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An Empirical Test of the Risk Aversion Hypothesis

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1. Introduction

The effects of uncertainty on producers' behaviour have been recently the subject of several studies.[1] These studies show that producers will change their behaviour under uncertainty and moreover the nature of the change in their behaviour will depend on their attitudes towards risk.[2]

The most common assumption about individuals' attitudes towards risk has been that individuals are risk averse. Theoretical support for this assumption, which is related to the boundedness of the Von-Neuman-Morgenstern utility function, is suggested by Arrow [1971]. Empirical support is given by observed phenomena such as purchase of insurance, portfolio diversification and other risk-sharing contracts. On the other hand, it has been also argued that since many firms are owned by a large number of shareholders whose portfolios are diversified, risk neutrality of the firm may be appropriate.

While the theoretical discussion on these issues is quite extensive and developed, there are very few empirical applications[3] of the theory of the firm under uncertainty. Many studies in applied production theory appeared recently in which new functional forms and new techniques are being used.[4] None of these studies, however, considers the existence and effects of uncertainty.

In this paper we develop a framework for the empirical analysis of production theory under uncertainty. Applying duality theory and using a flexible functional form for technology, we provide a model for production decisions under uncertainty that can be easily estimated and used to identify attitudes towards risk. Furthermore, we provide a measure of the degree of risk aversion (or preference) and a direct test for the risk aversion hypothesis.

In the empirical part we apply our framework to a study of the U.S. textile industry (1947-71) and find risk aversion behaviour to be significant.

In section 2 we provide the theoretical framework, in section 3 we discuss the econometric specification of the model and in section 4 we report the empirical results.

2.   Theoretical Framework

Consider a firm whose technology is given by the production function $y = F(x,v)$ where x in an n vector of variable inputs, v a vector of fixed inputs, y an output and F is a continuous, non-decreasing and quasiconcave function. The firm is assumed to be competitive in its input markets and faces the variable input price vector w with certainty. The firm is also assumed to be competitive in its output market, the output price P is, however, a random variable and is unknown when production decisions are made. The firm is, therefore, competitive in a probabilistic sense; it does not face a given output price, but it cannot affect its probability distribution.

The firm's objective is to maximize expected utility from profits and its attitude toward risk is described by a Von-Neuman-Morgenstern utility function $U = U(\pi)$ where U is utility, π profits and $U(\pi) > 0, U'(\pi) > 0$. The firm's utility function is concave, linear, or convex in π depending on whether the firm is risk averse, risk neutral or risk loving.

The firm's problem can, therefore, be written as

(1) $\max_{y,x} \{E[U(\pi)]: \pi = Py - wx, y = F(x,v), P \sim g(P) \ x,y \geq 0\}$

where E is the expectation operator and g(P) is the probability density function of P.

Since uncertainty enters the problem through output price uncertainty only, the problem can be rewritten as the following equivalent problem

(2) $\max_{y} \{E[U(\pi)]: \pi = Py - C(w,v,y), P \sim g(P) \ y \geq 0\}$

where

(3) $C(w,v,y) \equiv \min\limits_{x}\{wx: \; y = F(x,v), \; x \geq 0\}$

is the variable cost function which is dual to the production function F.[5]

Thus, the firm produces efficiently, although output price is uncertain.[6]

The input demand functions can be obtained from (3) by Shephard's Lemma[7] as

(4) $x = \nabla_w \, C(w,v,y)$

where ∇_w is the vector of derivatives of C with respect to w.

The optimality condition for output choice is obtained from (2) as

(5) $E[U'(\pi)(P - \frac{\partial C}{\partial y})] = 0$

which can be equivalently written as

(6) $E[U'(\pi)P] = \frac{\partial C}{\partial y} E[U'(\pi)]$

or, using the definition of the covariance;

(7) $\text{cov}[U'(\pi),P] + E[U'(\pi)] \, E[P] = \frac{\partial C(w,y,v)}{\partial y} E[U'(\pi)].$

Let us now define

(8) $\theta = \dfrac{\text{cov}[U'(\pi),P]}{E[U'(\pi)] \, E[P]} + 1.$

Then, the optimality condition can be written as

(9) $\theta \, E(P) = \partial \, C(w,y,v)/\partial y.$

Since $U'(\pi) > 0$ and $P > 0$, it is clear that $E[U'(\pi)]$, $E(P)$ and $E[U'(\pi)P]$ are all strictly positive and therefore, $\theta > 0$. Furthermore, $\text{cov}[U'(\pi),P]$ and $U''(\pi)$ have the same sign,

(10) $\text{cov}[U'(\pi),P] \gtreqless 0 \quad \text{iff} \quad U''(\pi) \gtreqless 0.$

In other words the covariance is positive, zero or negative depending on whether the firm is risk loving, risk neutral or risk averse respectively. Condition (10) can be expressed in terms of restrictions on θ

(11) $\theta \gtreqless 1 \quad \text{iff} \quad U'' \gtreqless 0.$

The optimality condition (9) and condition (11), imply that the firm will produce more, the same amount, or less output than an identical firm facing the price $E(p)$ with certainty depending on whether it is risk loving, risk neutral or risk averse respectively (assuming the cost function is convex in output).

Condition (10) provides us with a basis for testing the three alternative hypotheses about the firm's attitude towards risk. In other words, if given an appropriate framework we could test whether θ is greater, equal or smaller than one, we could then implicitly identify the firm's attitude towards risk.

For empirical implementation we have to specify a functional form for the firm's cost function. Given this functional form we can derive the input demand equations given by system (4) and the output optimality condition given by (9). If the expected price were known and if θ could be treated as an unknown parameter, we could simply estimate the system of equations given by (4) and (9) and carry out the desired tests about θ. The expected price is, however, unobservable and furthermore, θ is not a parameter, but some function of the observable data. It is necessary, therefore, to make specific assumptions about the relationship between expected and actual output prices and about the form of θ. We could assume for example, that $P = h(E(P),r)$ where r is some random variable and solve for $E(P)$ in terms of P and r; $E(P) = h^{-1}(P,r)$. The random variable r is then simply taken as part of the regression disturbance term.

The specific form of θ will depend, of course, on the characteristics of

the firm's utility function and the price distribution. It is, however, clear from the definition of θ that $\theta = \theta(w,v,y)$, i.e., it is some function of output and the exogenous variables (w,v).[8] We could, therefore, approximate θ by some function of (w,v,y) and use this approximation in the estimating system. Given the functional form for θ and the expression for $E(p)$, we could then estimate the system (4), (9) and investigate the nature of θ, i.e., the firm's attitude towards risk.

3. Empirical Implementation

Having discussed the theoretical framework we now apply it to the U.S. textile industry. We assume that there are two competitively priced "variable inputs" in the production of textile (y); labour x_L and intermediate goods x_M, whose prices are w_L, w_M respectively. In addition to labour and intermediate goods the industry uses a capital input K. In our study we treat the capital input as a "fixed input", in the sense that we do not explicitly consider the capital choice decision. It should be noted, however, that this does not mean that capital is actually held fixed, but that the firm's decision is conditional on the level of capital which changes over time. The reason for this treatment of capital is that by doing so we avoid the need to obtain price of capital data, which is usually quite difficult to construct.

The price and quantity series for labour, intermediate inputs and output are constructed from data published in various issues of the Survey of Current Business.

We assume that the industry's cost function is given by a translog function

$$(12) \quad \ln C = \alpha_0 + \sum_i \alpha_i \ln w_i + \frac{1}{2} \sum\sum \alpha_{ij} \ln w_i \ln w_j + \sum_i \beta_{iy} \ln w_i \ln y$$

$$+ \sum_i \beta_{iK} \ln w_i \ln K + \gamma_y \ln y + \frac{1}{2} \gamma_{yy}(\ln y)^2 + \gamma_{yK}\ln y \ln K + \delta_K \ln K$$

$$+ \delta_{KK}(\ln K)^2 \quad\quad i,j = L,M.$$

Linear homogeneity in prices and symmetry imply the following parameter restrictions

$$(13) \quad \Sigma_i \alpha_i = 1, \ \Sigma_i \alpha_{ij} = 0 \ \forall \ i, \ \Sigma_i \alpha_{ij} = 0 \ \forall \ i, \ \Sigma_i \beta_{iy} = 0, \ \Sigma_i \beta_{ik} = 0,$$

$$\alpha_{ij} = \alpha_{ji}.$$

Applying Shephard's Lemma to the cost function (12), we get the input cost share equations as

$$(14) \quad S_L = \alpha_L + \alpha_{LL} \ \ell n w_L + \alpha_{Ln} \ell n w_M + \beta_{Ly} \ell n y + \beta_{LK} \ell n K$$

$$S_M = \alpha_M + \alpha_{LM} \ell n w_L + \alpha_{MM} l n w_M + \beta_{My} l n y + \beta_{MK} l n K$$

where S_L, S_M are the cost shares of labour and materials respectively.

Given the cost function we also obtain the optimality condition (9) as

$$(15) \quad E(P) = (\gamma_y + \beta_{Ly} l n w_L + \beta_{My} l n w_M + \gamma_{yy} l n y + \gamma_{yK} l n K)/\theta$$

and assuming that $P = E(P) + r$, (i.e., actual price is given by expected price plus some random term) we get the condition

$$(16) \quad P = (\gamma_y + \beta_{Ly} l n w_L + \beta_{My} l n w_M + \gamma_{yy} l n y + \gamma_{yK} l n K)/\theta + r$$

As was indicated above, θ is not a parameter, but some function of the quantities of output and capital and of the prices of labour and materials. As a first approximation we take θ to be a linear function in these variables[9] and write it as

$$(17) \quad \theta = a_o + a_L w_L + a_M w_M + a_M K + a_y y$$

For equations (14) and (16) to be consistent with optimizing behaviour we have to impose the restrictions given by (13). It should also be noticed that since

cost shares sum to one, only one of the input share equations in (14) is independent. In the estimation we therefore drop one of the input share equations (the material share equation) and identify its parameters using the adding up (or homogeneity) restrictions. Our full model therefore, consists of the labour share equation in (14) and the price equation (16), with θ being defined by (17).

For empirical implementation the model has to be imbedded within a stochastic framework. To do this we assume that equations (14) and (16) are stochastic due to errors in optimization and define the error term in the labour share equation at time t as $e_L(t)$ and the error term in the price-marginal cost decision as $V_p(t)$. The disturbance term in equation (16) is therefore given by $e_p(t) = r(t) + V_p(t)$, i.e., it is the sum of the optimization error and the random deviation of price from expected price. We define the column vector of disturbances of time t as $e(t) \equiv [e_L(t), e_p(t)]'$ and assume that the vector of disturbances is identically and independently joint normally distributed with mean vector zero and non-singular covariance matrix Ω,

$$E[e(s)\,e(t)'] = \begin{bmatrix} \Omega & t=s \\ o & t \neq s \end{bmatrix} \quad \forall\ s,t$$

where Ω is a 2 x 2 positive definite matrix.

For estimation we use the full information maximum likelihood technique treating the labour cost share and the output price as endogenous variables. For statistical inference we use the likelihood ratio test.

4.   Empirical Results

We estimate the model given by the labour share and price equations with the symmetry and linear homogeneity restrictions (13) imposed. The model has 12 free parameters, the remaining parameters are identified using (13). The parameter estimates and standard errors are given in Table 1. An examination of this

table indicates that all the estimated parameters are statistically significant. Given the parameter estimates we calculate the estimated θ and report the figures in Table 2. An examination of these figures shows that the estimated θ is smaller than one at all sample points, indicating risk averse behaviour during the sample period.

To test for the significance of the risk aversion we have to test whether θ is significantly smaller than one. To carry out this test we first test the null hypothesis that θ is globally (i.e., for all w_L, w_M, K, y) greater or equal to one. This involves a test for the restrictions $a_o \geq 1\ a_L = a_M = a_K = a_y = 0$. The χ^2 statistics is 250.4 so that the null hypothesis is rejected ($\chi^2_{(5).01} = 15.1$) implying that we do not have global risk neutrality or preference. This global test is, however, a strong test and clearly, since θ is a function of the exogenous variables and not a constant, the rejection of the global restrictions does not necessarily imply the rejection of $\theta \geq 1$. The restrictions $a_o \geq 1$ $a_L = a_M = a_K = a_y = 0$ are sufficient but not necessary for $\theta \geq 1$. Thus, in order to test whether θ itself is significantly smaller than one we carry out a local test for risk aversion. We calculate the estimated θ and its standard deviation, both evaluated at the sample mean and test whether θ is locally significantly smaller than one. We find that at the sample mean θ is $\hat{\theta} = .865$ with a standard deviation of .019, so that a 99% one-sided confidence region is $\theta < .9091$. We therefore conclude that θ is significantly smaller than one, thus implying significant risk aversion.

5. Conclusion

In this paper we provided a framework for the empirical analysis of production theory under price uncertainty. We develop a flexible production model that can be easily estimated and provide a measure for the degree of risk aversion. We also suggest a direct test for the risk aversion hypothesis.

In applying our approach to the U.S. textile industry we find that risk aversion behaviour was statistically significant during the sample period.

Table 1

Parameter Estimates
(standard errors in parentheses)

α_L	.0492	(.0037)
α_{LL}	.5514	(.0102)
β_{Ly}	.0874	(.0047)
β_{LK}	-.0871	(.0159)
γ_y	.9265	(.0465)
γ_{yy}	.0092	(.0004)
γ_{yK}	-.1440	(.0059)
a_o	.6032	(.0073)
a_L	-.0633	(.0054)
a_M	.2698	(.0151)
a_K	.1822	(.0056)
a_y	-.1071	(.0050)

Table 2

Estimated Risk Aversion Measure $\hat{\theta}$

Year	$\hat{\theta}$
47	.863667
48	.891331
49	.889066
50	.906243
51	.960054
52	.939169
53	.910942
54	.910666
55	.893817
56	.891356
57	.897552
58	.884968
59	.866329
60	.865984
61	.857794
62	.846654
63	.847753
64	.831377
65	.818679
66	.809793
67	.819816
68	.817561
69	.812259
70	.815603
71	.801137

Footnotes

[1] See Sandmo [1971], Batra and Ullah [1974], Hartman [1976].

[2] See references in footnote 1.

[3] For an empirical application of choice under uncertainty see Parkin [1970] where a discount house portfolio model is developed.

[4] For references see Fuss and McFadden [1978], Diewert [1974].

[5] See Diewert [1971] for its regularity properties.

[6] This can be easily verified by observing that the first order conditions corresponding to (1) yield: $\frac{\partial F}{\partial x_i} E[U'(\pi)P] = w_i E[U'(\pi)]$ and thus, $[\partial F/\partial x_i]/[\partial F/\partial x_j] = w_i/w_j$.

[7] See Shephard [1970], Diewert [1971].

[8] For example, it can be shown that for quadratic utility and cost functions $\theta = \frac{Var(P)}{E(P)} E\left[\frac{1}{R_a}\right] y$ where $Var(P)$ is the variance of P and $R_a = -\frac{U''}{U'}$ is the measure of absolute risk aversion. Thus, it is clear that $\theta = \theta(w,v,y)$.

[9] We also tried a linear approximation in logarithms and the results were similar.

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