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Concessions and the Agenda in Bargaining*

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Abstract

While bargaining problems tend to require allocations for many items or issues, most extant models of bargaining fail to incorporate decisions regarding the bargaining agenda. Instead, they impose a fixed order in which issues are to be resolved: most often simultaneously. We show, in a model of endogenous agenda choice, that agenda selection can act as a means to signal type when there is (one-sided) incomplete information. Compared to bargaining with no agenda choice, bargaining where there is a choice between an issue-by-issue versus a simultaneous-offers agenda produces less frequent pooling on offers by the informed and introduces a separating equilibrium in which the "strong" bargainer signals type by use of issue-by-issue bargaining. The issue-by-issue agenda allows the strong bargainer to make sufficiently large concessions on initial issues as to credibly signal type, thereby generating large concessions from his opponent on later issues. This need to make initial concessions when issue-by-issue bargaining is used also means that the order in which issues are bargained over can be relevant.

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1 Introduction

In his 1950 paper on bargaining, John Nash wrote that:

> The economic situations of monopoly versus monopsony, of state trading between two nations and of negotiations between employer and labor union may be regarded as bargaining problems. ("The Bargaining Problem", p. 155)

Writing a few years later, Luce and Raiffa (1957) claim that:

> Most authors feel that, if such economic problems as duopoly, labor-management disputes, trade regulations between two countries, etc., can be treated as games at all, then it will have to be in the cooperative context. In like manner, one may hope that it will prove possible to formulate cooperative game models which reflect limited aspects of the diplomatic relations between two countries or of the political conflict between two parties within a single country. (*Games and Decisions,* p. 115)

Finally, Rubinstein (1982) writes:

> When I refer in this paper to the *Bargaining Problem* I mean the following situation and question:
>
> Two individuals have before them several possible contractual agreements. Both have interests in reaching agreement but their interests are not entirely identical. What "will be" the agreed contract, assuming that both parties behave rationally. ("Perfect Equilibrium in a Bargaining Model", p. 97)

A feature that almost all of the above examples of "bargaining problems" have in common, and that is common to most other settings that might be described as bargaining situations, is that bargaining must produce allocations/resolutions for many items or issues. In practice, the order in which issues are bargained over and settled (the agenda) is an important feature of any negotiations. In some instances, prior negotiations on the agenda are held before any actual bargaining takes place. Practitioners of the negotiating art even have theorized about how negotiations should be structured, some arguing that "easy" issues should be negotiated first to build negotiating momentum, others suggesting that "hard" issues should be first since failure on these matters makes other negotiations irrelevant. In spite of these facts, the "idealizations", as Nash called them, that are employed by economists to "solve" the bargaining problem typically ignore the agenda issue entirely. The stylized bargaining over a single "pie" popularized by Rubinstein does implicitly impose an agenda: one in which all issues are bargained over and resolved simultaneously. The adoption of alternative bargaining structures, however, is simply not at issue.

Given that the focus of many authors has been on a "solution" to the bargaining problem, the failure to address the issue of the bargaining agenda is perhaps understandable. With solutions now available, however, attention naturally has turned to the question of how, if at all, bargaining models inform us about situations like those listed above. It is our contention that, through the incorporation of such features as the bargaining agenda, a notion of hard versus easy issues, and learning about opponent characteristics, bargaining models can provide useful insights into the sorts of issues that originally motivated the study of bargaining. In what follows, we hope to illustrate this point by incorporating these features into the Rubinstein (1985) bargaining model. By so doing, we are able to show that an alternative (relative to the Rubinstein) agenda can be used by negotiators to signal information about "bargaining strength" and that, in some instances, the order in which issues are negotiated, in terms of "important" versus "less important" issues, is relevant.

Our paper is not the first to address issues relating to the bargaining agenda. Several authors have considered what effect different bargaining agendas may have on the bargaining solution. Within an axiomatic bargaining framework, Kalai (1977) considers an agenda in which issues can be bargained over one-by-one and agreement on all issues is not required for allocations to be made on those issues for which agreement does exist. (In what follows, we refer to this case as one of sequential implementation.) Under this sort of agenda, agreements on initial issues form disagreement points for bargaining on subsequent issues. Kalai seeks a bargaining solution that is invariant to the agenda and shows that only one satisfying a proportionality axiom will have this feature. Herrero (1989) and Fershtman (1990) consider similar questions within a non-cooperative bargaining context. Both introduce an agenda in which issues are bargained over sequentially but no allocations are implemented until agreement is reached on all issues. (This case will be referred to as simultaneous implementation.) The problem these authors address is the determination of those situations in which the agenda does or does not affect the bargained solution, where in this case the reference point is the solution under the Rubinstein (1982) agenda.

In all of the above papers, the agenda is imposed exogenously and the focus is on how the bargaining solution is affected by the agenda. No consideration is given to equilibrium agenda choice. Two papers that address the choice of agenda are Busch and Horstmann (1994) and Bac and Raff (1994). The former introduces an explicit pre-bargaining stage in which the agents bargain over the choice of agenda. It shows that, if agents differ in their evaluations of different issues, equilibrium can result in the adoption of an agenda in which issues are bargained one at a time. This result is derived within a complete information bargaining setting under sequential implementation. Bac and Raff consider an incomplete information setting like that in Rubinstein (1985) where, instead of a single pie, there are two pies. Rather than having a pre-bargaining stage, they allow agents to make offers either on both issues simultaneously or on only one issue at a time. In this latter case, allocations are implemented as soon as agreement on any one issue is achieved (sequential implementation). Bac and Raff find that in equilibrium the "patient" bargainer will follow an issue-by-issue agenda and make proposals on one issue at a time.

The current paper is similar to Bac and Raff in that it considers a two-pie version of Rubinstein (1985) and allows agents to make offers either on both pies or only on one pie. It differs from Bac and Raff in two fundamental ways. First, it assumes that no allocations are made until agreement is reached on both issues. This assumption is adopted not only because it conforms to many actual bargaining settings, especially collective bargaining, but also because it avoids any potential confusion regarding the role the agenda serves. In particular, under sequential implementation an issue-by-issue bargaining agenda might be important not for the agenda itself, but merely because it allows agents to consume the first pie while still negotiating on the second. Under simultaneous implementation, by contrast, since no allocations can be made until agreement is reached on all issues, the agenda choice can only be important because the agenda itself is valuable.

The second way our paper differs is that it assumes that it is in the "weak" bargainer's interest to mimic the "strong" bargainer. In Bac and Raff, the weak bargainer would prefer the full information outcome associated with his discount factor rather than delay agreement one period and receive the full information outcome for the strong bargainer.[1] This assumption means that sequential implementation arises in Bac and Raff not for any signaling reasons, but purely because it permits the strong bargainer to face delay only on one pie rather than two. If no allocations could be made until agreement were reached on all pies, then the strong bargainer would have no incentive to make anything other than simultaneous offers on both pies.

Within our framework, we show that the agenda itself can be used to signal information about bargaining strength. By making sufficiently large concessions on the first pie, a strong bargainer can signal his strength and so achieve large concessions on the second pie from his opponent. A weak bargainer is unwilling to mimic this strategy because the delay in obtaining concessions on the second pie makes the initial concessions too costly. Because the initial concessions must be sufficiently large to make mimicry costly, the pie on which the initial offer is made may also become relevant, if the pies are of different sizes. In cases where such is true, not only does a different (relative to Rubinstein) agenda arise in equilibrium, but we can make statements about the order in which issues are negotiated in this agenda. In particular, if bargainers have a preference, it is the large issue that is negotiated first.

In what follows, we lay out our results in more detail. The next section introduces the model and necessary notation. The third section provides the main results of the

[1] This is assumption A-2 in Bac and Raff, which contradicts assumption C-2 (reproduced later) in Rubinstein (1985), maintained in this paper.

paper while section four concludes.

2 Model Description and Notation:

There are two agents, called the informed and the uninformed. They bargain over the allocation of a jointly-owned surplus which is made up of two issues, X and Y. The size of issue X is X, while the size of issue Y is normalized to 1. Bargaining is via alternating offers, with the informed agent starting the game. An offer is a proposal of a share for the informed agent, implying that the remainder of the surplus goes to the uninformed agent. Offers can be made on either both issues simultaneously or on only one issue. We denote offers on a single issue by $x \in [0,1]$ or $y \in [0,1]$, where x is the informed's share of surplus X and y is the informed's share of surplus Y. A joint offer is denoted (x, y) in order to stress the fact that there are two separate issues making up the bargaining problem. If an offer is made on only one issue initially, neither agent is bound to continue to make offers on only this issue, or only one issue, for that matter. Similarly, a joint offer which is rejected can be answered by a joint or partial offer on either of the single issues. If an offer on only one issue is accepted, it becomes binding and not renegotiable. Further offers on this issue, including any joint offers, are thereby precluded. The game ends as soon as an agreement on both issues exists; that is, the game ends either after the acceptance of a joint offer or after the last single issue has been allocated.

Implementation of any agreements occurs only at the conclusion of all bargaining. Thus, the allocation of partial agreements does not occur until both issues are settled.[2] Agents' payoffs from an agreement on x at time t and y at time τ (where τ may or may not be equal to t) are given by

$$\begin{aligned} U_i((x,t),(y,\tau)) &= \delta_i^{\max\{t,\tau\}}(x \cdot X + y), \\ U_u((x,t),(y,\tau)) &= \delta^{\max\{t,\tau\}}((1-x) \cdot X + (1-y)). \end{aligned}$$

[2]This is in contrast to Bac and Raff (1994) where allocations are made as each issue is settled.

Here, $\delta \in (0,1)$ is the uninformed's discount factor and is public knowledge, while $\delta_i \in (0,1)$ is the informed's discount factor. The value of δ_i is private information of the informed agent. The uninformed's priors are that δ_i may take one of two values, δ_w or $\delta_s > \delta_w$, with the probability that $\delta_i = \delta_w$ given by $\omega_0 \in [0,1]$. If the informed agent's discount factor is δ_w, then that agent will be referred to as "weak". The informed agent will be referred to as "strong" if the discount factor is δ_s.

As in Rubinstein (1985), we impose a restriction on the relative magnitudes of the various values of δ. This restriction is defined in terms of the equilibrium shares under complete information. To be specific, let the values $\mathcal{S}_w$ and $\mathcal{S}_s$ be the full-information equilibrium shares received by the weak and strong informed agent, respectively, when it is that agent's turn to propose and all offers must be joint offers.

Definition 1

$$\mathcal{S}_w := \frac{1-\delta}{1-\delta\delta_w}, \qquad \mathcal{S}_s := \frac{1-\delta}{1-\delta\delta_s}.$$

We assume that discount factors are such that the weak informed agent is worse off making an offer of $\mathcal{S}_w$ that is accepted immediately rather than having one period of delay and accepting a counter-offer of $\delta_s\mathcal{S}_s$ (the share the strong informed receives when the uninformed offers and there is complete information). This assumption is (C-2) in Rubinstein (1985) and implies that:

Assumption 1 (Rubinstein 1985, (C-2))

$$\delta_w\delta_s(1-\delta\delta_w) > (1-\delta\delta_s).$$

The content of Assumption 1 is that the weak agent has an incentive to mimic the strong agent even if doing so requires delay. As a consequence, the strong agent cannot expect to obtain his full-information share simply by delaying agreement, nor can the uninformed agent expect to screen with an offer of the weak agent's full-information share.[3]

[3]This assumption stands in contrast to Bac and Raff (1994), who make the exact opposite assumption. Their assumption A-2 implies, in fact, that the weak prefers to get his full information share of both issues now, compared to getting the strong's full information share of one issue now, and of the other issue tomorrow.

Strategies and histories for the game are defined in the usual way.[4] A strategy for any agent will be a complete list of (type dependent) actions at each of that agent's decision nodes (after every history). This list includes both the decision to accept or reject an offer as well as the decision on what share to offer and on which issue(s). A history in this game will be a complete list of all such actions from the start of the game to the current node.

The equilibrium concept is bargaining sequential equilibrium as defined in Rubinstein (1985). This definition of equilibrium is chosen to permit direct comparison of our results with those of Rubinstein and so isolate the role of the agenda in bargained outcomes. For ease of reference, we re-produce Rubinstein's belief restrictions.

Assumption 2 (Once persuaded never dissuaded) *If $\omega_t = 0$ then $\omega_{t+k} = 0$ and if $\omega_t = 1$ then $\omega_{t+k} = 1$, $\forall k = 1, 2, \ldots$.*

Assumption 3 (Rubinstein 1985, (B-1)) *If $\omega_t \neq 1$, then after a sequence of actions which is weakly preferred by the strong type but strictly disliked by the weak type, the uninformed player is convinced that he faces the strong type, i.e., $\omega_{t+k} = 0$.*

Assumption 4 (Rubinstein 1985, (B-3)) *If players are indifferent between accepting a current offer and rejecting it, they accept.*

Assumption 5 (Rubinstein 1985, (B-4)) *Both types of the informed player will never demand more than the strong type's full information share of the surplus.*[5]

A final restriction not included in the above list relates to the updating of beliefs after deviations which are weakly preferred by both types. Rubinstein's assumption is that ω_t should not rise in this case. There remains some uncertainty on our part

[4] We omit the formal notation here as it is essentially the same as in Rubinstein (1985).

[5] This is the original assumption in Rubinstein 1985. On page 1163 he claims that this assumption is equivalent to assuming that the uninformed player will update beliefs to $\omega = 0$ if an offer which the weak type is supposed to accept and the strong type is supposed to reject is, in fact, rejected, independent of the counter-offer which is observed. It appears to us that at least in our formulation these two assumptions are not equivalent.

about how this restriction should be applied when there is more than one issue. We provide two interpretations of this restriction subsequently, listed as Assumptions 6 and 7.

Were only joint offers available in the above model, it would be precisely the model of Rubinstein (1985) with a pie size of $(1+X)$. We reproduce Rubinstein's equilibrium results for this case below. Since the equilibrium offers (x, y) are not unique in this model, the equilibrium outcomes have been expressed in terms of equilibrium payoffs.

Result 1 (Rubinstein 1985) *For a game starting with the uninformed player who holds initial beliefs* ω_t *and in which all players are restricted to make joint offers only:*

(i) (screening) If $\omega_0 \geq \omega^*$ *then the uninformed party offers a pair* (x, y) *such that*

$$(x \cdot X + y) = \delta_w \left(\frac{1 - \delta\omega_t - \delta^2(1-\omega_t)}{1 - \delta^2(1-\omega_t) - \delta\delta_w\omega_t} \right) \cdot (1 + X).$$

This offer is accepted only by the weak type. The strong type makes a counter-offer of

$$(\hat{x} \cdot X + \hat{y}) = \frac{1 - \delta\omega_t - \delta^2(1-\omega_t)}{1 - \delta^2(1-\omega_t) - \delta\delta_w\omega_t} \cdot (1 + X)$$

in the next period. This same offer is made by both types of informed player in periods when the informed has the offer.

(ii) (no screening) If $\omega_t \leq \omega^*$, *then the uninformed party offers a pair* (x, y) *such that* $(x \cdot X + y) = \delta_s S_s(1 + X)$ *and both informed types accept. In periods in which the informed player makes an offer, both types offer* $(\hat{x} \cdot X + \hat{y}) = S_s(1 + X)$. *Here*

$$\omega^* = \frac{(1 + \delta - \delta\delta_s - \delta_s)}{(1 + \delta - \delta\delta_s - \delta_w)}.$$

3 Agenda Effects

An implication of Result 1 is that, if agents are restricted to using only joint offers — the Rubinstein agenda — then no signaling is possible for the informed agent in equilibrium. We show in this section that, if agents are permitted to use single offers as well as joint offers — i.e., propose an issue-by-issue bargaining agenda — then the strong agent is able to signal by structuring proposals in this way. We also provide conditions under which the order in which the issues are structured is relevant.

We proceed by first establishing two preliminary results on bargaining equilibrium outcomes when agenda determination is part of the bargaining process. They are:

Lemma 1 *If $\omega = 0$, and agreement on $x^* \in [0,1]$ exists, then the uninformed will offer*

$$y^* = \begin{cases} 0 & \text{if} \quad x^*X \geq \delta_s \mathcal{S}_s(1+X), \\ \delta_s - (1-\delta_s)x^* \cdot X & \text{if} \quad x^*X \leq \mathcal{S}_s(1+X) - 1, \\ \delta_s \mathcal{S}_s(1+X) - x^* \cdot X & \text{otherwise,} \end{cases}$$

and both informed players will accept.

Proof: The proof is adapted from the standard full-information proof of Shaked and Sutton (1984). Let y denote the offer by the uninformed agent and $\hat{y}$ the offer by the informed agent. These offers solve the following system of inequalities:

$$\begin{aligned} (1-\hat{y}) + (1-x^*)X &\geq \delta(1-y) + \delta(1-x^*)X \\ y + x^*X &\geq \delta_s \hat{y} + \delta_s x^* X \end{aligned}$$

Solutions for this system can take one of three possible forms. (i) the uninformed agent is constrained and can demand no more than all of issue Y ; i.e., $y = 0$. This outcome occurs if the agreement x^* alone gives the informed a larger payoff than would result if the uninformed were to offer on both issues under full information, i.e., if $x^*X \geq \delta_s \mathcal{S}_s(1+X)$. (ii) the informed party is constrained and can demand no more than all of issue Y; i.e., $\hat{y} = 1$. This outcome occurs if the agreement x^* alone gives the uninformed a larger payoff than would result were the informed to offer on both issues under full information; i.e., if $1 + x^*X \leq \mathcal{S}_s(1+X)$. In this case only the second of the inequalities binds so that $y = \delta_s - (1-\delta_s)x^*X$. (iii) neither agent is constrained and so y and $\hat{y}$ are both strictly between zero and one. In this case both conditions are equalities, resulting in $y = \delta_s \hat{y} - (1-\delta_s)x^*X$ and $\hat{y} = \mathcal{S}_s(1+X) - x^*X$.

qed

Case (ii) of Lemma 1 will prove especially important in the construction of the signaling equilibrium to follow. In particular, if the strong agent is to signal successfully, he will have to make a concession on the first issue that results in the bargaining equilibrium on the second issue being given by case (ii).[6]

The next result modifies Rubinstein's Proposition 4. In the model without an agenda choice, Rubinstein has established that both types of informed agent make the same offer any time it is the informed's turn to propose. This result no longer holds if an agenda is available.

Lemma 2 *If* $X \geq (\delta_s^2(1-\delta\delta_w)-(1-\delta))/(1-\delta)$, *then in any Bargaining Sequential Equilibrium the informed players cannot both make a joint offer* (x, y) *such that*

$$x \cdot X + y \in \left[rS_w(1+X),\ \delta_s^2 S_s(1+X)\right)$$

which the uninformed player accepts.

Proof: Consider the uninformed with beliefs ω being confronted by a joint offer satisfying the condition of the lemma. We show that deviation from this offer by the strong type is unprofitable only if the uninformed agent's beliefs following the deviation violate Assumption 3. As a consequence, there cannot be a bargaining sequential equilibrium of this sort.

The deviation the strong type adopts is a single offer on X given by $\hat{x} = (x \cdot X + y - \delta_s^2)/(\delta_s^2 X) + \varepsilon$ for some $\varepsilon > 0$. It is easily checked that, for all joint offers satisfying the conditions of the lemma, there always exists a value of ε such that $\hat{x}$ falls within case (ii) of Lemma 1. Thus, the maximal settlement on Y that the informed can expect should the offer on X be accepted is $\delta_s - (1-\delta_s)\hat{x}X$. In this case, the payoff accruing to the informed agent in the period agreement is reached is $(x \cdot X + y)/\delta_s + \varepsilon X$. In present value terms, this payoff is strictly larger than the payoff under the proposed equilibrium for the strong informed agent but strictly less

[6]For either of the other two cases, the informed agent's equilibrium payoff is at least that obtained should the uninformed make a joint offer under full information. Assumption 1 then implies that any signaling attempt by the strong agent will be mimicked by the weak agent and so be unsuccessful.

for the weak (since $\delta_w/\delta_s < 1$). Assumption 3 then implies that $\omega' = 0$ following the deviation, thus making it profitable for the strong type to adopt the deviation.

As $(x \cdot X + y)/\delta_s \to \delta_s \mathcal{S}_s(1+X)$ this argument no longer holds, since the informed agent cannot obtain a payoff greater than $\delta_s \mathcal{S}_s(1+X)$ at the date of agreement under any circumstances. (It is the complete information payoff when the uninformed offers.) This gives the upper bound on payoffs from the joint offers in the lemma. Finally, the above deviation only makes sense if $\widehat{x} \geq 0$. For this to be the case for any ε, $xX + y \geq \delta_s^2$. Since the minimum possible value of $xX + y$ is $\mathcal{S}_w(1 + X)$, it must be that $\mathcal{S}_w(1 + X) \geq \delta_s^2$, yielding the condition on X. **qed**

Lemma 2 provides a first glimpse into the way that agenda choice makes signaling possible. In a model without agenda choice, the strong agent must obtain his full-information share of the surplus once his type is known. Since, by assumption, the weak agent prefers the payoff associated with this allocation to that obtained by admitting weakness, the weak agent would mimic any actions by the strong which are designed to reveal type. As a result, no signaling is possible. When agenda choice is available, proposing only on one issue at a time allows the strong agent to sacrifice a sufficient amount on the first issue to guarantee that the final agreement is unattractive to the weak agent. Because the strong agent is more patient, the payoff from doing so is still large enough as to be preferable to being pooled with the weak agent. Quite simply, by being able to make a concession on the first issue the strong agent is able to signal type credibly.

The signaling possibilities detailed above provide the basis for our first major result on the impact of agenda choice on bargained outcomes. We show that, for some environments, the no-signaling equilibrium in Rubinstein (1985) fails to survive when agenda choice is possible.

Proposition 1 *If* $X \geq (\delta_s^2(1-\delta\delta_w)-(1-\delta))/(1-\delta)$, *or* $X \leq (1-\delta)/[\delta_s^2(1-\delta\delta_w)-(1-\delta)]$, *then Rubinstein's (1985) equilibrium in which the informed players pool in* *their offers breaks down for all* $\omega_0 \in [\hat{\omega}, 1]$, *where*

$$\hat{\omega} = \frac{(1-\delta^2)(1-\delta_s^2 \mathcal{S}_s)}{\delta(1-\delta)+\delta\delta_s^2\mathcal{S}_s(\delta-\delta_w)} < 1.$$

Proof: In Rubinstein the informed agents pool their offers (see Result 1). We demonstrate that a deviation of the form given in Lemma 2 always exists for large enough values of ω. To see this, note that the pooling joint offer in Rubinstein is a decreasing function of ω. If the first condition on X is satisfied then, by Lemma 2, a deviation consisting of a single offer on X is profitable for the strong type but not the weak if the payoff from the pooling offer falls below $\delta_s^2\mathcal{S}_s(1+X)$. Setting the joint pooling offer of Rubinstein equal to this cutoff value, we obtain a critical value of ω given by:

$$\frac{1-\delta^2(1-\hat{\omega})-\delta\hat{\omega}}{1-\delta^2 1-\hat{\omega})-\delta\delta_w\hat{\omega}} = \delta_s^2\mathcal{S}_s.$$

Solution of this equation yields the $\hat{\omega}$ in the proposition. A simple calculation shows that, under the restrictions of Assumption 1, $\hat{\omega} \leq 1$. If the second condition on X is satisfied, then a deviation analogous to that of Lemma 2 but involving a single offer on Y exists. **qed**

Proposition 1 demonstrates that the introduction of an agenda choice has signaling value that destroys the pooling equilibrium of Rubinstein for some values of beliefs and relative sizes of issues. There remains, however, the question of whether or not a signaling equilibrium actually exists for this model when agenda choice is introduced. It is to this question that we now turn. Before demonstrating the existence of such an equilibrium, we provide one additional preliminary result.

Lemma 3 *For separation to occur in the offers of the informed agent, the weak type must make a joint offer* (x, y) *such that*

$$x \cdot X + y = \mathcal{S}_w(1+X),$$

while the strong player must make the partial offer

$$\hat{x} = \left(\frac{\mathcal{S}_w(1+X)}{\delta_w\delta_s} - 1\right)\frac{1}{X}.$$

Both types of offers are accepted by the uninformed.

Proof: Assume separation. Then, after observing the weak type's offer, the uninformed agent sets $\omega = 1$. Thus, the uninformed believes that an offer of $\hat{x} \cdot X + \hat{y} = \delta_w \mathcal{S}_w(1 + X)$ will be accepted next period. As a result, the uninformed will only accept an offer from the informed that yields at least $\delta(1 - \delta_w \mathcal{S}_w)(1 + X)$. Therefore, the largest offer the weak type can expect to have accepted in a separating equilibrium is

$$(1 - \delta(1 - \delta_w \mathcal{S}_w))(1 + X) = \mathcal{S}_w(1 + X).$$

This offer is also the lowest offer the weak type would ever make in any equilibrium, being equal to his full information offer. Finally, since this offer produces no delay, it dominates any revealing offer which would imply delay, since $\mathcal{S}_w(1 + X) \geq \delta_w^2 \mathcal{S}_w(1 + X)$. This proves the result for the weak type.

As for the strong type, notice that the weak type would deviate from the above strategy if the strong type's revealing offer led to a higher payoff. This fact rules out a joint offer which is immediately accepted as the revealing offer for the strong, as well as a joint offer which is rejected and followed by a joint counter offer (by Assumption 1.). A single issue offer $\hat{x}$ that satisfies the condition required for case (ii) of Lemma 1 remains as the only possibility. The largest of such offers will make the weak type just indifferent between following the prescribed equilibrium strategy and mimicking the strong type; that is, it will yield a payoff at date of agreement of $\mathcal{S}_w(1 + X)/\delta_w$. The value for $\hat{x}$ in the statement of the lemma is precisely this offer. Thus, the weak type will not deviate while the strong type prefers this offer to mimicking the weak type. **qed**

While Lemma 3 establishes the existence of separating strategies when agenda choice is available, it does not establish whether such strategies can form an equilibrium that satisfies Rubinstein's belief restrictions. To establish this fact, we must first specify belief restrictions following deviations that are weakly preferred by both informed types.[7] In Rubinstein (1985), the following restriction is imposed:

[7]Recall that this restriction was not included in the original list of assumptions.

Assumption 6 (Rubinstein 1985, (B-2)) *A sequence of actions which is weakly preferred by both types will not increase ω_t. (This includes the requirement that a deviation by the informed to higher offers will not increase ω_t.)*

Rubinstein's justification for this restriction was that deviations that lead to (potential) delay should not be more likely associated with the weak type, who dislikes delay, than the strong. In his model, deviations by the informed are either rejections of offers followed by counter offers, or offers that are so high as to lead to rejection. (B-2) in this context disallows optimistic beliefs (a weak opponent) after behavior that insists on a higher share at the cost of more time spent bargaining.

In our model, because the informed can make either joint or single offers, insistence on higher shares leading to (potential) delay can take multiple forms. One possible approach to constructing belief restrictions in this setting is simply to assume that a deviating offer by the informed is "more insistent" as long as it implies a higher total payoff, regardless of the form of the offer. Alternatively, one could assume that a single offer is "more insistent" than a joint offer because it implies greater (potential) delay. How this issue is resolved will determine the range of environments in which a separating equilibrium exists. We consider two possibilities: one in which the form of the deviating offer is irrelevant and only the total payoff matters (this is Assumption 6 above) and another in which the form of the deviating offer (single versus joint) matters (Assumption 7 below).

Under Assumptions 1-6, we can show the following:

Proposition 2 *Given Assumptions 1-6 and either $X \geq (\delta_s^2(1-\delta\delta_w)-(1-\delta))/(1-\delta)$ or $X \leq (1-\delta)/[\delta_s^2(1-\delta\delta_w)-(1-\delta)]$, a separating equilibrium exists in the game starting with the informed player's offer if*

$$\omega_0 \geq \frac{\delta_w(1-\delta^3)+(\delta^3-\delta_s)\mathcal{S}_w}{\delta\delta_w(1-\delta^2)+(\delta^3-\delta\delta_w^2)\mathcal{S}_w}.$$

Proof: In this separating equilibrium, the strategies implied by Lemma 3 are used if the first condition on X holds, while strategies which call for the corresponding single offer on Y first are used if the second condition on X holds. We will only prove the former case. For the strategies implied by Lemma 3 to be part of an equilibrium, it must be that: (i) the uninformed would reject any deviations to offers (joint or partial, under Assumption 6) that would give both informed types a higher ultimate payoff; (ii) rejection must be profitable for the uninformed given beliefs after such deviations are not allowed to increase above the initial value of ω. We provide, in turn, conditions such that each of these requirements is satisfied.

To begin, let z denote the share of total surplus under a hypothetical joint deviation by the informed. Such a z is accepted if it leads to a higher payoff for the uninformed than the equilibrium continuation payoffs after a rejection. The equilibrium continuation is a screening offer by the uninformed at the full information weak level that is accepted by the weak informed type and rejected by the strong informed type who counters with the same single offer as prescribed in Lemma 3. It also must be that the deviation to z yields both types of informed agents at least as much as the equilibrium path. Together, these two requirements imply that the a deviating offer, z, can only be successful if it satisfies the following conditions:

$$1 - z \geq \delta\left(\omega(1 - \delta_w \mathcal{S}_w) + \delta^2(1-\omega)\left(1 - \frac{\mathcal{S}_w}{\delta_w}\right)\right), \tag{1}$$

$$z \geq \delta_s \mathcal{S}_w / \delta_w. \tag{2}$$

By Assumption 6, beliefs in condition (1) continue to be given by ω. The first of these conditions is to guarantee that the uninformed accepts the offer z, while the second guarantees that the strong informed type prefers the offer z to the equilibrium offer (which, of course, implies that the weak type does also).[8]

For the proposed separating strategies to be equilibrium strategies, it must be that there exists no z which satisfies both (1) and (2) simultaneously. Combining (1) and (2), this requirement can be stated as

$$\left(1 - \frac{\delta_s \mathcal{S}_w}{\delta_w}\right) \leq \left(\omega(1 - \delta_w \mathcal{S}_w) + \delta^2(1-\omega)\left(1 - \frac{\mathcal{S}_w}{\delta_w}\right)\right). \tag{3}$$

[8]In the above conditions, the $(1+X)$ terms have been cancelled everywhere, leaving expressions in terms of shares of the total surplus only.

Solving for ω we obtain

$$\omega \geq \frac{\delta_w(1-\delta^3)+(\delta^3-\delta_s)S_w}{\delta\delta_w(1-\delta^2)+(\delta^3-\delta\delta_w^2)S_w}; \tag{4}$$

that is, as long as ω satisfies condition (4), no deviation by the informed to a joint offer can overturn the proposed equilibrium. It can be checked that, since $\delta_w < \delta_s$, the right hand side of this expression is strictly less than one for all discount factors under consideration. Thus, there is, in fact, a non-empty set of beliefs for which the separating strategies cannot be overturned by deviations of the above sort. Finally, note that deviations to a single offer by both types of informed agent are also handled by the above by simply interpreting z as a present value measure.

Next, we must check that the proposed screening continuation equilibrium assumed in the above is, indeed, an equilibrium. To do so, we must check whether or not there is a joint offer by the uninformed which is not screening and which is preferred by the uninformed to the screening offer. For there to be such a deviation, it must be that the strong agent accepts the offer. Since, by rejecting and making a partial offer that reveals his type, the strong agent can obtain a payoff of $\delta_s^3 S_s(1+X)$ in present value terms, the deviating offer by the uniformed must yield the informed at least this amount. A deviation to such a joint offer will not be profitable for the uninformed as long as

$$\left(1-\delta_s^3 S_s\right) \leq \left(\omega\left(1-\delta_w S_w\right)+\delta^2(1-\omega)\left(1-\frac{S_w}{\delta_w}\right)\right). \tag{5}$$

The right hand side of (5) is the same as in (1), save for a scaling by δ. Clearly then, if

$$\left(1-\frac{\delta_s S_w}{\delta_w}\right) \geq \delta\left(1-\delta_s^3 S_s\right)$$

the uninformed will not wish to deviate whenever ω satisfies the condition given by (4). A somewhat tedious manipulation shows that this last inequality does indeed hold under Assumption 1. As a result, the proposed separating strategy is an equilibrium whenever ω satisfies (4). **qed**

An alternative interpretation of Rubinstein's (B-2) is that a joint counter offer is deemed to stem from the weak type as long as there exists a partial counter offer that the strong finds preferable to this joint offer. Under this assumption, delay through a high joint offer is deemed less insistent than delay through a single offer: the joint offer implies less delay than the single offer. This interpretation allows for more optimistic beliefs than before, permitting the uninformed to set beliefs to $\omega = 1$ after certain deviations where this would not have been the case under Assumption 6.

Assumption 7 *A sequence of actions which is i) weakly preferred by both types and ii) such that the strong type has no alternative involving further delay which he prefers and the weak does not, will not increase* ω_t.

Under this more permissive rule on the updating of beliefs we obtain the following result.

Proposition 3 *Given Assumptions 1-5 and 7, and either* $X \geq (\delta_s^2(1-\delta\delta_w)-(1-\delta))/(1-\delta)$ *or* $X \leq (1-\delta)/[\delta_s^2(1-\delta\delta_w)-(1-\delta)]$, *a separating equilibrium exists in the game starting with the informed player's offer if*

$$\omega_0 \geq \frac{\delta_w(1-\delta^3)+\delta^3 S_w - \delta_w\delta_s^2 S_s}{\delta\delta_w(1-\delta^2)+(\delta^3-\delta\delta_w^2)S_w}.$$

Proof: The proof proceeds as above, with the only difference being that any deviating joint offer that is accepted by the uninformed now will have to yield not less than $z \geq \delta_s^2 S_s$ (from Lemma 2). The details are omitted. **qed**

Not surprisingly, the range of ω for which a separating equilibrium exists is larger under Assumption 7 than under Assumption 6. The reason is that, under Assumption 7, the uninformed is allowed to entertain more optimistic beliefs (that a deviation derived from the weak opponent) when confronted with a rejection and joint counter offer than under Assumption 6.

4 Discussion

What insights does the preceding analysis provide into the bargaining problem? The first, provided by Proposition 1, is that, in situations of incomplete information, the standard restriction in bargaining models that the agenda involve simultaneous bargaining over *all* issues severely limits the ability of the bargaining parties to signal bargaining strength. Indeed, in the model studied by Rubinstein, the lack of agenda choice rules out any signaling at all by the informed agent. Once agenda choice is introduced, however, the strong bargainer takes advantage of the ability to make issue-by-issue proposals to break the pooling equilibrium with the weak type. In this sense, the strong bargainer has an actual incentive to propose issue-by-issue bargaining rather than simultaneous bargaining as a means of differentiating himself from the weak bargainer.

The second insight, provided by Propositions 2 and 3, is that, even in situations in which no allocations are made until agreement is reached on all issues, agents want to, nonetheless, use their choice of agenda to signal bargaining strength. In particular, a strong bargainer can use an issue-by-issue bargaining process to signal strength, a simultaneous-offer process being deemed by the bargaining opponent to signal weakness. Thus, not only does the strong bargainer have an incentive to propose an issue-by-issue agenda, he can, in fact, successfully use this agenda to signal bargaining strength. Interestingly, the strong bargainer accomplishes this signaling by making a sufficiently large *concession* on the initial issue that it is not in the weak bargainer's interest to mimic. The strong bargainer makes-up for this concession by obtaining a large share of subsequent issues.[9]

Finally, Propositions 2 and 3 also shed some light on the controversy over the order in which issues are bargained. Specifically, the restrictions on the size of X in

[9]Technically, this result is driven by the fact that a large concession on the first issue allows the strong type to restrict his total share of the payoffs to lie below the full information level for the strong type. He can therefore commit not to exploit fully the information that he is strong, which makes the information credible.

these propositions imply, in some instances, a unique order of issues if the agenda is to successfully signal strength. In particular, when $X > (\delta_s^2(1-\delta\delta_w)-(1-\delta))/(1-\delta) > 1$, then successful signaling requires that offers first be made on X, the larger issue, and then on Y; when $X < (1-\delta)/(\delta_s^2(1-\delta\delta_w)-(1-\delta)) < 1$, signaling requires that offers first be made on Y, now the larger issue, and then on X. In both cases, the strong bargainer has a strict preference for negotiating the larger issue first and then the smaller one.[10] Thus, this model indicates that, if the bargaining parties have a preference, it is for negotiating the "large" issue first, not the "small" one. This preference is due to the strong bargainer's need to make sufficient concessions on the initial issue to make mimicry by the weak bargainer unprofitable.

These results also raise a number of additional issues not addressed by this analysis. For instance, we have chosen to distinguish between large and small issues in this model. It is not entirely clear that this distinction captures the spirit of what is meant in the negotiations literature by "hard" and "easy" issues. Alternative ways of differentiating between issues remain an open question. Also, we have not allowed the bargaining parties in this model to propose sequential implementation rather than simultaneous implementation. Nevertheless, one does observe both sorts of agenda in use, so that this issue is, presumably, also part of the agenda choice process. The impact on our model of allowing for such greater scope in agenda choice is unclear. We also have imposed issue size exogenously, when, in practice, the bargaining parties likely have some scope for determining this feature of bargaining as well, for example by bundling issues in various ways. We hope to pursue many of these issues in future research.

[10]In all other cases, the strong bargainer is indifferent about the order.

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