individuals to participate in unfair gambling, since this may reduce the implicit costs of the constraints.

Our explanation of gambling focuses on the constraints facing an individual rather than his preferences and consequently, it is clear that preferences alone are not sufficient to explain his behavior. This, of course, is not to say that we reject the Friedman-Savage hypothesis, but to suggest that under certain circumstances it may not be able to predict correctly.

Footnotes

[1]The word individual is, in general, used to denote economic units, including (especially) small/medium size firms.

[2]The term unfair lottery in the paper encompasses all investment opportunities offering acturially unfair risks.

[3]See Rothschild [1973] Spence [1973].

[4]See Stiglitz [1975] Spence [1973].

[5]Thus, for example, the additional services of capital that enable asset holders to overcome accessibility constraints, are explicitly discussed by Appelbaum and Harris [1978], who consider an investment problem of a firm facing credit rationing or borrowing constraints. They show that under such circumstances the constrained firm may over-accumulate capital (relative to a firm in a perfect capital market), since capital now plays an additional role by reducing the implicit costs of the borrowing constraints. By augmenting its capital stock in earlier periods the firm, therefore, reduces the amount by which it is bounded in subsequent periods, since the capital stock serves in effect as collateral against which it can borrow.

[6]See Hanoch and Levi [1969].

[7]This type of return function is discussed also in Blinder [1974] and Appelbaum and Harris [1978b].

[8]It does, however, seem reasonable that at least over some initial range (when R is not constant) $R'' > 0$ i.e., the function is convex. The convexity of R(A) at the initial range would, for example, follow from the convexity of the loan supply function derived by Jaffée and Modigliani [1968], Barro [1976] and Benjamin [1978].

[9]See, for example, Arrow [1965], [1970].

[10]The assumption of an instantaneous lottery is not necessary for our results and is made in the interests of expositional simplicity. The more general formulation would view the time elapsing between the purchase of the lottery ticket and the payment of the prize as (1/n)th of the period involved. This would introduce the complication arising from the fact that investment in a lottery may have an opportunity cost, for the duration of the lottery, in terms of ordinary investment possibilities. To avoid this modification, which does not alter the substance of our results, n has been set at infinity in our model.

[11]In general,

$$U[(A+B)(1+R(A+B))] = U + B\ [(1+R)U' + AR'U']$$
$$+ \tfrac{1}{2} B^2([\hat{U}''(1+\hat{R}+A\hat{R}')] + \hat{U}'[2\hat{R}' + A\hat{R}''])$$

where R, R', U and U' are valued at A and $\hat{R}$, $\hat{R}$, $\hat{R}''$, $\hat{U}'$ and $\hat{U}''$ are valued at $A + \gamma B$ and $0 \leq \gamma \leq 1$.

[12]See Arrow [1970] Green [1976].

[13]See Arrow [1970] Pratt [1964].

[14]See Hanoch and Levy [1969].

[15]This can be shown directly by examining the local curvature of $E[U(W')]$ with respect to (P,S). Using integration by parts and remembering that $\partial[\int f(R/A)\, dR]/\partial A > 0$ it can be shown that concavity of U is neither necessary nor sufficient for the concavity of $E[U(W')]$.

[16]It may be interesting to note that, we treated the asset and cash equally in terms of their effect on the conditional distribution of R. In general the effect may be different and will depend on the extent to which the two can "relax" the capital market constraints. For a discussion of these and related issues see Appelbaum and Katz [1978].

References

Appelbaum, E. and Harris, R. G. [1978a], "Optimal Capital Policy with Bounded Investment Plans," International Economic Review, 103-114.

Appelbaum, E. and Harris, R. G., [1978b], "Imperfect Capital Markets and Life-Cycle Saving," Canadian Journal of Economics, 11, 319-324.

Appelbaum, E. and Katz, E. [1978], "Portfolio Choice with Imperfect Capital Markets," Discussion paper, Department of Economics, University of Western Ontario.

Arrow, K. J., [1965], Aspects of the Theory of Risk-Bearing (Helsinki).

Arrow, K. J., [1970], Essays in the Theory of Risk-Bearing Chicago: Markham Publishing Co.

Barro, R. J. [1976], "The Loan Market, Collateral, and Rates of Interest," Journal of Money Credit and Banking, 439-56.

Benjamin, K. D. [1978], "The Use of Collateral to Enforce Debt Contracts," Economic Inquiry, 333-359.

Blinder, A. [1974], Toward an Economic Theory of Income Distribution, (Cambridge, Mass. MIT Press).

Eckstein, O. [1961], "A Survey of the Theory of Public Expenditure, Reply," in Conference of the Universities, National Bureau Committee for Economic Research, Public Finance: Needs, Sources and Utilization (Princeton: Princeton University Press).

Friedman, M. And Savage, L. J., [1948], "The Utility Analysis of Choices Involving Risk," Journal of Political Economy.

Green, J. H. A. [1976], Consumer Theory, The Macmillan Press Ltd.

Hanoch, G. and Levy, H. [1969], "The Efficiency Analysis of Choice Involving Risk," Review of Economic Studies.

Jaffée, D. M. and Modigliani, F. [1969], "A Theory and Test of Credit Rationing," American Economic Review, 850-872.

Nerlove, M. [1968], "Factors Affecting Differences Among Rates of Return on Investment in Individual Common Stocks," Review of Economics and Statistics, 312-331.

Pratt, J. [1964], "Risk Aversion in the Small and in the Large," Econometrica 32, 1-2.

Rothschild, M. [1973], "Models of Market Organization with Imperfect Information: A Survey," Journal of Political Economy, 1283-1308.

Spence, M. [1973], "Job Market Signaling," Quarterly Journal of Economics, 355-74.

Stiglitz, J. F. [1975], "The Theory of 'Screening', Education and the Distribution of Income," American Economic Review, 283-300.