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THE VALUE OF INFORMATION CONCERNING
HETEROGENEOUS TECHNOLOGIES IN A
PRINCIPAL-AGENT RELATIONSHIP

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1. Introduction

In this paper we define a technology to be the relationship between an agent's actions and the associated probability distribution of outcomes. Models of the principal and agent problem usually include either the assumption that agents have identical technologies (e.g., Harris and Raviv [1978], Holmstrom [1979], and Shavell [1979b]) or, if they differ, asymmetric information persists so that each agent knows all relevant parameters pertaining to him but principals are not privy to this information (e.g., Rothschild and Stiglitz [1976], Wilson [1977], and Spence [1974]). In this paper it is assumed that technologies are not identical across agents and that information concerning these differences is symmetric but initially not perfect. The impact of an increment of information concerning the matching of agents to their respective technologies is analyzed under various information structures (of principals) and the associated contracts. Both efficiency and equity effects are present¹ and the relative importance of these effects are shown to depend critically on the type of contract which exists between the principal and agent. Although an insurance model is used to illustrate the welfare effects of increasing information, alterations to the assumptions and extensions of the results to other problems of the principal and agent variety are indicated at convenient points throughout the paper.

An excellent set of examples to illustrate the importance of this problem is provided by the potentially revolutionary advances in "genetic prophecy", that is, the ability to determine an individual's susceptibility to a disease on the basis of genetic markers. These genetic markers range from hair colour to the level of a particular antigen in an individual's blood and provide information of varying degrees of accuracy on the diversity of

individuals' predispositions to various types of diseases. An important aspect of such information is the relationship between controllable environmental factors (i.e., those which can be affected by an agent's actions) and the uncontrollable genetic ones (i.e., those which cannot be altered).² The genetic differences embodied in the relationship between environmental factors and a particular disease can be described by using a different technology for each group of persons possessing a specific genetic trait. Such a technology can describe the relationship between an agent's actions concerning the control of his environment and the probability of contracting a particular disease. The precise forms of these technologies would differ according to the diversity of relevant genetic factors. The use of genetic markers represents imperfect information which improves the matching of individuals to their appropriate technologies.

In many instances knowledge of the relationship between genetic and environmental factors can lead to improvements in prevention or early treatment of relevant diseases (i.e., an improvement in efficiency). For example, the presence of the antigen HLA-Dw3 indicates a 278-fold increase in the probability of getting celiac disease if the individual possessing this marker habitually eats wheat and its products. The risk drops to normal levels if such an individual begins to avoid such foods early in life (Harsanyi and Hutton [1981, p. 128]). Since no strong relationship between the consumption of wheat products and susceptibility to celiac disease exists for persons without the marker, it is clear that the presence of this genetic trait indicates a different technology between environmental factors and disease rather than simply an exogenously determined higher probability of getting celiac disease. The value of such information in terms of efficiency may be substantial as those with the marker may find it worthwhile

to avoid wheat products while those without the marker may not. However, there is also the possibility that although information improves the accuracy of determining the incidence of disease, it may not lead directly to any efficiency gains. For example, the presence of the antigen HLA-B27 indicates an increased predisposition to the disease of arthritis but no preventive care is known (see Harsanyi and Hutton [1981, p. 70]).

It is interesting to note that the value of this type of information may depend on the existence or type of insurance contracts available in the market. Suppose, for example, that in the celiac case health insurance is available and contracts can be based on the agent's actions (i.e., the consumption of wheat products) so that moral hazard poses no second best problems. One can expect then that insurance premiums will be adjusted according to the presence of the marker HLA-Dw3 only if the insured does not avoid wheat products. However, if the increase in premiums is sufficiently high that the agent finds it worthwhile to avoid consumption of wheat products (and the associated higher insurance premium) then one may still conclude that efficiency has been improved by this information. However, if the agent's actions cannot be observed (the case of moral hazard) or, in addition, if neither can his total insurance purchases be observed (nonexclusivity of insurance provision) then the incentive effects of the increase in information cannot be as easily ascertained.

For the example of the marker HLA-B27 and arthritis it would appear that equity effects would dominate efficiency effects even in the presence of health insurance. Under a private scheme one might expect that such information would lead to the assessment of differential premiums on the basis of the presence of this genetic marker. Since no actions are known to affect the conditional probabilities of getting arthritis then individual's who possess

the marker cannot avoid paying a higher premium regardless of whether contracts are based on an agent's actions. Hence, in this example information would create equity effects but not any (direct) efficiency benefits.

As genetic testing becomes cheaper and more accurate the above mentioned efficiency and equity implications will likely become increasingly significant. Many of the examples discussed by Harsanyi and Hutton [1981] suggest that both effects will often be present with respect to the relationship between a genetic factor and the associated disease(s). Whether such information would improve welfare is shown in this paper to depend not only on the relationship between environmental and genetic factors but also on the structure of insurance contracts. Of particular importance is whether the insurer (principal) can observe the level of self-protection (the agent's actions) and/or the aggregate level of insurance protection purchased by a client.

In Section 2 the basic model for an insurance market with homogeneous agents is reviewed. The model and results closely correspond to those of Pauly [1974] and Shavell [1979b]. The method used in this paper to characterize the heterogeneity of agents is described in Section 3. In Section 4 the welfare implications of increases in information concerning the matching of individuals to their appropriate safety technologies when the level of self-protection is observed by firms is analyzed while the same is done in Section 5 for the situation in which neither the level of self-protection nor the aggregate amount of insurance purchases made by an agent is observed by firms. Since the analysis for the situation with self-protection observed by firms is not changed by the assumption concerning the observability or nonobservability of an agent's aggregate insurance purchases, the work in Section 4 covers

both possibilities. The welfare implications of increased information for the standard moral hazard situation (i.e., self-protection unobservable but aggregate insurance purchases observable) are investigated in Section 6. The Conclusion includes policy implications of the results of the various sections as well as a discussion of how the results of this paper fit into the general literature of the impact of information on the principal and agent problem.

2. The Model with Identical Agents: Existing Results

In this section the basic insurance model used throughout this paper is presented. The model is essentially the same as that used by Pauly [1974] and Shavell [1979b] and their results are reviewed. All notation is described as it is used and is also summarized in an appendix at the end of the paper.

2.1 The Consumer (Insured)

The model employs a two-state approach in which the consumer's wealth consists of a certain component of amount y . He sustains loss d if an accident occurs (bad state) but no loss otherwise (good state). Therefore, if no insurance or self-protection is used he receives wealth $y^g = y$ in the good state (no accident) and $y^b = y - d$ in the bad state (accident). It is assumed that $y > d$.

Self-protection in this model is treated as a financial expenditure although with appropriate normalization assumptions it could be treated as effort expended. The extent of self-protection is represented by expenditure s and the probability of an accident is $p(s)$ with $p' \leq 0$, $p'' \geq 0$, $p(0) = \underline{p} > 0$ and $0 < p(s) < 1$ Vs. Expenditure s is made before the revelation of the state of nature so that wealth inclusive of self-protection expenditure but exclusive of insurance is $y^g = y - s$ in the good state and $y^b = y - d - s$ in the bad state; these are sustained with probabilities $1 - p(s)$ and $p(s)$ respectively.

Insurance is modelled as a pay-out (net of premium) of amount α in the bad state and premium β in the good state. The consumer faces an insurance price schedule which may depend either on his total insurance purchases or his level of self-protection. The precise form of the pricing scheme, $\beta = \beta(\alpha, s)$, is an outcome of the information structure of the insurer; namely, can the firm observe either of α or s . The case for which s is directly observable by the firm is treated in Section 2.3 while the moral hazard problem, the case for which s is not observable, is analyzed in Section 2.4. The problem with both s and α unobservable by the firm is considered in Section 2.5.

Expected utility for the consumer is assumed to depend only on wealth; that is, it is state independent. Given that

$$y^g = y - \beta - s \quad (1)$$

$$y^b = y - d + \alpha - s \quad (2)$$

the consumer chooses α and s so as to maximize his expected utility (EU) where

$$EU = [1 - p(s)]u(y^g) + p(s)u(y^b) \quad (3)$$

He will, of course, take into account the effect that altering α or s has on β , if any.

2.2 The Firm (Insurer)

The insurer is assumed to be risk neutral and to operate in a perfectly competitive environment. Therefore, insurance contracts earn zero expected profit; that is

$$\beta(1 - p(s)) = \alpha p(s) \quad (4)$$

The form of the price schedule depends on the information structure of the firm. In particular, if the insurer can observe s then the price schedule will depend on this observation and the consumer will take into account the effect

that alterations in the level of self-protection will have on the price of insurance. This situation gives rise to a first-best solution (see Section 2.3). If s cannot be observed by the insurer (at sufficiently small cost) but the firm can observe a consumer's total insurance purchases then this latter information can be used to infer the level of s . It is in this situation, termed moral hazard, that the well-known second-best solution with partial insurance coverage occurs (see Section 2.4). Finally, if neither s nor the total insurance purchased by a consumer can be observed then the firm's price schedule will of necessity be linear and partial insurance coverage cannot be a result (see Section 2.5).

2.3 Insurer Able to Observe s : First-Best Solution

Suppose the insurer can observe s and, therefore, make α and β depend on s . The insurer's constraint becomes

$$\beta(s)(1 - p(s)) = \alpha(s)p(s) \quad (4)'$$

and the consumer takes into account the effect of altering s on α and β . Taking (4)' into account (that is, substituting for β) the consumer's maximization problem can be written as

$$\max_{\alpha, s} EU = [1 - p(s)]u(y^g) + p(s)u(y^b) \quad (5)$$

Using definitions (1) and (2) the first-order conditions for a maximum are

$$\begin{aligned} \frac{\partial EU}{\partial s} = p' [u(y^b) - u(y^g)] + (1 - p)u'(y^g) \left[\frac{-\alpha' p(1 - p) - p' \alpha}{(1 - p)^2} - 1 \right] \\ + pu'(y^b)(\alpha' - 1) = 0 \end{aligned} \quad (6)$$

$$\frac{\partial EU}{\partial \alpha} = p[u'(y^b) - u'(y^g)] = 0 \quad (7)$$

From equation (7) it is clear that $y^g = y^b$ which implies $\alpha = d - \beta$ so that from (4)' we get $\alpha = d(1 - p)$. Since $y^g = y^b$ implies $u(y^b) = u(y^g)$ and $u'(y^b) = u'(y^g)$ it is straightforward to show, using equations (5) and (6) that

$$p'(s) = -\frac{1}{d} \quad (8)$$

determines the optimal level of s for the consumer. This level of s maximizes expected per capita income³ and, since individuals face no risk with full coverage insurance ($y^b = y^g$) a first-best result is achieved.

It is easy to see that observability of s by firms makes the observation of total insurance purchases made by insureds redundant. Therefore, this section essentially covers both the cases with total insurance purchases observable and not observable to firms given that s is observable.

2.4 Insurer Not Able to Observe s but Able to Observe Total Insurance Purchases Made By a Consumer

If the insurer cannot observe s then the well-known tradeoff between risk-spreading and the efficient use of self-protection occurs (see Pauly [1974]). Complete risk-spreading requires that $y^g = y^b$. However, without the ability to monitor s full insurance eliminates the incentive to self-protect.⁴ This problem, moral hazard, leads to the result that the "socially optimal" insurance policy is one with less than full coverage. This also will be the market solution.

In this case the price of insurance from the consumer's perspective does depend on α but not on s ; that is, $\beta = \beta(\alpha)$ is not generally a linear function. The consumer's maximization problem is $\max_{s, \alpha} EU$ with β a function of α , $\beta(\alpha)$. The first order conditions are

$$\frac{\partial EU}{\partial s} = p' [u(y^b) - u(y^g)] - (1 - p)u'(y^g) - pu'(y^b) = 0 \quad (9)$$

$$\frac{\partial EU}{\partial \alpha} = (1 - p)u'(y^g) \left(-\frac{d\beta}{d\alpha}\right) + pu'(y^b) = 0 \quad (10)$$

From equation (9) it follows that the purchase of full insurance, $y^g = y^b$, implies a corner solution, $s = 0$. By continuity, $s = 0$ for some $y^b < y^g$ also. Therefore, since the consumer's choice of s depends on his choice of α we can write $s = s(\alpha)$ with $ds/d\alpha = 0$ for some α less than full insurance.

The insurer recognizes the interdependence of s and α and so we can write $p = p(s(\alpha))$. By observing α the insurer can infer s . The insurer, therefore, chooses a price schedule $\beta(\alpha)$ in order to maximize the consumer's expected utility subject to the zero expected profit condition. Noting that $y^g = y - \frac{\alpha p(s)}{1 - p(s)} - s$ and $y^b = y - d + \alpha - s$ ensures zero expected profit for the insurer this problem can be written as $\max_{\alpha} EU$ which gives the following first order condition.

$$\begin{aligned} & \frac{dp}{ds} \frac{ds}{d\alpha} [u(y^b) - u(y^g)] - \frac{ds}{d\alpha} [(1 - p)u'(y^g) + pu'(y^b)] \\ & + pu'(y^b) - pu'(y^g) - \frac{\alpha \frac{dp}{ds} \frac{ds}{d\alpha} u'(y^g)}{1 - p} = 0 \end{aligned} \quad (11)$$

Using the result of equation (9) we see that the first two terms of equation (11) combine to give zero. The optimal price schedule derived from equation (11) is characterized below in equation (12). The last equality is evident from equation (10).

$$\frac{p}{1 - p} + \frac{\alpha \frac{dp}{ds} \frac{ds}{d\alpha}}{(1 - p)^2} = \frac{pu'(y^b)}{(1 - p)u'(y^g)} = \frac{d\beta}{d\alpha} \quad (12)$$

Equation (12) can be interpreted as follows. Suppose effort were fixed at $s = \hat{s}$. Then the first term of equation (12) corresponds to $\left. \frac{d\beta}{d\alpha} \right|_{s=\hat{s}} = \frac{p(\hat{s})}{1 - p(\hat{s})}$, the actuarially fair price levied for an increase in insurance purchased by a consumer given $s = \hat{s}$ is his present level of self-protection. However, as

α increases the insurer must take into account the possibility that the consumer's choice of s will change and, hence, so will the probability of loss. This latter consideration explains the second term of equation (12). Moreover, notice that full insurance is optimal ($y^g = y^b$) only if $\frac{dp}{ds} \cdot \frac{ds}{d\alpha} = 0$.

2.5 Insurer Unable to Observe Either s or Total Insurance Purchases Made By a Consumer

Just as it is sometimes assumed that the monitoring of self-protection (or effort) is too costly for it to be practical for firms to observe s , it may also be too costly for a firm to observe the amount of insurance purchases made by a consumer from other sources. Although legislation generally excludes (or at least attempts to exclude) the possibility of insureds buying insurance in excess of the value of the potential loss, such a limit is not sufficient to rule out difficulties associated with the unobservability of a consumer's total insurance purchases. In a working paper Arnott and Stiglitz [August 1981] discuss the difficulties associated with the unobservability of "informal social institutions which serve to supplement insurance provided by the market"; for example, that provided among family members.

If a firm does not take into account "other" insurance coverage of its insureds then it may not make the correct inference concerning the level of self-protection being used. It will be assumed in this section that the total quantity of insurance purchased by a consumer cannot be observed by the insurer. This being the case, consumers will always claim to have purchased that level of insurance which places them at the lowest value for $\frac{d\beta}{d\alpha}$ on the insurer's schedule. The result is that price schedules will be effectively linear; that is

$$\beta = \frac{p(\hat{s})}{1 - p(\hat{s})} \alpha \quad (13)$$

where \hat{s} is the level of self-protection employed by the consumer.

From the consumer's first-order condition (equation (10)) we see that α is either zero (corner solution) or full insurance is purchased ($y^b = y^g$). Therefore, unless no insurance is purchased it follows that consumers employ no self-protection. The results of this section can be found in Pauly [1974, pp. 50-52].⁵

3. Heterogeneity and Safety Technologies

Before presenting specific details of the assumptions concerning the heterogeneity of the safety technologies for the insurance model a more general description of differential technologies with possible applications to various problems of the principal and agent variety is presented. By doing so the reader can more readily apply the particular results for the insurance market to other principal and agent problems.

Suppose an outcome x is a function of some choice variable e decided upon by the agent as well as some other characteristic ξ (i.e., $x = f(\xi, e)$ where x may denote a probability distribution of outcomes rather than a deterministic result).⁶ Although it is in general not necessary to assume that ξ is an endowed and unalterable characteristic this is done throughout this paper. For an insurance problem x might represent the probability of loss, e the level of self-protection employed and ξ some characteristic (e.g., genetic) that distinguishes one individual from another. In an employee-employer relationship x may refer to the agent's output, ξ to some characteristic such as "natural ability" and e being effort expended. Although ξ could be depicted as having values within an interval $[\underline{\xi}, \bar{\xi}]$, $\underline{\xi} < \bar{\xi}$, only discrete possibilities with $\xi \in \{\xi_1, \dots, \xi_n\}$ will be used.

If a more desirable outcome is associated with a larger value for x then some uniformity on the characterization of a problem can be imposed by assuming, for example, that for some $i \neq j$, $f(\xi_i, e) \geq f(\xi_j, e) \forall e$ with strict inequality for some e . This is equivalent to the statement that an individual of type i is in an absolute sense more productive than is an individual of type j . It may also be the case that $\frac{\partial}{\partial e} f(\xi_i, e) \cong \frac{\partial}{\partial e} f(\xi_j, e) \forall e$. If " $>$ " then the greater productivity of a type i individual is such that his productivity increases more as a result of an increase in e while if " $<$ " holds then the greater productivity of type i relative to j diminishes as both increase e . The difference is effectively constant if over different levels of e the equality holds.⁷

The description of the technological differences could be expanded to handle problems of increased sophistication. For example, consider the employer-employee problem with x output per unit time, ξ an index of innate ability, e effort, and E investment in education with $x = f(\xi, e, E)$. One could then investigate differences in the augmentation of individuals' productivity resulting from an increase in education for a given level of effort. For example, an increase in education for a given effort level may increase productivity more for an individual of type i than for an individual of type j (i.e., $\frac{\partial}{\partial E} f(\xi_i, e, E) > \frac{\partial}{\partial E} f(\xi_j, e, E)$) or it may increase the cross product in that marginal product of effort may be increased more by increases in education for type i than for type j individuals (i.e., $\frac{\partial^2 f}{\partial e \partial E}(\xi_i, e, E) > \frac{\partial^2 f}{\partial e \partial E}(\xi_j, e, E)$). The possibility of learning about the correlation between training (E) and the marginal productivity of effort may explain why in some cases on the job training (e.g., apprenticeship) may be preferred to the accumulation of education outside the work environment.

For the insurance problem let there be two types of individuals with personal characteristics denoted by $\xi = G_h$ or G_l . In the context of the health examples provided in the introduction of this paper G_h could refer to a genetic factor for a high risk type and G_l for a low risk type. Let $p_h(s) = f(G_h, s)$ and $p_l(s) = f(G_l, s)$ denote the self-protection technologies for high and low risk types respectively where $p_h(s)$ refers to the loss probability for a high risk type and $p_l(s)$ refers to the loss probability for a low risk type, both for a given level of self-protection, s .

In order to maintain ease of definition it is assumed that $f(G_h, s) \geq f(G_l, s) \forall s$. Three possible additional assumptions which are associated with the marginal productivities of using self-protection are entertained. These are:

- (i) Case A: $\frac{\partial f(G_h, s)}{\partial s} = \frac{\partial f(G_l, s)}{\partial s}, \forall s$
- (ii) Case B: $\frac{\partial f(G_h, s)}{\partial s} < \frac{\partial f(G_l, s)}{\partial s}, \forall s \leq s'$ with " \leq " otherwise $s' > 0$
- (iii) Case C: $\frac{\partial f(G_h, s)}{\partial s} > \frac{\partial f(G_l, s)}{\partial s}, \forall s \leq s'$ with " \geq " otherwise $s' > 0$

Since a reduction of a loss probability is associated with an improvement in productivity these conditions are interpreted as follows. Case A refers to the possibility that although $p_h > p_l$, the use of self-protection has no differential impact on the loss probabilities of high and low risk types. Although $p_h > p_l$, the use of self-protection is more productive for high risk types than it is for low risk types under Case B while for Case C the marginal productivity of self-protection is greater for low risk types than it is for high risk types.

The specific example concerned with celiac disease provides an excellent example of Case B (see Introduction). In this example let s represent the monetized cost of avoiding (at various levels) consumption of wheat and its products. Let G_h represent the genetic factor for those individuals with a high susceptibility to celiac disease and G_l for low risk types. If no effort is made to avoid consumption of wheat products early in life ($s=0$) then those susceptible ($\xi=G_h$) are significantly more likely to contract celiac disease than those who are not susceptible ($\xi=G_l$). However, the avoidance of wheat products by high risk types ($\xi=G_h$) improves the chances of not contracting celiac disease. The difference in productivity of s is sufficiently great that for "large enough" values of s the risk of contracting celiac becomes almost as small for high risk types as it is for low risk types.⁸ The relationship between the safety technologies of high and low risk types is illustrated below (see Figure 1). Note that $\frac{\partial f(G_h, s)}{\partial s} \leq \frac{\partial f(G_l, s)}{\partial s}$.

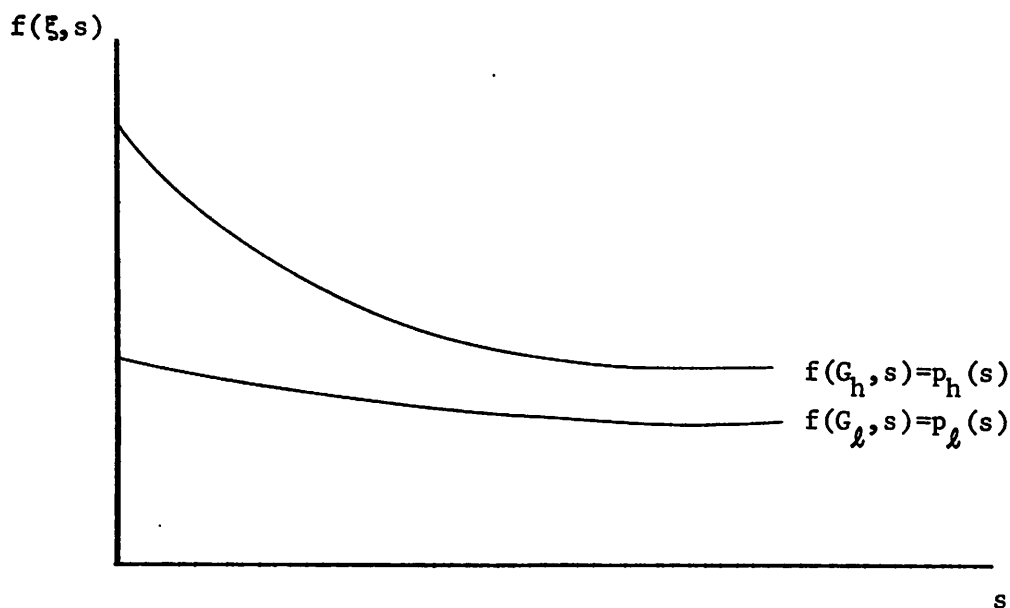


Figure 1

The example of being able to distinguish among individuals' predisposition to arthritis by use of the marker HLA-B27 provides a trivial example of Case A

with $f(G_h, s) > f(G_l, s)$ and $\frac{\partial f(G_h, s)}{\partial s} = \frac{\partial f(G_l, s)}{\partial s}$ ($= 0$) since no method of self-protection is known. An example of Case C would be provided by an instance where an individual's susceptibility to a disease cannot be predicted but different types may respond better than others to a treatment which avoids certain other medical costs, that is, $f(G_h, 0) = f(G_l, 0)$ but $\frac{\partial f(G_l, s)}{\partial s} < \frac{\partial f(G_h, s)}{\partial s}$.

In order to highlight the importance of some interesting comparative statics results which are carried out later some further restrictions are placed on the technologies (in particular for Cases B and C). These additional restrictions are noted below and the resulting characterizations (Cases \hat{A} , \hat{B} , and \hat{C}) are illustrated in Figures 2, 3, and 4.

Case \hat{A} : $p_h(s) = p_l(s) + \delta_o$, $\delta_o > 0$

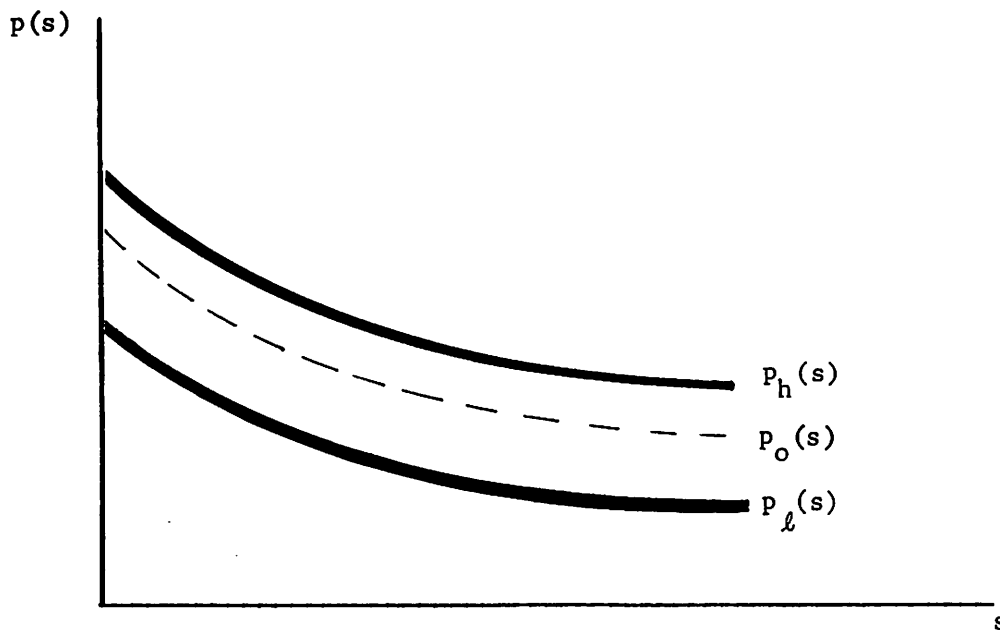


Figure 2

Case \hat{B} : $p_h(s) = p_l(s) + \delta(s)$
 $\delta'(s) < 0$ for $s < \bar{s}$, $\bar{s} > 0$
 $\delta(s) = 0$ and $\delta'(s) = 0$ for $s \geq \bar{s}$

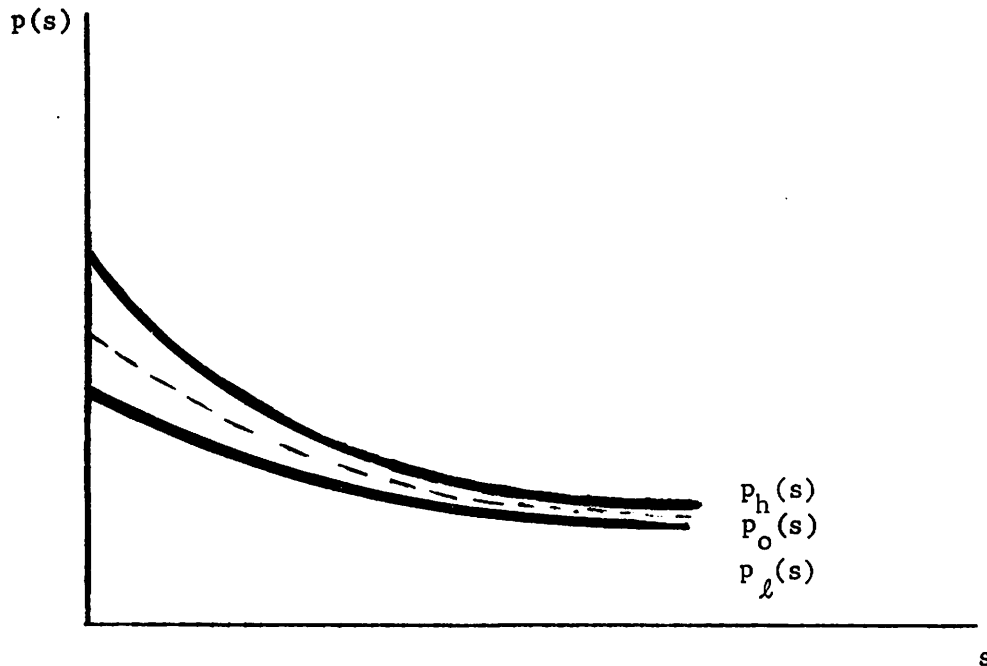


Figure 3

Case \hat{C} : $p_h(s) = p_l(s) + \delta(s)$
 $\delta'(s) > 0$ for $s < \bar{s}$ and $\delta'(s) = 0$ for $s \geq \bar{s}$, $\bar{s} > 0$
 $\delta(0) = 0$, i.e., $p_h(0) = p_l(0)$.

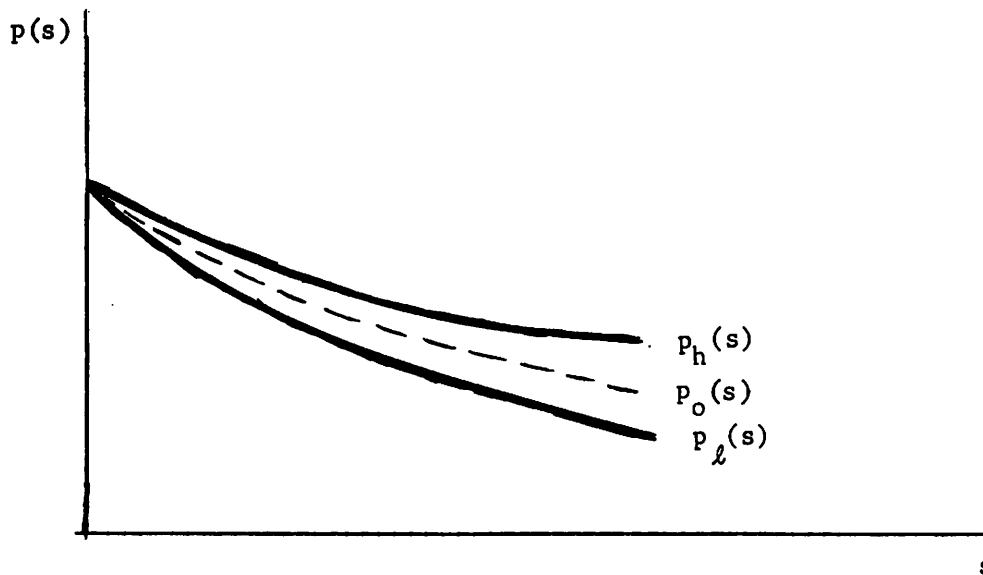


Figure 4

Notice that the added restrictions for Cases \hat{A} , \hat{B} and \hat{C} do not lead to violations of the somewhat more general conditions stated earlier in this section. The added restrictions of $\delta(s) = 0$, $s \geq \bar{s}$ for Case \hat{B} and $\delta(0) = 0$, $s \geq \bar{s}$ for Case \hat{C} are relaxed later.

It is not always the case that individuals possess perfect information concerning their risk type. Therefore, knowledge about safety technologies may be of a "pooled" or "mixed" type. In particular, consider the situation in which no information concerning the differences in safety technologies is available; that is, all economic agents (firms and consumers) perceive the safety technology to be that which would be observed on the basis of pooled data analysis. This perceived technology is represented by $p_0(s) = q_h p_h(s) + q_l p_l(s)$ where q_i is the proportion of risk type i individuals in the population (see Figures 2, 3, and 4).

Since $p_0(s)$ is consistent with aggregative analysis of actual loss experience, expectations are rational with respect to the information structure available. Although throughout this paper implications of changes in the amount of information available are studied, it is always assumed that expectations are rational conditional on the state of information. Moreover, information is assumed to be symmetric in this model; that is, consumers and firms are equally knowledgeable about the extent of information concerning the risk type of an individual.⁹ In the following section the task of deriving implications concerning the welfare effects of increased information relating individuals to risk types is begun by considering the situation in which firms can observe the level of self-protection employed by insureds.

4. Value of Information with Self-Protection Observable by Firms

Since it is assumed in this section that s can be observed by the firm, the analysis of consumer behavior from Section 2.3 is applicable. For example, the relevant safety technology, represented by $p(s)$ in Section 2.3, is $p_0(s)$ if no information concerning the risk type of individuals is available.

From Section 2.3 it follows that

$$p_0(s) = q_h p_h(s) + q_l p_l(s) = p_l(s) + q_h \delta(s) \quad (14)$$

The first-order condition for the consumer's maximization problem gives rise to the optimal choice of self-protection being s_0^* satisfying $p'_0(s_0^*) = -\frac{1}{d}$ (see equation (8)). Therefore,

$$p'_0(s_0^*) = p'_l(s_0^*) + q_h \delta'(s_0^*) = -\frac{1}{d} \quad (15)$$

This choice of s_0^* is illustrated in Figures 5, 6, and 7.

The method of incorporating imperfect information into this model is to assume that some characteristic, which is imperfectly correlated with each individual's risk type, is observed by both consumers and firms. For simplicity, assume it is a single dichotomous characteristic which defines two risk categories denoted by the index $c = H, L$ for the high and low risk categories respectively. Assume there are K_c individuals in category c while there are N_i ($i=h, l$) individuals of risk type i in the population. If N is the total number of individuals in the population then $K_H + K_L = N_L + N_l = N$. Therefore, if n_{ic} represents the number of risk type i individuals belonging to category c , the following relationships hold.

$$\begin{aligned}
n_{hH} + n_{hL} &= N_h & n_{hH} + n_{lH} &= K_H \\
n_{lH} + n_{lL} &= N_l & n_{lL} + n_{hL} &= K_L
\end{aligned}
\tag{16}$$

Recall that q_h and q_l represent the proportions of high and low risk types in the population. By using q_{ic} to denote the proportion of individuals of risk type i in category c (i.e., $q_{ic} = \frac{n_{ic}}{K_c}$), a categorization scheme of information value e can be defined as one with $q_{hH} = q_h + e$. That is, for $0 < e < 1 - q_h$, a categorization scheme of value e is one for which the proportion of high risk types in the high risk category is greater than that in the aggregate population by amount e . Since $q_{lL} + q_{hL} = 1$, $q_{hH} + q_{lH} = 1$ and $q_{hH} + q_h + e$, it is easy to show that, with $k = \frac{K_H}{K_L}$,

$$\begin{aligned}
q_{hH} &= q_h + e \\
q_{lL} &= q_l + ek \\
q_{hL} &= q_h - ek \\
q_{lH} &= q_l - e
\end{aligned}
\tag{17}$$

The level of self-protection employed when there is no information concerning insureds' risk type is determined by equation (15) above. With information of value e available the perceived safety technology for a member of category c is $p_c(s) = q_{lc}p_l(s) + q_{hc}p_h(s)$. Therefore, the level of self-protection employed, s_c^* , is determined by the first-order condition $p'_c(s) = -\frac{1}{d}$. Using the definition $p_h(s) = p_l(s) + \delta(s)$ and the results of equation set (17) the first-order conditions which determine the levels of self-protection for members of the high and low risk categories are given by equations (18) and (19) respectively.

$$F_H(s_H^*, e) = (-p'_l(s_H^*) - (q_h + e)\delta'(s_H^*))d - 1 = 0 \quad (18)$$

$$F_L(s_L^*, e) = (-p'_l(s_L^*) - (q_h - ek)\delta'(s_L^*))d - 1 = 0 \quad (19)$$

In order to determine the welfare implications of increased information we will differentiate various expressions with respect to e . It is of independent interest to differentiate expressions (18) and (19) with respect to e in order to determine the impact of increased information on the level of self protection used by members of the high and low risk category. For members of the high risk category it can easily be shown that

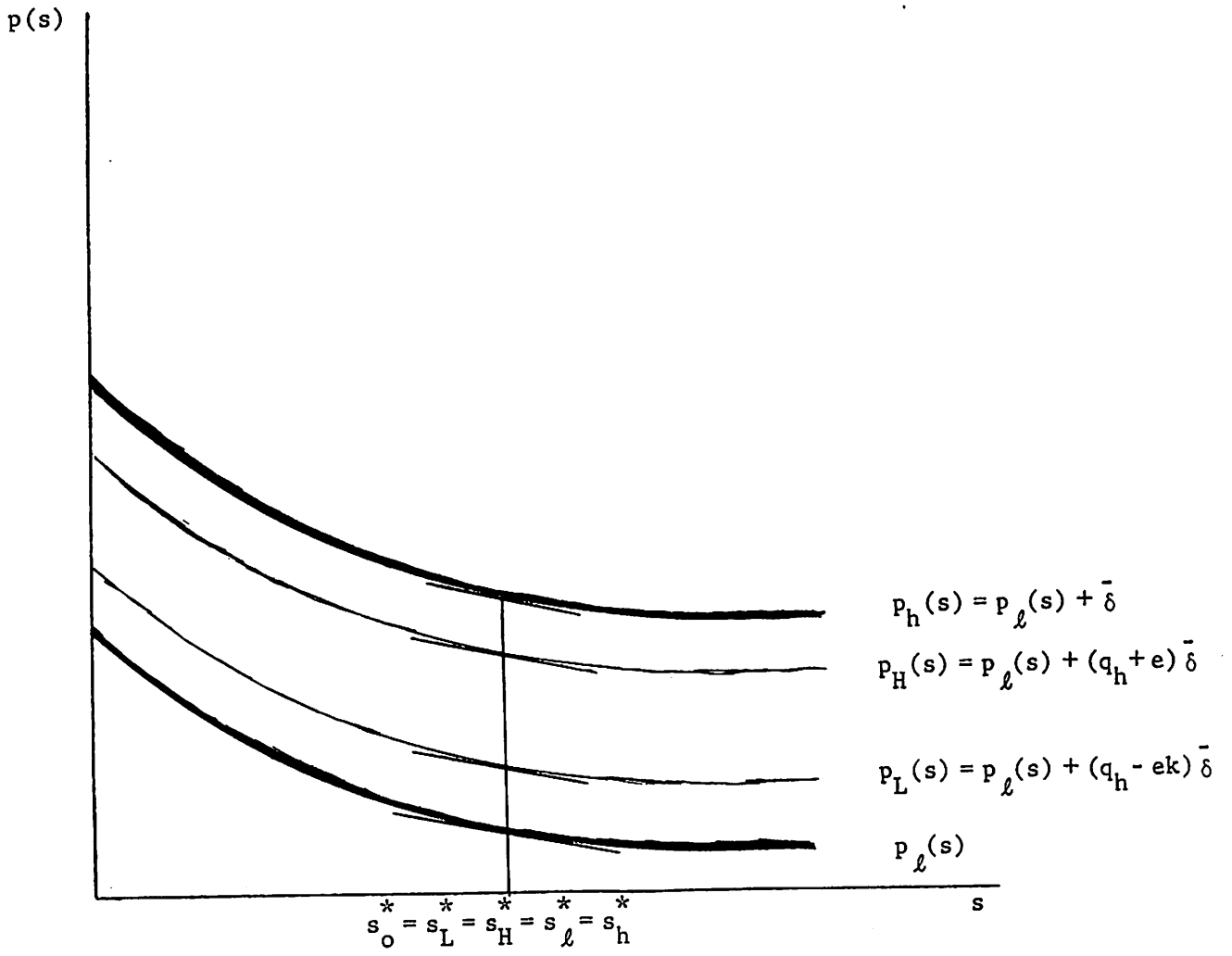
$$\begin{aligned} \frac{ds_H^*}{de} &= 0 \text{ Case } \hat{A} \\ &> 0 \text{ Case } \hat{B} \\ &< 0 \text{ Case } \hat{C} \end{aligned} \quad (20)$$

These results follow since $\frac{\partial F_H}{\partial s_H^*} < 0$ according to the second order condition (i.e., $p''_H > 0$) and, therefore, $\frac{ds_H^*}{de} = -\frac{\partial F_H/\partial e}{\partial F_H/\partial s_H^*}$ has the opposite sign to that of $\delta'(s_H^*)$. For members of the low risk category it can be shown that

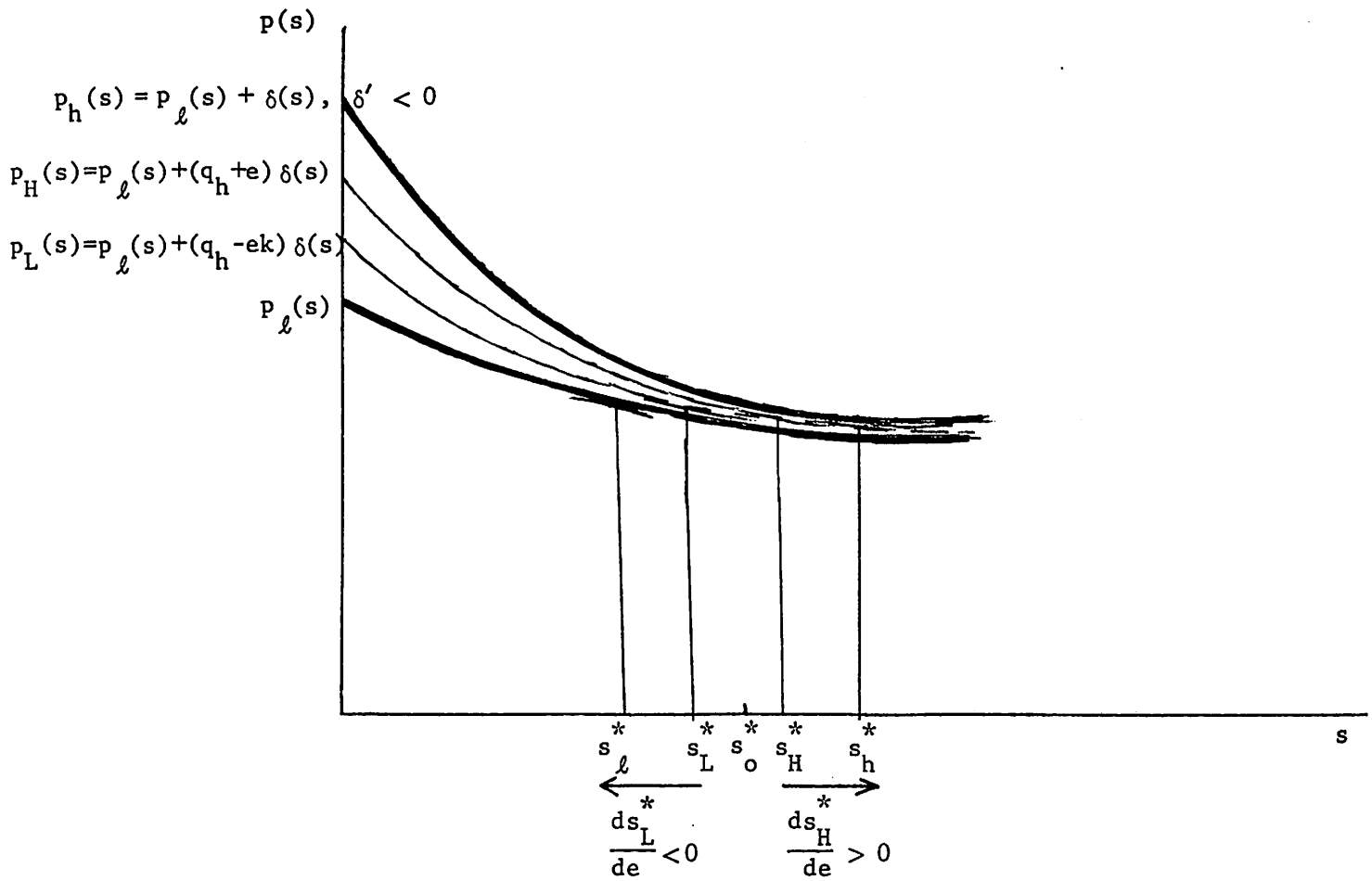
$$\begin{aligned} \frac{ds_L^*}{de} &= 0 \text{ Case } \hat{A} \\ &< 0 \text{ Case } \hat{B} \\ &> 0 \text{ Case } \hat{C} \end{aligned} \quad (21)$$

These results follow since $\frac{\partial F_L}{\partial s_L^*} < 0$ according to the second-order condition (i.e., $p''_L > 0$) and, therefore, $\frac{ds_L^*}{de} = -\frac{\partial F_L/\partial e}{\partial F_L/\partial s_L^*}$ has the same sign as $\delta'(s_L^*)$.

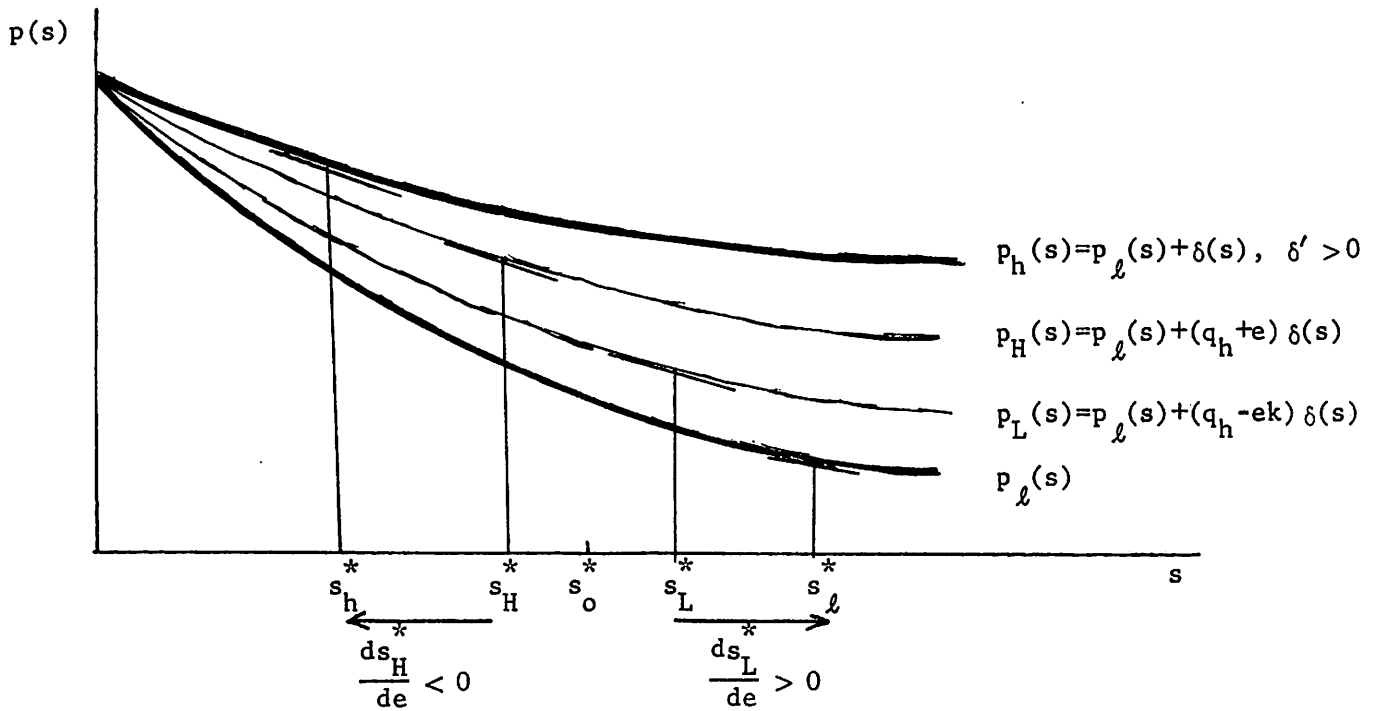
Using equations (18) through (21) we can obtain the relative sizes of s_O^* , s_H^* and s_L^* for Cases \hat{A} , \hat{B} , and \hat{C} . Also, the derivatives for $\frac{ds_L^*}{de}$ and $\frac{ds_H^*}{de}$ can be noted. This is done in Figures 5, 6, and 7 below. The perfect information solutions are indicated by using s_l^* and s_h^* .



Case \hat{A} : Figure 5



Case \hat{B} : Figure 6



Case \hat{C} : Figure 7

The three cases used to characterize the difference between high and low risk types is shown here to be quite rich. Upon comparison of the situations where there is no information and perfect information about each individual's risk type specific safety technology it is possible that (i) low risk types underutilize self-protection ($s_o^* < s_l^*$) while high risk types overutilize self-protection ($s_o^* > s_h^*$) as in Case \hat{C} , (ii) low risk types overutilize self-protection ($s_o^* > s_l^*$) while high risk types underutilize self-protection ($s_o^* < s_h^*$) as in Case \hat{B} or (iii) no overutilization or underutilization occurs ($s_o^* = s_l^* = s_h^*$) as in Case \hat{A} . It is also interesting to note that even though $p_h(s) \geq p_l(s) \forall s$, it may nevertheless occur that high risk types will sustain a smaller probability of loss if $s_h^* > s_l^*$, as is possible for the characterization provided in Figure 6 (Case \hat{B}).

It is easy to show that aggregate expected income is maximized by use of perfect information (if it is available).¹⁰ The implications of an increasing amount of imperfect information (e) is less intuitive. An increase in information improves the properly classified individuals' perceptions of their safety technologies as well as increasing the number of "correct matches". However, the misclassified individuals' perceptions of their safety technologies are worsened. Nevertheless, the net effect of an increase in e is an increase in aggregate expected income. This result is derived below in Section 4.1.

Whether information is perfect or imperfect there will be distributional consequences associated with its use. Given risk aversion of consumers this is an important phenomenon. This issue is analyzed by consideration of the Pareto welfare implications of increases in e (see Section 4.2 below) and by use of a Utilitarian social welfare function (see Section 4.3 below). The results are summarized at the end of this section.

4.1 The Effect of Information on Expected Aggregate Income

Let y_{ic} denote the expected income generated by a risk type i individual in category c . Since e affects an individual's perception of his safety technology and, hence, his choice of s , the expected income generated via the use of s is also affected by e . We can write

$$Ey_{ic} = (1 - p_i(s_c^*)) (y - s_c^*) + p_i(s_c^*) (y - d - s_c^*) \quad (22)$$

Consider the expected income generated by a high risk individual in the high risk category, Ey_{hH} . Since $p_h(s_H^*) = p_l(s_H^*) + \delta(s_H^*)$ we get

$$\frac{dEy_{hH}}{de} = (-dp_l'(s_H^*) - d\delta'(s_H^*) - 1) \frac{ds_H^*}{de} \quad (23)$$

Using the first-order condition for the optimal choice of self-protection by members of the high risk category (see equation (18)), it follows that

$$\frac{dEy_{hH}}{de} = d\delta'(q_h + e - 1) \frac{ds_H^*}{de} \geq 0 \quad (24)$$

The sign of equation (24) is found by using the results of equation (20) with associated $\delta' \geq 0$ and the fact that $q_h + e - 1 < 0$.

In a similar fashion the following expressions can be found and signed.

$$\frac{dEy_{hL}}{de} = d\delta'(q_h - ek - 1) \frac{ds_L^*}{de} \leq 0 \quad (25)$$

$$\frac{dEy_{lH}}{de} = d\delta'(q_h + e) \frac{ds_H^*}{de} \leq 0 \quad (26)$$

$$\frac{dEy_{lL}}{de} = d\delta'(q_h - ek) \frac{ds_L^*}{de} \geq 0 \quad (27)$$

In each of the equations (24) through (27) the equality holds for Case \hat{A} . Otherwise, the expected income generated by properly classified individuals rises as information increases while it falls for misclassified individuals, as expected. There is a further effect of information and that is on the number of misclassified individuals. It follows from the definition of the value of information that the number of misclassified individuals falls as e increases. This factor must be taken into account in order to determine the aggregate efficiency effect of an increase in e .

Expected income generated by members of category c is written below:

$$Ey_c(s_c^*, e) = q_{hc} Ey_{hc} + q_{lc} Ey_{lc}, \quad c = H, L \quad (28)$$

Ey_c is a function of e (as well as of s_c^*) since the q_{ic} 's are a function of e (see equation set (17)) and represent the impact that e has with respect to the matching between risk type and risk category. Therefore, aggregate expected income, EY , is a function of s_H^* , s_L^* and e and can be written as below.

$$EY(s_H^*, s_L^*, e) = K_H Ey_H(s_H^*, e) + K_L Ey_L(s_L^*, e) \quad (29)$$

It follows that

$$\frac{dEY}{de} = \frac{\partial EY}{\partial e} + K_H \frac{\partial Ey_H}{\partial s_H^*} \frac{ds_H^*}{de} + K_L \frac{\partial Ey_L}{\partial s_L^*} \frac{ds_L^*}{de} \quad (30)$$

Only the first term of equation (30) can be non-zero since s_c^* is chosen to maximize Ey_c (i.e., $\frac{\partial Ey_c}{\partial s_c^*} = 0$ and the envelope theorem applies).

Using equations (29), (28) and equation set (17) we get $\frac{\partial EY}{\partial e} = K_H (Ey_{hH} - Ey_{hL} + Ey_{lL} - Ey_{lH})$. From equation (22) and the fact that $p_h(s) = p_l(s) + \delta(s)$ it follows that¹¹

$$\frac{dEY}{de} = K_H [\delta(s_L^*) - \delta(s_H^*)]d \geq 0 \quad (31)$$

Therefore, the impact on the efficiency gain resulting from an increase in information is derived from the improved matching of risk types to appropriate risk categories and this efficiency gain is greater the greater is the difference between the two technologies. This result accords well with intuition. In Case A $\delta(s_L^*) = \delta(s_H^*)$ so no production efficiency occurs.

4.2 Pareto Welfare Implications of an Increase in Information

Since an increase in information leads to an increase in aggregate expected income, it is natural to investigate the possibility of an increase in information leading to a Pareto improvement in welfare. While an increase in e implies an increase in the proportion of high-risk types in the high risk category (and hence an increase in the price of insurance), one might expect that in some circumstances this adverse effect may be compensated for by a more efficient use of self-protection.¹² However, such a result is shown here not to be possible. That is, members of the high risk category will always be made worse off as a result of an increase in e .

To demonstrate the above claim recall that everyone buys full coverage insurance. Members of risk category $c(=H,L)$ receive income $y_c = y - P_c(s_c^*) - s_c^*$ where $P_c(s_c^*)$ is the price of (full-coverage) insurance, which depends on the level of self-protection employed. It is shown below that $\frac{dy_H}{de} < 0$ and $\frac{dy_L}{de} > 0$; that is, a Pareto improvement does not occur.

For members of the high risk category $P_H(s_H^*) = q_{hH}P_h(s_H^*)d + q_{lH}P_l(s_H^*)d$.

Therefore,

$$\begin{aligned} \frac{dP_H(s_H^*)}{de} = & \left[\frac{dq_{hH}}{de} p_h(s_H^*) + \frac{dg_{lH}}{de} p_l(s_H^*) \right] d \\ & + \left[q_{hH} \frac{dp_h(s_H^*) ds_H^*}{ds_H^*} \frac{ds_H^*}{de} + q_{lH} \frac{dp_l(s_H^*)}{ds_H^*} \frac{ds_H^*}{de} \right] d \end{aligned} \quad (32)$$

The first term of equation (32) refers to the direct effect on the price of insurance resulting from the changing proportion of high (and low) risk types due to the change in e . The second term refers to the change in price resulting from the change in the use of self-protection which results from a change in e (i.e., improved knowledge of the safety technology). Using equation set (17) and $p_h(s) - p_l(s) = \delta(s)$, we can rewrite equation (32) as

$$\frac{dP_H(s_H^*)}{de} = \delta(s_H^*) d + (p_l'(s_H^*) + (q_h + e) \delta'(s_H^*)) \frac{ds_H^*}{de} d \quad (33)$$

From the first-order condition for optimal choice of s_H^* by members of the high risk category (see equation (18)) we get $\frac{dP_H(s_H^*)}{de} = \delta(s_H^*) d - \frac{ds_H^*}{de}$ which gives

$$\frac{dy_H}{de} = -\delta(s_H^*) d < 0 \quad (34)$$

It can be shown in a similar fashion that

$$\frac{dy_L}{de} = k\delta(s_L^*) d > 0 \quad (35)$$

Therefore, the improved perception of the safety technology by members of the high risk category does not counterbalance the direct effect on price caused by the increased proportion of high risk types. The following example

using a discrete change in the safety technology more intuitively illustrates this result.

Suppose, as illustrated in Figure 8, that $p_l(s) = \bar{p}$ and $p_h(s) = \bar{p} + \bar{\delta}, s < \bar{s}$
 $= \bar{p}, s \geq \bar{s}$.

Assume that, with $p_o(s) = q_h p_h(s) + q_l p_l(s)$, it is initially not economic for all persons to employ self-protection $s = \bar{s}$; that is $q_h p_h(0)d + q_l p_l(0)d < \bar{p}d + \bar{s}$.

However, suppose if perfect information is available, it follows that $p_h(\bar{s})d + \bar{s} < p_h(0)d$ so that perfect information leads to high risk types employing $s = \bar{s}$.

Although this is a more efficient solution (i.e. fewer total resources are used up) high risk types cannot be better off after information is available.

Otherwise, it would be the case that the price of insurance with $s = \bar{s}$ for all individuals before categorization would have been cheaper than for $s = 0$. Therefore, some distributive risk must exist.

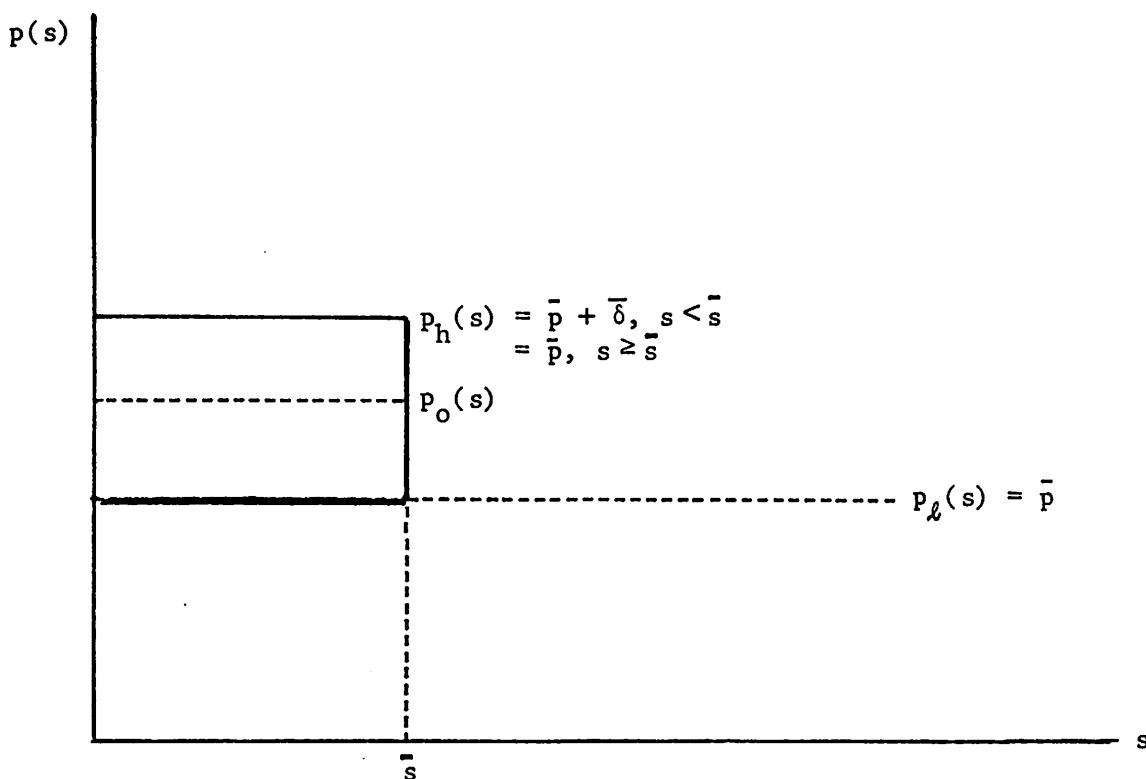


Figure 8

4.3 The Effect of Information on the Utilitarian Social Welfare Function

A risk averse individual always purchases full insurance when s is observed by firms. Therefore, income in the bad state equals income in the good state for each member of a particular risk category (i.e., $y_c = y - P_c(s_c^*) - s_c^*$ as defined in the previous subsection). Letting EU_c denote expected utility for an individual in category c ($=H,L$) it follows that $EU_c = u(y_c)$. Therefore, with K_c individuals in risk category c it follows that the utilitarian social welfare function can be written as below.

$$W = K_H u(y_H) + K_L u(y_L) \quad (36)$$

Upon differentiation of W with respect to e we get, with use of equations (34) and (35),

$$\frac{dW}{de} = K_H \{ \delta(s_L^*) u'(y_L) - \delta(s_H^*) u'(y_H) \} d \quad (37)$$

Upon comparison of equation (37) with equation (31) it is seen that for the case of risk neutrality $\frac{dW}{de} > 0$ as is $\frac{dY}{de} > 0$.¹³ However, risk aversion on the part of insureds implies that $u'(y_L) < u'(y_H)$ so that $\delta(s_L^*) > \delta(s_H^*)$ is not sufficient to guarantee $\frac{dW}{de} > 0$; that is, the distributive risk associated with increases in information is reflected by risk aversion.

Upon consideration of equation (37) the following condition can be obtained:

$$\frac{dW}{de} < 0 \quad \text{iff} \quad \frac{\delta(s_L^*)}{\delta(s_H^*)} < \frac{u'(y_H)}{u'(y_L)} \quad (38)$$

The fact that $\delta(s_L^*) \geq \delta(s_H^*)$ represents the efficiency effect of information

while the fact that $u'(y_H) > u'(y_L)$ represents the equity effect. Therefore, by looking at factors which tend to make $\delta(s_L^*)$ smaller relative to $\delta(s_H^*)$ and/or $u'(y_H)$ larger relative to $u'(y_L)$ one can characterize parameter changes which increase the relative importance of equity considerations (over efficiency ones) and more likely lead to $\frac{dW}{de} < 0$.

In the specific situation of Case \hat{A} (see Figure 5) there is no efficiency effect whatsoever (i.e., $s_L^* = s_H^* \Rightarrow \delta(s_L^*) = \delta(s_H^*) \Rightarrow \frac{dW}{de} < 0$) as the difference between high and low risk types' safety technology is independent of s (i.e. $\delta'(s) = 0$) and, hence, will be referred to as an autonomous difference. This being the case, information has no role to play in the efficient use of self-protection and $dW/de < 0$ is assured. This result, which is a corollary to the more general problem discussed and proved below, can be extended to analyze situations similar to Cases B and C below.

Suppose we begin with a pair of safety technologies represented by $p_h(s)$ and $p_l(s)$ and generate a new set of safety technologies $p_h^*(s)$ and $p_l^*(s)$ which differ from the initial technologies according to an increase in the autonomous part of the difference between them but without altering the aggregate loss probability (i.e. a mean preserving spread between safety technologies). This procedure can be characterized by a parameter $\Delta > 0$ with $p_h^*(s) = p_h(s) + \frac{1}{q_h} \Delta$ and $p_l^*(s) = p_l(s) - \frac{1}{q_l} \Delta$. Since $\frac{d\Delta}{ds} = 0$ its inclusion has no effect on the choice of s_c^* and hence no effect on the ratio $\frac{\delta(s_L^*)}{\delta(s_H^*)}$. However, an increase in Δ increases, ceteris paribus, the price of insurance for members in the high risk category and decreases the price of insurance for those in the low risk category. Therefore, y_L falls and y_H rises so that, given risk aversion, an increase in information is more likely to lead to a decrease in welfare as measured by W .¹⁴

The effects of an increase in the autonomous difference between safety technologies is illustrated for Case \hat{C} in Figure 8 below. As Δ increases there is an increase in $p_h^*(s)$ and $\hat{p}_H^*(s)$ while $p_l^*(s)$ and $\hat{p}_L^*(s)$ fall.¹⁵ Since Δ has no effect on the choice of safety (s_L^* or s_H^*) the only relevant impact is on the price of insurance and hence the values of y_H which falls and y_L which increases.

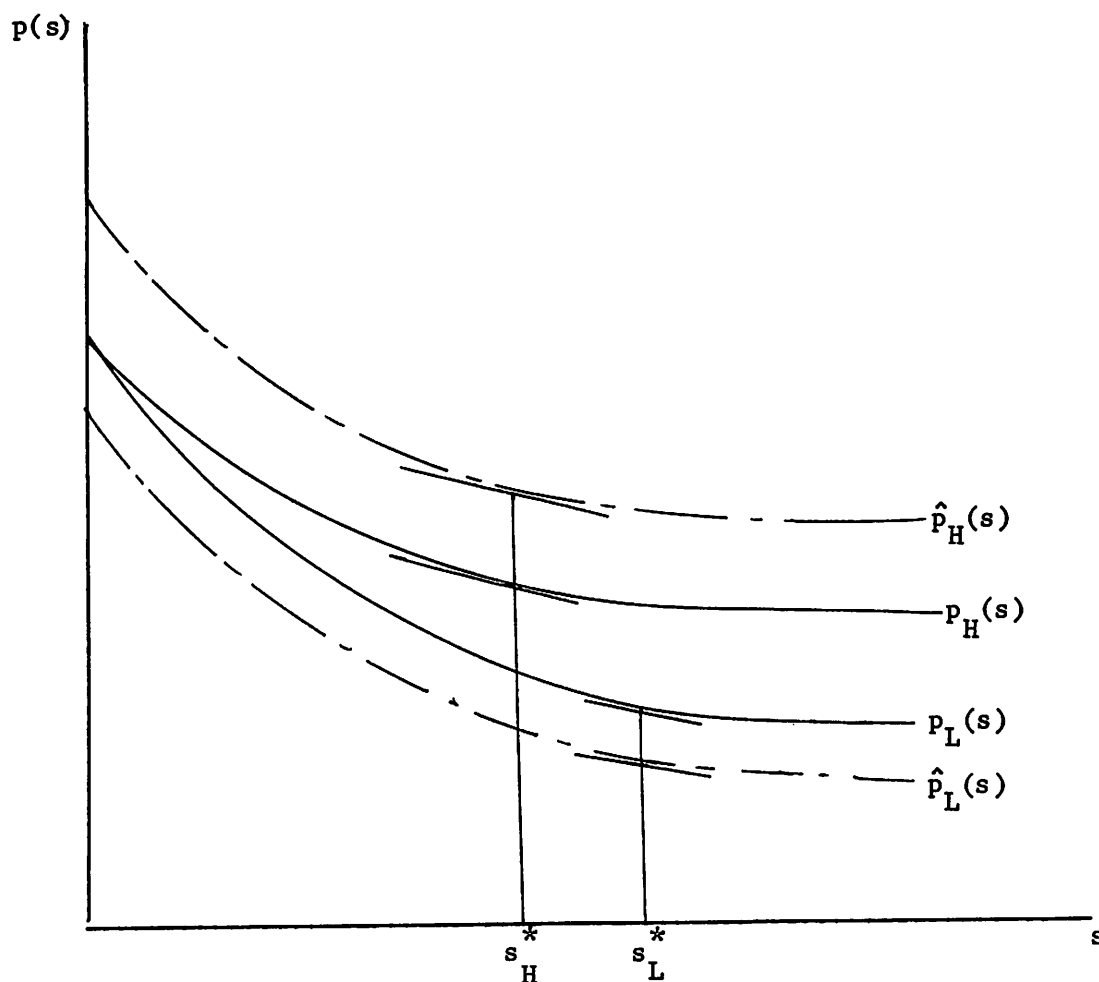


Figure 9: Case C

Since individuals purchase full insurance and choose s so as to maximize expected wealth when the level of self-protection is observable to firms, it would seem an intuitive result that equity considerations become more important the more risk averse are individuals. This is shown below to be the case for individuals who possess constant absolute risk aversion (CARA) or constant relative risk aversion (CRRA).

For the CARA utility function $u(w) = -e^{-\alpha w}$, $\alpha > 0$. Therefore,

$$\left[\frac{u'(y_H)}{u'(y_L)} \right] = \frac{\gamma e^{-\alpha y_H}}{\gamma e^{-\alpha y_L}} = e^{\alpha(y_L - y_H)} \quad \text{and so} \quad \frac{d[\cdot]}{d\alpha} = (y_L - y_H) e^{\alpha(y_L - y_H)} > 0. \quad \text{For the}$$

$$\text{CRRA utility function } u(w) = \frac{w^{1-\gamma}}{1-\gamma}, \quad \gamma > 0, \quad \gamma \neq 1. \quad \text{Therefore, } \left[\frac{u'(y_H)}{u'(y_L)} \right] = \frac{y_H^{-\gamma}}{y_L^{-\gamma}}$$

$$= \left(\frac{y_L}{y_H} \right)^\gamma. \quad \text{Taking logs, } [\cdot]^* = \ln[\cdot] = \gamma \ln \frac{y_L}{y_H} \quad \text{it follows that } \frac{d[\cdot]^*}{d\gamma} > 0 \quad \text{since}$$

$y_L > y_H$ and so $\frac{d[\cdot]}{d\gamma} > 0$ also. To summarize, an increase in the degree of risk

aversion as defined above leads to an increase in $\left[\frac{u'(y_H)}{u'(y_L)} \right]$. Since the degree

of risk aversion has no effect on the level of self-protection there is no

counterbalancing effect on $\frac{\delta(s_L^*)}{\delta(s_H^*)}$. Therefore, an increase in the degree of risk

aversion will, ceteris paribus, increase the likelihood that a greater extent of information will cause a decrease in welfare (W).

The results of this section are summarized in the following multipart theorems.

Theorem 4.1 When self-protection is observable to firms an increase in information concerning insureds' risk type has the following consequences:

- (i) aggregate expected income either rises or remains constant;
- (ii) a Pareto-improvement in welfare will not occur;
- (iii) social welfare as measured by the utilitarian social welfare function may be improved or worsened.

Theorem 4.2 When self-protection is observable to firms the following conditions increase the likelihood that an increase in information will lead to a worsening in social welfare as measured by the utilitarian social welfare function:

- (i) an increase in the autonomous difference between safety technologies
- (ii) an increase in the degree of risk aversion for either the constant absolute or constant relative risk aversion utility functions.

In light of Theorem 4.1 (especially results (i) and (ii)) one should note that a Pareto-improvement is possible provided an appropriate redistribution scheme is used. Therefore, social insurance schemes may be able to extract the efficiency value of information while circumventing the adverse equity effects.

5. The Value of Information with Self-Protection and the Aggregate Insurance Purchases Unobservable by Firms

If the insurer is unable to observe either the level of self-protection or the aggregate insurance purchases chosen by a consumer then either the consumer purchases full insurance and uses no self-protection or he purchases no insurance and chooses s accordingly (see Section 2.5). Before considering the impact that an increase in information has on these decisions for the model with heterogeneous agents, the effect that an increase in information (e) has on the optimal choice of self-protection (s_H^*, s_L^*) when no insurance is purchased is considered. This exercise is of independent interest and provides results which are of use later in this section.

If an individual belonging to risk category c purchases no insurance then his optimal choice of self-protection, s_c^* , is determined by the problem

$$\max_s EU_c = [1 - p_c(s_c)]u(y - s_c) + p_c(s_c)u(y - d - s_c)$$

which gives rise to the following first-order conditions for members of the high and low risk categories respectively.

$$\begin{aligned} g_H(s_H^*; e) &= p_H'(s_H^*) [u(y - d - s_H^*) - u(y - s_H^*)] \\ &\quad - [1 - p_H(s_H^*)]u'(y - s_H^*) - p_H(s_H^*)u'(y - d - s_H^*) = 0 \end{aligned} \quad (39)$$

$$\begin{aligned} g_L(s_L^*; e) &= p_L'(s_L^*) [u(y - d - s_L^*) - u(y - s_L^*)] \\ &\quad - [1 - p_L(s_L^*)]u'(y - s_L^*) - p_L(s_L^*)u'(y - d - s_L^*) = 0 \end{aligned} \quad (40)$$

Therefore, $\frac{ds_c^*}{de} = -\frac{\partial g_c / \partial e}{\partial g_c / \partial s_c^*}$. The second-order condition for utility maximization requires that $\partial g_c / \partial s_c^*$ be negative so that $\frac{ds_c^*}{de}$ has the same sign as $\frac{\partial g_c}{\partial e}$. Noting

that $p_H(s_H) = p_h(s_H) + (q_h + e)\delta(s_H)$ and $p_L(s_L) = p_l(s_L) + (q_h - ek)\delta(s_L)$ it follows that

$$\begin{aligned} \frac{\partial g_H}{\partial e} &= \delta'(s_H^*) [u(y - d - s_H^*) - u(y - s_H^*)] \\ &\quad + \delta(s_H^*) [u'(y - s_H^*) - u'(y - d - s_H^*)] \end{aligned} \quad (41)$$

$$\begin{aligned} \frac{\partial g_L}{\partial e} &= -k\delta'(s_L^*) [u(y - d - s_L^*) - u(y - s_L^*)] \\ &\quad - k\delta(s_L^*) [u'(y - s_L^*) - u'(y - d - s_L^*)] \end{aligned} \quad (42)$$

From the above considerations it can be shown that if $\delta'(s) \geq 0$, which holds for Cases A ($\delta' = 0$) and C ($\delta' > 0$), then $\frac{ds_H^*}{de} < 0$ (since $\frac{\partial g_H}{\partial e} < 0$) and

$\frac{ds_L^*}{de} > 0$ (since $\frac{\partial g_L}{\partial e} > 0$). For Case B the signs of $\frac{ds_H^*}{de}$ and $\frac{ds_L^*}{de}$ are indeterminate.

The implications of an increase in information on the choice of s_c^* is demonstrated

for Case A below by comparing the choice of self-protection before and after imperfect categorization.

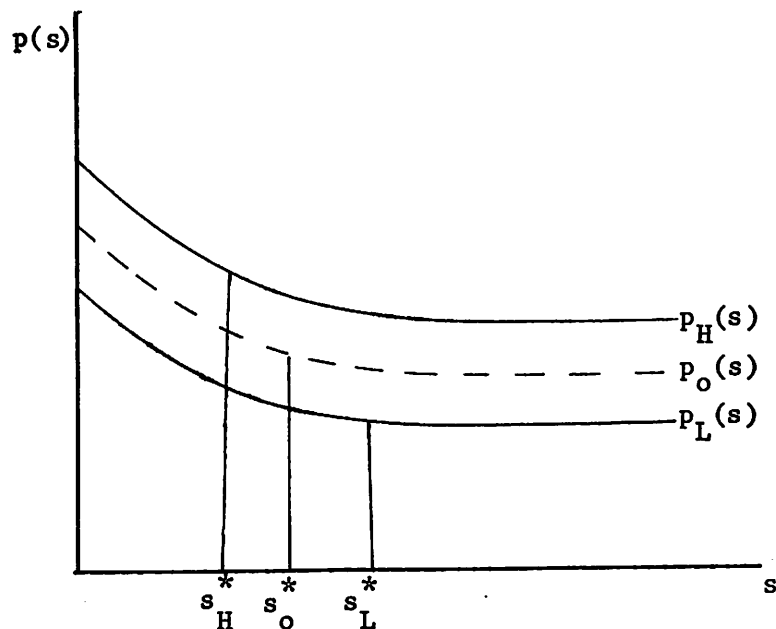


Figure 10: Case A

The result that members of the high-risk category tend to use less self-protection than do members of the low-risk category may seem counter-intuitive at first glance;¹⁶ that is, it initially seems strange that individuals who perceive an increase in their loss probability would reduce their level of self-protection. However, an increase in s_H^* leads to a reduction in the probability of the bad state occurring ($p_H \downarrow$) and a rise in the probability of the good state ($(1 - p_H) \uparrow$) occurring. This represents a shift in probability from the state with a relatively large marginal utility of income to a state with a relatively small marginal utility of income (i.e., $u'(y - s_H^* - d) > u'(y - s_H^*)$). However, an increase in the level

of self-protection, s_H^* , would lead to an increase in the difference between these marginal utilities. Since this difference is relatively more important to members of the high risk category it is not so surprising that they reduce expenditure on self-protection while, mutatis mutandis, members of the low risk category increase expenditure on self-protection. In Case C this phenomenon is further supported by the fact that the use of self-protection is relatively more productive for low risk types (i.e. $\delta'(s) > 0 \Rightarrow p_l'(s) < p_h'(s)$ which means an increase in the use of s reduces the loss probability more for a low risk type than for a high risk type). For case B the use of self-protection is relatively more productive for high risk types so that the two above-mentioned effects conflict and the signs of $\frac{ds_c^*}{de}$, $c = H, L$ are indeterminate.¹⁷

We can now investigate the implications that an increase in information has with respect to the Utilitarian social welfare function. With both self-protection and aggregate insurance purchases unobservable by firms, individuals choose one of two possible strategies to maximize their expected utility (see Section 2.5). An individual either purchases full insurance and uses no self-protection or purchases no insurance and chooses an optimal level of self-protection as described by equations (39) and (40) for high and low risk category members, respectively. The case where no insurance is purchased before or after the increase in information is considered first.

Letting y_c^j denote income received by an individual belonging to category $c (=H, L)$ in state $j (=g, b)$ it follows that $y_c^g = y - s_c^*$ and $y_c^b = y - s_c^* - d$ if no insurance is purchased. Expected utility for a risk type i individual in category c can be written as below.

$$EU_{ic} = [1 - p_i(s_c^*)]u(y_c^g) + p_i(s_c^*)u(y_c^b) \quad (43)$$

Using q_{ic} , the proportion of risk type i individuals in category c we can write average expected utility for an individual in category c as follows:

$$EU_c = q_{hc} EU_{hc} + q_{lc} EU_{lc} \quad (44)$$

Therefore, the Utilitarian social welfare function can be written as in either of the following two equations.¹⁸

$$W(s_H^*, s_L^*; e) = K_H EU_H + K_L EU_L \quad (45)$$

$$W(s_H^*, s_L^*; e) = K_H [q_{hH} EU_{hH} + q_{lH} EU_{lH}] \\ + K_L [q_{hL} EU_{hL} + q_{lL} EU_{lL}] \quad (46)$$

Using equation (47) it follows that

$$\frac{dW}{de} = \frac{\partial W}{\partial e} + K_H \frac{\partial EU_H}{\partial s_H} \frac{ds_H}{de} + K_L \frac{\partial EU_L}{\partial s_L} \frac{ds_L}{de} \quad (47)$$

However, s_c^* is chosen to maximize EU_c (see equations (39) and (40)) so that the last two terms of equation (47) are zero (according to the envelope theorem).

Therefore, using equation (46) and equation set (17) it follows that

$$\frac{dW}{de} = K_H [EU_{hH} - EU_{lH}] + K_L [kEU_{lL} - kEU_{hL}] \quad (48)$$

Using the definitions $p_h(s) = p_l(s) + \delta(s)$ and $k = \frac{K_H}{K_L}$ it follows that

$$\frac{dW}{de} = K_H \{ \delta(s_H^*) [u(y - s_H^* - d) - u(y - s_H^*)] \\ + \delta(s_L^*) [u(y - s_L^*) - u(y - s_L^* - d)] \} \quad (49)$$

Equation (49) can be used to determine the welfare implications of an increase in information as is illustrated below:

$$\frac{dW}{de} \stackrel{\Delta}{\approx} 0 \text{ as } \frac{\delta(s_L^*)}{\delta(s_H^*)} \stackrel{\Delta}{\approx} \frac{u(y - s_H^*) - u(y - d - s_H^*)}{u(y - s_L^*) - u(y - d - s_L^*)} \quad (50)$$

Now, for Cases A and C we have from our previous analysis concluded that

$s_L^* > s_H^*$ which implies that $u(y - s_H^*) > u(y - s_L^*)$ and $u(y - s_H^* - d) > u(y - s_L^* - d)$.

Also, $y_H^g - y_H^b = y_L^g - y_L^b = d$ and $y - s_H^* > y - s_L^*$. Therefore, risk aversion assures that $u(y - s_H^*) - u(y - s_H^* - d) < u(y - s_L^*) - u(y - s_L^* - d)$ so that the right side of equation (50) is less than 1. Since $\delta(s_L^*) \geq \delta(s_H^*)$ the left side of equation (50) is greater than 1 so that $\frac{dW}{de} > 0$. The result for Case B appears to be indeterminate.

Theorem 5.1 If no insurance is purchased either before or after an increase in information the said increase leads to an improvement in social welfare as measured by the utilitarian social welfare function at least for Cases A and C.

The above results suggest that an increase in information may improve welfare when a consumer's level of self-protection and aggregate insurance purchases are not observable by the firm. The improvement occurs as a result of a change in the "ex ante" optimal level of self-protection. Emphasis on the ex ante nature of the improvement in the efficient use of self-protection is important and is illustrated by the result that an increase in information leads to a reduction in the level of self-protection for members of the high risk category and an increase for members of the low risk category even when the marginal productivity of self-protection is identical for both groups.

As mentioned at the beginning of this section, however, it is also possible that an individual will purchase full coverage insurance and employ

no self-protection. This being the case, it is also important to incorporate the effects that changing information has on the price of insurance and the decision to rely exclusively on insurance or exclusively on self-protection. Rather than providing a complete taxonomy of results for each of Cases A, B and C and all the possibilities of full coverage insurance versus no insurance only, two groups of possibilities are considered. These two possible results are sufficiently comprehensive for our purposes as they provide opposing conclusions and offer the insight desired.

Firstly, suppose that both before and after the extent of information increases all individuals purchase full coverage insurance. This being the case, no self-protection is employed in any situation so that expected aggregate income remains unchanged. It is clear that for Cases A and B members of the high risk category must pay a higher price for insurance after the increase in information while members of the low risk category pay less. Therefore, information provides no efficiency benefits and worsens the distribution of income.¹⁹ The consequence is a reduction in the level of welfare (W).

Alternatively, suppose that before the extent of information increases everyone purchases full-coverage insurance while after information increases members of one of the two categories do not purchase insurance and instead rely exclusively on self-protection. For Case C (see Figure 7) it is clear that the expected utility obtained by members of the low risk category is greater than that obtained by members of the high risk category when individuals rely exclusively on self-protection (i.e., purchase no insurance). Since they experience the same expected utility when full insurance is purchased and

self-protection is zero (i.e., before the increase in e) then as information increases continuously members of the low risk category are the first to switch from purchasing insurance to using self-protection. Since expected utility doesn't change for those who continue to buy full insurance (since $\delta(0) = 0$) an increase in information leads to a Pareto-improvement in welfare and, a fortiori, an increase in W . This result is preserved as high risk category members also change their decision. However, the same cannot be said for Cases A and B as $\delta(0) \neq 0$ and an increase in e leads to an increase in the insurance premium for members of the high risk category.

The former of the above two results highlights the possibility of an adverse distributive effect that an increase in information can cause when insurance is purchased. The second of the results emphasizes the efficiency potential of information with respect to the dichotomous choice of relying exclusively on self-protection or insurance. This provides an interesting augmentation to Theorem 4.1(i) which takes account of the benefit of information in improving the use of self-protection in a marginal context. The following theorems summarize these results.

Theorem 5.2 Suppose full coverage insurance (implying $s_H^* = s_L^* = 0$) is purchased before and after an increase in information. This being the case, welfare as measured by the Utilitarian social welfare function is either reduced (Cases A and B) or unchanged (Case C).

Theorem 5.3 Suppose full coverage insurance (implying $s_H^* = s_L^* = 0$) is purchased before an increase in information but that members of one risk category (at least) do not purchase insurance afterwards. This being the case, a Pareto-improvement in welfare occurs for Case C and, a fortiori, social welfare (W) increases. Some distributive risk will generally be associated with Cases A and B.

6. The Value of Information with Moral Hazard

If a firm cannot observe the level of self-protection employed by a consumer but can observe the consumer's aggregate insurance purchases then the firm can infer the level of self-protection (s) from the amount of insurance purchased (α). Nevertheless, a change in α leads to a change in the optimal level of s chosen by the consumer and since s cannot be directly monitored only a second-best contract can be constructed. This is an example of the well-known problem of moral hazard (see Section 2.4).

Since contracts are not made contingent on the level of self-protection, information concerning differences in individuals' safety technology cannot be used directly to improve the efficiency of contracts. However, such information will have an impact on the price of insurance offered to various types of individuals and so indirectly will affect the level of self-protection chosen by individuals. Therefore, information has both equity and efficiency effects. As is seen below, the analysis for this section demonstrates an interesting combination of effects analogous to those found in Sections 4 and 5. Comparisons are reserved for the end of this section.

Using, once again, y_c^j to denote income received by members of category $c(=H,L)$ in state $j(=g,b)$ and letting (α_c, β_c) represent the insurance contract (i.e., the price schedule) for individuals in risk category c it follows that

$$y_c^g = y - \beta_c - s_c \quad (51)$$

$$y_c^b = y - d + \alpha_c - s_c \quad (52)$$

The insurer chooses α_c, β_c such that

$$\beta_c = \frac{\alpha_c p_c(s_c)}{1 - p_c(s_c)} \quad (53)$$

The insurer cannot observe directly the level of s_c but can infer it from the individual's choice of α_c .

A consumer in risk category c chooses the level of self-protection (s_c) and insurance coverage (α_c) (taking equation (53) into account) so as to maximize expected utility; that is

$$\max_{\alpha_c, s_c} EU_c = (1 - p_c(s_c))u(y_c^g) + p_c(s_c)u(y_c^b) \quad (54)$$

which gives rise to the following first-order conditions:

$$\begin{aligned} \frac{\partial EU_c}{\partial s_c} &= p_c'(s_c) [u(y_c^b) - u(y_c^g)] - (1 - p_c(s_c))u'(y_c^g) \\ &\quad - p_c(s_c)u'(y_c^b) = 0 \end{aligned} \quad (55)$$

$$\frac{\partial EU_c}{\partial \alpha_c} = - \frac{\partial \beta}{\partial \alpha_c} (1 - p_c(s_c))u'(y_c^g) + p_c(s_c)u'(y_c^b) = 0 \quad (56)$$

Expected utility for a member of risk category c can be written as a function of the parameter e , representing the extent of information, and the endogenous variables α_c and s_c ; that is, $EU(\alpha_c, s_c; e)$. Therefore,

$$\frac{dEU_c}{de} = \frac{\partial EU_c}{\partial e} + \frac{\partial EU_c}{\partial \alpha_c} \frac{d\alpha_c}{de} + \frac{\partial EU_c}{\partial s_c} \frac{ds_c}{de} \quad (57)$$

According to the envelope theorem (i.e., equations (55) and (56)) the first two terms on the right side of equation (57) vanish. Therefore,

$$\frac{dEU_H}{de} = \delta(s_H^*) [u(y_H^b) - u(y_H^g)] - \left. \frac{d\beta_H}{de} \right|_{\alpha_H \text{ constant}} (1 - p_H(s_H))u'(y_H^g) \quad (58)$$

$$\frac{dEU_L}{de} = k\delta(s_L^*) [u(y_L^g) - u(y_L^b)] - \left. \frac{d\beta_L}{de} \right|_{\alpha_L \text{ constant}} (1 - p_L(s_L))u'(y_L^g) \quad (59)$$

We can now consider the impact of an increase in information on the Utilitarian social welfare function which can be written as follows:

$$W(s_H^*, s_L^*, \alpha_H^*, \alpha_L^*; e) = K_H EU_H + K_L EU_L \quad (60)$$

Using equations (57), (58) and (59) it follows that

$$\begin{aligned} \frac{dW}{de} = & K_H \{ \delta(s_H^*) [u(y_H^b) - u(y_H^g)] + \delta(s_L^*) [u(y_L^g) - u(y_L^b)] \} \\ & - \left(\frac{d\beta_H}{de} \right)_{\alpha_H \text{ constant}} (1 - p_H(s_H^*)) u'(y_H^g) K_H \\ & - \left(\frac{d\beta_L}{de} \right)_{\alpha_L \text{ constant}} (1 - p_L(s_L^*)) u'(y_L^g) K_L \end{aligned} \quad (61)$$

Upon comparison of equation (61) with equation (49) we see that the first term of equation (61) represents an efficiency effect of increased information. One noteworthy difference persists, however, since y_c^j in equation (61) takes into account insurance purchases (i.e., $y_c^g = y - s_c^* - \beta_c^*$ and $y_c^b = y - s_c^* + \alpha_c^* - d$). The next two terms of equation (61) represent the price effects of increasing information. Using equation (53) and the definition $p_L(s_L) = p_l(s_L) + (q_h - ek) \delta(s_L)$ and $p_H(s_H) = p_h(s_H) + (q_h + e) \delta(s_H)$ we get the following result for the price effects:

$$\text{price effects} = K_H \left\{ \frac{\alpha_L \delta(s_L^*)}{[1 - p_L(s_L^*)]} u'(y_L^g) - \frac{\alpha_H \delta(s_H^*)}{[1 - p_H(s_H^*)]} u'(y_H^g) \right\} \quad (62)$$

Therefore, although we can combine previous results from which comparisons can be made, the issue of the value of information when moral hazard persists is more difficult to analyze than for the previously considered information structure. At least we can identify the various effects.

7. Conclusions

The problem of moral hazard in the principal-agent relationship has been studied extensively in the recent economics literature. However, most of these studies assume that agents are homogeneous. Models depicting adverse selection, in which individuals are heterogeneous, generally ignore the problem of moral hazard (e.g., see Rothschild and Stiglitz [1976], Wilson [1977], and Hoy [1982]). In this paper a model of an insurance market in which individuals differ according to safety technologies rather than simply on the basis of exogenously determined probabilities is presented. The analysis does not, however, represent a merging of the problems of adverse selection and moral hazard since information concerning differences between individuals' safety technologies is assumed to be symmetric. Therefore, the analysis here is perhaps better thought of as an extension of the moral hazard literature than of the adverse selection literature. However, if one were to alter this model by assuming that the insured (agent) knows his own safety technology but the insurer (principal) is not, at least initially, privy to this information then the problems of adverse selection and moral hazard would be truly combined.

Holmström [1979] and Shavell [1979a] have recently considered the value of imperfect information in the context of a principal and agent model with moral hazard. However, their analysis is quite different from that used in this paper. In their models agents are homogeneous and information relates to the imperfect observation of the level of effort taken by the agent or, in this case, the level of self-protection. In the models presented in this paper agents are heterogeneous and information relates to the imperfect

matching of individuals to safety technologies.

Besides analyzing the effects of increasing information used to match individuals to their appropriate safety technologies when moral hazard persists, this paper also considers this phenomenon for related models in the principal and agent domain. These other instances include the cases where self-protection (the agent's actions) is observable to the firm as well as the case where an agent's aggregate insurance purchases (as well as his level of self-protection) are unobservable to the firm. From a policy perspective all of these results are of interest since any particular information structure is possible; which one of these that will actually persist in a particular situation depends on the costs of making the relevant observations.

Although this paper uses the example of an insurance problem, the results can be extended in a straightforward manner to the general principal-agent problem. For example, MacDonald [1982] presents a model where information which improves the matching of workers' skills to appropriate tasks becomes available symmetrically to worker (agent) and employer (principal). Using the results of this paper one could investigate further this phenomenon when implicit or explicit insurance on wage variations is present (e.g., provided by the employer or a union) and where the observability or nonobservability of effort provided by the worker is also a relevant possibility. The particular information structure which is assumed has an important impact on the model.

There are further interesting aspects of the problem presented in this paper which could be considered. The extent of information concerning the matching of individuals to their safety technologies is assumed in this paper to

be determined exogenously. Interesting insights might be provided by a model which treated information endogenously. The extent to which firms will invest in such information acquisition may depend on such factors as market structure.²¹ It may also be interesting to try various explicit methods for modelling information acquisition. Suppose, for example, that we use past experience as a method of acquiring information on an individual's safety technology. If complete technologies are unknown (i.e., $p_h(s)$, $p_l(s)$ or $p_H(s)$, $p_L(s)$) then individuals will probably change their behavior (level of s) as they gain more experience concerning their loss probabilities. The result may be an identification problem; namely, is a change in one's loss experience an indication of risk class membership or an effect resulting from an alteration in the level of self-protection chosen. Such problems are eliminated in this paper as information is treated as if it were obtained by controlled experimentation.

References

- Arnott, R. and J. Stiglitz (1981), "Equilibrium in Competitive Insurance Markets: The Welfare Economics of Moral Hazard: Parts I and II," unpublished mimeo.
- Arnott, R. and J. Stiglitz (1982), "Equilibrium in Competitive Insurance Markets, The Welfare Economics of Moral Hazard I: Basic Analytics," Discussion Paper No. 465, Queen's University, Kingston, Canada.
- Arrow, K. J. (1971), "A Utilitarian Approach to the Concept of Equality in Public Expenditures," Quarterly Journal of Economics, 409-15.
- Grossman, S. J. and J. E. Stiglitz (1980), "The Impossibility of Informationally Efficient Markets," American Economic Review, 393-408.
- Harris, M. and A. Raviv (1978), "Some Results on Incentive Contracts," American Economic Review, 20-30.
- Harsanyi, Z. and R. Hutton (1981), Genetic Prophecy: Beyond the Double Helix [Rawson, Wade].
- Hirshleifer, J. (1971), "The Private and Social Value of Information and the Reward to Inventive Activity," American Economic Review, 561-74.
- Holmström, B. (1979), "Moral Hazard and Observability," Bell Journal of Economics, 74-91.
- Hoy, M. (1982), "Categorizing Risks in the Insurance Industry," Quarterly Journal of Economics, 321-336.
- MacDonald, G. (1982), "A Market Equilibrium Theory of Job Assignment and Sequential Accumulation of Information," American Economic Review, 1038-55.
- Pauly, M. V. (1974), "Overinsurance and Public Provision of Insurance: The Roles of Moral Hazard and Adverse Selection," Quarterly Journal of Economics, 44-62.
- Rothschild, M. and J. Stiglitz (1976), "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," Quarterly Journal of Economics, 629-649.

Shavell, S. (1979a), "Risk Sharing and Incentives in the Principal and Agent Relationship," Bell Journal of Economics, 55-73.

Shavell, S. (1979b), "On Moral Hazard and Insurance," Quarterly Journal of Economics, 541-562.

Spence, A. M. (1974), Market Signaling [Harvard University Press, Cambridge, Mass.].

Wilson, C. (1977), "A Model of Insurance with Incomplete Information," Journal of Economic Theory, 167-207.

Appendix ASummary of Notation

s	insured's expenditure on self-protection
d	loss sustained in state with accident
α	payout (net of premium) paid in state with accident
β	premium paid if no accident occurs
$p(s)$	probability of accident given level of self-protection s
	assume: $p' < 0$, $p'' > 0$, $p(0) = \underline{p} > 0$, $p(\infty) = \bar{p} < 1$, $0 < p(s) < 1 \quad \forall s$.
y	certain income, $y > d$
$y^g = y - \beta - s$	income if no accident (good state)
$y^b = y - d + \alpha - s$	income if accident occurs (bad state) Note: In some instances $\beta = \alpha = 0$ or $s = 0$.
$i = h, \ell$	index for risk types: $i=h$ for high risk types, $i=\ell$ for low risk types
$c = H, L$	index applies to risk category: $c = H$ for high risk category, $c = L$ for low risk category.
e	extent of information (defined below)
N_i	number of individuals of risk type i in population
K_c	number of individuals in category c
N	total number of individuals ($N_h + N_\ell = K_H + K_L = N$)
n_{ic}	number of individuals of risk type i assigned to category c
	$\therefore n_{hH} + n_{hL} = N_h \quad n_{hH} + n_{\ell H} = K_H$
	$n_{\ell H} + n_{\ell L} = N_\ell \quad n_{\ell L} + n_{hL} = K_L$

q_i	Proportion of individuals who are of risk type i in the aggregate population ($q_i = N_i/N$)
q_{ic}	Proportion of individuals who are of risk type i in category c ($0 < q_{ic} < 1$).
	$\therefore q_{\ell} + q_h = 1, \quad q_{\ell L} + q_{hL} = 1, \quad q_{hH} + q_{\ell H} = 1$
e	is defined so that
	$q_{hH} = q_h + e, \quad 0 \leq e \leq 1 - q_h$
	$\Rightarrow q_{\ell L} = q_{\ell} + ek$
	$q_{hL} = q_h - ek$
	$q_{\ell H} = q_{\ell} - e, \quad k = \frac{K_H}{K_L}$
s_L, s_H	amount of self-protection employed by members of the low and high risk categories respectively
$p_i(s)$	probability of loss for risk type i define: $p_h(s) = p_{\ell}(s) + \delta(s), \quad \delta(s) \geq 0$
$p_c(s)$	perceived probability of loss for members of risk category c
	$\therefore p_c(s) = q_{\ell c} p_{\ell}(s) + q_{hc} p_h(s)$
(α_c, β_c)	insurance contract offered to members of category c
P_c	price of full coverage insurance to members of category c
\hat{y}_c^j	income received by members of risk category c in state j ; $c = H, L$; $j = g, b$
Ey_{ic}	expected income generated by a risk type i individual assigned to category c
$\bar{E}y$	average per capita expected income
EU_{ic}	actual expected utility for an individual of risk type i in category c
EU_i	actual expected utility for an individual of risk type i
EU_c	perceived expected utility for a member of risk category c

Appendix B

The following comparative statics exercise refers to Section 6. The exercise is completed only for members of the high risk category.

$$\text{Consumer: } \max_{s_H, \alpha_H}^m EU = [1 - p_H(s_H)]u(y_H^g) + p_H(s_H)u(y_H^b)$$

$$\text{where } y_H^g = y - \beta_H - s_H$$

$$y_H^b = y - d + \alpha_H - s_H$$

$$\beta_H = f(\alpha_H, s_H(\alpha_H; e); e)$$

$$\text{with } \frac{df}{d\alpha_H} = \frac{p_H}{1-p_H} + \frac{\alpha_H \frac{dp_H}{ds_H} \frac{ds_H}{d\alpha_H}}{(1-p_H)^2}, \text{ from eqn. (13)}$$

f.o.c.

$$\frac{\partial EU}{\partial s_H} = F^1(s_H, \alpha_H; e) = p_H'(s_H)[u(y_H^b) - u(y_H^g)] - [1-p_H(s_H)]u'(y_H^g) - p_H u'(y_H^b) = 0$$

$$\frac{\partial EU}{\partial \alpha_H} = F^2(s_H, \alpha_H; e) = [1-p_H(s_H)](-\frac{df}{d\alpha_H})u'(y_H^g) + p_H(s_H)u'(y_H^b) = 0$$

$$\therefore \frac{ds_H}{de} = \frac{\begin{vmatrix} -\frac{\partial F^1}{\partial e} & \frac{\partial F^1}{\partial \alpha_H} \\ -\frac{\partial F^2}{\partial e} & \frac{\partial F^2}{\partial \alpha_H} \end{vmatrix}}{|J|}, \quad \frac{d\alpha_H}{de} = \frac{\begin{vmatrix} \frac{\partial F^1}{\partial s_H} & -\frac{\partial F^1}{\partial e} \\ \frac{\partial F^2}{\partial s_H} & -\frac{\partial F^2}{\partial e} \end{vmatrix}}{|J|}$$

$$\text{where } |J| = \begin{vmatrix} \frac{\partial F^1}{\partial s_H} & \frac{\partial F^1}{\partial \alpha_H} \\ \frac{\partial F^2}{\partial s_H} & \frac{\partial F^2}{\partial \alpha_H} \end{vmatrix} > 0 \text{ from second order conditions}$$

Now,

$$\frac{\partial F^1}{\partial s_H} = p_H''(s_H)[u(y_H^b) - u(y_H^g)] + 2p_H'(s_H)[u'(y_H^g) - u'(y_H^b)] + [1-p_H(s_H)]u''(y_H^g) + p_H(s_H)u''(y_H^b)$$

$$\frac{\partial F^1}{\partial \alpha_H} = p_H'(s_H)[u'(y_H^b) + \frac{df}{d\alpha_H} u'(y_H^g)] + (1-p_H)\frac{df}{d\alpha_H} u''(y_H^g) - p_H u''(y_H^g) - p_H u''(y_H^b)$$

$$\frac{\partial F^2}{\partial s_H} = p_H'(s_H) \frac{df}{d\alpha_H} u'(y_H^g) + (1-p_H(s_H)) \frac{df}{d\alpha_H} u''(y_H^g) + p_H'(s_H)u'(y_H^b) - p_H(s_H)u''(y_H^b)$$

$$\frac{\partial F^2}{\partial \alpha_H} = -\frac{d^2 f}{d\alpha_H^2} [1-p_H(s_H)]u'(y_H^g) + \left(\frac{df}{d\alpha_H}\right)^2 [1-p_H(s_H)]u''(y_H^g) + p_H(s_H)u''(y_H^b)$$

$$\frac{\partial F^1}{\partial e} = \delta'(s_H)[u(y_H^b) - u(y_H^g)] + \delta(s_H)[u'(y_H^g) - u'(y_H^b)] + p_H'(s_H)u'(y_H^g) \frac{df}{de} + [1-p_H(s_H)]u''(y_H^g) \frac{df}{de}$$

$$\frac{\partial F^2}{\partial e} = \delta(s_H) \frac{df}{d\alpha_H} u'(y_H^g) - [1-p_H(s_H)] \frac{d^2 f}{d\alpha_H de} u'(y_H^g) + [1-p_H(s_H)] \left(\frac{df}{d\alpha_H}\right) u''(y_H^g) \frac{df}{de} + \delta(s_H)u'(y_H^b)$$

where $\frac{\partial F^1}{\partial s_H}$ and $\frac{\partial F^2}{\partial \alpha_H} < 0$ for S.O.C.

However, $\frac{\partial F^1}{\partial e}$, $\frac{\partial F^1}{\partial \alpha_H}$, $\frac{\partial F^2}{\partial s_H}$, $\frac{\partial F^2}{\partial e}$ cannot be signed.

Footnotes

*Chin Lim, Glenn MacDonald and John McMillan provided helpful suggestions to an earlier version of this paper.

¹The possibility that information may have undesirable equity effects is illustrated by Hirshleifer [1971]. MacDonald [1982] provides a notable exception to the use of models with homogeneous technologies and concentrates on the efficiency effects of information which improves the match between agents' skills and their job assignments. A richer set of information structures is considered in this paper as well as explicit concern for equity effects.

²In a book which provides numerous interesting examples of such relationships, Harsanyi and Hutton [1981, p. 22] note that "Every disease has both environmental and genetic components... (and that)... the relative weights of (these) factors vary according to the disease."

³Per capita expected income is $E\bar{y} = [1-p(s)](y-s) + p(s)(y-d-s)$. Choosing s to maximize $E\bar{y}$ gives $p'(s) = -\frac{1}{d}$.

⁴If utility is state-dependent this conclusion does not necessarily follow.

⁵In the papers by Pauly [1974], Shavell [1979b] and in this paper it is assumed that the model is "well-behaved". In particular it is assumed that indifference curves in α - β space are quasi-concave and the consumer's second order conditions hold for interior solutions. Arnott and Stiglitz [1982] demonstrate that with the presence of self-protection (effort in their case) this condition does not follow necessarily from the assumption of risk aversion. Nevertheless, we will treat the problem here as if it were well-behaved mathematically without giving the explicit conditions (see Arnott and Stiglitz [1982]).

⁶ Arrow [1971] provides an example of such a technique for distinguishing among individuals with f being utility, e being educational attainment and ξ being ability.

⁷ Arrow [1971] uses an analogous set of conditions to determine an optimal expenditure rule (e.g., on education) over a heterogeneous group of individuals.

⁸ The presence of the antigen HLA-Dw3 is a marker (predictor) of the characteristic $\xi = G_h$. The analysis of this paper allows for any degree of accuracy of such markers or other types of information.

⁹ This is likely to be the case when information is embodied by a statistical correlation between a (costlessly) observable personal characteristic and an individual's risk type.

¹⁰ Average per capita expected income, which equals actual average income when full insurance is purchased, is $E\bar{y} = q_h [(1-p_h(s_h))(W-s_h) + p_h(s_h)(W-d-s_h)] + q_\ell [(1-p_\ell(s_\ell))(W-s_\ell) + p_\ell(s_\ell)(W-d-s_\ell)]$. Solving the problem $\max_{s_\ell, s_h} E\bar{y}$ gives rise to choices of $s_\ell = s_\ell^*$ and $s_h = s_h^*$ to maximize $E\bar{y}$.

Therefore, in case \hat{A} information has only distributive consequences (i.e., $s_o^* = s_\ell^* = s_h^*$).

¹¹ It is easily seen from Figures 5, 6 and 7 that $\delta(s_L^*) \geq \delta(s_H^*)$ with equality holding for Case \hat{A} . The term K_H has little significance since the actual improvement in matching depends simultaneously on K_H and the size of the change in e . Taking K_H as fixed implies that all relevant measurement of information improvements depends on the size of the change in e . Note, however, that perfect information (i.e., $q_{hH} = 1$, $q_{\ell H} = 0$, $q_{hL} = 0$, $q_{\ell L} = 1$) is possible only if $K_H = N_h$.

¹² That is, an increase in e has a direct effect on p_H , the perceived probability of loss for individuals in the high risk category. Since an increase in e increases p_H it also results in a higher price of insurance. However, an increase in e also leads to a better perception of their safety technology which may lead to a more efficient use of s which either reduces p_H or eliminates some nonproductive expenditure on s .

¹³ It is not surprising that an increase in information leads to an increase in welfare for the utilitarian social welfare function under conditions of risk neutrality since aggregate expected income rises.

¹⁴ As y_L rises and y_H falls the ratio $\frac{u'(y_H)}{u'(y_L)}$ increases. An increase in Δ increases $\delta(s_L^*)$ and $\delta(s_H^*)$ by equal amounts. Since $\delta(s_L^*) > \delta(s_H^*)$ it follows that $\frac{\delta(s_L^*)}{\delta(s_H^*)}$ falls as a result of an increase in Δ . From equation (39) it is obvious that both of these effects increase the likelihood that $\frac{dW}{de}$ will be negative.

¹⁵ From previous definitions it follows that the modified loss probability perceived by individuals in risk category c is $p_c^*(s) = q_{hc} p_h^*(s) + q_{lc} p_l^*(s)$. Substitution for $p_h^* = p_h(s) + \frac{1}{q_h} \Delta$ and $p_l^* = p_l(s) - \frac{1}{q_l} \Delta$ and use of equation set (17) leads to $p_H^*(s) = p_H(s) + e(\frac{1}{q_h} + \frac{1}{q_l})\Delta$ and $p_L^*(s) = p_L(s) - eK(\frac{1}{q_h} + \frac{1}{q_l})\Delta$. The price of insurance is $P_H^*(s_H^*) > P_H^*(s_H^*)$ and $P_L^*(s_L^*) < P_L^*(s_L^*)$ where $P_c^*(s_L^*)$ is the price of insurance in the modified case. Therefore, an increase in Δ increases the price of insurance for members of the high risk category and decreases it for members of the low risk category. The results discussed in footnote 14 then follow.

¹⁶ Notice that an increase in information for Case A is like an increase in the autonomous difference between safety technologies.

¹⁷ If for Case B an increase in information leads to a greater use of s for low risk types and a reduction in s for high risk types then ex post efficiency may be worsened since s is relatively more effective in reducing losses for high risk types.

¹⁸ Note that actual probabilities rather than perceived probabilities are used in construction of EU_{ic} . This convention is uncontroversial here since the utilitarian social welfare function is used and expectations are rational in an overall frequency sense. The use of perceived versus actual probabilities may otherwise be viewed as controversial if a different social welfare function is used. For an explanation, see Hammond [1981].

¹⁹ Information has no distributive impact in Case C since $p_l(o) = p_h(o) = p_L(o) = p_H(o)$.

²⁰ The comparative statics for this section are presented in Appendix B and demonstrate the difficulty of signing the expressions.

²¹ The extent to which a firm's information is reflected in prices may influence the incentive to acquire information (see Grossman and Stiglitz [1980]). Market structure is another factor which is likely to be important.