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ABSTRACT

In this paper we propose an alternative method of estimating equations with combined moving average errors under rational expectations. The procedure is computationally tractable, with the variance-covariance calculation involving simple matrix manipulations. As an illustration, we examine weekly data in the thirty-day Canadian/United States forward foreign exchange market, and test the unbiasedness hypothesis of the forward exchange rate. In the particular examples, the estimated standard errors for the proposed method are smaller than the Hansen and Hodrick (1980) calculation.

KEY WORDS: Autocorrelation; Unbiasedness; Generated Regressors.

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Estimating Equations with Combined Moving Average Processes Under Rational Expectations

1. Introduction

One prominent obstacle in estimating equations under rational expectations is the complex serial correlation processes that often arise. For example, in the forward foreign exchange market serial correlation will occur in testing whether the forward rate is an unbiased linear predictor of the future spot rate (referred to as the unbiasedness hypothesis). That is, if the sampling interval is finer than the forward-contract interval (see Hansen and Hodrick, 1980), it can be shown that the disturbance for the test relation of unbiasedness is composed of a linear combination of moving average error terms. While, under the unbiasedness hypothesis, ordinary least squares does yield consistent parameter estimates, the usual least squares estimate of the variance-covariance matrix is not consistent. Unfortunately, standard techniques based upon generalized least squares transformations are not appropriate in this environment and will, in general, lead to inconsistent parameter estimates. Similar serial correlation problems under rational expectations are also encountered in testing the unbiasedness of the forward and future stock and bond markets as well as testing the term structure of interest rates.

Recently Hansen and Hodrick (1980) proposed a simple calculation using the least squares residuals which produces a consistent estimate of the variance-covariance matrix.[1] The unbiasedness hypothesis may be tested using this adjusted variance-covariance matrix together with the consistent least squares estimates of the test relation. Although the serial correlation of the error term is taken into account when constructing the variance-covariance matrix, it plays no role in actual parameter estimation. That is, unlike most traditional treatments of serial correlation in econometrics, the Hansen-Hodrick calculation does not yield estimates of the coefficients of the moving average error process. For some applications, the researcher may also be interested in such coefficients (see, for example, Stockman, 1978).

The purpose of this paper is to provide an alternative method which estimates both the systematic and moving average error coefficients of the test relation under rational expectations. The approach has a good deal in common with the generated regressor models discussed by Pagan (1984). Specifically we obtain estimates of the moving average errors in the test relation by estimating an auxiliary vector autoregression. These generated regressors are substituted into the test relation for the moving average errors and the modified equation is then consistently estimated by ordinary least squares. The procedure developed here is computationally tractable, involving simple matrix manipulations to calculate the variance-covariance matrix. As an illustration, we consider weekly data in the thirty-day Canadian/United States forward foreign exchange market. Interestingly, for this particular example, the estimated standard errors from the proposed method are smaller than those obtained from the Hansen-Hodrick calculation.

Although the analysis can be made quite general, it is perhaps better to choose a specific but familiar example. Therefore, we shall derive the results in terms of the well-known tests of unbiasedness of the forward foreign exchange market. However, it should be obvious that the arguments can be modified to handle tests of other markets. In Section 2 we set up the problem and discuss the estimation procedure outlined by Hansen and Hodrick. In Section 3, an alternative estimation strategy based upon generated regressors is developed. We present a numerical example in Section 4 and a brief conclusion follows in Section 5.

2. Testing the Unbiasedness Hypothesis in the Forward Foreign Exchange Market

Possibly in light of the widespread adoption of flexible exchange rates recent investigations have concentrated upon the efficiency of the forward foreign exchange market. A popular regression-based test in this literature is the test whether the forward rate is an unbiased linear predictor of the future spot rate (see, for example, Baillie, Lippens, and McMahon, 1983; Longworth, 1981; and Stockman, 1978). This has been referred to as a 'speculative efficiency' hypothesis by Bilson (1981) and as a 'simple efficiency' hypothesis by Hansen and Hodrick (1980 and 1981). Despite the fact that there are serious limitations upon interpreting these tests as tests of market efficiency (see, for example, Gregory and McCurdy, 1984), Bilson (1981) has nevertheless argued that the unbiasedness hypothesis may be worthy of study in its own right. It is to this test that we now turn.

Denote the levels of the spot exchange rate and the forward rate k periods ahead at time t+k as S_{t+k} and F^k_{t+k}, respectively. We assume that the deviations from the population means, $s_{t+k} = S_{t+k} - \mu_S$ and $f^k_{t+k} = F^k_{t+k} - \mu_F$ (where μ_S and μ_F are the population means of the spot and forward rates) are jointly stationary and ergodic processes with the following Wold decomposition:[2]

$$s_{t+k} = \sum_{i=1}^{\infty} \theta_i v_{t+k-i} + \sum_{i=1}^{\infty} \varphi_i w_{t+k-i} + v_{t+k}$$

(1)

$$f^k_{t+k} = \sum_{i=1}^{\infty} \psi_i v_{t+k-i} + \sum_{i=1}^{\infty} \eta_i w_{t+k-i} + w_{t+k} .$$

Let v_{t+k} and w_{t+k} be processes such that $v_{t+k} \overset{iid}{\sim} (0,\sigma^2_v)$, $w_{t+k} \overset{iid}{\sim} (0,\sigma^2_w)$ and $E(v_t \; w_r) = \sigma_{vw}$ for all t=r and 0 for all t≠r. The conditional expectation at time t of the spot exchange rate process given $v_t, v_{t-1}\ldots$ and $w_t, w_{t-1},\ldots$ is:

$$(2) \qquad E_t \, s_{t+k} = \sum_{i=k}^{\infty} \theta_i v_{t+k-i} + \sum_{i=k}^{\infty} \varphi_i w_{t+k-i} ,$$

where E_t is the conditional expectation operator. The forward rate at time t is:

$$(3) \qquad f_t^k = \sum_{i=k+1}^{\infty} \psi_i \, v_{t+k-i} + \sum_{i=k+1}^{\infty} \eta_i \, w_{t+k-i} + w_t \ .$$

Under rational expectations the k-period forward rate is an unbiased linear predictor of s_{t+k} so that $f_t^k = E_t \, s_{t+k}$ and the following restrictions apply:

$$(4) \qquad \theta_k = 0, \ \varphi_k = 1, \ \psi_i = \theta_i \ \text{ and } \ \eta_i = \varphi_i, \qquad i = k+1, k+2\ldots \ .$$

Under these rational expectations restrictions, the corresponding forecast error, e_{t+k}, is:

$$(5) \qquad e_{t+k} = s_{t+k} - f_t^k = v_{t+k} + \theta_1 \, v_{t+k-1} + \cdots + \theta_{k-1} \, v_{t+1} + \varphi_1 w_{t+k-1} + \cdots + \varphi_{k-1} \, w_{t+1} \ .$$

In order to make the discussion specific and to aid in the description of the estimation method, we will examine the case of k=4. That is, F_t will be the four-week (30-day) forward rate, with the time period between t and t+1 being one week (the same as Hansen and Hodrick, 1980). Thus we may rewrite equation (5) in <u>level</u> form as the regression equation of the test relation:

$$(6) \qquad S_t = \alpha + \beta F_{t-4} + \theta_1 v_{t-1} + \theta_2 v_{t-2} + \theta_3 v_{t-3} + \varphi_1 \, w_{t-1} + \varphi_2 \, w_{t-2} + \varphi_3 \, w_{t-3} + v_t \ ,$$

where $\alpha = \mu_S - \mu_F$. With the problem set up in this way, many empirical investigations have tested the null hypothesis of unbiasedness H_0:α=0 and β=1. The coefficient α has often been interpreted as capturing a time invariant risk premium and, as has been demonstrated theoretically by several authors, this may be positive, negative or zero (see, for example, Frenkel and Razin, 1980). Let us rewrite (6) as:

$$(7) \qquad S_t = P_{t-4} \, \psi + \mu_t, \qquad t = 1 \ldots T$$

where $P_{t-4} = [1 \ F_{t-4}]$, $\psi^T = [\alpha \ \ \beta]$ and

$$u_t = \theta_1 v_{t-1} + \theta_2 v_{t-2} + \theta_3 v_{t-3} + \varphi_1 w_{t-1} + \varphi_2 w_{t-2} + \varphi_3 w_{t-3} + v_t \ .$$

The disturbance term u_t in equation (7) is a composite error composed of two moving average processes. Since F_{t-4} is orthogonal to u_t under rational expectations (that is under the hypothesis that $E_{t-4}S_t = F_{t-4}$), ordinary least squares (OLS) estimation of ψ in equation (7) would yield consistent parameter estimates. However, in view of the serial correlation of u_t, the associated least squares estimate of the variance-covariance of ψ would not be consistent, and hence statistical inference based upon this estimate would not be valid.[3]

Cumby and Obstfeld (1982) and Hansen and Hodrick (1980) have argued that generalized least squares (GLS) estimation to take into account this serial correlation would not, in general, be consistent. That is, while it is true that u_t may be written as a moving-average process of at most order three:

(8) $$u_t = \xi_t + \rho_1\xi_{t-1} + \rho_2\xi_{t-2} + \rho_3\xi_{t-3} \ ,$$

where ξ_t is independently and identically distributed (see Ainsley, Spivey and Wrobleski, 1977), there is no guarantee that the application of a GLS transformation would result in non-zero correlations between the filtered disturbances and the filtered regressors. This is despite the fact that under rational expectations, F_{t-4} is orthogonal to each individual element to equation (7) (see Cumby and Obstfeld, 1982 and Hayashi and Sims, 1983).

Hansen and Hodrick (1980), hereafter referred to as HH, have proposed a simple procedure whereby valid hypotheses tests of ψ are possible. Their method is to: (i) estimate equation (7) by OLS (thus obtaining consistent parameter estimates); and (ii) use a consistent estimate of the variance-covariance matrix given by:

(9) $$\hat{\nu}(\hat{\psi}) = (P^TP)^{-1} P^T\hat{\Omega}P(P^TP)^{-1},$$

where P is a (T$\times$2) matrix defined from equation (7) and $\hat{\Omega}$ is a (T$\times$T) symmetric matrix constructed as follows.[4] First calculate:

$$\hat{R}_u(j) = \frac{1}{T} \sum_{t=j+1}^{T} \hat{u}_t \, \hat{u}_{t-j} \qquad j=0,\ 1,\ 2,\ 3 \tag{10}$$

where $\hat{u}_t$ is the OLS residual from (7) for observation t with sample size T, and then form:

$$\hat{\Omega} = \begin{bmatrix} \hat{R}_u(0) & & & & & & & & \\ \hat{R}_u(1) & \hat{R}_u(0) & & & & & & & \\ \hat{R}_u(2) & \ddots & \ddots & & & & & & \\ \hat{R}_u(3) & \ddots & \ddots & \ddots & & & & & \\ 0 & \ddots & \ddots & \ddots & \ddots & & & & \\ \cdot & & \ddots & \ddots & \ddots & \ddots & & & \\ \cdot & & & \ddots & \ddots & \ddots & \ddots & & \\ \cdot & & & & \ddots & \ddots & \ddots & \ddots & \\ 0 & \cdot & \cdot & \cdot & 0 & \hat{R}_u(3) & \hat{R}_u(2) & \hat{R}_u(1) & \hat{R}_u(0) \end{bmatrix} . \tag{11}$$

3. <u>An Alternative Estimating Strategy Using Generated Regressors</u>

The method of estimation outlined in this section is based upon the work of Pagan (1984) for models with generated regressors. Consider equation (7) in matrix form:

$$S = P\,\psi + v + \theta_1 v_{-1} + \theta_2 v_{-2} + \theta_3 v_{-3} + \phi_1 w_{-1} + \phi_2 w_{-2} + \phi_3 w_{-3} . \tag{12}$$

Our goal is to replace the lagged unobservable errors by estimated values (denoted by ^). Equation (12) may be rewritten as:

$$S = P\,\psi + v + \theta_1 \hat{v}_{-1} + \theta_2 \hat{v}_{-2} + \theta_3 \hat{v}_{-3} + \varphi_1 \hat{w}_{-1} + \varphi_2 \hat{w}_{-2} + \varphi_3 \hat{w}_{-3} + \sum_{j=1}^{3} (v_{-j} - \hat{v}_{-j})\theta_j + \sum_{j=1}^{3} (w_{-j} - \hat{w}_{-j})\varphi_j . \tag{13}$$

We will assume that we can approximate the infinite vector moving average process (1) by a finite vector autoregressive process in s and f.[5] The exact choice of the order of the process may be determined on the basis of such criteria as likelihood ratio tests and tests for serial correlation (see Baillie, Lippens and McMahon, 1983). We will assume a p^{th}-order vector process:

$$\begin{aligned} s_{-1} &= Z_{-1}\gamma + v_{-1} \\ f_{-1} &= Z_{-1}\delta + w_{-1}, \end{aligned} \tag{14}$$

where $Z_{-1} = [s_{-2}, s_{-3}, \ldots, s_{-p-1}, f_{-2}, f_{-3}, \ldots, f_{-p-1}]$. We will continue to assume that v and w are only correlated contemporaneously. The vector autoregressive process is written with a one-period lag in order to simplify the calculation of the variance-covariance matrix. Each of the $\hat{v}_j$ and $\hat{w}_j$ is obtained as:

$$\begin{aligned} \hat{v}_{-j} &= s_{-j} - Z_{-j}\hat{\gamma} = v_{-j} + Z_{-j}(\gamma-\hat{\gamma}) \\ \hat{w}_{-j} &= f_{-j} - Z_{-j}\hat{\delta} = w_{-j} + Z_{-j}(\delta-\hat{\delta}) \end{aligned} \qquad j=1,\ 2,\ 3 \tag{15}$$

where $\hat{\gamma}$ and $\hat{\delta}$ are OLS estimates of (14) over T observations. Substituting (15) into (13) yields:

$$S = P\psi + v + \sum_{j=1}^{3} \theta_j \hat{v}_{-j} + \sum_{j=1}^{3} \varphi_j \hat{w}_{-j} + \sum_{j=1}^{3} Z_{-j}(\gamma-\hat{\gamma})\theta_j + \sum_{j=1}^{3} Z_{-j}(\delta-\hat{\delta})\varphi_j . \tag{16}$$

Grouping (16) together as:

$$S = X\lambda + \epsilon, \tag{17}$$

where $x = [P:\hat{v}_{-1} : \hat{v}_{-2} : \hat{v}_{-3} : \hat{w}_{-1} : \hat{w}_{-2} : \hat{w}_{-3}]$, $\lambda = [\psi,\ \theta_1, \theta_2, \theta_3,\ \varphi_1, \varphi_2, \varphi_3]$

and $\epsilon = v + \sum_{j=1}^{3} Z_{-j}(\gamma-\hat{\gamma})\theta_j + \sum_{j=1}^{3} Z_{-j}(\delta-\hat{\delta})\varphi_j$.

Estimating (17) by OLS gives:

$$\hat{\lambda} = (X^TX)^{-1} X^TS$$

(18)

$$= \lambda + (X^TX)^{-1}X^Tv + (X^TX)^{-1} \sum_{j=1}^{3} X^TZ_{-j}(\gamma-\hat{\gamma})\theta_j + (X^TX)^{-1} \sum_{j=1}^{3} X^TZ_{-j}(\delta-\hat{\delta})\varphi_j \ .$$

If we let $D_1 = Z_{-1}\theta_1 + Z_{-2}\theta_2 + Z_{-3}\theta_3$ and $D_2 = Z_{-1}\varphi_1 + Z_{-2}\varphi_2 + Z_{-3}\varphi_3$

and note that $(\hat{\gamma}-\gamma) = (Z_{-1}^T Z_{-1})^{-1} Z_{-1}^T v_{-1}$ and $(\hat{\delta}-\delta) = (Z_{-1}^T Z_{-1})^{-1} Z_{-1}^T w_{-1}$,

equation (18) becomes:

(19) $$\hat{\lambda} = \lambda + (X^TX)^{-1} X^Tv - (X^TX)^{-1} X^TD_1(Z_{-1}^T Z_{-1})^{-1} Z_{-1}^T v_{-1} - (X^TX)^{-1} X^TD_2(Z_{-1}^T Z_{-1})^{-1} Z_{-1}^T w_{-1} \ .$$

If we assume:

(i) $\operatorname*{plim}_{T\to\infty} T^{-1} X^Tv = 0$ (ii) $\operatorname*{plim}_{T\to\infty} T^{-1} Z_{-1}^T v_{-1} = 0$

(iii) $\operatorname*{plim}_{T\to\infty} T^{-1} Z_{-1}^T w_{-1} = 0$ (iv) $\operatorname*{plim}_{T\to\infty} T^{-1}(X^TX) = A > 0$

(v) $\operatorname*{plim}_{T\to\infty} T^{-1}(Z_{-1}^T Z_{-1}) = \beta > 0$ (vi) $\operatorname*{plim}_{T\to\infty} T^{-1} X^TD_i = C_i > 0$ $i = 1,2,$

then

$$\operatorname*{plim}_{T\to\infty} \hat{\lambda} = \lambda$$

Therefore OLS estimation of (17) yields consistent parameter estimates.[6] The key to consistent estimation of λ is that under the hypothesis that forward rate is an unbiased predictor of the future spot rate, the moving average errors must be uncorrelated with any lagged variable. This permits us to construct estimates of the v's and w's which are not correlated with the error term ϵ and also ensures that P is not correlated with ϵ. Nevertheless, since ϵ is serially correlated, the least squares estimates of the variance-covariance matrix are not consistent. Under the earlier assumptions regarding v and w, the variance-covariance matrix of the asymptotic distribution of $\sqrt{T}(\hat{\lambda}-\lambda)$ is:

$$(20)\quad \operatorname*{plim}_{T\to\infty} T\{\sigma_v^2(X^TX)^{-1} + \sigma_v^2(X^TX)^{-1} X^T D_1(Z_{-1}^T Z_{-1})^{-1} D_1^T X (X^TX)^{-1}$$
$$+ \sigma_w^2(X^TX)^{-1} X^T D_2(Z_1^T Z_{-1})^{-1} D_2^T X(X^TX)^{-1}$$
$$+ \sigma_{vw}(X^TX)^{-1}X^T D_1(Z_{-1}^T Z_{-1})^{-1} D_2^T X(X^TX)^{-1}$$
$$+ \sigma_{vw}(X^TX)^{-1}X^T D_2(Z_{-1}^T Z_{-1})^{-1} D_1^T X(X^TX)^{-1}\}$$

A consistent estimate of the asymptotic covariance matrix may be obtained as:

$$(21)\quad T\{\hat{\sigma}_v^2(X^TX)^{-1} + \hat{\sigma}_v^2\, \hat{\pi}_1(Z_{-1}^T Z_{-1})^{-1} \hat{\pi}_1^T + \hat{\sigma}_w^2\, \hat{\pi}_2(Z_{-1}^T Z_{-1})^{-1} \hat{\pi}_2^T$$
$$+ \hat{\sigma}_{vw}\hat{\pi}_1(Z_{-1}^T Z_{-1})^{-1} \hat{\pi}_2^T + \hat{\sigma}_{vw}\hat{\pi}_2(Z_{-1}^T Z_{-1})^{-1} \hat{\pi}_1^T\},$$

where $\hat{\pi}_1 = (X^TX)^{-1} X^T\hat{D}_1$, $\hat{\pi}_2 = (X^TX)^{-1} X^T\hat{D}_2$ $\hat{D}_1 = Z_{-1}\hat{\theta}_1 + Z_{-2}\hat{\theta}_2 + Z_{-3}\hat{\theta}_3$, $\hat{D}_2 = Z_{-1}\hat{\varphi}_1 + Z_{-2}\hat{\varphi}_2 + Z_{-3}\hat{\varphi}_3$ and θ_j and $\varphi_j (j=1,2,3)$ are the OLS estimates from (17). The estimates of $\hat{\sigma}_v^2$, $\hat{\sigma}_w^2$ and $\hat{\sigma}_{vw}$ may be obtained from estimating each equation of (14) by OLS. The calculation of this should pose no computational problems since all that is required are simple matrix manipulations. It does not seem possible to make an analytical comparison regarding the relative efficiency of the estimates of α and β from this procedure with those from HH. This ambiguity in relative efficiency is similar to that encountered by Hayashi and Sims (1983). The difficulty here is due, in part, to the unknown sign of θ's and φ's and the appearance of Z_{-1} in (21). In the next section we find that for a specific numerical example there is some gain in precision for the generated regressor technique. However, it would not be appropriate to conjecture what might happen for different data sets. An area of current research is to investigate analytically the finite sample properties of these estimators for much simpler models.

One advantage of this method over HH approach is that it allows us to guage the relative contributions of the innovations v_{-j} and w_{-j} $(j=1,2,3)$ to the forecast error of the forward rate in predicting the future spot. A recent focus in macroeconometrics has been to examine the source of innovations in aggregate time series. However comparisons of this sort are not possible for the HH procedure.

Finally, we mention that an alternative approach to testing the unbiasedness of the forward rate which has been followed in the market efficiency literature is to examine whether the forecast error, $S_t - F^k_{t-k}$ is orthogonal to public information that was available when the forward contract was drawn (see, for example, Hansen and Hodrick, 1980 and 1981). The procedure outlined here can easily be extended to handle these kinds of orthogonality tests.[7]

4. Results

The weekly Canadian/American foreign exchange data are from a daily data tape that has been kindly supplied by the Bank of Canada. The estimation period is 1978-1981 giving two hundred and nine weekly observations. In view of the stationarity requirement, we estimate the test relation as in Gregory and McCurdy (1984):

$$(22) \qquad \frac{S_t - S^T_{t-4}}{S^T_{t-4}} = \alpha + \beta \left[\frac{F_{t-4} - S^T_{t-4}}{S^T_{t-4}}\right] + u_t,$$

where $u_t = \theta_1 v_{t-1} + \theta_2 v_{t-2} + \theta_3 v_{t-3} + \varphi_1 w_{t-1} + \varphi_2 w_{t-2} + \varphi_3 w_{t-3} + v_t$, F_{t-4} and S^T_{t-4} are Tuesday closing rates of the forward and spot exchange rates, respectively, and S_t is the Thursday closing spot rate four weeks and two business days ahead. This ensures that there are thirty days between F_{t-4} and S_t, while at the same time the forward premium contains rates which are available to market participants.[8] In this form we test whether the normalized forward premium is an unbiased linear predictor of the rate of change of the corresponding spot rate.

To carry out the estimation procedure using generated regressors, we must first estimate the vector autoregressive system (14) by OLS. For this example, on the basis of likelihood ratio tests and tests for serial correlation (Godfrey, 1978), we found that a fourth-order vector process best described movements in the rate of change of the spot and the normalized forward premium.

The OLS estimates of equation (7) with the HH variance calculation (see equation (9)) are given in Table 1. Beside these in Table 1 are the estimates of equation (22) using the generated regressors (the estimated residuals from the vector autoregressive process (14)) and the estimated covariance matrix (21) (referred to here as GR). Three features of the results are as follows:

(i) While the coefficient estimates of α and β are very similar for both methods, the standard errors for the GR estimates are smaller.

(ii) The calculated Wald (W) statistic for the joint hypothesis that $\alpha = 0$ and $\beta = 1$ based upon the GR estimates is more than three times larger than that obtained from the HH calculation. At the one percent level of confidence, this null hypothesis is rejected on the basis of the GR estimates but is retained for the HH estimates.

(iii) The GR results indicate that only the innovations in the change of the spot rate are important in the sense of yielding significant coefficient estimates in the test relation equation. The effects of these innovations appear to be fairly uniform and significant over the three lag values. As for the forward premium, the innovative coefficient estimates for all three lags are numerically large but not significant.

Thus, it is evident from this example that there can be a gain in precision for the GR calculation over the HH. In addition, we tried several other estimation periods which were based upon the sample intervals of Gregory and

McCurdy (1984).[10] We found that for all cases considered, the proposed method produced quite similar parameter estimates of α and β as HH but with smaller standard errors.

5. Conclusion

In this paper, we have developed an alternative estimation strategy to test the hypothesis that the (normalized) forward premium is an unbiased linear predictor of the rate of change of the spot rate. The procedure involves estimating the moving average structure by obtaining estimates of the innovation series from an auxiliary vector autoregression equation. We have found that this approach is not only easy to implement but performed well in a particular example. Clearly the method may be usefully applied to test relations in the stock and bond markets where combined moving average error processes under rational expectations may also arise.

Table 1

The Forward Premium as Unbiased Linear Predictor

	HH	GR
α	0.001463 (0.0017)	0.001590 (0.00055)
β	-1.4860 (0.95)	-1.4623 (0.63)
θ_1	-	0.8373 (0.070)
θ_2	-	0.7273 (0.18)
θ_3	-	0.5424 (0.34)
φ_1	-	-1.0440 (77.88)
φ_2	-	-2.2293 (4.49)
φ_3	-	-4.4392 (5.53)
R^2	0.04	0.56
S	0.012	0.0085
W	7.26	25.93

Note: Standard errors are given in parentheses, S is the standard error of the regression, R^2 is the coefficient of determination and W is the Wald test of the joint hypothesis that $\alpha = 0$ and $\beta = 1$. The critical value of the chi-square distribution at the one percent level of significance with two degrees of freedom is 9.21.

FOOTNOTES

[1]The Hansen and Hodrick (1980) solution to the estimation problem is based upon the more general discussion in Hansen (1982).

[2]Since the joint stationarity of the levels of the spot and forward foreign exchange rates is doubtful, we have in the empirical example of Section 4 transformed the variables as in Gregory and McCurdy (1984).

[3]Gregory and McCurdy (1984) have advised that before any parameter significance testing is undertaken, a careful specification analysis of the test relation is required. Since we are primarily interested in developing an alternative testing procedure, we will assume that the test structures are constant. The evidence in Gregory and McCurdy (1984) suggests that over a large sample this is not likely to be true.

[4]In this derivation, we are assuming conditional homoscedasticity of the disturbances (following Hansen and Hodrick, 1980). The case of conditional heteroscedasticity has been considered in Cumby and Obstfeld (1982), Hansen and Hodrick (1981), and Hsieh (1983).

[5]This is the same type of approximation that Baillie, Lippens and McMahon (1983) have employed.

[6]As is true with all least square methods there is no guarantee that the estimated moving average process is invertible.

[7]Cumby, Huizinga, and Obstfeld (1982) and Hayashi and Sims (1983) have suggested two different types of two-stage least squares estimators which exploit the orthogonality between the forecast errors and the instrument (information) set.

[8] We note that since there is a two-day separation of the observations for the forward and spot rates, a moving average process of order four is possible in equation (7). See Hakkio (1981). For this example, we consider only the third-order moving average process for the test relation. However, allowing for fourth-order serial correlation in equation (4) is a reasonably straightforward extension for both procedures.

[9] In order to facilitate comparison with Hansen and Hodrick, we have calculated the variances and covariances of (14) by dividing by T. In small samples, it may be preferable to make some adjustments for degrees of freedom.

[10] These results, as well as the data set, are available from the author upon request.

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