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OPTIMAL TENURE AND THE TIMING
OF FACULTY MEETINGS

by

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Do Not Quote

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"All good things must come to a bad end."

- Swami Mhakhaphi

Introduction

Economists are generally rather quick to point out the drawbacks of extant institutions. When one considers government interference in markets, government subsidies of the aged or unemployed, "fair trade" or "closed shop" legislation or any of thousands of institutions that protect individuals from the painful incentives to reach a Pareto optimum, a vast economic literature is available for reference. Economists have rightfully and thoroughly dissected the restraint of free trade involved in the American Medical Association, the labor laws, Regulation Q, the SEC, FTC, CAB, DOE, ... and the resultant welfare losses.¹ In some cases, the criticism has even provided a refreshing irony in a profession justifiably accused of taking itself too seriously.² Furthermore, every serious economist has understood that distortions of the free market generally result in welfare losses and an economy that could not be realistically described as even second best.³ Economists correctly argue that externalities far too often justify market interference that protects one group at the expense of another.⁴ For some obscure reason, however, they are often quite willing to see externalities lurking in education.⁵ While they may privately disparage public education, they rarely reach the vehemence given, say, agricultural price supports, at least in print. But examples exist of attacks on public education, while one can search extensively without finding derogatory references to the institution of academic tenure.⁶ Again, economists tend to point out the obvious deficiencies and deadwood loss associated with tenure in private discussion, but in print, tenure is taboo.

It is the purpose of this paper to consider tenure in a formal model and estimate the costs and benefits associated with tenure. While this is a path-breaking, seminal article, there are, of course, articles that should be cited in case their authors turn out to be referees for this manuscript.

The early models of the effects of tenure were partial equilibrium models that considered only the response of the individual to tenure, without considering the tenure granting committee's response. The first appearance of such a model dates to Tance (1936). This was followed by Morris (1938), Kamil (1938), Kaizer (1939),⁷ and Dimple (1939). All of these studies emphasize the importance of academic freedom as an input to academic productivity. This stream of the literature died out during the Second World War, when the authors, denied tenure, were killed in action.

The first equilibrium models of tenure emerged from The Institute for Advanced Psychiatric Applications.⁸ Unfortunately, most of these were not translated until recently, and many of them remain classified. However, they are certainly the first papers that incorporate the notion that the tenure granting committee will respond to alterations in the behavior of tenure seeking agents. Of the Soviet authors whose works have been translated, certainly the most profound include Khamsin (1964, 1965), Zhumkoff (1965), and Orlov (1965).

Denied access to Soviet manuscripts, the Western rediscovery of the tenure controversy took a different approach. The counterfactual method was used in the first tenure revival papers by Wallace (1979), Wallace and Colonel (1980), and Barren-Grossmann (1981).⁹ Meanwhile, other authors considered the alterations of the accumulation of knowledge, with and without

tenure. Most of this enquiry focused on the papers of early physicists in the presence of tenure, for example, Michlego (1982). Also see my own paper (1984).¹⁰

The paper is structured as follows. In the section you have just read, the literature is surveyed. In the paragraph you are currently reading, a discussion of the paper's structure is presented. In the upcoming section, a Nash equilibrium model is developed. The third section presents some econometric work that bears little relation to the formal model. Finally, some concluding remarks and suggestions for future research¹¹ are given.

Nash Equilibrium Tenure

Suppose there are two types of professors, A and B. The professors have utility functions $U^i(r,p)$, where p is the amount of time spent talking to the dean¹² and r is the amount of research done, $i \in \{A,B\}$. Let a subscript j refer to the derivative with respect to the j^{th} argument.

For tractability, let

$$U^A = e^{ar} \log(1-p), \text{ and} \quad (1)$$

$$U^B = e^{br} \log(p), \text{ where } a \leq b. \quad (2)$$

The university awards tenure at time T if

$$\int_0^T r(t)dt \geq r_0. \quad (3)$$

Furthermore, the university chooses $p(t)$, $0 \leq t \leq T$, while the individual chooses $r(t)$, $0 \leq t$ and $p(t)$, $T \leq t$, if tenured. If the individual is denied tenure, they go into "private industry", where type A individuals experience $p=1$ and type B individuals experience $p=0$.

Neither does any work r in this case. Index the individual type choices by r_A, p_A, r_B, p_B . If the subscript is omitted, the claim holds for both individuals. Let the types both have discount rate δ .

While this model is certainly a simplification of reality, it captures most of the desirable attributes. First, there are type A people who have lower cost of production, in the sense that

$$\frac{U_1^A}{U^A} = a \leq b = \frac{U_1^B}{U^B}. \quad \text{Second, type A individuals find administrative duties}$$

lower their utility, while the other type is the missing link between professors and deans.

Theorem 1: Given $p(t)$, $0 \leq t \leq T$, T and r_0 , we have

$$\int_0^T r(t) dt = r_0. \quad \text{For } t > T, \text{ (ii) } r(t) = 0, p_A(t) = 0 \text{ and } p_B(t) = 1.$$

Proof: Examining (1) and (2), we note that private industry is just too boring. Thus, the incentive to achieve tenure is infinite, and (3) must hold. As $\log(x)$ for $x \in [0,1]$ is non-positive, (3) holds with equality.

Theorem 2: Given $p(t)$, T and r_0 , we have

$$r_A(t) = \frac{1}{a} \{ \delta t - \log K_A - \log \{-\log(1-p)\} \} \quad (4)$$

$$\log K_A = \frac{\delta T}{2} - \frac{ar_0}{T} - \frac{1}{T} \int_0^T \log \{-\log(1-p)\} dt \quad (5)$$

and

$$r_B = \frac{1}{b} \{ \delta t - \log K_B - \log \{-\log(p)\} \} \quad (6)$$

where

$$\log K_B = \frac{\delta T}{2} - \frac{br_0}{T} - \frac{1}{T} \int_0^T \log \{-\log(p)\} dt \quad (7)$$

Proof: Consider the choice of r_A to maximize

$$\int_0^T U^A(r_A, p) e^{-\delta t} dt \quad \text{s.t.} \quad \int_0^T r_A(t) dt = r_0.$$

It follows immediately that $U_1^A e^{-\delta t}$ is constant, or thus that, for some K ,

$$e^{-\delta t} e^{ar} \log(1-p) = -K \leq 0$$

Thus

$$e^{-ra} = \frac{1}{K} e^{-\delta t} (-\log(1-p)), \text{ or}$$

$$r_A(t) = -\frac{1}{a} [\log K - \delta t + \log \{-\log(1-p)\}]$$

Thus

$$r_o = \int_0^T r_A(t) dt = \frac{1}{a} \left[\frac{\delta T^2}{2} - T \log K - \int_0^T \log \{-\log(1-p)\} dt \right]$$

or

$$T \log K = \frac{\delta T^2}{2} - ar_o - \int_0^T \log \{-\log(1-p)\} dt$$

The case for r_B is symmetric.

This completely describes the behavior of professors. To procure an equilibrium, we suppose that the university is unable to recognize whether a given employee is type A or B, and thus must choose a p that will be imposed on both types. Strong empirical evidence for the hypothesis that universities are unable to recognize talented faculty is provided (Minn (1978)). If α is the proportion of type A individuals, then, the university will choose p to

$$\max \int_0^T e^{-\delta t} [\alpha r_A(t) + (1-\alpha)r_B(t)] dt \quad (8)$$

subject to Theorem 2.

Theorem 3: p is constant, invariant to T and r_o , and solves the equation

$$p^{\alpha p} = (1-p)^{(1-\alpha)(1-p)} .$$

Proof: See Appendix 1.

Suppose x is a signal that takes on values in the unit interval, with 0 meaning an assertion is false, while 1 means the assertion is true. A measure of information content of the signal is, then, $x \log x$ (see Logos (1972)), called the entropy measure. By rewriting this expression, we see

$$\alpha p \log p = (1-\alpha)(1-p) \log(1-p)$$

and thus the universities' choice of p equates the expected content of p . That is to say, the university chooses administrative tasks to make both types of professors proportionally obscure.

Corollary: $\alpha = 0 \Rightarrow p = 1$; $\alpha = 1 \Rightarrow p = 0$, $\frac{dp}{d\alpha} < 0$.

Thus, Type A departments minimize administrative interference, and conversely.

Theorem 4: $\dot{r}_A = \frac{\delta}{a} \geq \frac{\delta}{b} = \dot{r}_B \geq 0$, and the present value of per capita output is:

$$PV = r_0 + \left(\frac{\alpha}{a} + \frac{1-\alpha}{b} \right) \left[\frac{1}{\delta} (1 - e^{-\delta T}) - \frac{T}{2} (1 + e^{-\delta T}) \right]$$

Proof: The first assertion follows from Theorems 2 and 3. The second is proved in the appendix.

Now suppose r_0 is set exogenously, but the university chooses T^* .

Then

Theorem 5: $T^* = 0$, $PV = r_0$.

Proof: In appendix.

Now consider an exogenously fixed tenure time T and production level r_0 . Then a higher proportion of Type A individuals increases the present value of output. PV is decreasing in T . Furthermore, PV is decreasing in δ - patience is rewarded. Thus, a characterization of a high productivity department is:

- i) high standards for tenure
- ii) low proportion of preadministrative type B
- iii) brief interval prior to the tenure decision
- iv) low discount rate
- v) balanced administrative interference.

These are certainly stylized facts concerning good departments. Note that, as $\alpha \rightarrow 1$, $p \rightarrow 0$, so that the best departments have no faculty meetings, committees, and so forth.

Furthermore,

- vi) professorial output goes up at the discount rate
- vii) administrative interference is constant (incessant) over time, and for all but $\alpha = 1$ departments, is nonzero¹³
- viii) $\dot{r}_A = \frac{\delta}{a} \geq \frac{\delta}{b} = \dot{r}_B$

That is, Type B individuals lag behind Type A's.

Certainly these predictions are in agreement with most individual's view of the profession.

Econometric Application

The model was tested by examining efficiency units of output prior and posterior to tenure. Efficiency units of output were established by the journal ranking scheme¹⁴ of Milcsaap (1983).¹⁵ The results are presented in Table 1. The hypothesis that tenure affects productivity was overwhelmingly accepted. Various tests of the hypothesis were used, and most readers will be familiar with them, so I won't describe their mechanics here.¹⁶

Institution Data	CHICAGO	HARVARD	MIT	NORTHWESTERN	UCLA	BERKELEY	UBC	PRINCETON	U HAWAII	U MOSCOW
Pre Tenure Output ^a	17.0 (3.2)	10.1 (20)	14.3 (10.1)	17.3 (5.5)	NA ^c	NA ^d	0 ^e (0.0)	15.1 (1.5)	20.9 (1.0)	173.9 ^f (0.0)
Post Tenure Output ^a	12.1 (5.7)	0 (0.0)	-7.2 (6.5)	NA ^b	-8 (1.1)	NA ^d	0 ^e (0.0)	15.3 (1.5)	0 (0.0)	173.9 ^f (0.0)
t Test Confidence Level	72%	99.9	99.9	-	-	-	100%	0%	100%	0%
Wald M Test	8	8	8	8	8	8	8	8	8	8
Mixed Metaphor Test	2.7	80.1	68.2	18.3	100.0	29.1	100.0	5.1	92.6	100.0
Brookings Institute Test	0	94	100	42	40	55	-	45	99	100
Hoover Institute Test	100	6	0	58	60	45	-	55	1	0
Peano Space Filling Test	1	2	4	8	16	32	64	128	256	512

TABLE 1: Output in efficiency units at various economic departments, before and after tenure. The last 6 rows refer to the hypothesis: Accept a differential.

Notes:

^aStandard deviation in parentheses.

^bNot available.

^cNot clear what productivity could mean.

^dWe were scared to visit there.

^eToo boring to evaluate, due to aggregation problems.

^fSoviet data.

Conclusion

Why is there tenure? What is the difference between tenure for professors and unemployment insurance for factory workers? Noting that every economics department has the incentive to continue to employ professors doing controversial but high quality work, can we conclude that 'academic freedom' is precisely the freedom to have incompetence protected?

In the seminal work by Nosfu (1984), the issue of the optimal elimination of tenure was considered. Nosfu argues that tenure should be eliminated as an option for all new entrants into the academic market, for "those currently in the market entered it presuming the existence of tenure, and, thus, to eliminate tenure would be pulling the rug out from under them". Presumably Nosfu is expecting to quit working.

Many have argued that tenure is irrelevant, for a department can allow deadwood salaries to lag behind the inflation rate (see Filmore (1977)). In the presence of faculty unions, this is unclear. In any case, it seems to me that tenure at public universities is precisely analogous to civil service job security, and an equal boondoggle.

Certainly more work remains on this topic. First, a model of the competitive determinants of r_0 and T should be given. Also, this model focused on serious economists, but of course there are those on the margin between working for a university and elsewhere.¹⁷ Thus, a competitive model of T and r_0 , with labor substitution, would be of interest. A continuum of disutilities of effort, with some having an effort blisspoint associated with positive output, would generalize the model in a valuable way.

Second, there is obviously the incentive to self select into high or low p universities. Utilizing a continuum model, one might generate an evolutionary selection process along the lines of MacDonald's 1984 job/worker typing. Of course, one should capture differential returns,

so that type B's tendency to mimic type A's, in the early stages of their careers, is fully captured.

In addition, this model has not captured the phenomena observed by Zlumbre (1981), who observed that tenure causes, in the Granger sense, serious economists to spout random policy prescriptions unconnected with their formal analyses. Future work should attempt to derive the Type A's spontaneous metamorphosis into Type B's as an optimization.

FOOTNOTES

¹It is ironic that the existence of unequated marginals has become known as a dead weight loss. What could be preferable to the loss of dead weight?

²For example, the analyses of the regulated monopoly, and various associated welfare losses, that are to be found in the late Bell Journal of Economics.

³Unlike medical doctors, the term economist is singularly unregulated. Thus, some self-styled "economists" may take issue with this statement. The sentence is best seen as a definition of the term "serious economist". Being pragmatic forces me to conclude that those "economists" that speak on T.V. about downside risks in the rapeseed futures market, or, worse still, appear to know what a convertible debenture is, cannot be seriously described as economists.

⁴The inventor of the infant industry argument deserves to be burned in effigy. In Canada, where I reside, every industry is an infant industry, and growing younger by the minute.

⁵Anyone who has driven through the south side of Chicago will understand why a University of Chicago economist might see these, and other lurking market failures. Given the conditions, it is surprising that they don't see more.

⁶Even if there were such articles, would you want me to discuss them much less cite them: reference space is a scarce resource.

⁷Evidently, Morris merely asked sociology professors "On a scale of one to ten, how productive are you?" Non tenured professors had an average response of 3.7, while tenured professors averaged 4.1, and Morris concluded that tenured professors were more productive. Kaizer pointed out that Morris could not legitimately use a student's t distribution to test his hypothesis, for data on a [1,10] scale

cannot be normally distributed. Kaizer assumed instead a uniform distribution of sample responses, and accepted the hypothesis of "no difference" at 95%, but not at 90%.

⁸Located in Lubyanka, Moscow.

⁹The Barren-Grossman paper is perhaps the most innovative of these. In this monograph, she discusses the cause of her own Ph.D. dissertation, and whether it would have been written, if tenure did not exist. She eventually concludes: "the model was almost certainly created so that I might prove my superiority to other economists. After all, how better to accomplish this than by getting the profession to take my paper seriously, and then be the first to point out how stupid it is?" She continues: "[this proved] that economists are closet Keynesians, even today." Also, see the brief comment by Short (1983).

¹⁰This is a trick to take advantage of journal ranking procedures. Because they tend to count references in a fan-like search, a paper that references itself will obtain an unbounded number of references. In addition, a journal publishing a self-referential article will obtain an infinity of references.

¹¹And, of course, I have tried all these suggestions, and they are all entirely intractable. But it can't hurt me for you to waste your time with them, can it?

¹²Without loss of generality, one could let p be the number of faculty meetings, the probability one must serve as Dean, or the probability one gets interested in economic policy. This important functional form correctly assumes professors have marginal income equal to zero. Why else did they go into academics? For evidence in support of these functional forms, see Eden (1978) and Anthrax (1980).

¹³Corroboration of this model was received indirectly from comments on it. One group of people found the optimality of positively many faculty meetings astounding, while the rest thought it was obvious to the most casual observer. Calling these groups type A and type B, we have found the model's agents.

¹⁴And numerology.

¹⁵For a survey, see Palmist (1983). Also see Skuaalid (1979).

¹⁶Besides, only the t-test is understood by anyone. Note that, unlike most papers, the econometric discussion section is brief, without loss of comprehensibility.

¹⁷Of curcial importance, in a study of this kind, is an answer to the question: how do we deter those on the margin between academics and elsewhere from entering academics? It is evident that low salaries are an insufficient deterrant--you should see the papers I am expected to referee! Did you ever notice how refereeing seems like one of those cards that say, on both sides, "how to keep a moron occupied--see other side". You send one back and they send you another. Anyway, it would be nice to deter them without using low salaries, don't you think?

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Appendix 1

Define $\dot{u} = \log\{-\log(p)\}$

$\dot{v} = \log\{-\log(1-p)\}$

Then $r_A = \frac{1}{a} \left\{ \delta(t - \frac{T}{2}) + \frac{ar_0}{T} + \frac{1}{T} v - \dot{v} \right\}$, from (4) and (5),

$r_B = \frac{1}{b} \left\{ \delta(t - \frac{T}{2}) + \frac{br_0}{T} + \frac{1}{T} u - \dot{u} \right\}$, from (6) and (7),

and $e^{-(e^{\dot{v}})} + e^{-(e^{\dot{u}})} = 1$. Thus, we may rewrite (8) as

$$\max \int_0^T e^{-\delta t} \left\{ \left(\frac{\alpha}{a} + \frac{1-\alpha}{b} \right) \delta(t - \frac{T}{2}) + \frac{r_0}{T} + \alpha \left(\frac{1}{T} v - \dot{v} \right) + (1-\alpha) \left(\frac{1}{T} u - \dot{u} \right) \right\} dt \quad (9)$$

$$\text{s.t. } e^{-e^{\dot{v}}} + e^{-e^{\dot{u}}} = 1. \quad (10)$$

As (9) coincides with

$$\max (1 - e^{-\delta T}) \left[\frac{r_0}{T} + \left(\frac{\alpha}{a} + \frac{1-\alpha}{b} \right) \left(\frac{1}{\delta} - \frac{T}{2} \right) \right] + \int_0^T \left\{ \alpha \left(\frac{1}{T} v - \dot{v} \right) + (1-\alpha) \left(\frac{1}{T} u - \dot{u} \right) \right\} e^{-\delta t} dt$$

we may consider

$$\max \int_0^T \left\{ \alpha \left(\frac{1}{T} v - \dot{v} \right) + (1-\alpha) \left(\frac{1}{T} u - \dot{u} \right) \right\} e^{-\delta t} dt \quad (11)$$

subject to (11).

Now, the proof of Theorem 3.

Proof: Write

$$H = \alpha \left(\frac{1}{T} v - \dot{v} \right) + (1-\alpha) \left(\frac{1}{T} u - \dot{u} \right) - \lambda (1 - e^{-e^{\dot{v}}} - e^{-e^{\dot{u}}})$$

$$\frac{\alpha}{T} = \frac{\partial H}{\partial v} = \frac{d}{dt} \frac{\partial H}{\partial \dot{v}} = \frac{d}{dt} [-\alpha - \lambda e^{-e^{\dot{v}}} e^{\dot{v}}] \text{ and}$$

$$\frac{1-\alpha}{T} = \frac{d}{dt} [- (1-\alpha) + \lambda e^{-e^{\dot{u}}} e^{\dot{u}}]$$

But $e^{-e^{\dot{u}}} = p$, so $e^{\dot{u}} = -\log p$, yielding

$$1 - \frac{\alpha}{T} = \frac{d}{dt} - \lambda p (\log p) = \dot{\lambda} p \log p + \lambda \dot{p} (\log p + 1) \quad (12)$$

$$\frac{\alpha}{T} = \frac{d}{dt} - \lambda(1-p) \log(1-p) = \lambda(1-p) \log(1-p) - \lambda p (\log(1-p) + 1) \quad (13)$$

In addition, we have, by transversality:

$$\alpha = -\lambda(1-p) \log(1-p) \Big|_{t=T}$$

$$(1-\alpha) = -\lambda p \log p \Big|_{t=T}$$

Let p_T , λ_T solve these equations. Then, from (12) and (13),

$$\frac{d}{dt} - \lambda p \log p = \frac{1-\alpha}{T}, \text{ so}$$

$$-\lambda p \log p \Big|_t = -\lambda p \log p \Big|_T + \frac{1-\alpha}{T} (t-T) = (1-\alpha) \frac{t}{T}$$

Similarly

$$-\lambda(1-p) \log(1-p) = \alpha \frac{t}{T}$$

This forces

$$\frac{p \log p}{(1-p) \log(1-p)} = \frac{1-\alpha}{\alpha}$$

or

$$\alpha \log p^p = (1-\alpha) \log((1-p)^{1-p})$$

$$\begin{aligned} \int_0^T \delta e^{-\delta t} \left(t - \frac{T}{2}\right) dt &= \\ -\left[t - \frac{T}{2}\right] e^{-\delta t} \Big|_0^T + \int_0^T e^{-\delta t} dt &= \\ -\frac{T}{2} (1 + e^{-\delta T}) + \frac{1 - e^{-\delta T}}{\delta} \end{aligned}$$

$$\text{Thus } PV = r_0 + \left\{ \frac{\alpha}{a} + \frac{1-\alpha}{b} \right\} \left[\frac{1}{\delta} (1 - e^{-\delta T}) - \frac{T}{2} (1 + e^{-\delta T}) \right]$$

PV is maximized when

$$h(T) = -\frac{1}{\delta} e^{-\delta T} - \frac{T}{2} (1 + e^{-\delta T}) \text{ is maximized.}$$

$$h'(T) = e^{-\delta T} - \frac{1}{2} (1 + e^{-\delta T}) + \frac{T}{2} \delta e^{-\delta T}$$

$$= \frac{1}{2} [-1 + e^{-\delta T} (1 + \delta T)]$$

$$h''(T) = \frac{1}{2} [-\delta e^{-\delta T} (1 + \delta T) + \delta e^{-\delta T}] = -\frac{\delta^2 T}{2} e^{-\delta T} < 0.$$

Thus PV is strictly concave in T , and $h'(T) = 0$ yields a unique optimum.

But $h'(0) = 0$.