

1979

# Secular Inflation under Endogenous National Money Supplies: An Adaptation and Extension of the Cagan-Bailey Model

Geoffrey Kingston

Follow this and additional works at: <https://ir.lib.uwo.ca/economicsresrpt>

 Part of the [Economics Commons](#)

---

## Citation of this paper:

Kingston, Geoffrey. "Secular Inflation under Endogenous National Money Supplies: An Adaptation and Extension of the Cagan-Bailey Model." Department of Economics Research Reports, 7905. London, ON: Department of Economics, University of Western Ontario (1979).

20146

ISSN: 0318-725X  
ISBN: 0-7714-0079-9

RESEARCH REPORT 7905

SECULAR INFLATION UNDER ENDOGENOUS  
NATIONAL MONEY SUPPLIES: AN ADAPTATION  
AND EXTENSION OF THE CAGAN-BAILEY MODEL

by

Geoffrey Kingston

January, 1979

SECULAR INFLATION UNDER ENDOGENOUS NATIONAL MONEY SUPPLIES:  
AN ADAPTATION AND EXTENSION OF THE CAGAN-BAILEY MODEL\*

Geoffrey Kingston  
Department of Economics  
The University of Western Ontario

January 1979

\* This paper is based on part of Chapter IV of my doctoral dissertation. I wish to thank my supervisor, Professor S. J. Turnovsky, and my examination committee, Professors D. W. Henderson and J. M. Parkin. The usual disclaimer applies.

## ABSTRACT

This paper investigates the secular effects of exogenously-determined national budget deficits on endogenously-determined growth rates of money and prices, real per capita levels of international reserves, and percentage movements in exchange rates. Therefore it departs from the traditional Cagan-Bailey analysis (extended to  $n$  countries by Mundell (1972)), which postulates an exogenous growth rate of money.

One interesting finding is that unless the country in question is on the periphery of a reserve-currency system, a given percentage rise in the real per capita domestic deficit will generally induce an amplified increase in the growth rates of domestic money and prices. In particular, and to a first approximation, the response will exceed proportionality by the absolute interest elasticity of domestic money demand.

## 1. INTRODUCTION AND SUMMARY

The theoretical literature provides straightforward answers to questions of the general form: what are the long-run effects on inflation at home and abroad of a sustained increase in monetary growth at home? But it has less to say on questions of the kind: what are the long-run effects on inflation and monetary growth at home and abroad of a sustained increase in the gap between public purchases of home output and net explicit taxes on home incomes? The present paper addresses questions of this sort, along with various related questions concerning the secular effects of exogenous increases in government deficits. To this end, a model is presented which utilizes essentially the Cagan (1956) specification of money demand. Following Bailey (1956), the emphasis is on comparative steady states, although stability issues also receive attention. In common with both the foregoing contributions, real per capita output is assumed fixed.

Apart from an extension of the analysis to the multi-country case, and a more explicit treatment of the role of bonds and (to a lesser extent) of equity, the main departure from the Cagan-Bailey analysis is extensive use of the Dutton (1971)-Sargent and Wallace (1973) hypothesis concerning the relationship between money supply growth and government deficits. On this view, public sectors choose to peg the gap between their real per capita purchases and net explicit tax receipts. Under fixed real per capita output, and if real per capita government purchases are also pegged, then the hypothesis in question is equivalent to the standard macroeconomic assumption that the rate of tax on factor incomes is pegged. In this sense the hypothesis is just a standard element of conventional macro theory.

If noninterest-bearing debt (i.e., high-powered money) is the only asset issued by the government, then the rate of money issue must be permitted to be endogenous. Even in the event that a government also issues bonds, it

turns out that nominal monetary growth can be pegged only if the real rate of return on bonds permanently diverges from the golden rule.<sup>1</sup> The present paper assumes that there is no such divergence (see Section 2), so that endogenous monetary growth is retained, as postulated by the original Dutton-Sargent-Wallace hypothesis. The golden rule is not part of conventional positive macro theory. Yet it has several attractive features in a positive (as well as a normative) context.

Another key assumption of the present paper is that inflation and monetary growth rates are "moderate" in the sense that they are not sufficiently high to drive the absolute value of the interest elasticity of money demand above unity. The reason is that any fixed government deficit which is feasible, in the sense of not requiring more than the maximum attainable revenue from money creation, is generally associated with two equilibria, one above and the other below the point on the money demand schedule at which the interest elasticity of money demand is unity.<sup>2</sup> It turns out that both the stability and the steady-state characteristics of the "hyperinflation" equilibrium are anomalous; see Section 2. Moreover, the chief motivation for the analysis is the "moderate" postwar inflation of the Western industrial countries. Hence our focus on the other equilibrium.

The main assumptions of the open-economy analysis are as follows. First, for simplicity we focus on the two-country case, (see Section 3), although further extension to the n-country case is straightforward (see Appendix). Second, in the fixed-rates case we postulate a reserve-currency standard whereby non-reserve-currency countries--"peripheral" countries--hold all the reserves required to defend the rate, for this purpose drawing on a stockpile held in the form of interest-bearing debt issued by the public sector of the reserve-currency country. For evidence that the predominant international asset in postwar official portfolios has been the interest-bearing debt of the United States public sector, see, e.g.,

the surveys by Whitman (1974), (1975). And for future reference, recall theoretical investigations showing that such a regime leads to strong national asymmetries in the international transmission of monetary disturbances; see, e.g., Roper (1973), Girton and Henderson (1976), and Girton and Roper (1977). Third, in keeping with the present paper's long-run orientation, deficit finance policies are specified in a way which rules out "quasi equilibria" of the kind analyzed by, e.g., Mundell (1968) and Swoboda (1972). That is, it is not possible for a public sector permanently to finance part of its deficit by foreign borrowing. A sufficient condition to rule out this case is that in each country, the proportion of the government deficit financed by money creation is stationary (see Section 3). Fourth, a single exogenously-given rate of population and real output growth is assumed to prevail in all countries.

The main results of the analysis concern comparative steady states, and can be summarized as follows:

1. The long-run inflationary and balance-of-payments effects of increased budget deficits are independent of: (i) relative economic size; (ii) the extent to which goods and capital markets are integrated internationally, providing such integration has proceeded past a certain level; (iii) the share of money in deficit finance at home and abroad, again provided that the parameters in question exceed certain critical minima; and (iv) the extent to which peripheral countries attempt to sterilize balance-of-payments surpluses, providing such policies aim for less than complete sterilization. These provisos will be recognized as being necessary merely to ensure stability.

2. Either in a reserve-currency country, or in any country under flexible exchange rates, higher budget deficits will induce a more than proportional increase in the own-country rates of monetary and price growth. To a first approximation, the increase is given by unity plus the absolute value of the domestic interest elasticity of money demand. This turns out to be the most surprising result, inasmuch as there would appear to be no counterpart in the traditional theory of the secular domestic and foreign effects of monetary disturbances.<sup>3</sup>

3. Price and monetary growth rates abroad will increase *pari passu* or remain unchanged according as whether the exchange rate is fixed or flexible. In contrast to the preceding result, here we have exact analogues of familiar propositions in the relevant monetary theory.

4. Higher budget deficits in peripheral countries will not affect rates of price and monetary growth anywhere. This too is an analogue of a known monetary finding. In particular, recall the foregoing discussion of the literature on reserve-currency standards with interest-bearing international reserves.

5. If the public sector of a reserve-currency country or any country under flexible rates were to opt for a pegged rate of monetary growth, then it would also have to unpeg its fiscal deficit (cf. the foregoing discussion of the closed-economy case); and if the public sector of a peripheral country were to peg its international reserve ratio (i.e., the proportional international backing of the monetary base), then it too would have to unpeg its deficit. Thus national public sectors have fewer degrees of freedom than might be supposed.<sup>4</sup>



6. Successive increases in the budget deficit of a peripheral country will induce successive reductions in that country's real per capita international reserves, paralleling the familiar monetary result on the effect of exogenous increases in domestic credit. On the other hand, successive increases in the budget deficit of a reserve-currency country will first raise, then reduce, international liquidity in real intensive terms. The real per capita international reserves of any given peripheral country are maximized at the point where its interest elasticity of money demand and international reserve ratio together sum to unity. Finally, an increase in the budget deficit of any flexible-rate country will raise the rate at which its exchange rate depreciates (or slow down the rate of appreciation).

## 2. A CLOSED ECONOMY

This section sets out and analyzes a single-country model which will constitute the building block for the multi-country analysis of the sequel. Clearly, such a model must be simple. Accordingly, in steady state the closed-economy model reduces to just two equations. One describes portfolio equilibrium, and the other links the growth rate of money to the government deficit. These equations jointly determine the steady states of real intensive money balances and nominal monetary growth.

### The Model

Let  $m \equiv M/PN$  = monetized public debt in real per capita terms, so that  $M$  is the nominal monetary base, and  $P$  and  $N$  denote price and population levels respectively. Let  $\mu \equiv \dot{M}/M$  = rate of nominal monetary growth. Denote the pegged states and steady states of variables by an overbar and an

asterisk respectively. In steady state,  $m$  and  $\mu$  are determined by the pair of equations

$$m^* = k \exp(-\alpha \mu^*) \quad k, \alpha > 0 \quad (1)$$

$$\mu^* = \bar{\delta}/m^* \quad (2)$$

Here  $\delta \equiv G/PN - T/PN \equiv g - t =$  real per capita flow of government purchases,  $g$ , minus the real per capita flow of net explicit tax receipts,  $t$ .  $\delta$  will be termed the "explicit" deficit. That this variable is assumed pegged, with an endogenous growth rate of money, is the main difference between the above system and its Cagan-Bailey counterpart. For a quick preview of how this modification of the conventional model is reflected in the standard diagrammatic presentation--of the welfare costs of anticipated inflation and so on--the reader is referred forward to Figure 1. At this stage, however, it is expositionally convenient to proceed to a short-run counterpart of the system. This clarifies the "behind-the-scene" role of bonds, equity and stability in the subsequent long-run analysis (our main concern).

Assume that the assets in the economy are money, bonds, and equity. Bonds are traded at par<sup>5</sup> and are perfect substitutes for equity. Their common rate of return is  $r$  in ex post nominal terms, and  $\rho \equiv r - \pi$  in ex post real terms, whence we have  $\pi \equiv \dot{P}/P =$  ex post rate of inflation. Denote ex ante magnitudes by a superscript (<sup>e</sup>), so that  $r$ ,  $\pi$  and  $\rho$  have ex ante counterparts  $r^e$ ,  $\pi^e$ , and  $\rho^e$  respectively. Postulate the following short-run counterpart of (1):

$$m = k \exp(-\alpha r^e) \quad (3)$$

which of course is closely related to the point-in-time condition for money-market equilibrium in the Cagan model.

Next, suppose that both the anticipated and actual real rates of return are tied to the exogenously-given golden-rule rate of return:

$$\rho^e = \rho = \gamma \quad (4)$$

where  $\gamma \equiv \dot{N}/N$  = exogenously-given rate of population growth. Clearly, this equation represents a strong assumption, and it warrants further comment.

The assumption is essential only in the steady state, so that justification of (4) can be confined to its steady-state counterpart  $\rho^{e*} = \rho^* = \gamma$ .

In this context it is known that a positive optimizing analysis which postulates fairly general utility and production functions for the private sector, and a Friedman (1969)-style helicopter for the public sector, will yield the modified golden rule<sup>6</sup> in steady state; see Sidrauski (1967). This suggests that if the public sector were also to purchase output, raise explicit taxes and issue bonds, yet on a scale which is sufficiently "small" in the sense that its activities do not significantly affect the real rate of return on equity, then the modified golden rule would continue to hold in steady state.

The foregoing argument is not as satisfactory as would be an explicit reformulation and extension of the Sidrauski model, but such an exercise would be exceedingly difficult even for a single-country world. Two other arguments for (4) are as follows. First, in contrast to some models it ensures that the secular nominal return on bonds will reflect the Fisher effect, in line with the findings of, e.g., Fama and Schwert (1977) on the postwar rates of return yielded by U.S. treasury bills. Second, it enables one to avoid resorting to a popular but questionable alternative simplifying assumption, namely, that debt service--and no other class of government outlays-- is wholly financed at each instant by explicit taxes.<sup>7</sup>

Equations (3) and (4) together imply an alternative short-run counterpart of the steady-state portfolio balance condition (1), namely,

$$m = k \exp[-\alpha(\gamma + \pi^e)] \quad (1)'$$

which turns out to be more useful than (3) in the subsequent stability analysis (see below).

Consider next the question of a suitable instantaneous counterpart of the steady-state money supply rule (2). In nominal terms, the government budget constraint is

$$\dot{M} + \dot{B} = G - T + rB \quad (5)$$

where  $B$  is the nominal stock of interest-bearing public debt. Define the share of money in deficit finance,  $\theta$ , by  $\theta \equiv \dot{M}/(\dot{M} + \dot{B})$ , and follow various authors<sup>8</sup> in assuming that this variable is pegged at some value between zero and unity, so that  $\theta = \bar{\theta}$ ,  $0 < \bar{\theta} < 1$ . Deflate (5) by  $PN$  and recall the definition of  $\delta$ , and equation (4), which yields:

$$\dot{m} + \dot{b} = \delta - (\pi + \gamma)m.$$

Introduce the initial condition  $m_0/(m_0 + b_0) = \bar{\theta}$  (a simplifying assumption which is innocuous in the present context), and recall that  $\delta$  is assumed pegged, which enables us to rewrite the foregoing equation as:

$$\dot{m} = \bar{\theta}[\bar{\delta} - (\pi + \gamma)m]. \quad (2)'$$

This turns out to be the most useful short-run counterpart of (2) for our purposes. However it is worth pointing out that the familiar identity<sup>9</sup>

$$\mu \equiv \frac{\dot{m}}{m} + \pi + \gamma \quad (6)$$

leads to the following equivalent of (2)':

$$\mu = \bar{\theta} \frac{\bar{\delta}}{m} + (1 - \bar{\theta})(\pi + \gamma).$$

That is, equation (2)', the assumed short-run counterpart of the money supply rule (2), asserts that the rate of monetary growth at each instant is a fixed-weighted average of the explicit deficit expressed as a proportion of the monetary base,  $\bar{\delta}/m$ , and the inflation-cum-growth rate,  $\pi + \gamma$ . Note that the short-run, proximate contribution of the normalized explicit deficit to nominal monetary growth could be quite small--in marked contrast to the long-run relationship (2).

To close the short-run model it is necessary to explain the formation of inflation expectations. In this regard we require an hypothesis which is (i) simple; (ii) consistent with rational expectations at least in a long-run sense, i.e.,  $\pi^{e*} = \pi^*$ ; and (iii) not destabilizing in the neighborhood of the equilibrium associated with "moderate" inflation, i.e., with equilibrium monetary growth falling short of the revenue-maximizing rate. The upshot of a process of elimination<sup>10</sup> is that we retain (without enthusiasm) the traditional adaptive specification:

$$\dot{\pi}^e = \beta(\pi - \pi^e), \quad \beta > 0. \quad (7)$$

The preferred short-run counterpart of the system (1) - (2), therefore, is summarized by equations (1)', (2)', and (7). The associated endogenous variables are the anticipated and actual rates of inflation, i.e.,  $\pi^e$  and  $\pi$  respectively, and the real per capita money stock,  $m$ .

### Stability

Equations (1)', (2)' and (7) are easily reduced to the following first-order equation in terms of  $m$ :

$$\dot{m} = - \frac{\beta \bar{\theta}}{\bar{\theta} - \alpha\beta} [\alpha\bar{\delta} + m \log(\frac{m}{k})] . \quad (8)$$

Linearize (8) in the neighborhood of equilibrium, recall (1)', and note the relationships  $\gamma + \pi^* = \rho^* + \pi^* = r^*$ , to obtain the following eigenvalue ( $\lambda$ ):

$$\lambda = - \frac{\beta \bar{\theta} (1 - \alpha r^*)}{\bar{\theta} - \alpha \beta} \quad (9)$$

It follows that under the parameter restrictions assumed thus far, the system is locally stable if and only if

$$\left. \begin{array}{l} \alpha r^* < 1 \\ \alpha \beta < \bar{\theta} \end{array} \right\} \quad (10a)$$

or

$$\left. \begin{array}{l} \alpha r^* > 1 \\ \alpha \beta > \bar{\theta} \end{array} \right\} \quad (10b)$$

That is, the system is locally stable if and only if either the absolute value of the interest elasticity of money demand is less than unity, and the product of the expectational and money-demand parameters is less than money's share in the deficit; or if both these inequalities are reversed.

Noting the steady-state relationship  $r^* = \mu^*$ , the condition  $\alpha r^* < 1$  can also be interpreted as stating that the equilibrium rate of nominal money creation,  $\mu^*$ , is less than that associated with maximal revenue from money creation,  $1/\alpha$ . (For more on this unit-elasticity formula, see, e.g., Cagan (1956), Bailey (1956), and Friedman (1971).) Noting the steady-state relationship  $\mu^* = \pi^* + \gamma$  (yielded by (6)), and since the present paper deals with "moderate" inflations, assume henceforth that conditions (10a) are satisfied, whence conditions (10b) are not satisfied. Accordingly, the present model will be stable insofar as the product of the money-demand and expectational parameters ( $\alpha\beta$ ) is small, and deficit finance is money intensive (i.e.,

$\bar{\theta}$  is large). Related models yield entirely similar conclusions; for more on the roles of  $\alpha\beta$  and  $\bar{\theta}$  in stability, see, e.g., Cagan (1956) and Dornbusch (1977) respectively.

### Steady States

We now turn to a detailed investigation of the steady-state equations (1) and (2). A preliminary observation is that the system in question is independent of the share of money in the deficit. Although this parameter has just been shown to exert a critical influence on the transitional states of nominal rates of inflation, interest and monetary growth, and real intensive money balances, it has no effect on the steady states of those variables. In particular, an increase in  $\bar{\theta}$  will not raise  $\mu^*$  or  $\pi^*$  ( $=\mu^*-\gamma$ ). For further discussion relevant to this counter-intuitive proposition, see Dornbusch (1977), who analyzes a system which can be viewed as a more general version of the present model, and concludes that "the growth rate of money...is not necessarily raised by a shift towards money finance" (p. 148). This is not to say that  $\bar{\theta}$  exerts no long-run influence whatsoever, since the fact that money and bonds must eventually stand in the relation  $m^* = \bar{\theta}(m^*+b^*)$  implies that  $(db^*/b^*)/(d\bar{\theta}/\bar{\theta}) = -1$ , that is, a shift towards money finance will induce a proportional decrease in the steady-state real per capita bond stock.

Consider next the geometrical representation of (1) and (2).

Figure 1 below is the relevant diagram; it is comparable to a diagram in Turnovsky (1978), although Turnovsky depicts a linear money demand function. Using his notation, the LL and GG schedules portray (1) and (2) respectively in  $(m^*, \mu^*)$  space. The LL schedule shows the combinations of  $m^*$  and  $\mu^*$  that are consistent with portfolio balance; it belongs to the exponential family. The GG schedule shows the combinations of  $m^*$  and  $\mu^*$  that are consistent with public-sector budget balance (taking into account the revenue from money creation) and is a rectangular hyperbola.

If the explicit deficit is positive yet less than its maximal value, i.e., if  $0 < \bar{\delta} < k/\alpha e$ , where  $k/\alpha e = \max_{\mu^*} [\mu^* m(\mu^*)]$ , then GG will cut LL twice, as shown in Figure 1. To see this, consider first the rate of monetary growth which induces a unitary interest elasticity of money demand, i.e.,  $\mu^* = 1/\alpha$ . Then the associated point on the GG schedule, call it R, must be to the left of its counterpart on the LL schedule, call it R', since  $\bar{\delta}$  is assumed to be less than maximal. Consider next the relative slopes of the schedules. It is easily shown that

$$\left. \frac{d\mu^*}{dm^*} \right|_{LL} \quad \lessgtr \quad \left. \frac{d\mu^*}{dm^*} \right|_{GG}$$

according as whether

$$\mu^* \quad \lessgtr \quad 1/\alpha.$$

That is, the LL schedule is steeper (flatter) than the GG schedule according as whether the absolute value of the interest elasticity of money demand is less than (greater than) unity.<sup>12</sup> It follows that GG must cut LL twice. Call these lower and upper cutpoints P and Q respectively.

Under our dynamic assumptions, P is the only rest point of the system-- recall the foregoing stability analysis. Hence the shaded rectangle in Figure 1 shows the public sector's "explicit" deficit--or, in conventional terminology, its steady-state revenue from money creation. Following later developments of the Cagan-Bailey analysis, e.g., Laidler (1977), this steady-state revenue,  $\mu^* m^*$ , can be split into an inflation-tax component,  $\pi^* m^*$ , and a growth-tax component,  $\gamma m^*$ . Similarly, the shaded triangle-like area in Figure 1 shows the deadweight loss which ensues when the public sector issues money at a rate faster than that prescribed by the full-liquidity rule.<sup>13</sup>



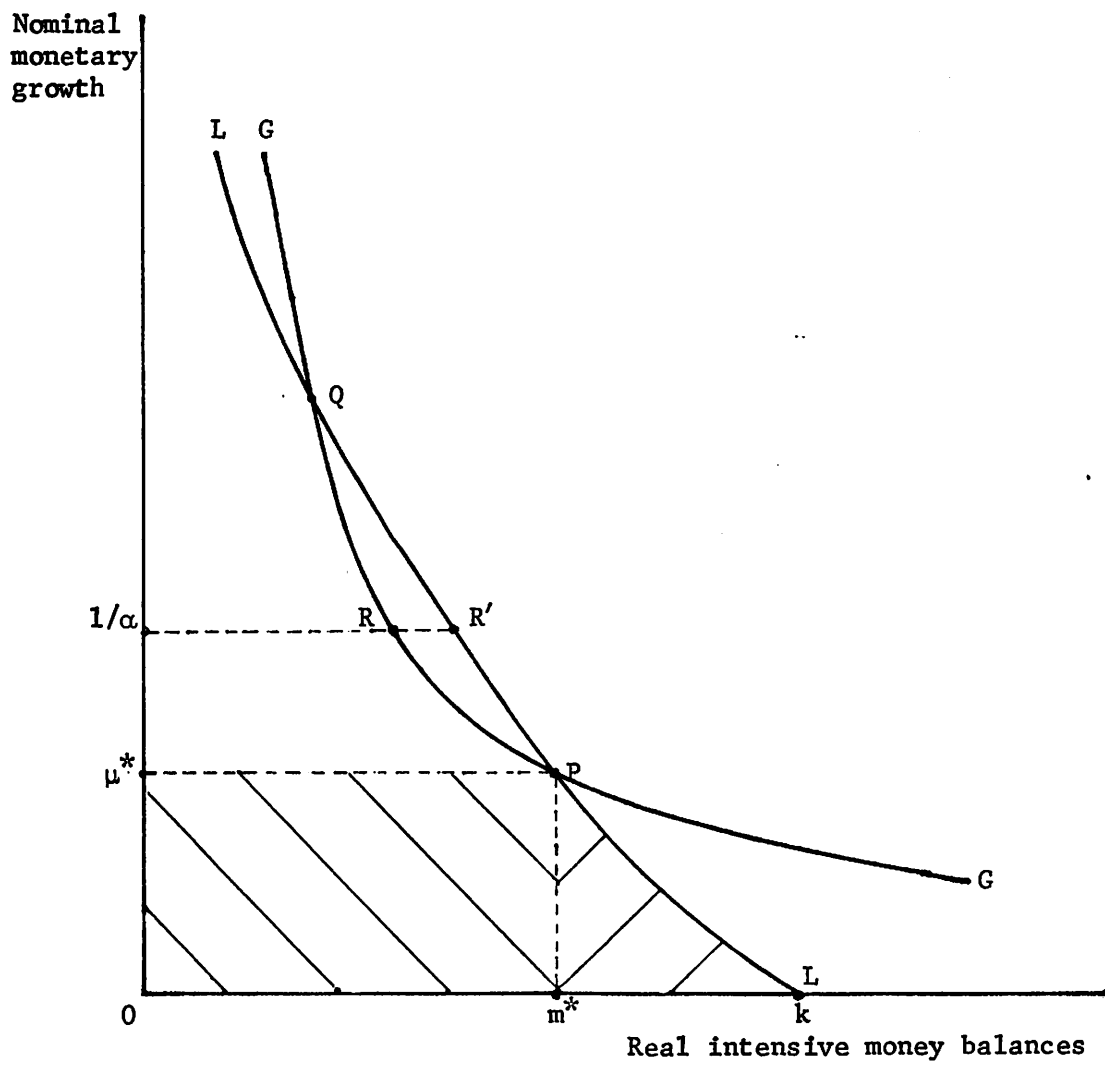


Figure 1

For points such as P, namely, for  $0 < \bar{\delta} < k/\alpha\epsilon$  and  $\mu^* < 1/\alpha$ , equations (1) and (2) have the following interesting implication:

$$\left. \frac{d\mu^*/\mu^*}{d\bar{\delta}/\bar{\delta}} \right|_{0 < \mu^* < 1/\alpha} = \frac{1}{1-\alpha\mu^*} > 1 \quad (11)$$

That is, the restoration of budget balance after an increase in the explicit deficit requires a more than proportional increase in the growth rate of money. Since  $\mu^* = \pi^* + \gamma$ , this holds a fortiori if the dependent variable is price growth rather than monetary growth, providing  $\gamma > 0$ . In either case, and in the terminology of Mundell (1972), there is a secondary "volume" effect in addition to the primary "value" effect. In more detail, from the expansion and decomposition

$$\frac{1}{1-\alpha\mu^*} = 1 + \alpha\mu^* + (\alpha\mu^*)^2 + \dots = 1 + \frac{\alpha\mu^*}{1-\alpha\mu^*}$$

it may be seen that the value effect--the required increase in the inflation-cum-growth tax rate for a given inflation-cum-growth tax base--is unity; and the volume effect--the additional increase called for by the induced reduction in the aforesaid tax base--is  $\alpha\mu^*$  approximately, and  $\alpha\mu^*/(1-\alpha\mu^*)$  exactly. The geometric representation of these two effects is straightforward, and may be left to the reader.

Although points such as P are the most interesting, four other cases also warrant attention. First, at points such as Q, namely, for  $0 < \bar{\delta} < k/\alpha\epsilon$  and  $\mu^* > 1/\alpha$ , the interest elasticity of money demand exceeds unity, so that the volume effect operates perversely. Specifically, the government could reduce monetary growth and inflation by raising its spending and/or by lowering its net explicit taxes. This is part of the aforementioned "anomaly".<sup>14</sup> Second, if  $\bar{\delta} = 0$ , then  $\mu^* = 0$ , and there is a single equilibrium, at point k on the abscissa of Figure 1. As already suggested, this is the state which would arise

if the public sector were following the full-liquidity rule. Third, if  $\bar{\delta} = k/\alpha e$ , i.e.  $\mu^* = 1/\alpha$ , there is again a single equilibrium, described in this instance by the tangency of GG to LL, so that points R and R' would coincide. From (11) we observe that

$$\lim_{\mu^* \rightarrow \frac{1}{\alpha^-}} \left( \frac{d\mu^*/\mu^*}{d\bar{\delta}/\bar{\delta}} \right) = \infty$$

which is suggestive in the light of findings by Cagan (1956), and subsequent investigators, that hyperinflations tend to be highly volatile in the neighborhoods of their revenue-maximizing rates. Fourth, if  $\bar{\delta} > k/\alpha e$ , no steady-state equilibrium exists. In terms of Figure 1, GG and LL do not intersect anywhere.

### 3. TWO COUNTRIES

This section extends the foregoing closed-economy analysis to the case of a two-country world. Both fixed and flexible exchange rates are considered.

#### The Model

Assume perfect integration of national commodity markets and zero international integration of national markets for money stocks. Our strong assumption concerning goods-market integration is purely for expositional convenience; the relevant steady-state relationships will be found to depend only upon relative purchasing power parity, although the extent to which national commodity markets are integrated will obviously affect transients.<sup>15</sup> Similarly, the steady-state relationships are invariant with respect to the extent to which capital is mobile internationally. Indeed, the analysis neither contains nor requires an explicit model of either the current or the capital accounts of the balance of payments. On the other hand, it is essential that each national economy grows at a common exogenously-given rate,  $\gamma$  say, where  $\gamma \geq 0$ . Assume too that in each country, real interest rates conform to the golden rule (see Section 2).

Consider next the assumptions relating to national public-sector policies. Under either exchange-rate regime, each public sector pegs its real per capita spending and net explicit tax receipts. There are no public-sector imports, nor are there any net explicit taxes raised from non-residents. Each public sector denominates its debt issues in terms of its own noninterest-bearing debt, i.e., its high-powered money. These assumptions are amenable to various straightforward generalizations.

Under fixed rates, the peripheral country holds international reserves, kept in the form of the interest-bearing debt of the reserve-currency country. In conjunction with the golden-rule assumption, an immediate implication is the absence of any international "seignorage problem" of the kind extensively investigated by, e.g., Mundell (1972). Assume further that the peripheral-currency equivalent of the associated flow of interest payments is continuously rebated to the peripheral country's private sector, by distributionally neutral means (e.g., by helicopter drop). To assume otherwise would be to encumber the analysis with a partial sterilization rule that is analytically awkward. But the present paper does allow for another, "proportional" rule concerning partial sterilization (see below).

Under flexible exchange rates, neither country intervenes in the foreign exchange or holds international reserves. At the cost of slight extra complexity it would be possible to allow for non-zero reserves under floating rates.

At this stage it is expositionally convenient formally to set out the short-run counterpart of the two-country model. Call the two countries A and B, where A is the reserve-currency country in the fixed-rate case, and denote country-specific variables by a subscript wherever this is necessary to avoid ambiguity:

$$c_a = k_a \exp[-\alpha_a (\pi_a^e + \gamma)], \quad c_b + f = k_b \exp[-\alpha_b (\pi_b^e + \gamma)] \quad (12a), (12b)$$

$$\dot{c}_a = \bar{\theta}_a [\bar{\delta}_a - (\pi_a + \gamma) c_a], \quad \dot{c}_b = \bar{\theta}_b [\bar{\delta}_b - (\pi_b + \gamma) c_b] - \bar{s} \dot{f} \quad (13a), (13b)$$

$$\dot{\pi}_a^e = \beta_a (\pi_a - \pi_a^e), \quad \dot{\pi}_b^e = \beta_b (\pi_b - \pi_b^e) \quad (14a), (14b)$$

$$\pi_a = \pi_b + \mathcal{E} \quad (15)$$

$$\mathcal{E} = 0 \quad \text{or} \quad f = 0 \quad (16)$$

This system may be seen broadly to resemble its single-country counterpart (1)' - (2)' - (7). Equation (12a) reflects the absence of a foreign-source component of A's money supply, as a consequence of B's reserve-asset preferences. Thus  $c_a \equiv C_a/P_a N_a$  denotes the real per capita domestic source component of A's monetary base,  $m_a$ , and we have  $c_a = m_a$ . By contrast, the real per capita foreign source component of B's monetary base,  $f \equiv F/P_b N_b$ , is zero only under flexible exchange rates, as is clear from (12b) together with (16). Equation (13a) is precisely the same as the closed-economy equation (2)', whereas (13b) reflects the open-economy assumption that the real per capita domestic source component of B's monetary base,  $c_b$ , is manipulated partly to offset the impact of B's real intensive balance of payments,  $\dot{f}$ , on B's overall real per capita monetary base,  $m_b \equiv c_b + f$ .<sup>16</sup> Assume  $0 < \bar{\theta}_a, \bar{\theta}_b < 1$ , and postpone consideration of the magnitude of  $\bar{s}$ . Equation (15) constitutes an elementary "absolute" purchasing power parity explanation of the price of A's currency,  $E$ , as is required when domestic and foreign goods are perfect substitutes. Define  $\pi_a \equiv P_a/P_a$ ,  $\pi_b \equiv \dot{P}_b/P_b$ ,  $\mathcal{E} \equiv \dot{E}/E$ , and  $\dot{N}_a/N_b = \dot{N}_b/N_b = \gamma$ . In summary, there are 8 equations to determine the 8 endogenous variables  $c_a$ ,  $c_b$ ,  $\pi_a$ ,  $\pi_b$ ,  $\pi_a^e$ ,  $\pi_b^e$ ,  $\mathcal{E}$ , and  $f$ .

Flexible Exchange Rates

Since it is simpler, we begin with the flexible-rates case ( $f=0$ ). In this case the foregoing short-run equations may be summarized by the following two first-order equations, in terms of  $m_a (=c_a)$  and  $m_b (=c_b)$  respectively:<sup>17</sup>

$$\dot{m}_a = -\frac{\beta_a \bar{\theta}_a}{\bar{\theta}_a - \alpha_a \beta_a} [\alpha_a \bar{\delta}_a + m_a \log(\frac{m_a}{k_a})] , \quad (17a)$$

$$\dot{m}_b = -\frac{\beta_b \bar{\theta}_b}{\bar{\theta}_b - \alpha_b \beta_b} [\alpha_b \bar{\delta}_b + m_b \log(\frac{m_b}{k_b})] . \quad (17b)$$

For future reference, write down the eigenvalue yielded by the differential equation describing the motion of A's real intensive money balance (cf. eq. (9)):

$$\lambda_a = -\frac{\beta_a \bar{\theta}_a (1 - \alpha_a r^*)}{\bar{\theta}_a - \alpha_a \beta_a} .$$

Inspection reveals that the foregoing closed-economy stability analysis carries over to the present case with no modification whatsoever, so that no further comment on stability under flexible rates is required. Similarly, the steady-state equations reduce to the following two pairs of equations describing each national economy in isolation,

$$m_a^* = k_a \exp(-\alpha_a \mu_a^*), \quad m_b^* = k_b \exp(-\alpha_b \mu_b^*) \quad (18a), (18b)$$

$$\mu_a^* = \bar{\delta}_a / m_a^*, \quad \mu_b^* = \bar{\delta}_b / m_b^* \quad (19a), (19b)$$

where  $\mu_a \equiv \dot{C}_a / C_a$  and  $\mu_b \equiv \dot{C}_b / C_b$ , so that the foregoing analysis of comparative steady states of a closed economy continues to apply. Thus each country's inflation rate is wholly independent of exogenous variables and parameters

in the other country. That is, the classical "insulation property" of flexible rates continues to hold if expansionary public policy is characterized by a money-accommodated fiscal expansion (as is the case here), rather than a fiscal-accommodated monetary expansion (as is the case in conventional monetary models).

Finally it is worth pointing out that stability of the system summarized by (17a) and (17b) will imply a stationary rate of inflation of the price of A's currency, and not necessarily a stationary level of the exchange rate, i.e., we have  $\dot{\epsilon}^* = 0$  rather than  $\dot{E}^* = \epsilon^* = 0$ . In the light of this consideration, and noting that the present paper allows for intervention in the foreign exchange market only in the fixed-rates case, a reader who is more interested in a flexible-rates system characterized by public policies which lead to both  $\epsilon^* = 0$  and endogenous non-zero reserves should consult the present paper's fixed-rates model, that being our next topic.

#### Fixed Exchange Rates

Under fixed rates ( $\epsilon = 0$ ), the short-run equations (12) to (16) may be summarized by (17a) (which holds irrespective of whether the exchange rate is fixed or flexible), together with<sup>18</sup>

$$\dot{m}_b = \frac{\alpha_b \beta_b m_b}{\bar{\theta}_a - \alpha_a \beta_a} \left[ \frac{\bar{\theta}_a \bar{\delta}_a}{m_a} + \beta_a \cdot \log \left( \frac{m_a}{k_a} \right) + \frac{(\bar{\theta}_a - \alpha_a \beta_a)}{\alpha_b} \cdot \log \left( \frac{m_b}{k_b} \right) \right] \quad (20)$$

$$\dot{f} = \frac{[m_b (\bar{\theta}_b - \alpha_b \beta_b) - \bar{\theta}_b f]}{(1-s) (\bar{\theta}_a - \alpha_a \beta_a)} \cdot \left[ \frac{\bar{\theta}_a \bar{\delta}_a}{m_a} + \beta_a \cdot \log \left( \frac{m_a}{k_a} \right) \right] - \frac{\beta_b}{1-s} \cdot m_b \cdot \log \left( \frac{m_b}{k_b} \right) - \frac{\bar{\theta}_b \bar{\delta}_b}{1-s} \quad (21)$$

Equations (17a), (20) and (21) define a recursive third-order system in terms of  $m_a (\equiv c_a)$ ,  $m_b (\equiv c_b + f)$ , and  $f$ . Linearize it in the neighbourhood of equilibrium, and recall the eigenvalue of (17a), to construct the characteristic polynomial:

$$\begin{vmatrix} -(\lambda + |\lambda_a|) & 0 & 0 \\ \cdot & -(\lambda + \beta_b) & 0 \\ \cdot & \cdot & -(\lambda + \frac{\mu_a^* \bar{\theta}_b}{1-\bar{s}}) \end{vmatrix} ,$$

where the elements below the main diagonal need not be reported, since the associated eigenvalues are immediately seen to be

$$\lambda_1 = -|\lambda_a| = \lambda_a, \quad \lambda_2 = -\beta_b, \quad \lambda_3 = -\frac{\mu_a^* \bar{\theta}_b}{1-\bar{s}} .$$

The necessary and sufficient stability conditions are threefold. First, the reserve-currency country A must be stable in isolation--or, equivalently, it must be stable under floating rates. Second, to ensure the stability of real intensive money balances and anticipated inflation in B, the speed of expectations adjustment must be positive there. Third, assuming a positive growth rate of money in A, i.e.,  $\mu_a^* > 0$ , and that deficit finance by B's public sector is money-intensive to some degree, i.e.,  $\bar{\theta}_b > 0$ , then stability of B's real per capita reserves requires that the sterilization policy implemented by B's public sector is no more than partial, i.e.,  $\bar{s} < 1$ .

In contrast to the flexible-rates case, the overall fixed-rates model cannot be split into subsystems describing each national economy



in isolation. But it is still conveniently recursive. This has already been noted in the foregoing analysis of transients, and is also apparent from the following summary of the steady-state relationships:

$$m_a^* = k_a \exp(-\alpha_a \mu_a^*) \quad (22a)$$

$$\mu_a^* = \bar{\delta}_a / m_a^* \quad (23a)$$

$$c_b^* = \bar{\delta}_b / \mu_a^* \quad (23b)$$

$$f^* = k_b \exp(-\alpha_b \mu_a^*) - c_b^* \quad (22b)$$

The nature of the recursivity is quite simple. The world rate of growth of nominal variables is simultaneously determined by A's portfolio balance schedule (22a) and its budget balance schedule (23a). B must accept this rate  $\mu_a^*$ , which together with B's explicit deficit  $\bar{\delta}_b$ , determines the domestic source component of B's monetary base  $c_b^*$ ; see (23b). Finally, these relationships, together with B's portfolio balance condition (22b), determine real per capita reserves,  $f^*$ .

In terms of the traditional terminology, there is a "burden-of-adjustment problem". Indeed, B's loss of policy autonomy is more severe than is often thought to be the case, inasmuch as B would have to unpeg its explicit deficit  $\delta_b$  if it sought to exert more proximate control over international reserves by, for example, pegging the proportional international backing of its monetary base,  $\psi$  ( $\equiv F/M_b$  or  $f/m_b$ ). This holds even if B is not a small country, and is contrary to what would appear to be a fairly widespread impression, namely, that peripheral-country status involves at most a loss of monetary autonomy.

On the other hand, B retains some scope for discretionary fiscal policy

in the sense that if the minimal and maximal values of  $\psi$  consistent with successful defense of the parity are  $\psi_l$  and  $\psi^u$  respectively, then  $\delta_b$  can take any value in the interval:

$$[(1 - \psi^u)\mu_a^* \exp(-\alpha_b \mu_a^*), (1 - \psi_l)\mu_a^* \exp(-\alpha_b \mu_a^*)].$$

Consider next the fixed-rates counterpart of Figure 1. The left-hand panel of Figure 2 below requires no further comment. In the right-hand panel, the  $L_B L_B$  schedule shows the combinations of  $f^*$  that are consistent with B's private sector balancing its portfolio, for given values of  $\mu_a^*$  and  $c_b^*$ . The  $G_B G_B$  schedule shows the combinations of  $c_b^*$  that are consistent with B's public sector balancing its steady-state budget (taking into account the revenue from money creation), for given values of  $\mu_a^*$  and  $\delta_b$ . The significance of points  $R_B$  and  $R'_B$  is explained below.

Consider finally the comparative-static properties of equations (22) and (23). To avoid excessive taxonomy, we confine attention to the intermediate case  $0 < \bar{\delta}_a < k_a/\alpha_a e = \max_{\mu_a^*} [\mu_a^* m_a(\mu_a^*)]$ , and  $0 < \mu_a^* < \min(1/\alpha_a, 1/\alpha_b)$ . (It

follows that  $\delta_b$  cannot be pegged at a level less than zero.) The effect of an increase in A's explicit deficit on the world rate of growth of nominal variables is the same as the effect of an increase in  $\delta_a$  on  $\pi_a^*$  or  $\mu_a^*$  under flexible rates (see above), and therefore requires no further discussion.

Other effects of an increase in  $\delta_a$  include:

$$\frac{dc_b^*/c_b^*}{d\bar{\delta}_a/\bar{\delta}_a} = -\frac{1}{1 - \alpha_a \mu_a^*} < -1, \quad \text{sgn}(df^*/d\bar{\delta}_a) = \text{sgn}(1 - \psi^* - \alpha_b \mu_a^*) > 0.$$

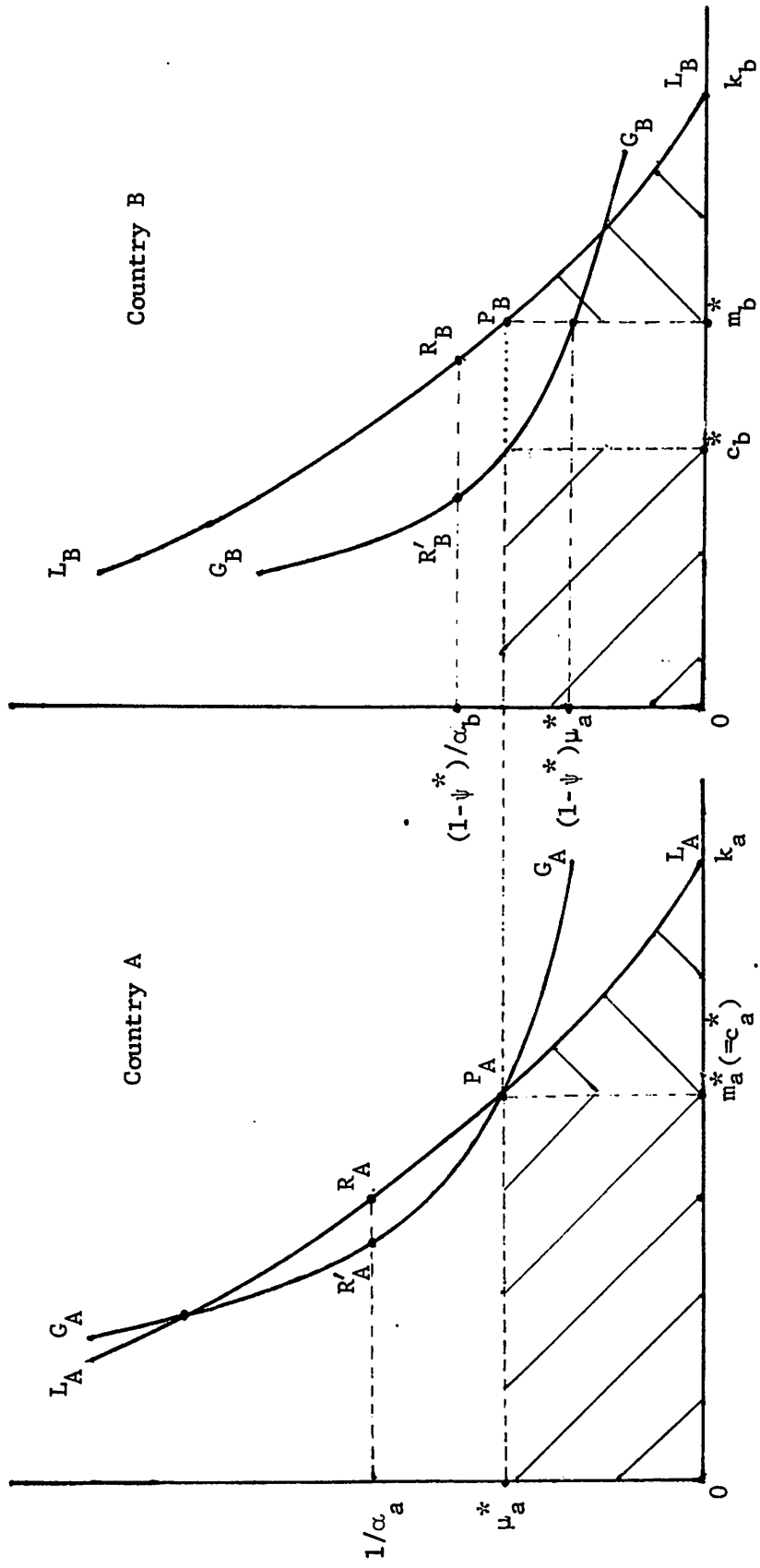


Figure 2

The first result is straightforward. An increase in A's explicit deficit will raise B's inflation-cum-growth tax rate,  $\mu_b^*$  ( $=\mu_a^*$ ), which necessitates a reduction in B's inflation-cum-growth tax base  $c_b^*$  in order to preserve budget balance in B. The second result is a little more complicated. An increase in A's explicit deficit will raise the opportunity cost of holding money balances in B--specifically,  $r_b^*$  ( $=\mu_a^*$ ) will rise. Hence  $m_b^*$  will decline. Offsetting this negative effect on B's real intensive reserves is a decline in the domestic source component  $c_b^*$  of B's overall monetary base  $m_b^*$ . The overall effect is ambiguous, and will depend on the initial relative magnitudes of the steady-state proportion of domestic assets in B's monetary base,  $1 - \psi^*$ , and the steady-state interest elasticity of money demand by B residents,  $\alpha_b \mu_a^*$ . Specifically, the effect of  $\bar{\delta}_a$  on  $f^*$  will be negative if and only if  $\psi^* + \alpha_b \mu_a^*$  exceeds unity. This ambiguity is readily deduced from Figure 2; note that points  $R_B$  and  $R'_B$ , defined by the property that their associated tangents are parallel to one another, correspond to the borderline case  $df^*/d\bar{\delta}_a = 0$ .

Putting this another way, successive increases in A's real intensive budget deficit will first raise, then reduce, world international liquidity in real intensive terms. Maximal international liquidity obtains when B's interest elasticity of money demand and international reserve ratio together sum to unity.

Finally, the non-zero effects of a change in B's explicit deficit  $\delta_b$  are:

$$\frac{dc_b^*/c_b^*}{d\bar{\delta}_b/d\delta_b} = 1, \quad \frac{df^*/f^*}{d\bar{\delta}_b/d\delta_b} = -\frac{1 - \psi^*}{\psi^*} < 0.$$

That is, an increase in B's explicit deficit will raise the domestic source component of B's monetary base, and lower the foreign source component by an identical (absolute) amount, thereby leaving B's overall high-powered money

stock unchanged. This is just the familiar monetary approach to the balance of payments (albeit at one remove) and therefore requires no further explanation.

#### 4. CONCLUDING COMMENT

We conclude by drawing attention to the main limitations of the foregoing analysis. First, our justification for applying the simple golden rule was heuristic rather than rigorous. Second, on standard correspondence-principle lines, the steady-state analysis was preceded by a stability analysis, essentially in order to show that stability is possible. In common with most such exercises, however, this involved recourse to a somewhat ad hoc model of short-run price adjustment.

## APPENDIX

This appendix extends the one- and two-country analysis of the main text to the n-country case. Recalling the assumptions set out there, and also the short-run equations (12) to (16) based on those assumptions, we have:

$$c_i + f_i = k_i \exp[-\alpha_i (\pi_i^e + \gamma)] \quad , \quad k_i, \alpha_i > 0 \quad (\text{A.1})$$

$$\dot{c}_i = \bar{\theta}_i [\bar{\delta}_i - (\pi_i + \gamma)c_i] - \bar{s}_i \dot{f}_i, \quad 0 < \bar{\theta}_i < 1 \quad (\text{A.2})$$

$$\dot{\pi}_i = \beta_i (\pi_i - \pi_i^e) \quad , \quad \beta_i > 0 \quad (\text{A.3})$$

$$\pi_n = \pi_i + \varepsilon_i \quad (\text{A.4})$$

$$\varepsilon_j = 0 \quad \text{or} \quad f_j = 0 \quad (\text{A.5})$$

$$f_n = 0 \quad (\text{A.6})$$

where  $i=1, \dots, n$ ;  $j=1, \dots, n-1$ ; and the subscript n denotes the reserve-currency country.

The 5n endogenous variables are:  $c_i \equiv c_i/P_i N_i$  = real intensive domestic source component of (country) i's monetary base,  $f_i \equiv F_i/P_i N_i$  = real intensive foreign source component of i's monetary base,  $\pi_i \equiv \dot{P}_i/P_i$  = actual rate of inflation of i's price level  $P_i$ ,  $\pi_i^e$  = anticipated rate of inflation of  $P_i$  by the residents of i, and  $\varepsilon_i \equiv \dot{E}_i/E_i$  = rate of inflation of the price of n's currency in terms of the price of i's currency.

These variables are determined by the 5n equations (A1) to (A6). Additionally, for future reference note that  $\mu_i \equiv \dot{C}_i/C_i$ ,  $m_i \equiv c_i + f_i$ ,

and  $r_i = \gamma + \pi_i$ , where  $\gamma = \dot{N}_i/N_i$ . Thus  $\gamma$  denotes both the exogenously-given world rate of population and real output growth, and the world's actual and anticipated real rate of interest (see main text). Finally, pegged government policy variables are denoted by an overbar, and are:  $\delta_i \equiv (G_i - T_i)/P_i \cdot N_i$ , where  $G_i = i^{\text{th}}$  public sector's nominal domestic spending,  $T_i = i^{\text{th}}$  public sector's nominal net explicit tax receipts from domestic residents;  $\theta_i \equiv \partial c_i / \partial [\delta_i - (\pi_i + \gamma)c_i] = i^{\text{th}}$  public sector's propensity to finance budget deficits by money creation;  $s_j \equiv \partial f_j / \partial \dot{c}_j = j^{\text{th}}$  public sector's propensity to sterilize balance-of-payments surpluses.

In the pure flexible-rates case, i.e., all  $f_j$ 's equal to zero, express  $\pi_i^e$  and  $\pi_i$  in terms of  $m_i (= c_i)$ , using (A.1) and (A.2) respectively. Together with (A.3) this yields a decomposable system summarized by  $n$  first-order differential equations:

$$\dot{m}_i = -\frac{\beta_i \theta_i}{\Delta_i} [\alpha_i \delta_i + m_i \log \left( \frac{m_i}{k_i} \right)], \quad i = 1, \dots, n, \quad (\text{A.7})$$

where  $\Delta_i \equiv \bar{\theta}_i - \alpha_i \beta_i$ , with  $\Delta_i > 0$  under stable, "moderate" inflations (see main text).

The eigenvalues in the neighborhood of equilibrium are given by:

$$\lambda_i = -\frac{\beta_i \bar{\theta}_i}{\Delta_i} (1 - \alpha_i r_i^*), \quad i = 1, \dots, n, \quad (\text{A.8})$$

where an asterisk denotes a steady-state value, and  $r_i^* = \mu_i^* = \pi_i^* + \gamma$ ,  $i = 1, \dots, n$ , under fixed or flexible rates.

In steady state the flexible-rates system may be summarized by  $2n$  equations:

$$\left. \begin{aligned} m_i^* &= k_i \exp(-\alpha_i \mu_i^*) \\ \mu_i^* &= \bar{\delta}_i / m_i^* \end{aligned} \right\} \quad i = 1, \dots, n.$$

These yield all the results discussed in the main text. Thus

$$d(\log \mu_1^*)/d(\log \bar{\delta}_1) = 1/(1 - \alpha_1 \mu_1^*), \text{ etc.}$$

Turning to the pure fixed-rates case, i.e., all  $\varepsilon_j$ 's equal to zero, it is convenient for stability analysis to reduce the short-run system to  $2n - 1$  equations. Towards this end, observe that (A.4) implies the "law of one price",

$$\pi_j = \pi_n, \quad j = 1, \dots, n-1, \quad (\text{A.9})$$

and that

$$\dot{m}_n = -\frac{\beta_n \bar{\theta}_n}{\Delta_n} [\alpha_n \delta_n + m_n \log(\frac{m_n}{k_n})] \quad (\text{A.10})$$

under fixed as well as flexible rates. Equations (A.2) under  $i=n$ , (A.6), (A.9), and (A.10) together imply:

$$\pi_i + \gamma = \frac{\bar{\theta}_n \bar{\delta}_n}{\Delta_n m_n} + \frac{\beta_n}{\Delta_n} \log(\frac{m_n}{k_n}), \quad i = 1, \dots, n. \quad (\text{A.11})$$

Use (A.1) and the identities  $m_j \equiv c_j + f_j$ ,  $j = 1, \dots, n-1$ , to obtain

$$\pi_j^e + \gamma = -[\log(m_j/k_j)]/\alpha_j, \text{ deduce that } \pi_j^e = -\dot{m}_j/\alpha_j m_j, \text{ and recall (A.3)}$$

and (A.11) to obtain:

$$\dot{m}_j = -\frac{\alpha_j \beta_j}{\Delta_n} \left[ \frac{\bar{\theta}_n \bar{\delta}_n}{m_n} + \beta_n \log(\frac{m_n}{k_n}) + \frac{\Delta_n}{\alpha_j} \log(\frac{m_j}{k_j}) \right], \quad j = 1, \dots, n-1. \quad (\text{A.12})$$

Equation (A.10) and (A.12) together constitute  $1 + n-1 = n$  equations in the  $2n - 1$  endogenous variables  $m_i$ ,  $i = 1, \dots, n$ , and  $f_j$ ,  $j = 1, \dots, n-1$ . To close the system, consider  $j = 1$  to  $n-1$  of the  $n$  equations defined by (A.2), add  $\dot{f}_j$  to both sides of those equations, recall equations (A.11) and the identities  $m_j \equiv c_j + f_j$  to eliminate the  $\pi_j$ 's and  $c_j$ 's respectively, and use (A.12) to eliminate the  $\dot{m}_j$ 's. This yields the  $n-1$  equations:





$$\mu_n^* = \bar{\delta}_n / m_n^*$$

$$m_n^* = k_n \exp(-\alpha_n \mu_n^*).$$

Again, the economic interpretation is essentially the same as for the case  $n = 2$ . One point worth mentioning is that now we are able to see that for the variables under consideration here, there are no interactions whatsoever between peripheral countries; instead there are only recursive pairwise interactions between each such country and the reserve-currency or " $n^{\text{th}}$ " country (with causation running from the latter to the former, as when  $n = 2$ ).

The foregoing analysis easily extends to cases wherein some countries operate under flexible rates, and others peg their currencies to a single reserve currency.

- Bailey, M. J., 1956, "The Welfare Cost of Inflationary Finance," Journal of Political Economy 64, 93-110.
- Cagan, P., 1956, "The Monetary Dynamics of Hyperinflation," in: M. Friedman (ed.), Studies in the Quantity Theory of Money (University of Chicago Press: Chicago), 25-117.
- Christ, C. F., 1978, "Some Dynamic Theory of Macroeconomic Policy Effects on Income and Prices under the Government Budget Restraint," Journal of Monetary Economics 4, 45-70.
- Dornbusch, R., 1977, "Inflation, Capital and Deficit Finance," Journal of Money, Credit and Banking 9, 141-150.
- Dutton, D. S., 1971, "A Model of Self-Generating Inflation: The Argentine Case," Journal of Money, Credit and Banking 3, 245-262.
- Fama, E. G., and G. W. Schwert, 1977, "Asset Returns and Inflation," Journal of Financial Economics 5, 115-146.
- Frenkel, J. A., 1975, "Inflation and the Formation of Expectations," Journal of Monetary Economics 1, 403-421.
- Friedman, B., 1978, "Stability and Rationality in Models of Hyperinflation," International Economic Review 18, 45-64.
- Friedman, M., 1969, "The Optimum Quantity of Money," in: The Optimum Quantity of Money and other Essays (Aldine Publishing Co.: Chicago), 1-50.
- Friedman, M., 1971, "Government Revenue from Inflation," Journal of Political Economy 79, 846-856.
- Girton, L. and D. W. Henderson, 1976, "Financial Capital Movements and Central Bank Behavior in a Two-Country Short-Run Portfolio Balance Model," Journal of Monetary Economics 2, 33-61.
- Girton, L. and D. Roper, 1977, "A Monetary Model of Fixed and Flexible Exchange Rates Applied to the Postwar Canadian Experience," American Economic Review 67, 537-548.
- Kingston, G. H. and S. J. Turnovsky, 1978, "A Small Open Economy in an Inflationary World: Monetary and Fiscal Policies under Fixed Exchange Rates," Economic Journal 88, 18-43.
- Kingston, G. H., 1978, "On the Theory of Inflation in Open Economies," unpublished doctoral dissertation, Australian National University, 1978.
- Laidler, D. E. W., 1977, "The Welfare Costs of Anticipated Inflation in the Neoclassical Theory," in: E. Lundberg (ed.), Inflation Theory and Anti-Inflation Policy (Macmillan: London).

- Mundell, R. A., 1968, International Economics (Macmillan, New York), Part III.
- Mundell, R. A., 1972, "The Optimal Balance of Payments Deficit," in E. Claassen and P. Salin (eds.): Stabilization Policies in Interdependent Economies (North Holland: Amsterdam-London) 69-87.
- Niehans, J., 1978, The Theory of Money (Johns Hopkins University Press: Baltimore).
- Parkin, J. M. and R. Bade, 1978, "Central Bank Laws and Monetary Policies: A Preliminary Investigation," Economic Record, forthcoming supplement.
- Roper, D., 1973, "Implications of the Gold-Exchange Standard for Balance-of-Payments Adjustment," Economica Internazionale 26, 3-22.
- Sargent, T. J., 1977, "The Demand for Money During Hyperinflations under Rational Expectations: I," International Economic Review 18, 59-82.
- Sargent, T. J., and N. Wallace, 1973a, "Rational Expectations and the Dynamics of Hyperinflation," International Economic Review 14, 328-350.
- Sargent, T. J., and N. Wallace, 1973b, "The Stability of Models of Money and Growth with Perfect Myopic Foresight," Econometrica 41, 1043-1048.
- Sidrauski, M., 1967, "Rational Choice and Patterns of Growth in a Monetary Economy," American Economic Review Supplement 57, 534-544.
- Swoboda, A. K., 1972, "Equilibrium, Quasi-Equilibrium and Macroeconomic Policy Under Fixed Exchange Rates," Quarterly Journal of Economics 86, 190-209.
- Swoboda, A. K., 1977, "Monetary Approaches to Worldwide Inflation," in: L. Krause and W. Salant (eds.), Worldwide Inflation (Brookings: Washington, D. C.), 9-62.
- Tobin, J. and W. H. Buiter, 1976, "Long-Run Effects of Fiscal and Monetary Policy in Aggregate Demand," in: J. L. Stein, (ed.), Monetarism (North-Holland, Amsterdam), 273-309.
- Turnovsky, S. J., 1978, "Macroeconomic Dynamics and Growth in a Monetary Economy: A Synthesis," Journal of Money, Credit and Banking 10, 1-26.
- Weiszäcker, C. C. v., 1965, "Existence of Optimal Programs of Accumulation for an Infinite Time Horizon," Review of Economic Studies 32, 85-104.
- Whitman, M. v. N., 1974, "The Current and Future Role of the Dollar: How Much Symmetry?" Brookings Papers on Economic Activity 3, 539-583.
- Whitman, M. v. N., 1975, "Global Monetarism and the Monetary Approach to the Balance of Payments," Brookings Papers on Economic Activity 4, 491-536.

## Notes

<sup>1</sup>Christ (1978) tacitly allows for such a divergence, but then shows that the policy regime in question is unstable, so we stress that the aforesaid divergence is a necessary rather than sufficient condition for the policy regime to be secularly feasible.

<sup>2</sup>Turnovsky (1978) presents a model which exhibits a similar two-equilibria property.

<sup>3</sup>See, e.g., Mundell (1972), Whitman (1975) and Swoboda (1977). Of course the traditional theory furnishes numerous examples of "overshooting" as a transient response to monetary expansion; see, e.g., Friedman (1969) and Frenkel (1975).

<sup>4</sup>If, on the other hand, the public sector of a "peripheral" country does not peg its international reserve ratio, then it retains some leeway for discretionary fiscal policy; see Section 3.

<sup>5</sup>Alternatively, if we were to assume that interest-bearing public debt consisted of perpetuities then the dynamics of the short-run model would be a little more complex, and its steady state would continue to be described by (1) and (2); see Tobin and Buiter (1976) for an analagous observation.

<sup>6</sup>I.e., the real rate of return on capital is tied to the sum of the exogenously-given rates of population growth, discount, and depreciation. The simple golden rule raises convergence problems for the Sidrauski model, but presumably could be recovered by invoking the "overtaking" optimality criterion; see von Weizsäcker (1965).

<sup>7</sup>The debt service item in the government budget constraint can easily lead to unfruitful complications. For contributions which seek to avoid such complications by invoking the asymmetric tax assumption just mentioned, see, e.g.,

Tobin and Buiter (1976) and Niehans (1978). Formally, our alternative assumption is equivalent to asserting that bonds are "over-indexed" to an extent  $\gamma$ .

<sup>8</sup>See, e.g., Dornbusch (1977).

<sup>9</sup>Defined only if M and P (as well as N) are continuous with respect to time; that will be assumed henceforth. With regard to the disturbances considered by the present paper, this assumption is innocuous.

<sup>10</sup>An obvious candidate is the continuous deterministic version of point-in-time rational expectations, in other words, perfect myopic foresight ( $\pi^e = \pi$ ). This hypothesis obviously satisfies the first and second of the foregoing criteria, but it fails the stability criterion. Such instability (whose verification is straightforward) is reminiscent of the findings of Sargent and Wallace (1973b) on the stability of the Cagan (1956) model under fixed monetary growth and perfect myopic foresight. An alternative, stochastic version of rational expectations is part of the aforementioned Sargent and Wallace (1973a) reformulation of the Cagan model. But Friedman (1978) establishes that this reformulation leads to an implausible model of price formation. One candidate which meets all three criteria is the special second-order autoregressive hypothesis proposed by Frenkel (1975). In fact this scheme can be shown to impart somewhat more stability to the present model than does Cagan's first-order scheme, in keeping with Frenkel's findings concerning the relative stability of the Cagan model under fixed monetary growth and first-order versus second-order autoregressive specifications. On the other hand, the first-order case has the advantage of simplicity, as it also does vis-à-vis the Sargent and Wallace (1976b) hypothesis. In this context we re-emphasize that transients are not the main object of interest in this paper.

<sup>11</sup>In the Cagan-Bailey model, by contrast, the appropriate policy schedule is a horizontal line, reflecting their alternative policy assumption that the public sector pegs the growth rate of money. Observe that the system would be overdetermined in the event that both the deficit and the money growth rate were pegged.

<sup>12</sup>Hence the tangent to R has the same slope as the tangent to R'.

<sup>13</sup>Cf. Bailey (1956). See also Laidler (1977) for a review of the special assumptions underlying simple geometric measures of consumer's surplus. On the full-liquidity rule, see Friedman (1969); in the present context, "full-liquidity" monetary growth simply means  $\mu^* = 0$ .

<sup>14</sup>See Section 1. The other element of anomaly, of course, is that stability of solutions such as Q would require  $\bar{\theta} < \alpha\beta$  which, in the light of previous findings on stability, reverses the expected inequality.

<sup>15</sup>See Kingston and Turnovsky (1978) for a model which resembles the present analysis in that it incorporates the government budget constraint with fixed spending and taxes under nonzero inflation, and which shows that low price elasticities of demand for importables are destabilizing influences, as is always the case in the standard barter model of international trade.

<sup>16</sup>Parallel to our alternative presentation of equation (2)' (see Section ), equation (13b) can be rewritten as:

$$\mu_b = \bar{\theta}_b \frac{\bar{\delta}_b}{c_b} + (1 - \bar{\theta}_b + \frac{\bar{s}_\psi}{1 - \psi}) \cdot (\pi_b + \gamma) - (\frac{\bar{s}_\psi}{1 - \psi})\varphi$$

where  $\mu_b \equiv \dot{C}_b / C_b$  = growth rate of B's nominal domestic credit,  $\psi \equiv f/m_b \equiv F/M_b$  = proportional international backing of B's monetary base and  $\varphi \equiv \dot{F}/F$  = growth rate of B's nominal international reserves. Such a formulation is more closely (but by no means precisely) related to those underlying typical estimated central-bank reaction functions; see Parkin and Bade (1978) for a cross-country survey of the relevant econometric work.

<sup>17</sup>To obtain (17a), rearrange (12a) and (13a) so that  $(\pi_a^e + \gamma)$  and  $(\pi_a + \gamma)$  respectively are the variables on the left-hand side; then use the resulting equations to eliminate  $\pi_a^e$  and  $\pi_a$  in (14a). The derivation of (17b) is analagous.

<sup>18</sup>As a preliminary to the derivation of (20) and (21), note that  $c_a = m_a$  which, together with (13a) and (17a), yields:

$$\pi_a + \gamma = \frac{\bar{\theta}_a \bar{\delta}_a}{(\bar{\theta}_a - \alpha_a \beta_a) m_a} + \frac{\beta_a}{\bar{\theta}_a - \alpha_a \beta_a} \cdot \log \left( \frac{m_a}{k_a} \right).$$

Turning to the detailed derivation of (2), first note that (12b) and the identity  $c_b + f \equiv m_b$  together imply  $\pi_b^e + \gamma = -[\log(m_b/k_b)]/\alpha_b$ , whence  $\dot{\pi}_b^e = -\dot{m}_b/\alpha_b m_b$ . Next, noting that (15) with  $\varepsilon = 0$  is just "the law of one price"  $\pi_a = \pi_b$ , and recalling (14b),

$$\dot{\pi}_b^e = \beta_b [(\pi_a + \gamma) - (\pi_b^e + \gamma)],$$

$$\text{i. e., } -\frac{\dot{m}_b}{\alpha_b m_b} = \beta_b \left[ \frac{\bar{\theta}_a \bar{\delta}_a}{(\bar{\theta}_a - \alpha_a \beta_a) m_a} + \frac{\beta_a}{(\bar{\theta}_a - \alpha_a \beta_a)} \log \left( \frac{m_a}{k_a} \right) + \frac{1}{\alpha_b} \log \left( \frac{m_b}{k_b} \right) \right]. \text{ Rearrange}$$

this expression to obtain (20). Finally, to derive (21), add  $\dot{f}$  to both sides of (13b); recall  $c_b + f \equiv m_b$  and  $\pi_b + \gamma = \pi_a + \gamma$ , to eliminate  $c_b$  and  $\pi_b + \gamma$  respectively; and use (20) to eliminate  $\dot{m}_b$ .