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HEDGING AND OUTPUT DECISIONS UNDER PRICE AND OUTPUT UNCERTAINTY*

by

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Abstract

If, in addition to price uncertainty, a risk averse firm's end of period output is uncertain then the presence of a futures market does not imply that its output (scale) decision can be determined independently of its price expectations or attitude towards risk. Therefore, the appropriate price variable(s) to be used in an econometric model of the production decision should include both the futures price and proxies to capture the effect of the firm's subjective probability distribution concerning the spot price to be realized at the sale date.

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One of the important issues concerning the theory of the competitive firm in an environment of uncertainty is the effect of price uncertainty on the production decisions of firms if the sales price of their output is unknown at the time that input decisions must be taken. In an important contribution Agnar Sandmo (1971) has demonstrated that risk averse firms are affected by price uncertainty in that their output decisions depend upon their expectations, according to a subjective probability distribution of the sales (future spot) price, as well as their attitudes towards risk taking. A further important development was provided by Duncan Holthausen (1979) and illustrated that if a futures market exists for the output of the firm then production decisions can be determined independently of expectations about price or attitudes towards risk taking.[1] Holthausen shows, in particular, that expected utility maximizing firms choose that level of output which equates marginal cost to the futures price. Therefore, as noted by Holthausen (1979, p. 990), the appropriate econometric supply model should use futures prices as the appropriate variable rather than trying to generate a proxy to represent price expectations whenever a futures market is available. Moreover, attitudes towards risk should not play any role.

The models used by Sandmo and Holthausen are particularly attractive for applications to markets characterized by substantial price uncertainty with many small owner-operated enterprises, the latter characteristic suggesting that risk aversion is likely to be a relevant factor. Markets for agricultural products represent very good candidates for application of these models. As Holthausen notes, futures trading exists for many of these markets. In many of these cases, however, producers also are faced with significant output uncertainty. Moreover, comprehensive crop insurance is rarely available to producers.[2] In this paper an example is provided which illustrates that if output uncertainty persists then producers' price expectations and attitudes towards risk are relevant factors in the production decision even if futures trading is possible. Therefore, these factors should not be excluded from econometric models of such problems. Since both the futures price and expectations concerning the spot price are relevant to the output decision, the results obtained from the model in this paper represent in some sense a synthesis of the Sandmo and Holthausen results.[3]

In Section I of this paper the model is presented and the results analyzed. A mean-variance model is used rather than one with general distributions since the purpose here is simply to show that the presence of output uncertainty requires that price expectations and attitudes towards risk be included in modelling production decisions even when futures markets exist. Conclusions and some extensions are provided in the final section.

I. The Model

A firm must decide upon a scale decision, x, before the sale (spot) price is known and before a random shock which determines the level of output, conditional on x, occurs. Output uncertainty enters in a multiplicative fashion with $X = x(1+\varepsilon)$ where ε is a random variable, normally distributed as $N(0,\sigma_\varepsilon^2)$.[4] Cost depends on x, c(x), and is certain. The usual conditions $c'(x) > 0$ and $c''(x) > 0$ pertain. A producer's expectations regarding the end-of-period spot price (P) are characterized by the subjective probability distribution $N(EP,\sigma_P^2)$. Random variables P and ε are assumed to be independent of each other.[5] The (certain) futures price is b with h being the amount hedged. It follows that income (Y) is a random variable equal to

$$Y = P(x(1+\epsilon) - h) + bh - c(x) \tag{1}$$

with expected value and variance given by equations (2) and (3), respectively.

$$EY = EP(x-h) + bh - c(x) \tag{2}$$

$$\mathrm{var}\, Y = x^2\sigma_P^2\sigma_\epsilon^2 + x^2(EP)^2\sigma_\epsilon^2 + (x-h)^2\sigma_P^2 \tag{3}$$

Equation (3) follows since if X and Y are independently distributed random variables then var $XY = \mathrm{var}\, X\, \mathrm{var}\, Y + (EX)^2\, \mathrm{var}\, Y + (EY)^2\, \mathrm{var}\, X$. For a proof of this result see Mood, et al (1974, p. 180).

Assume that firms are risk averse with constant absolute risk aversion of degree γ. This, in conjunction with the normality assumptions, implies that expected utility can be written as $EU = EY - m\, \mathrm{var}\, Y$ where $m = \frac{\gamma}{2}$. Choosing x and h so as to maximize expected utility gives, along with equations (2) and (3), the following first order conditions.

$$\frac{\partial EU}{\partial x} = EP - c' - 2m[x\sigma_P^2\sigma_\epsilon^2 + x(EP)^2\sigma_\epsilon^2 + (x-h)\sigma_P^2] = 0 \tag{4}$$

$$\frac{\partial EU}{\partial h} = -EP + b + 2m(x-h)\sigma_P^2 = 0 \tag{5}$$

Addition of (4) and (5) gives

$$c'(x) = b - 2mx\sigma_\epsilon^2[\sigma_P^2 + (EP)^2] \tag{6}$$

while from (5) we get

$$h - x = \frac{b - EP}{2m\sigma_P^2} \tag{7}$$

According to equation (6) we see that if output were certain ($\sigma_\epsilon^2 = 0$ and $x = X$) then the result of Holthausen that producers choose a level of output such that marginal cost equals the futures price obtains. Even if $EP > b$ the firm produces $x^* = X^*$ such that $c'(X^*) = b$. Further production in the light of "optimistic" price expectations does not occur since it is cheaper to hedge by taking a long position in the futures market than it is to produce more output (i.e., $c'(X) > b$ for $X > X^*$). However, in the presence of output uncertainty ($\sigma_\epsilon^2 \neq 0$) we see that the production decision depends upon both the futures price and variables representing price expectations. Moreover, in comparison to the case with price certainty--in particular, for $EP = b$--we see that the effect of uncertainty concerning price (or output) does have a contracting effect on the firm's optimal level of output. The effects of price uncertainty noted by Sandmo are therefore re-established even for markets with futures trading. Moreover, differing levels of risk aversion by firms may be an important determinant when investigating the ownership structure of the market.

Equation (7) illustrates the optimal heding rule and is in accord with Holthausen's results in that $h \gtreqqless x$ as $b \gtreqqless EP$. In a paper which investigates the hedging decision Losq (1982) has shown that under general assumptions concerning preferences and expectations, the above relationship does not necessarily hold. In that paper, however, no input (scale) decision is included. Moreover, the point of this paper has to do with the issue of whether the introduction of output uncertainty in addition to price uncertainty changes the appropriate price variable which should be used in modelling a firm's production decision. The example used is sufficient to illustrate this point.

Although this model uses very special assumptions, the comparative statics results do nevertheless provide some insight as to the importance that price expectations and attitudes towards risk have on the production decision of firms. From equation (6) it follows that a greater degree of risk aversion implies a smaller size for the scale variable (x) and, hence, expected output of a firm. Likewise, an increase in output variance (σ_ε^2) or price variance (σ_p^2) leads to a reduction in output (i.e., scale).

II. Conclusion

The use of models which employ the assumption of price uncertainty in a market with futures trading but do not include output uncertainty (e.g., Holthausen (1979) and Hey (1981b)) leads to a separation theorem with production decisions depending upon only the futures price and the cost function while the hedging decision depends upon firms' expectations of the spot price as well as their attitudes towards risk taking. Such models, however, seem for the most part applicable to agricultural markets where significant output uncertainty persists. Inclusion of output uncertainty in these models destroys the separation result as production decisions depend not only on the futures price and cost conditions but also on price expectations and attitudes towards risk-taking. Therefore, one obtains a synthesis of the Sandmo (1971) and Holthausen (1979) results in that econometric models of production decisions must include both the futures price as well as some proxies to capture the factors of price expectations and risk-taking attitudes.

As Hey (1981a) has noted, the formal modelling of a labour-managed firm is identical to that for a profit maximizing firm. All that is needed to obtain appropriate results is a reinterpretation of the choice variables x and h. In this case we should write $X = \frac{x}{L}$, scale per worker, and $H = \frac{h}{L}$, amount hedged per worker. The shock to output, represented by ε, is a firm specific shock which affects identically each worker's productivity within a specific firm. Therefore, the results for the modified profit maximizing model, which includes both output and price uncertainty, can be applied to the labour-managed firm as well. The application is sketched below.

The labour-managed firm maximizes expected utility per worker instead of expected utility of gross profit. Comparative static results usually lead to conclusions which are opposite to those for the profit (utility) maximizing firm. In particular, with output certain the first order condition $c'(X) = b$ implies, with $c'' > 0$, that an increase in b (the futures price) would lead to an increase in output per worker. This result entails a decrease in employment and total output. Upon the introduction of output uncertainty it follows that an increase in any one of output variance, price variance, or expected price leads to an increase in output and employment for the labour-managed firm.

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Footnotes

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[1] Hedging decisions, however, do depend upon price expectations and attitudes towards risk. Interesting analyses of hedging decisions are available in R. McKinnon (1967), Jacques Rolfo (1980), John Hey (1981a) and Etienne Losq (1982). These papers, however, do not include an output decision which provides the motivation for this paper.

[2] For documentation of the fact that all-risk crop insurance is rarely provided by the private market, as well as for some suggested reasons for the nonexistence of such coverage, see John Kurian (1977) and Syed Ahsan, et al (1982).

[3] Sandmo's results do not require substantial alteration if a futures market is not available, as he implicitly assumes.

[4] Note that x is a variable representing the scale of operation which is measured in units so as to be equal to expected output. A multiplicative shock seems realistic for the producer of an agricultural commodity, the yield of which is influenced by weather factors which affect the yield of all his crops uniformly.

[5] For the agricultural application of this model the assumption that X and P are independent requires (at least) that weather conditions have "independent effects" across many "small" producers.