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THE PERCEPTION AND EFFICIENCY OF LABOUR SUPPLY CHOICES BY PIGEONS

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ABSTRACT

Hunter and Davison (1982) observed how pigeons supplied labour to concurrent schedules allowing access to food. The rates of return to labouring on either schedule were widely varied by altering both the force needed to emit each response and the scheduled access to food for any response rate. This paper models the pigeons' choice problems as a neo-classical type labour allocation problem and estimates the problems' expansion paths and the efficiency levels achieved by the pigeons. The estimated efficiency levels are high, supporting the neo-classical model. The analysis provides an opportunity to examine the model's sensitivity to the assumption of perfect information.

1. INTRODUCTION

A commonly voiced complaint directed at microeconomists is that too much effort goes into extending their theories and too little goes into testing the theories themselves. A common reply to the criticism is that microeconomists would test their theories more extensively if only appropriate data sets were available. Relevant data now come from three sources. First, and foremost, are the statistical collection agencies. The second source is data collected by experimental economists. More recently still, a third data source has been used to test basic aspects of microeconomic theory. The work of Rachlin et.al. (1976) and Battalio et.al. (1979, 1981a, 1981b) are examples of a synthesis of aspects of microeconomics with aspects of operant psychology to test fundamentals of the neo-classical theory of consumer choice. The current paper is also in this vein; data from experiments involving pigeons are used to test a neo-classical model of labour supply and allocation.

Operant psychology is the study of how organisms interact with, or operate upon, their environments. For the past half century operant psychologists have conducted a great variety of experiments to try to discover what influences the choices of individuals. Some subjects are human; others are animals. A vast operant experimental literature now exists. One striking finding is that, in simple experimental situations, animal subjects such as pigeons or apes achieve steady-state performances which are as good as those achieved by humans (see de Villiers and Herrnstein, 1976). From this, psychologists have concluded that much can be learned about human decision making from observing the problem solving of lower species. It appears that while humans, with their greater intelligence, may be quicker to learn about and to solve the problems used in operant research, the approach used by humans to solve these problems is not fundamentally different to that used by lower species.

Some brief comments on the history of concurrent schedule experiments will provide the reader with necessary background information. Beginning in the 1930's, experiments were conducted which offered subjects alternative ways of achieving a given reward. Solomon (1948) reviewed these experiments and concluded that subjects' behaviours obeyed the "Law of Least Effort" which, as its name suggests, states that, ceteris paribus, a subject will prefer the least effortful way of achieving a given reward. A second class of experiments involves the use of 'schedules of reinforcement'. Such a schedule is akin to the economist's idea of a production function. A subject provides an input by 'responding' to the schedule (e.g. pecking at a disc, pressing a lever). A 'reinforcer' is a welfare improving output provided as a consequence of responding (e.g. access to food or drink, receiving money, or relief from pain). The reinforcement schedule is the functional relationship between the response rate and the rate at which reinforcers are received. A great variety of these schedules have been devised by psychologists (see Ferster and Skinner (1957) for details). An experiment in which a subject can choose to allocate its responses to two or more simultaneously available schedules of reinforcement is called a 'concurrent schedule experiment'. This paper is concerned with explaining subjects' choices in a particular type of concurrent schedule experiment. Since the explanatory model advanced in this paper is a constrained utility maximization model of choice, testing this model against experimental data offers economists insights into the robustness of the usual constrained utility maximization model of choice routinely employed by microeconomists.

The structure of the paper is as follows. The next section expounds a model of labour supply and work allocation. Section 3 describes the data generated by the Hunter and Davison (1982) concurrent schedule experiment. Section 4 describes the characteristics of the reinforcement schedules used by Hunter and Davison. The role of perception in solving the model is discussed in Section 5. Expansion paths, which are predictors of the experimental subjects' choices, are derived in Section 6 and, in Section 7, are compared to the actual choices. Some conclusions are presented in Section 8.

2. THE MODEL

Let R_i denote the rate at which a subject receives reinforcers from schedule i when it responds to this schedule at a rate P_i. The subject forms a perception (estimate) of the ith reinforcement schedule denoted by

$$R_i = g_i(P_i), \text{ for } i = 1,\ldots,n \tag{2.1}$$

where $n > 1$ is the number of schedules concurrently accessible to the subject. Each response to schedule i requires an amount of work w_i so a response rate P_i to schedule i implies the subject's rate of work in responding to this schedule is w_iP_i. Thus a response rate vector $(P_1,\ldots,P_n)$ implies a total work rate of

$$W = w_1P_1 + \ldots + w_nP_n. \tag{2.2}$$

Every mortal's work rate is bounded above by some total work rate, $\bar{W}$, which is the largest within the subject's capacity. $\bar{W}$ will vary from one species to another (humans can work harder than pigeons) and from one member of a species to another (a bigger or better nourished subject should have a larger $\bar{W}$).

Finally, the subject's tastes are represented by a utility function

$$V = f(R_1,\ldots,R_n,W). \tag{2.3}$$

In this paper we consider only the case in which the same reinforcer is delivered by each of two schedules. In this case the subject's utility function simplifies to $V = f(\sum_{i=1}^{2} R_i, W)$, allowing the subject's choice problem to be written as

$$\max_{P_1, P_2} V = f(\sum_{i=1}^{2} g_i(P_i), \sum_{i=1}^{2} w_i P_i), \qquad (2.4)$$

subject to $\sum_{i=1}^{2} w_i P_i \leq \overline{W}$

and $P_1, P_2 \geq 0$.

This paper assesses whether the experimental subjects can solve (2.4).[2] Data from an experiment conducted by Hunter and Davison (1982) are used to answer this question. The procedure used is as follows. The functional forms of the reinforcement schedules are estimated. Using these estimates together with the optimality conditions for (2.4) allows the estimation of the experiments' expansion paths. The data are consulted to see how efficient the subjects' choices are relative to these paths. Poor performance would call into question the usefulness of a neo-classical model such as (2.4) in modelling choice.

The subjects of the Hunter and Davison experiment are very hungry pigeons (maintained at 80% of their normal body weights) for whom the reinforcer, food, is a good. Hence, assuming f is differentiable,

$$\frac{\partial f}{\partial R_i} > 0, \quad \text{for } i=1,2. \qquad (2.5)$$

Also assumed is that as the subject's total work rate, W, approaches the highest work rate of which it is physically capable, $\overline{W}$, further increments to W have very large negative utility. More formally,

$$\frac{\partial f}{\partial W} \to -\infty \text{ as } W \to \overline{W}. \qquad (2.6)$$

Forming the Lagrangean function for (2.4) gives

$$L(P_1, P_2, \lambda, \mu_1, \mu_2) = f(g_1(P_1) + g_2(P_2),\ w_1P_1 + w_2P_2)$$
$$+ \lambda(\overline{W} - w_1P_1 - w_2P_2) - \mu_1P_1 - \mu_2P_2. \qquad (2.7)$$

(2.6) ensures that $\lambda = 0$ at an optimum so the necessary optimality conditions obtained from differentiating (2.7) are

$$\frac{\partial f}{\partial(R_1+R_2)} \cdot \frac{\partial g_i}{\partial P_i} + \frac{\partial f}{\partial W} w_i - \mu_i = 0; \quad i=1,2 \qquad (2.8a)$$

$$\overline{W} - w_1P_1 - w_2P_2 > 0 \qquad (2.8b)$$

$$\mu_1P_1 = \mu_2P_2 = 0 \qquad (2.8c)$$

$$\mu_1, \mu_2, P_1, P_2 \geq 0 \qquad (2.8d)$$

(It is explained in the following sections that, in the context of the Hunter and Davison experiment, the feasible set for problem (2.4) is non-empty and convex, so the Kuhn-Tucker conditions (2.8) are defined at a maximum--see Takayama 1974, Theorem 1.D.4.) From condition (2.8a) we see that the model views the subject as choosing a response rate pair (P_1^*, P_2^*) such that, for whichever of P_1^* or P_2^* is positive, the marginal utility of the reinforcement rate achieved by incrementing P_i slightly from P_i^* is exactly offset by the marginal disutility of the work rate required by the increment to P_i; $i=1,2$. Each response rate pair (P_1^*, P_2^*) is efficient in the sense of providing the highest reinforcement rate for a given total work rate. We note, in passing, that the dual of this statement is the "Law of Least Effort" observed by operant psychologists (see Solomon (1948)).

3. THE HUNTER AND DAVISON EXPERIMENT

The data needed for a searching test of the above model must be generated from an experiment which uses both a variety of reinforcement schedules, g_1 and g_2, and a variety of work requirements per response, w_1 and w_2. To our knowledge the largest set of such data is that reported by Hunter and Davison (1982).

A brief summary of Hunter and Davison's experiment is given here for the reader's convenience. Full details of the experimental methodology is given in Hunter and Davison (1982). The experimental subjects were five homing pigeons, numbered 42, 43, 44, 45 and 46. Each subject was exposed to 42 experimental conditions; 38 were distinct and 4 were replications. In each condition the subject could peck on two lighted disks, called keys, each of which required a given force to depress. Each proper depression of a key was accompanied by an audible click and counted as a single response. A response could result in the subject obtaining "reinforcement" from a "variable-interval schedule of reinforcement" in the form of access to a food bin. The experimental manipulations which distinguished one condition from another consisted of either varying the forces required to depress the keys or changing the schedules so as to vary the mean arrival rates of reinforcer availability. The range of each type of variation was large. The forces variously required to respond to a key were 13, 22, 31, 40 and 49 newtons and the mean arrival rates programmed for the individual variable-interval schedules were chosen from the values of approximately $\frac{1}{8}$, $\frac{1}{2}$, 1, 2 and 4 reinforcers/minute. Each experimental condition was maintained for a subject until its behaviour was replicated across five sessions. Then measurements were made of the number of responses to each schedule, the number of reinforcers received from each schedule and the time spent responding to each schedule.

4. THE VI-SCHEDULES

A variable interval schedule, denoted by VI-Δt, is meant to make a reinforcer available in a manner which is both time-dependent and stochastic. Δt denotes the expected time interval (in seconds) between arrivals of reinforcer availability. For example, a VI-15 schedule is meant to make a reinforcer available every 15 seconds on average. The first response emitted by the subject following the arrival of a reinforcer availability causes the delivery of the reinforcer. The rate of reinforcement delivery is thus no greater and may be less than the arrival rate of reinforcement availability. The expected reinforcement rate delivered to the subject is zero if $P = 0$ and monotonically increasing w.r.t. P up to an upper bound of $S = \frac{1}{\Delta t}$ reinforcers/minute.

The construction of Hunter and Davison's VI-schedules requires a brief comment. Each VI-schedule contains a loop of film with holes punched in it at carefully measured intervals. The loop is cycled at a constant speed. Each time a hole passes a fixed position a reinforcer becomes available; the loop stops until the reinforcer is delivered as a consequence of the subject's next response. The length of the loop of film, the lengths of the intervals between neighbouring holes in the film, the rate of cycling of the film and the subject's response rate jointly determine the characteristics of the arrival process of reinforcer availability.

Since Ferster and Skinner (1957) first introduced the VI-schedule, there has been discussion about the most appropriate way of determining the intervals to use. For example, Fleshler and Hoffman (1962) provide an exponential series of interval lengths for constructing VI-schedules that keep the probability of an arrival of reinforcement availability nearly constant over time. While many experiments have followed the Fleshler and Hoffman construction, many others, including Hunter and Davison, have used an arithmetic series to decide interval lengths. Only very recently have operant psychologists asked whether subject's responses are affected by the manner in which the schedules' intervals are determined; Taylor and Davison (1983) compared subjects' responses to arithmetic and exponential VI-schedules and found noticeable differences in response patterns. The recent survey of Nevin and Baum (1980) of the sizeable literature on estimating VI-schedules shows the absence of a consensus on their functional forms. We are therefore faced with the task of estimating how the VI-schedules used by Hunter and Davison relate the rate at which reinforcers are received by subjects to the subjects' response rates.

We begin by assuming that a Hunter and Davison VI-Δt schedule makes a reinforcer available with a probability which is constant across time and with a mean arrival rate of reinforcer availability of $S = \frac{1}{\Delta t}$ reinforcers/minute. In other words, the VI-Δt schedule generates a Poisson arrival process for reinforcer availability with parameter $S = \frac{1}{\Delta t}$ reinforcers/minute. Suppose the subject responds at a constant rate of P responses/minute. The probability that any response is successful in being rewarded by receiving a reinforcer is the probability that a reinforcer availability has arrived in the P^{-1} minutes interval between responses. That is,

$$
\begin{aligned}
&\Pr(\text{a reinforcement is received from a response} \mid S,P) \\
&= 1 - \Pr(\text{no reinforcement availability arrives in an interval of length } P^{-1} \text{ minutes} \mid S,P) \\
&= 1 - e^{-S/P}. \qquad (4.1)
\end{aligned}
$$

Responding to an "ideal" VI-Δt schedule at a steady rate of P responses/minute is therefore equivalent to conducting a sequence of

$\frac{1}{P}$ independent Bernouilli trials, each with a probability of "success" of $1 - e^{-\frac{S}{P}}$. Let R be the random variable which is the number of reinforcers received per minute from this binomial experiment. Then

$$Pr(R = r | S,P) = {}^{P}C_{r}\, e^{S(1-\frac{r}{P})}(1 - e^{-\frac{S}{P}})^{r} \; ; \; r = 0,\ldots,P. \tag{4.2}$$

The mean and variance of the binomial process (4.2) are

$$E[R | S,P] = P(1 - e^{-\frac{S}{P}}) \tag{4.3}$$

and

$$var(R | S,P) = Pe^{-\frac{S}{P}}(1 - e^{-\frac{S}{P}}). \tag{4.4}$$

(4.3) has been footnoted by Prelec (1982, p. 197). It is easily shown from (4.3) and (4.4) that

$$E[R | S,P = 0] = var(R | S,P = 0) = 0 \tag{4.5a}$$

and

$E[R | S,P]$ is strictly increasing and strictly concave w.r.t. P (4.5b)

and

$var(R | S,P)$ is strictly increasing w.r.t. P (4.5c)

and

$$\lim_{P \to \infty} E[R | S,P] = S \tag{4.5d}$$

and

$$\lim_{P \to \infty} var(R | S,P) = S. \tag{4.5e}$$

(4.2) was used to calculate maximum likelihood estimates of S for each of the VI-15, VI-30, VI-60, VI-120 and VI-480 schedules employed by Hunter and Davison. Given $n > 1$ observations $(r_1,P_1),\ldots,(r_n,P_n)$, the likelihood function constructed from (4.2) is

$$L(S|r_1,P_1,\ldots,r_n,P_n) = \prod_{i=1}^{n} {}^{P_i}C_{r_i}\, e^{S(1-\frac{r_i}{P_i})} (1 - e^{-\frac{S}{P_i}})^{r_i}. \qquad (4.6)$$

Taking logs and differentiating w.r.t. S reveals the maximum likelihood estimator $\hat{S}$ of S is implicitly defined by

$$\sum_{i=1}^{n} (1 - e^{-\frac{\hat{S}}{P_i}})^{-1} \cdot \frac{r_i}{P_i} = n. \qquad (4.7)$$

Applying (4.7) to the Hunter and Davison data and solving iteratively for $\hat{S}$ in each case resulted in the estimates of S given in Table 1.

Insert Table 1 here

The values of $\hat{S}$ were substituted into (4.3) and $E[R|\hat{S},P]$ plotted against P. Figures 4.1 and 4.2 display the VI-15 and VI-60 schedule estimates as dashed lines and are typical of the plots for the five schedules. The data

Insert Figures 4.1 and 4.2 here

used for the estimations are also shown in the figures and are rates computed from the data presented in Hunter and Davison (1982, Appendix). For example, the data used in the estimation of the VI-15 schedule were gathered from the reported data for each subject for each experimental condition utilizing a VI-15 schedule. Note, from Figures 4.1 and 4.2, that the received reinforcement rate varies considerably for any given response rate.

Figures 4.1 and 4.2 also indicate that (4.3) is a misspecification of the true functional form of the mean of the VI-schedules. This suggests that the Hunter and Davison schedules do not, in fact, maintain a Poisson arrival process for reinforcement availability. There are two possible reasons for the error: the arithmetic series used by Hunter and Davison to generate the VI intervals may be discriminably different to the constant probability or exponential series that was recommended by Fleshler and Hoffman; or the subjects may not respond at a steady rate P on the keys, but may respond in bursts. The Taylor and Davison (1983) study indicates the first reason as the most likely source of the specification error.

The Hunter and Davison VI-Δt schedules must possess the properties (4.5a), (4.5b) and (4.5d). With this in mind we considered the following generalization of (4.3) to

$$E[R|k,S,P] = kP(1 - e^{-\frac{S}{kP}}) \qquad (4.8)$$

where $k > 0$ is a constant. (4.8) also possesses the properties (4.5a), (4.5b) and (4.5d). Since the mixed binomial/Poisson process (4.2) can no longer be considered to be an adequate description of the underlying reinforcement arrival process, the maximum likelihood estimation procedure (4.7) is no longer appropriate. Instead we chose to use a non-linear least squares procedure to estimate k and S for each of the five cases.[3] The results are given in Table 2. Bracketed quantities are standard errors. $\tilde{k}$ and $\tilde{S}$ denote the non-linear least squares estimates.

Insert Table 2 here

The reader should note that the $\tilde{k}$ values differ considerably from the value of unity which constrains (4.8) to (4.3) and that the non-linear least squares estimates $\tilde{S}$ are all noticeably higher than their corresponding "maximum likelihood" estimates $\hat{S}$. (Asymptotic tests rejected (4.2) in each of the five cases for significance levels well below 1%; see footnote 3). The above values for k and S were substituted into (4.8) and $E[R|\tilde{k},\tilde{S},P]$ plotted against P. The VI-15 and VI-60 plots are the solid lines drawn in Figures 4.1 and 4.2; these are typical of the VI-30, VI-120 and VI-480 cases also.[4] These estimates of the Hunter and Davison VI schedules were considered to be adequate for the task of Section 6; calculating an estimate of the expansion path for each of the experimental conditions.

5. PERCEPTION

To find at least a local optimum for its choice problem (2.4), it is enough that a subject can experiment by altering its response rate choices and discerning if a reallocation of responses results in an improvement or worsening in the mean total rate at which reinforcers are received. By a succession of such comparisons the subject eventually arrives at a pair of response rates from which no further perceptible improvement in the mean total reinforcement rate is locally achievable for the chosen work rate. If the subject's perceptive abilities are perfect then the chosen response rates will be exactly locally optimal. The weaker are his perceptive abilities, the less he is able to discriminate between response rates which are nearly, rather than exactly, optimal. Of course the controller of the experiment, who is completely informed and able to discriminate perfectly, can solve the problem exactly. A subject using the same decision making process, but who cannot discriminate perfectly, can only approximate the controller's optimal solution.

The reality is that problem (2.4) presents some difficulties for the subject in its efforts to perceive the exact nature of the problem. Again consider Figures 4.1 and 4.2. Two points should be noted. Firstly, the considerable scatter of the data about the estimated mean relationships reveals the high variability of R for any given P. Secondly, the slope of each of the estimated means of the VI schedules decreases rapidly towards zero as the response rate increases. For example, the estimated marginal mean reinforcement rate productivities of responding for the VI-60 schedule are 0.0324, 0.0050 and 0.0019 reinforcers/minute/response for respective response rates of 10, 30 and 50 responses/minute. Together, these points reveal the considerable difficulty faced by the subject in using a marginalist approach to solving problem (2.4); the margins are not only small but as well are only expected values. Perceiving if a reallocation of, or change to, its total work rate alters its mean total reinforcement rate is difficult. This facet of the Hunter and Davison experiment should be borne in mind by the reader when interpreting the results displayed in Section 7.

6. THE EXPANSION PATHS

The Hunter and Davison VI schedules provide reinforcer availability in a manner which appears to the subject to arrive stochastically over time. The model described in Section 2 assumes a deterministic relationship between reinforcement rate and response rate for each schedule. We continue throughout the following analysis by supposing the subjects are risk-neutral w.r.t. the reinforcement rate/work rate gambles represented by the VI reinforcement schedules, allowing the substitution of (4.8) for the functional forms g_1 and g_2 in (2.4). We note, from (4.8), that the estimated VI schedules are strictly concave w.r.t. P, justifying the comment made below (2.8d) that conditions (2.8) are necessarily satisfied at a local maximum. Substituting from (4.4) into (2.8) shows that, for given k_1, k_2, S_1, S_2, w_1 and w_2, the locus of efficient response rate pairs (P_1^*, P_2^*) satisfies

$$\frac{k_1\left(1 - e^{-\frac{S_1}{k_1 P_1^*}} \cdot \left(1 + \frac{S_1}{k_1 P_1^*}\right)\right)}{k_2\left(1 - e^{-\frac{S_2}{k_2 P_2^*}} \cdot \left(1 + \frac{S_2}{k_2 P_2^*}\right)\right)} \gtreqless \frac{w_1}{w_2} \qquad (6.1)$$

as
$P_1^* > 0$ and $P_2^* = 0$
$P_1^* > 0$ and $P_2^* > 0$
$P_1^* = 0$ and $P_2^* > 0$.

If the model is to be valuable, the expansion paths computed from solving (6.1) for each of the Hunter and Davison conditions must be paths which imply response rate pairs "close" to those actually chosen by the experimental subjects. The meaning of "close" in this context is as follows: a subject's <u>chosen</u> response rate pair (P_1, P_2) implies a total work rate of $W' = w_1 P_1 + w_2 P_2$ and a mean total reinforcement rate of

$$E[TR' | P_1, P_2] = k_1 P_1 \left(1 - e^{-\frac{S_1}{k_1 P_1}}\right) + k_2 P_2 \left(1 - e^{-\frac{S_2}{k_2 P_2}}\right). \qquad (6.2)$$

Let the response rate pair (P_1^*, P_2^*) in the work-rate-expansion path be the pair which implies a total work rate of W' and denote the associated mean total reinforcement rate by

$$E[TR^* | P_1^*, P_2^*] = k_1 P_1^* \left(1 - e^{-\frac{S_1}{k_1 P_1^*}}\right) + k_2 P_2^* \left(1 - e^{-\frac{S_2}{k_2 P_2^*}}\right). \qquad (6.3)$$

Obviously $E[TR^*|P_1^*,P_2^*] > E[TR'|P_1,P_2]$. The "closeness" of (P_1,P_2) to (P_1^*,P_2^*) is measured as the size of the percentage loss in the mean total reinforcement rate due to choosing (P_1,P_2) instead of (P_1^*,P_2^*); that is,

$$d = 100|TR' - TR^*|/TR^* . \quad (6.4)$$

The model will be supported if the d-values revealed by the data are small.

Note that we are not concerned if conditions (6.1) yield multiple local maxima to problem (2.4). Local concavity in the region of any local maximum implies that d should be small for any maximum solution to (2.4), be it local or global.

7. THE RESULTS

Hunter and Davison used 42 experimental conditions and five subjects so 210 observations were obtained. Only 207 of these are presented here because Subject 42 made no responses to conditions 10, 11 and 13.

Table 3 presents the results. For each experimental condition, it tabulates the experimental parameter values, the response rates (P_1,P_2) chosen by each subject, its total work rate W,[5] the estimated mean total reinforcement rate TR' obtained by choosing (P_1,P_2), the estimate (P_1^*,P_2^*) of the response rates which represent the mean total reinforcement rate maximizing allocation of the total work rate W, the estimated mean total reinforcement rate TR^* obtained by choosing (P_1^*,P_2^*), the percentage difference d between TR' and TR^*, the estimated marginal mean reinforcement rate productivities MRP_1 and MRP_2 of responding for each schedule at the chosen rates P_1 and P_2, and the ratio $\frac{MRP_1}{w_1}/\frac{MPR_2}{w_2}$.

Insert Table 3 here

The most important feature of the results is that Hunter and Davison's subjects all succeeded in allocating their chosen work rates very efficiently, despite the perceptual difficulties mentioned in Section 5. Figure 7.1 is a frequency plot of the d-values presented in Table 3.

Insert Figure 7.1 here

The median d-value is only 0.28%, 74.9% of the d-values are smaller than 1% and 87.0% are smaller than 2%. It should be noted here that the least efficient choices of response rates imply d-values in excess of 45% for all conditions and as large as 95% (i.e. 5% efficiency) for some conditions. The subjects therefore have ample latitude to be grossly inefficient, but were not.

Figures 7.2 and 7.3 are presented to assist the reader in interpreting Table 3 by illustrating conditions 2 and 27 respectively. The figures show various iso-total-reinforcement-rate (iso-TR) lines, iso-total-work-rate (iso-W) lines and the expansion path for each condition.

Insert Figures 7.2 and 7.3 here

Consider Figure 7.2. In condition 2 both reinforcement schedules are VI-60's, $w_1 = 40$ and $w_2 = 13$. Each subject's chosen response rate pair is denoted by a plain dot indexed by the subject's number on Figure 7.2 and by (P_1, P_2) on Table 3. For example, subject 44's chosen response rate pair is (62.9, 46.8). The work rate chosen is thus $W = (40)(62.9) + (13)(46.8) = 3124$. The iso-W line for which $W \equiv 3124$ intersects with the estimated expansion path for condition 2 at the response rate pair (49.3, 88.6). This pair is denoted by a starred dot indexed by the subject's number; it is the response rate pair yielding the estimated maximal total reinforcement rate given the subject 44's chosen work rate is $W = 3124$. Using (6.3) and substituting for k_1, k_2,

S_1 and S_2 from Table 2, this reinforcement rate is estimated to be

$$E[TR^*|P_1^* = 49.3,\ P_2^* = 88.6] = 2.584 \text{ reinforcers/minute.} \tag{7.1}$$

The subject 44's chosen total reinforcement rate is, from (5.2), estimated to be

$$E[TR'|P_1 = 62.9,\ P_2 = 46.8] = 2.555 \text{ reinforcers/minute,} \tag{7.2}$$

so the estimated efficiency loss due to selecting (62.9, 46.8) rather than (49.3, 88.5) is d = 1.11%. These values are listed in Table 3. Figure 7.2 also shows the estimated positions of the TR ≡ 2.555 and the TR ≡ 2.584 iso-TR curves. A further point which the reader should note from Figure 7.2 is that the subjects' choices are all in the region of very sharp diminishing returns to scale in the production of reinforcers. For example, although subject 43's chosen work rate of 3823 exceeds subject 42's chosen rate of 2082 by 84%, the estimated efficient mean total reinforcement rate for subject 43 is 2.613 reinforcers/minute, which is only 4.2% larger than subject 42's rate of 2.507 reinforcers/minute. This is a direct consequence of the point noted in Section 5 that, for response rates of 30 or more per minute, the marginal mean reinforcement rate productivities of responding to a VI-60 schedule are very small. It follows that, in condition 2, a subject needs to stray a significant distance from the efficient response rate pair before a perceptible decrement results to the expected total reinforcement rate. For instance, the Euclidean distance between subject 44's chosen and efficient response rate pairs is 44.0 responses/minute while the efficiency loss is only 1.11%. With schedules such as the Hunter and Davison VI schedules, therefore, the model could be anticipated to be an accurate predictor of response rates only if the subjects had extremely accurate powers of perception. It should be further noted, however, that the iso-W line for

$W \equiv 3124$ contains a range of response pairs which, at the extremeties of this line, imply efficiency losses of 47.8% and 49.6%. As Table 3 informs us, no estimated efficiency loss even approaching these magnitudes is observed so, in this case, the data give us information on the extent of the subjects' perceptive abilities.

Figure 7.3 contains information of the same nature as does Figure 7.2. The schedules used in condition 27 are a VI-60 and a VI-480. Once again the subjects' choices place them in the region of severe diminishing returns to scale in the production of reinforcers. An obvious difference between the two figures is the shape of the iso-TR curves of each. The iso-TR curves of Figure 7.2 are symmetric about a 45°-line from the origin; those of Figure 7.3 are sharply skewed towards the P_1-axis since schedule 1 is the (more productive) VI-60 schedule. This skewness indicates the very sharp decline of the TR-surface for response rates near to the P_1 axis. Notice that all the subjects chose response rates close to the expansion path. This outcome is typical of all those conditions which generated iso-TR curves sharply skewed near one of the P-axes, suggesting the subjects are able to perceive that the TR-surface is less flat above any iso-W line in condition 27 where the two schedules differ in their productivities, than in condition 2 where the two schedules are equally productive.

At any interior point on any condition's expansion path, it is necessarily true that $\frac{MRP_1}{w_1}/\frac{MRP_2}{w_2} = 1$. We have computed the estimated values of $\frac{MRP_1}{w_1}/\frac{MRP_2}{w_2}$ for each of the chosen response rate pairs. These are listed in Table 3 under the heading "Ratio". Even a casual glance at the Ratio column shows that the values contained therein usually differ markedly from unity. In fact, only 43% of the values are in the interval $[\frac{1}{2}, 2]$. (Two values in the "Ratio" column of Table 3 are bracketed to denote that, in these two instances, the efficient response rate pairs (P_1^*, P_2^*) are at corner points.) To those used to neo-classical choice theory, these frequent and often substantial divergences of the ratio from unity may at first suggest subjects were rather inefficient in allocating their total work rates. But, as we saw above from our examination of the d-values, these allocations were, in fact, very efficient. Figures 7.2 and 7.3 show the sources of the apparent conflict. Consider Figure 7.2 first. Subject 44's chosen response rates correspond to an estimated value of $\frac{MRP_1}{w_1}/\frac{MRP_2}{w_2} = 0.19$, even though the estimated efficiency loss is only 1.11%. The reader should note the TR-surface is nearly flat in the region of subject 44's chosen response rates. Even though the value of $\frac{MRP_1}{MRP_2}$ differs, at 44's chosen response rates, by a factor of 0.19 from the value of $\frac{MRP_1}{MRP_2}$ at 44's efficient response rates, the estimated rate of increase in the mean total rate of reinforcement caused by altering the chosen rates by transferring one response/minute from schedule 1 to schedule 2 and keeping $W \equiv 3124$ is only $MRP_2 - MRP_1 = 0.0009$ reinforcers/minute. This is why 44's chosen response rates, which are distant from its efficient rates by 44 responses/minute, cost only an estimated 0.029 reinforcers/minute, a change so small as to be difficult for the subject to perceive under this condition. The apparent conflict between small d-values and ratio values substantially different from unity is, therefore, no real conflict at all.

Similar comments apply to condition 27, illustrated in Figure 7.3 where, for example, subject 43 gains only about $MRP_2 - MRP_1 = 0.0015$ reinforcers/minute by altering its chosen response rates by transferring one response per minute from schedule 1 to schedule 2, keeping $W \equiv 937$. Even though $\frac{MRP_1}{MRP_2}$ values for these rates differ by a factor of 0.43, the estimated total reinforcement rate loss is only 0.002 reinforcers/minute, a quantity which, once again, is arguably at the limits of the subject's perceptive abilities under this condition. We therefore suggest that the departures from unity displayed in the "Ratio" column of Table 3 should be interpreted, for the Hunter and Davison experiment, as evidence of the great sensitivity of the first-order optimality conditions to departures from optimality rather than as evidence of substantial sub-optimality in the allocations of total work rate.

The next question asked of the data in Table 3 was if they revealed any statistically significant differences in the perceptual abilities of Hunter and Davison's five subjects. We constructed the contingency table shown below and conducted a χ^2-test for independence.

Insert Table 4 here

The calculated χ^2_{16}-value is $29.8 > 26.30 = \chi^2_{16,0.05}$, suggesting significant differences between subjects. Removing subject 46 from the contingency table and testing for independence amongst subjects 42, 43, 44 and 45 yields $\chi^2_{12} = 8.73 < 21.03 = \chi^2_{12,0.05}$. Subject 46's performance is erratic. A frequency plot of 46's d-values shows that 22 of the 42 values are smaller than 0.5% but, even so, subject 46 is responsible for 5 of the 6 d-values above 5% (see Figure 7.1; subject 43 generated the d-value of 6.79%). On the basis of these tests it seems reasonable to conclude that subjects 42, 43, 44 and 45 were similar in their abilities to achieve efficient work rate allocations while subject 46 was significantly different in this regard to its colleagues.

The final question addressed was whether or not the subjects displayed systematic differences in their performances over total work rates. For each of the 42 experimental conditions, the subjects' work rates were ranked in increasing order, with 1 assigned to the subject working most slowly and 5 assigned to the subject working most rapidly. Ranks within conditions are used rather than actual work rates because work rates vary across experimental conditions, due to the different VI schedules and response force requirements used in the various conditions. Figure 7.4 displays the frequency plots of each subject's rankings. (Subject 42, which did not respond to conditions 10, 11 and 13, was assigned a rank of 1 for these cases.) The figure shows some

Insert Figure 7.4 here

substantial differences between subjects. These are confirmed by pairwise Kendall's τ-tests, conducted with 5% significance levels, which show an ordering over the subjects of $42 < 43 = 46 < 45 = 44$, where $<$ denotes a significantly different work rate ranking distribution and $=$ denotes no significant difference between distributions. The Hunter and Davison data therefore reveal significant differences in the subjects' individual preferences over total work rates.

Finally, it is of interest to know if the subjects' preferences were stable over the duration of the full experiment. The Hunter and Davison data contain a small amount of information useful for a direct answer to the question. In their experiment Hunter and Davison replicated some conditions; in particular, conditions 24, 30 and 32 are replicas, as are conditions 29 and 21 and conditions 38 and 43. Note that each replica is separated from its predecessor by at least one other condition. Figure 7.5 displays the estimated mean total reinforcement rate/total work rate (i.e. TR^* and W) frontiers for each of these three distinct conditions and, as well, presents the chosen work rates W and estimated mean total reinforcement rates, TR', achieved for each subject in each of the seven conditions.

Insert Figure 7.5 here

For conditions 24, 30 and 32 and conditions 29 and 31, the pattern in essentially the same. Subjects 42, 44, 45 and 46 replicate their individual choices quite closely, while Subject 43 is more erratic. Conditions 38 and 43 show a similar pattern to the above, but more weakly; only subjects 42, 44 and 46 closely replicate their choices, while subject 45's two choices are well separated. Thus there is some weak evidence of temporal stability of preferences from the responses to the replicated conditions, given the perceptual difficulties hindering the subject's task.

Finally, for these seven conditions, Figure 7.5 illustrates quite graphically to the reader the impressive efficiency levels achieved by the pigeons.

8. CONCLUSIONS

The Hunter and Davison (1982) experiment exposed five pigeons to a wide variety of choice problems. In each problem each bird chose the rate of responding to each of two concurrently available variable-interval schedules of reinforcement which allowed access to food in a manner contingent upon the bird's response rates. Hunter and Davison varied both the reinforcement schedules and the force needed to complete each single response to either schedule. Each subject's choices were noted over forty two separate experimental conditions.

This paper used the Hunter and Davison data set to test a neo-classical model of labour supply and allocation. The expansion path containing the response rate pairs yielding the maximum rate of expected access to food for every work rate from zero upwards was estimated for each experimental condition. Each subject's chosen response rates for that condition were used to calculate its total work rate and were then compared to the efficient response rate pair in the expansion path for this work rate. The estimated loss of expected access to food was computed as a percentage of the access achievable by choosing the efficient response rate pair instead of the pair actually chosen by the subject. It was found that these estimated efficiency losses are almost always small; 87% of them are smaller than 2%. This indicates that the experimental subjects do very well in solving their labour supply and allocation problems and supports the use of neo-classical models of labour supply and allocation in analyzing these types of choice problem.

The Hunter and Davison experiment was not designed to test the model advanced in this paper but, instead, to test an alternative model of choice called "matching", which is popular amongst some operant psychologists.[6] The matching phenomenon is often noticed when subjects are faced with variable-interval schedules of reinforcement. Interest in matching led Hunter and Davison to employ variable-interval schedules for their experiment. From the point of view of testing the model of this paper, however, variable-interval schedules are far from ideal. Firstly, a variable-interval schedule generates a stochastic relationship between response rate and the rate at which access to food becomes available. Secondly, for even quite low response rates, increments to the response rate cause only small increments to the mean rate at which access to food becomes available from variable-interval schedules. These two factors make it difficult for the pigeons to perceive the nature of their choice problems. Given this, it is something of a tribute to the pigeons' perceptual and decision making abilities that they achieve the high estimated efficiency levels observed. A more searching test of the model presented in this paper would be made possible by replacing the variable-interval schedules with non-stochastic schedules that did not cause marginal response rate productivities to decline so sharply towards zero with increasing response rates.

The results section of the paper contains some examples of two cautions for those accustomed to using microeconomic choice models which presume decision makers are perfectly informed. The first of these is that, although subjects' efficiency levels were high, their chosen response rates were sometimes quite distant from the experimental conditions' expansion paths. The explanation offered for this is that the marginal response rate productivities were so small in these instances that even substantial deviations from the expansion paths of these conditions caused only imperceptible losses to the mean rate of access to food. Thus, even though the subjects displayed considerable perceptive powers, the chosen response rates were sometimes far enough from the efficient response rates to give the initial, erroneous, impression that the subjects' chosen rates were rather inefficient. This is a direct consequence of the properties of the VI schedules used by Hunter and Davison; in this case, substantial departures from an expansion path do still correspond to very efficient solutions of the choice problem.

The second caution concerns the interpretation of the ratio of the marginal returns, $\frac{MRP_1}{w_1}/\frac{MRP_2}{w_2}$. If it is assumed that the decision maker is perfectly informed, then finding that the ratio deviates markedly from unity might be interpreted as meaning either the theory does not accurately model the decision making process or the behaviour of the decision maker is not rational. Neither of these conclusions is appropriate in the present case. In this case, the marginal returns were very small; and while the differences between the marginal returns on the two schedules were also small, the differences were none the less substantial relative to the marginal returns, resulting in large ratios. In the present case, the ratios of the marginal returns were divergent from unity probably because the subjects experienced difficulty in discriminating the small marginal returns from each schedule.

Finally, the study presented in this paper is a complement to those by Rachlin et.al. (1980) and Battalio et.al. (1981a, 1981b) who have explored the effects of other types of constraints on choice; in particular, time constraints and budget constraints.

Footnotes

1. We are most grateful to Michael Davison for supplying us with details of his 1982 experiment with Ian Hunter and for valuable discussions relating to the model presented in the current paper. We also thank Aman Ullah for econometric assistance and thank Robin Carter and Peter Wagstaff for helpful comments. Any errors are solely our responsibility. We are also grateful to Mary Cherin, Suzanne Dolman the U.W.O. Economics Department's typing pool for their typing services.

2. We wish to note here that, while it is common in microeconomics to regard production technologies like the reinforcement schedules in (2.4) as constraints on achievable consumption sets, this is a view advanced only recently in the operant psychology literature by Staddon (1979) and Allison (1983) who suggest "hill-climbing" and "minimum distance" hypotheses containing the idea.

3. The estimation procedure employed for each of the five VI schedules is as follows. The model estimated is

$$\dot{R}_t = kP_t(1 - e^{-\frac{S}{kP_t}}) + \epsilon_t \qquad \text{(A)}$$

where

$$\text{var}(\epsilon_t | P_t) \propto P_t^{\alpha} \cdot e^{-\frac{\beta}{P_t}} \cdot (1 - e^{-\frac{\beta}{P_t}}), \text{ for } t=1,\ldots,n. \qquad \text{(B)}$$

The form of the heteroscedasticity is analogous to the variance of a binomial variable with a "success" probability of $1 - e^{-\beta/P}$. For instance, if the original model (4.2) is to be supported by the data we would observe acceptance of the joint hypothesis that $k=\alpha=1$ and $\beta=S$. Also, the above heteroscedasticity form satisfies (4.5a), admits monotonic increasing variance up to S (e.g. $\alpha=1$, $\beta=1$) and admits variance increasing initially and then decreasing (e.g. $\alpha=\frac{1}{2}$, $\beta=0.9$). (A) was estimated by non-linear least squares. The residuals $\hat{\varepsilon}_t$ and the standard error s of this regression were used to construct the dependent variable for the estimation by non-linear least squares of α and β from

$$\frac{\hat{\varepsilon}_t^2}{s^2} = P_t^{\alpha} \cdot e^{-\frac{\beta}{P_t}} \cdot (1 - e^{-\frac{\beta}{P_t}}) \quad \text{for } t=1,\ldots,n. \qquad \text{(C)}$$

The estimates $\tilde{\alpha}$ and $\tilde{\beta}$ obtained from (C) were substituted for their true values and used to transform (A) to

$$\frac{R_t}{\theta_t} = k \cdot \frac{P_t}{\theta_t} \cdot (1 - e^{-\frac{S}{kP_t}}) + e_t \qquad \text{(D)}$$

where

$$\theta_t = (P_t^{\tilde{\alpha}} \cdot e^{-\frac{\tilde{\beta}}{P_t}} \cdot (1 - e^{-\frac{\tilde{\beta}}{P_t}}))^{1/2}, \quad \text{for } t=1,\ldots,n.$$

(D) was estimated by non-linear least squares; the estimates so obtained are the $\tilde{k}$ and $\tilde{S}$ values listed in Table 2. For each of the five cases the hypothesis $H_o: k=1, \alpha=1$ was tested against $H_1: k \neq 1$ or $\alpha \neq 1$ by repeating the above estimation procedure restricted by setting $k=1$ and $\alpha=1$. The sum of squared residuals from the restricted procedure, RSS, and from the unrestricted procedure, USS, were used to calculate the test statistic $(RSS - USS)/s^2$ which, for this problem, is asymptotically distributed as an $F_{2,n-2}$ statistic. A summary of the results is as follows, with standard errors in brackets.

Schedule	VI-15	VI-30	VI-60	VI-120	VI-480
n	100	25	102	95	94
$\tilde{\alpha}$	0.554 (0.165)	1.346 (0.545)	1.020 (0.351)	1.107 (0.441)	0.910 (0.886)
$\tilde{\beta}$	7.705 (6.76)	0.253 (0.545)	0.083 (0.106)	0.702 (1.20)	1.064 (2.66)
s^2	1.140	0.0315	0.0321	0.0032	0.0017
USS	111.8	0.726	3.21	0.2996	0.1567
RSS	180.2	1.375	50.67	0.6618	5.379
$\frac{RSS-USS}{2s^2}$	30.0	10.3	739	56.2	1533

The last row of the above table shows the asymptotic test statistic rejects H_o: $k=\alpha=1$ (and thus rejects (4.2)) at significance levels well below 1% for each of the five VI schedules.

4. We would like to note here that, in an important respect, the labels VI-15, VI-30, VI-60, VI-120 and VI-480 for the Hunter and Davison schedules are no longer particularly appropriate. Recall that, for a VI-Δt schedule, Δt = 60/S seconds/reinforcer where S is the mean reinforcement arrival rate, measured in reinforcers/minute. From Table 2, the respective estimates of Δt are 8.7, 23.1, 43.7, 83.2 and 301 seconds/reinforcer. The differences between the estimated and stated values of Δt are considerable, ranging from 23% to 42% of the stated values.

5. The units of measurement of W are unknown. This is because the work rate on either key is the rate of responding on the key multiplied by the force required to depress the key multiplied by the distance by which the key is depressed. This last quantity is not reported by Hunter and Davison. Consequently, we have defined this distance as one unit and implicitly defined the units of work rate in this way. Nothing is lost by doing so since the units implicitly defined here are directly proportional to those in common use.

6. The "direct matching law" was advanced by Herrnstein (1961, 1974) and states that subjects allocate their response rates P_1 and P_2 in approximately direct proportion to the received reinforcement rates; that is, $\frac{P_1}{P_2} \approx \frac{R_1}{R_2}$. Systematic deviations from this empirical "law" resulted in Baum (1974) advancing a weaker statement, called the "generalized matching law", which states that $\frac{P_1}{P_2} \approx a(\frac{R_1}{R_2})^b$ where, typically, $a \approx 1$ and $b \approx 0.9$. Both forms of the matching "law" are inductive rather than deductive. An alternative form of matching also considered by operant psychologists is time matching where subjects are described as allocating the times T_1 and T_2 spent responding to the individual schedules so as to satisfy $\frac{T_1}{T_2} \approx a'(\frac{R_1}{R_2})^{b'}$. Worth noting in the context of the present paper is that, in analyzing the same data used here, Hunter and Davison (1982) note that even Baum's generalization of the matching law is not supported by their data and examine a further inductive extension of the matching law to the form $\frac{P_1}{P_2} \approx a(\frac{R_1}{R_2})^b(\frac{W_1}{W_2})^c$. The mean of Hunter and Davison's ordinary least squares estimates of the parameters of the log-linear form of this equation across their five subjects were $a = 1.08$, $b = 0.88$ and $c = -0.71$ (see Hunter and Davison, 1982, Table 2).

References

Allison, J., Behavioural Economics, Praeger Publishers, 1983.

Battalio, R.C., Green, L., and Kagel, J.H. "Income-Leisure Tradeoffs of Animal Workers." American Economic Review 71 (1981a): 621-632.

Battalio, R.C., Kagel, J.H., and Green, L. "Consumer Demand Behavior with Pigeons as Subjects." Journal of Political Economy 89 (1981b): 67-91.

Battalio, R.C., Kagel, J.H., and Green, L. "Labor Supply Behaviour of Animal Workers: Towards an Experimental Analysis." Research in Experimental Economics 1 (1979): 231-253.

Baum, W.M. "On Two Types of Deviation from the Matching Law: Bias and Undermatching." Journal of the Experimental Analysis of Behavior 22 (1974): 231-242.

de Villiers, P.A., and Herrnstein, R.J. "Toward a Law of Response Strength." Psychological Bulletin 83 (1976): 1131-1153.

Ferster, C.B., and Skinner, B.F. Schedules of Reinforcement. New York, Appleton-Century Crofts, 1957.

Fleshler, M., and Hoffman, H.S. "A Progression for Generating Variable-Interval Schedules." Journal of the Experimental Analysis of Behavior 5 (1962): 529-530.

Herrnstein, R.J. "Relative and Absolute Strength of Response as a Function of Frequency of Reinforcement." Journal of the Experimental Analysis of Behavior 4 (1961): 267-272.

Herrnstein, R.J. "Formal Properties of the Matching Law." Journal of the Experimental Analysis of Behavior 21 (1974): 159-164.

Hunter, I.W., and Davison, M.C. "Independence of Response Force and Reinforcement Rate on Concurrent Variable-Interval Schedule Performance." Journal of the Experimental Analysis of Behavior 37 (1982): 183-198.

Nevin, J.A., and Baum, W.M. "Feedback Functions for Variable-Interval Reinforcement." Journal of the Experimental Analysis of Behavior 34 (1980): 207-217.

Prelec, D. "Matching, Maximising, and the Hyperbolic Reinforcement Schedule Function." Psychological Review 89 (1982): 187-225.

Rachlin, H.C., Green, L., Kagel, J.H., and Battalio, R.C. "Economic Demand Theory and Psychological Studies of Choice." In The Psychology of Learning and Motivation, edited by G.H. Bower. Vol. 10. New York: Academic Press, 1976.

Rachlin, H.C., Kagel, J.H., and Battalio, R.C. "Substitutability in Time Allocation." Psychological Review 87 (1980): 355-374.

Solomon, R.L. "The Influence of Work on Behavior." Psychological Bulletin 45 (1948): 1-40.

Staddon, J.E.R. "Operant Behavior as Adaption to Constraint." Journal of Experimental Psychology: General 108 (1979): 48-67.

Takayama, A. Mathematical Economics. Dryden Press, 1974.

Taylor, R., and Davison, M. "Sensitivity to Reinforcement in Concurrent Arithmetic and Exponential Series." Journal of the Experimental Analysis of Behavior 39 (1983): 191-198.

TABLE 1

Schedule	VI-15	VI-30	VI-60	VI-120	VI-480
$\hat{S}$ (reinf./min.)	5.32	2.23	1.19	0.667	0.160
n	100	25	102	95	94

TABLE 2

Schedule	VI-15	VI-30	VI-60	VI-120	VI-480
$\tilde{S}$ (reinf./min.)	6.93 (0.2726)	2.60 (0.1037)	1.37 (0.0307)	0.721 (0.0091)	0.199 (0.0123)
$\tilde{k}$	0.214 (0.0256)	0.180 (0.0304)	0.174 (0.0231)	0.164 (0.0158)	0.143 (8.8×10^{-7})
n	100	25	102	95	94

TABLE 3

Condition	Subject	(P_1,P_2)	(P_1^*,P_2^*)	W	TR'	TR*	d	MRP_1	MRP_2	Ratio
No. 2	42	(36.7, 47.2)	(32.7, 59.5)	2082	2.501	2.507	0.24	0.0035	0.0022	0.52
VI-60 - VI-60	43	(67.7, 85.8)	(60.5, 108.1)	3823	2.609	2.613	0.13	0.0011	0.0007	0.51
w_1 = 40	44	(62.9, 46.8)	(49.3, 88.6)	3124	2.555	2.584	1.11	0.0013	0.0022	0.19
w_2 = 13	45	(74.8, 95.3)	(66.9, 119.5)	4231	2.622	2.625	0.12	0.0009	0.0006	0.52
	46	(45.6, 87.6)	(46.8, 84.1)	2963	2.575	2.575	0.01	0.0023	0.0007	1.14
No. 3	42	(31.2, 27.1)	(34.0, 25.5)	1002	2.406	2.407	0.06	0.0047	0.0061	1.31
VI-60 - VI-60	43	(74.2, 81.5)	(92.7, 70.6)	2758	2.613	2.617	0.16	0.0009	0.0008	2.03
w_1 = 13	44	(72.1, 43.2)	(63.6, 48.2)	1888	2.557	2.559	0.09	0.0010	0.0026	0.64
w_2 = 22	45	(104.0, 48.1)	(81.0, 61.7)	2410	2.590	2.599	0.33	0.0005	0.0021	0.38
	46	(90.5, 43.8)	(72.0, 54.7)	2140	2.573	2.581	0.30	0.0006	0.0025	0.42
No. 4	42	(40.3, 30.8)	(41.6, 30.1)	2119	2.460	2.460	0.01	0.0029	0.0048	1.10
VI-60 - VI-60	43	(57.4, 30.4)	(48.5, 35.3)	2479	2.493	2.499	0.22	0.0015	0.0049	0.55
w_1 = 22	44	(67.7, 33.3)	(55.2, 40.2)	2821	2.520	2.527	0.30	0.0011	0.0042	0.48
w_2 = 40	45	(80.8, 22.8)	(52.6, 38.3)	2690	2.470	2.517	1.87	0.0008	0.0083	0.17
	46	(78.7, 24.1)	(52.7, 38.4)	2695	2.479	2.518	1.55	0.0008	0.0075	0.20
No. 5	42	(0.0, 0.3)	(0.0, 0.3)	4	0.053	0.053	0.00	0.1725	0.1725	(0.27)
VI-60 - VI-60	43	(10.6, 44.5)	(14.3, 30.5)	1098	2.228	2.266	1.71	0.0298	0.0024	3.25
w_1 = 49	44	(45.6, 40.4)	(36.7, 73.9)	2760	2.509	2.539	1.17	0.0023	0.0029	0.21
w_2 = 13	45	(40.2, 36.4)	(32.5, 65.6)	2443	2.482	2.514	1.25	0.0029	0.0035	0.22
	46	(11.3, 76.4)	(20.4, 42.2)	1547	2.294	2.392	4.10	0.0270	0.0009	8.27
No. 6	42	(23.7, 17.2)	(20.5, 20.5)	1636	2.270	2.279	0.41	0.0078	0.0136	0.57
VI-60 - VI-60	43	(43.3, 25.5)	(34.4, 34.4)	2752	2.437	2.455	0.72	0.0026	0.0068	0.38
w_1 = 40	44	(56.1, 32.9)	(44.5, 44.5)	3560	2.503	2.518	0.59	0.0016	0.0043	0.37
w_2 = 40	45	(47.2, 26.9)	(37.1, 37.1)	2964	2.456	2.475	0.77	0.0022	0.0062	0.35
	46	(20.2, 9.8)	(15.0, 15.0)	1200	2.081	2.136	2.61	0.0103	0.0336	0.31
No. 7	42	(20.7, 20.7)	(20.7, 20.7)	1656	3.744	3.744	0.00	0.0279	0.0279	1.00
VI-30 - VI-30	43	(30.0, 21.8)	(25.9, 25.9)	2072	3.965	3.986	0.53	0.0152	0.0257	0.59
w_1 = 40	44	(53.5, 25.7)	(39.6, 39.6)	3168	4.267	4.357	2.05	0.0055	0.0197	0.28
w_2 = 40	45	(44.7, 21.6)	(33.2, 33.2)	2652	4.118	4.215	2.31	0.0076	0.0261	0.29
	46	(41.5, 22.0)	(31.8, 31.8)	2540	4.102	4.178	1.81	0.0087	0.0253	0.34
No. 8	42	(33.0, 9.9)	(25.8, 11.8)	914	2.169	2.186	0.77	0.0043	0.0331	0.48
VI-60 - VI-60	43	(35.3, 10.1)	(26.8, 12.4)	954	2.184	2.205	0.95	0.0038	0.0321	0.44
w_1 = 13	44	(61.8, 17.3)	(44.9, 21.8)	1651	2.393	2.412	0.81	0.0013	0.0134	0.37
w_2 = 49	45	(66.9, 12.8)	(40.9, 19.7)	1497	2.321	2.381	2.52	0.0011	0.0222	0.19
	46	(68.3, 7.1)	(34.1, 16.2)	123	2.127	2.313	8.08	0.0011	0.0531	0.08
No. 9	42	(26.6, 40.7)	(33.7, 33.7)	875	4.195	4.228	0.78	0.0186	0.0090	2.08
VI-30 - VI-30	43	(37.5, 34.2)	(35.9, 35.9)	932	4.278	4.280	0.03	0.0103	0.0121	0.85
w_1 = 13	44	(65.7, 32.0)	(48.9, 48.9)	1270	4.426	4.501	1.67	0.0038	0.0136	0.28
w_2 = 13	45	(57.8, 85.8)	(71.8, 71.8)	1867	4.692	4.709	0.36	0.0048	0.0023	2.09
	46	(109.3, 92.7)	(101.0, 101.0)	2626	4.841	4.843	0.05	0.0014	0.0020	0.73
No. 10	43	(6.3, 57.0)	(5.1, 61.5)	1050	6.067	6.084	0.28	0.0620	0.0238	0.69
VI-60 - VI-15	44	(20.0, 106.0)	(13.3, 131.3)	2358	7.103	7.174	0.99	0.0105	0.0082	0.34
w_1 = 49	45	(11.6, 93.6)	(9.8, 100.5)	1785	6.851	6.863	0.17	0.0259	0.0102	0.67
w_2 = 13	46	(3.3, 98.2)	(7.6, 82.0)	1438	6.422	6.576	2.35	0.1201	0.0094	3.40
No. 11	43	(2.3, 9.1)	(0.9, 10.5)	559	2.278	2.300	0.93	0.1491	0.1861	0.80
VI-60 - VI-15	44	(19.7, 45.2)	(11.4, 53.5)	3180	6.077	6.187	1.78	0.0107	0.0345	0.31
w_1 = 49	45	(6.7, 49.9)	(9.9, 46.7)	2773	5.902	5.942	0.67	0.0573	0.0296	1.94
w_2 = 49	46	(1.9, 24.9)	(4.5, 22.3)	1313	4.201	4.301	2.33	0.1600	0.0798	2.00
No. 12	42	(10.7, 29.6)	(14.6, 28.6)	1590	5.183	5.204	0.42	0.0294	0.0639	1.74
VI-60 - VI-15	43	(5.1, 11.1)	(7.8, 10.4)	610	2.945	2.987	1.41	0.0797	0.1686	1.78
w_1 = 13	44	(33.7, 69.2)	(31.9, 69.7)	3829	6.756	6.756	0.01	0.0041	0.0172	0.89
w_2 = 49	45	(24.6, 30.3)	(16.3, 32.5)	1805	5.430	5.474	0.80	0.0073	0.0619	0.44
	46	(26.0, 24.9)	(14.4, 28.0)	1558	5.060	5.161	1.96	0.0066	0.0798	0.31
No. 13	43	(3.8, 15.6)	(9.7, 9.7)	951	3.731	4.002	6.79	0.2135	0.1313	1.63
VI-15 - VI-15	44	(28.9, 56.7)	(42.8, 42.8)	4194	9.442	9.717	2.83	0.0660	0.0241	2.74
w_1 = 49	45	(20.2, 23.7)	(22.0, 22.0)	2151	7.227	7.242	0.21	0.1018	0.0848	1.20
w_2 = 49	46	(9.8, 33.7)	(21.8, 21.8)	2132	6.470	7.205	10.20	0.1801	0.0534	3.37

TABLE 3 (cont'd.)

Condition	Subject	(P_1, P_2)	(P_1^*, P_2^*)	W	TR'	TR*	d	MRP_1	MRP_2	Ratio
No. 14	42	(45.4, 51.7)	(70.5, 41.2)	2193	10.099	10.349	2.42	0.0343	0.0279	2.93
VI-15 - VI-15	43	(62.2, 71.0)	(95.0, 57.2)	3010	10.961	11.158	1.77	0.0206	0.0165	2.98
w_1 = 13	44	(69.0, 92.7)	(117.9, 72.2)	3771	11.373	11.637	2.27	0.0173	0.0104	3.98
w_2 = 31	45	(95.2, 49.3)	(87.7, 52.5)	2766	10.948	10.959	0.10	0.0099	0.0301	0.78
	46	(51.7, 70.9)	(90.8, 54.5)	2870	10.707	11.047	3.08	0.0279	0.0165	4.03
No. 15	42	(19.3, 30.2)	(20.0, 29.1)	1882	7.606	7.609	0.04	0.1069	0.0622	1.09
VI-15 - VI-15	43	(47.4, 29.2)	(35.5, 48.0)	3228	9.203	9.581	3.94	0.0321	0.0651	0.31
w_1 = 49	44	(61.4, 83.2)	(62.4, 81.6)	5588	11.120	11.120	0.01	0.0211	0.0125	1.06
w_2 = 31	45	(36.6, 53.9)	(38.2, 51.4)	3464	9.803	9.808	0.05	0.0475	0.0261	1.15
	46	(16.4, 35.4)	(20.2, 29.4)	1901	7.559	7.649	1.17	0.1256	0.0498	1.59
No. 16	42	(2.0, 40.6)	(1.0, 42.2)	1357	4.916	4.942	0.54	0.0221	0.0407	0.34
VI-480 - VI-15	43	(4.6, 52.1)	(1.4, 57.2)	1841	5.330	5.414	1.55	0.0054	0.0276	0.12
w_1 = 49	44	(10.9, 126.5)	(3.7, 137.9)	4456	6.297	6.339	0.65	0.0011	0.0059	0.11
w_2 = 31	45	(3.2, 99.2)	(2.6, 100.1)	3232	6.070	6.072	0.02	0.0102	0.0092	0.70
	46	(0.7, 62.6)	(1.5, 61.3)	1975	5.494	5.511	0.31	0.0845	0.0204	2.62
No. 17	42	(3.9, 73.9)	(2.6, 75.2)	1011	5.775	5.782	0.11	0.0072	0.0154	0.47
VI-480 - VI-15	43	(8.5, 100.7)	(3.6, 105.6)	1420	6.106	6.130	0.38	0.0017	0.0089	0.19
w_1 = 13	44	(9.3, 146.5)	(5.2, 150.6)	2025	6.399	6.407	0.12	0.0015	0.0045	0.32
w_2 = 13	45	(4.8, 132.5)	(4.6, 132.7)	1785	6.317	6.317	0.00	0.0050	0.0054	0.92
	46	(5.5, 121.2)	(4.2, 122.5)	1647	6.254	6.256	0.03	0.0039	0.0064	0.61
No. 18	42	(2.6, 61.0)	(3.4, 60.7)	1925	5.528	5.530	0.04	0.0145	0.0213	1.62
VI-480 - VI-15	43	(4.5, 69.8)	(3.9, 70.0)	2222	5.713	5.713	0.01	0.0056	0.0170	0.78
w_1 = 13	44	(8.5, 124.5)	(6.9, 125.2)	3970	6.282	6.283	0.01	0.0017	0.0061	0.67
w_2 = 31	45	(5.4, 99.5)	(5.5, 99.4)	3155	6.087	6.087	0.00	0.0040	0.0091	1.05
	46	(3.8, 50.6)	(2.9, 51.0)	1618	5.282	5.285	0.04	0.0076	0.0289	0.62
No. 19	42	(3.4, 17.8)	(1.6, 18.3)	916	3.353	3.378	0.72	0.0092	0.1161	0.30
VI-480 - VI-15	43	(3.1, 21.2)	(1.8, 21.5)	1079	3.711	3.722	0.30	0.0108	0.0965	0.42
w_1 = 13	44	(4.3, 56.3)	(4.2, 56.3)	2815	5.437	5.437	0.00	0.0061	0.0243	0.94
w_2 = 49	45	(5.3, 52.3)	(3.9, 52.7)	2632	5.339	5.341	0.05	0.0042	0.0274	0.57
	46	(13.8, 24.1)	(2.2, 27.2)	1360	3.999	4.195	4.66	0.0007	0.0831	0.03
No. 20	42	(1.1, 62.7)	(0.8, 63.8)	869	5.522	5.526	0.07	0.0517	0.0204	0.67
VI-480 - VI-15	43	(1.0, 92.2)	(1.3, 91.0)	1248	5.948	5.951	0.05	0.0580	0.0105	1.47
w_1 = 49	44	(4.2, 144.3)	(2.4, 150.9)	2082	6.373	6.385	0.18	0.0063	0.0046	0.36
w_2 = 13	45	(1.7, 139.7)	(2.2, 137.8)	1899	6.318	6.320	0.03	0.0284	0.0049	1.53
	46	(1.0, 118.5)	(1.8, 115.5)	1590	6.168	6.178	0.17	0.0580	0.0067	2.31
No. 21	42	(54.7, 3.3)	(52.0, 4.0)	873	1.442	1.443	0.08	0.0016	0.0097	0.64
VI-60 - VI-480	43	(74.7, 4.9)	(71.8, 5.7)	1211	1.477	1.478	0.03	0.0009	0.0048	0.71
w_1 = 13	44	(29.1, 0.3)	(23.9, 1.7)	393	1.246	1.305	4.57	0.0054	0.1351	0.15
w_2 = 49	45	(138.8, 2.3)	(113.2, 9.1)	1917	1.485	1.512	1.77	0.0003	0.0177	0.06
	46	(97.9, 4.8)	(89.2, 7.1)	1508	1.493	1.496	0.19	0.0005	0.0050	0.41
No. 22	42	(17.6, 3.0)	(17.8, 2.8)	1009	1.266	1.267	0.01	0.0131	0.0114	1.15
VI-60 - VI-480	43	(31.8, 1.2)	(28.5, 4.5)	1617	1.334	1.371	2.69	0.0046	0.0462	0.10
w_1 = 49	44	(1.1, 0.0)	(1.1, 0.0)	54	0.191	0.191	0.02	0.1730	0.1429	1.21
w_2 = 49	45	(52.2, 3.6)	(48.1, 7.7)	2734	1.440	1.449	0.63	0.0018	0.0083	0.22
	46	(21.7, 2.8)	(21.2, 3.3)	1201	1.309	1.310	0.07	0.0091	0.0128	0.71
No. 23	42	(11.8, 0.7)	(12.5, 0.0)	613	2.448	2.473	1.02	0.1624	0.0845	1.92
VI-15 - VI-480	43	(38.4, 0.5)	(37.7, 1.2)	1906	4.746	4.766	0.41	0.0442	0.1096	0.40
w_1 = 49	44	(72.9, 3.4)	(73.8, 2.5)	3739	5.756	5.759	0.06	0.0158	0.0092	1.72
w_2 = 49	45	(46.3, 0.7)	(45.5, 1.5)	2303	5.069	5.085	0.32	0.0333	0.0845	0.39
	46	(52.2, 0.9)	(51.4, 1.7)	2601	5.262	5.274	0.23	0.0275	0.0655	0.42
No. 24	42	(40.6, 1.6)	(45.3, 0.3)	606	4.905	4.998	1.85	0.0407	0.0310	4.95
VI-15 - VI-480	43	(99.6, 4.8)	(111.3, 1.7)	1530	6.086	6.145	0.97	0.0091	0.0050	6.91
w_1 = 13	44	(135.9, 4.1)	(142.7, 2.3)	1968	6.331	6.346	0.22	0.0052	0.0066	2.96
w_2 = 49	45	(178.4, 1.0)	(171.6, 2.8)	2368	6.441	6.469	0.42	0.0031	0.0580	0.20
	46	(88.5, 0.7)	(86.5, 1.2)	1185	5.887	5.896	0.17	0.0113	0.0845	0.50

TABLE 3 (cont'd.)

Condition	Subject	(P_1, P_2)	(P_1^*, P_2^*)	W	TR'	TR*	d	MRP_1	MRP_2	Ratio
No. 25	42	(46.2, 3.9)	(48.5, 1.6)	651	5.146	5.186	0.76	0.0334	0.0072	4.62
VI-15 - VI-480	43	(70.7, 3.3)	(71.6, 2.4)	962	5.719	5.722	0.06	0.0166	0.0097	1.72
$w_1 = 13$	44	(145.6, 3.9)	(144.5, 5.0)	1944	6.377	6.378	0.02	0.0046	0.0072	0.63
$w_2 = 13$	45	(166.3, 2.2)	(162.9, 5.6)	2191	6.441	6.457	0.25	0.0036	0.0190	0.19
	46	(91.3, 4.1)	(92.2, 3.2)	1240	6.000	6.002	0.04	0.0107	0.0066	1.61
No. 26	42	(43.3, 10.7)	(55.6, 5.6)	895	1.443	1.457	0.98	0.0026	0.0011	5.49
VI-60 - VI-480	43	(76.8, 8.8)	(78.8, 8.0)	1271	1.490	1.490	0.01	0.0009	0.0016	1.27
$w_1 = 13$	44	(99.5, 8.8)	(97.0, 9.9)	1566	1.505	1.505	0.01	0.0005	0.0016	0.77
$w_2 = 31$	45	(116.3, 15.7)	(123.6, 12.6)	1999	1.519	1.519	0.04	0.0004	0.0005	1.72
	46	(79.1, 2.1)	(67.8, 6.8)	1093	1.453	1.477	1.63	0.0008	0.0205	0.09
No. 27	42	(37.7, 8.5)	(39.9, 6.3)	601	1.423	1.425	0.13	0.0033	0.0017	1.93
VI-60 - VI-480	43	(65.4, 6.7)	(62.2, 9.9)	937	1.474	1.476	0.14	0.0012	0.0027	0.43
$w_1 = 13$	44	(83.7, 12.5)	(83.0, 13.2)	1251	1.499	1.500	0.00	0.0007	0.0008	0.88
$w_2 = 13$	45	(127.2, 23.6)	(130.0, 20.8)	1960	1.525	1.526	0.01	0.0003	0.0002	1.34
	46	(78.7, 9.5)	(76.1, 12.1)	1147	1.492	1.493	0.05	0.0008	0.0014	0.59
No. 28	42	(24.5, 10.0)	(25.0, 8.1)	1331	1.360	1.361	0.05	0.0073	0.0013	1.53
VI-60 - VI-480	43	(37.4, 21.4)	(39.7, 12.7)	2111	1.431	1.435	0.22	0.0034	0.0003	3.08
$w_1 = 49$	44	(54.1, 15.0)	(53.6, 17.0)	2846	1.468	1.469	0.01	0.0017	0.0006	0.77
$w_2 = 13$	45	(68.0, 11.6)	(65.6, 20.7)	3483	1.485	1.487	0.15	0.0011	0.0010	0.30
	46	(18.6, 27.2)	(23.8, 7.8)	1265	1.314	1.351	2.78	0.0119	0.0002	17.4
No. 29	42	(43.4, 2.1)	(43.1, 3.3)	2154	5.026	5.031	0.10	0.0368	0.0205	0.48
VI-15 - VI-480	43	(36.4, 3.8)	(36.7, 2.8)	1833	4.754	4.757	0.06	0.0478	0.0076	1.68
$w_1 = 49$	44	(103.7, 4.9)	(103.1, 7.3)	5145	6.122	6.125	0.04	0.0085	0.0048	0.47
$w_2 = 13$	45	(75.1, 3.5)	(74.6, 5.4)	3725	5.790	5.794	0.06	0.0150	0.0087	0.46
	46	(33.5, 3.2)	(33.7, 2.6)	1683	4.601	4.603	0.03	0.0539	0.0102	1.40
No. 30	42	(57.3, 1.4)	(59.9, 0.7)	813	5.417	5.437	0.37	0.0236	0.0375	2.37
VI-15 - VI-480	43	(80.7, 0.4)	(78.1, 1.1)	1069	5.760	5.782	0.37	0.0132	0.1232	0.40
$w_1 = 13$	44	(145.3, 2.2)	(144.8, 2.3)	1997	6.356	6.356	0.00	0.0046	0.0190	0.91
$w_2 = 49$	45	(162.7, 1.7)	(159.3, 2.6)	2198	6.416	6.421	0.08	0.0037	0.0284	0.49
	46	(75.6, 0.5)	(73.7, 1.0)	1007	5.701	5.712	0.21	0.0148	0.1096	0.51
No. 31	42	(24.9, 2.8)	(25.1, 2.1)	1257	4.032	4.035	0.09	0.0798	0.0128	1.66
VI-15 - VI-480	43	(57.7, 3.9)	(57.6, 4.2)	2878	5.468	5.468	0.00	0.0234	0.0072	0.86
$w_1 = 49$	44	(108.3, 6.1)	(107.9, 7.7)	5386	6.164	6.165	0.01	0.0079	0.0032	0.65
$w_2 = 13$	45	(71.9, 1.1)	(70.8, 5.1)	3537	5.689	5.733	0.77	0.0162	0.0517	0.08
	46	(30.9, 1.4)	(30.6, 2.4)	1532	4.418	4.427	0.20	0.0603	0.0375	0.43
No. 32	42	(58.4, 0.9)	(59.2, 0.7)	803	5.418	5.420	0.04	0.0229	0.0655	1.32
VI-15 - VI-480	43	(11.2, 0.0)	(11.2, 0.0)	146	2.263	2.263	0.02	0.1677	0.1429	(4.42)
$w_1 = 13$	44	(136.8, 3.2)	(140.4, 2.2)	1935	6.329	6.334	0.08	0.0051	0.0102	1.89
$w_2 = 49$	45	(167.2, 0.3)	(158.6, 2.6)	2188	6.339	6.418	1.24	0.0035	0.1351	0.10
	46	(63.0, 1.1)	(64.1, 0.8)	873	5.528	5.532	0.07	0.0202	0.0517	1.47
No. 33	42	(42.7, 7.6)	(45.9, 6.8)	928	0.867	0.868	0.05	0.0008	0.0021	1.44
VI-120 - VI-480	43	(63.1, 10.9)	(66.7, 9.9)	1354	0.884	0.884	0.02	0.0004	0.0011	1.33
$w_1 = 13$	44	(96.3, 14.1)	(95.5, 14.3)	1943	0.895	0.895	0.00	0.0002	0.0007	0.95
$w_2 = 49$	45	(135.4, 9.6)	(109.5, 16.5)	2231	0.895	0.898	0.32	0.0001	0.0014	0.23
	46	(61.3, 5.0)	(51.5, 7.6)	1042	0.870	0.873	0.40	0.0004	0.0046	0.33
No. 34	42	(16.8, 11.0)	(17.0, 10.2)	966	0.822	0.822	0.01	0.0047	0.0011	1.19
VI-120 - VI-480	43	(43.2, 29.1)	(44.1, 25.7)	2495	0.880	0.880	0.01	0.0008	0.0002	1.32
$w_1 = 49$	44	(51.1, 21.1)	(49.1, 28.6)	2778	0.884	0.884	0.05	0.0006	0.0003	0.51
$w_2 = 13$	45	(41.6, 29.9)	(42.9, 25.0)	2427	0.879	0.879	0.02	0.0009	0.0002	1.50
	46	(9.3, 33.2)	(15.6, 9.4)	887	0.770	0.814	5.39	0.0134	0.0001	29.1
No. 35	42	(8.3, 43.2)	(6.6, 49.6)	968	2.770	2.782	0.44	0.0163	0.0081	0.54
VI-120 - VI-30	43	(11.7, 49.3)	(8.4, 61.7)	1214	2.855	2.879	0.86	0.0090	0.0064	0.38
$w_1 = 49$	44	(21.9, 90.6)	(16.1, 112.6)	2251	3.056	3.070	0.46	0.0029	0.0021	0.37
$w_2 = 13$	45	(12.5, 69.6)	(10.7, 76.5)	1517	2.956	2.960	0.15	0.0080	0.0034	0.63
	46	(3.6, 72.3)	(7.7, 56.9)	1116	2.773	2.845	2.54	0.0566	0.0031	4.78

TABLE 3 (cont'd.)

Condition	Subject	(P_1, P_2)	(P_1^*, P_2^*)	W	TR'	TR*	d	MRP_1	MRP_2	Ratio
No. 36	42	(18.4, 34.9)	(18.0, 36.4)	1355	1.319	1.320	0.00	0.0040	0.0012	0.89
VI-120 - VI-120	43	(12.1, 44.5)	(15.5, 31.6)	1171	1.292	1.302	0.76	0.0085	0.0007	3.02
$w_1 = 49$	44	(38.8, 36.2)	(31.7, 63.0)	2372	1.361	1.370	0.66	0.0010	0.0011	0.23
$w_2 = 13$	45	(22.1, 44.7)	(22.2, 44.5)	1664	1.341	1.341	0.00	0.0028	0.0007	1.02
	46	(5.6, 61.9)	(14.3, 29.2)	1079	1.196	1.291	7.31	0.0305	0.0004	20.5
No. 37	42	(24.2, 17.7)	(31.9, 15.7)	1182	1.298	1.303	0.34	0.0024	0.0043	2.11
VI-120 - VI-120	43	(62.4, 11.6)	(37.0, 18.3)	1380	1.297	1.322	1.86	0.0004	0.0092	0.16
$w_1 = 13$	44	(84.4, 18.9)	(53.8, 27.0)	2023	1.347	1.358	0.87	0.0002	0.0038	0.21
$w_2 = 49$	45	(95.4, 11.0)	(47.5, 23.7)	1779	1.300	1.347	3.54	0.0002	0.0101	0.06
	46	(64.6, 3.5)	(27.4, 13.4)	1011	1.108	1.281	13.50	0.0004	0.0586	0.02
No. 38	42	(31.3, 36.9)	(34.1, 34.1)	887	1.353	1.354	0.04	0.0015	0.0011	1.37
VI-120 - VI-120	43	(49.2, 24.6)	(36.9, 36.9)	959	1.351	1.360	0.69	0.0006	0.0023	0.27
$w_1 = 13$	44	(87.5, 52.0)	(69.8, 69.8)	1814	1.395	1.398	0.21	0.0002	0.0006	0.36
$w_2 = 13$	45	(143.8, 63.1)	(103.5, 103.5)	2690	1.407	1.412	0.37	0.0001	0.0004	0.20
	46	(74.8, 67.2)	(71.0, 71.0)	1846	1.399	1.399	0.01	0.0003	0.0003	0.81
No. 39	42	(8.1, 13.7)	(10.9, 10.9)	1068	1.174	1.188	1.13	0.0170	0.0068	2.48
VI-120 - VI-120	43	(43.8, 19.8)	(31.8, 31.8)	3116	1.333	1.347	1.05	0.0008	0.0035	0.22
$w_1 = 49$	44	(57.7, 21.6)	(39.7, 39.7)	3886	1.347	1.366	1.34	0.0005	0.0030	0.15
$w_2 = 49$	45	(34.9, 18.2)	(26.6, 26.6)	2602	1.319	1.330	0.82	0.0012	0.0041	0.29
	46	(8.6, 4.2)	(6.4, 6.4)	627	1.012	1.044	3.03	0.0153	0.0461	0.33
No. 40	42	(25.1, 26.5)	(25.8, 25.8)	2064	1.326	1.327	0.00	0.0022	0.0020	1.11
VI-120 - VI-120	43	(24.1, 10.7)	(17.4, 17.4)	1392	1.251	1.275	1.88	0.0024	0.0106	0.23
$w_1 = 40$	44	(61.0, 34.0)	(47.5, 47.5)	3800	1.373	1.378	0.39	0.0004	0.0013	0.32
$w_2 = 40$	45	(44.1, 14.5)	(29.3, 29.3)	2344	1.309	1.340	2.30	0.0008	0.0062	0.12
	46	(17.4, 8.1)	(12.8, 12.8)	1022	1.194	1.220	2.14	0.0044	0.0170	0.26
No. 41	42	(23.3, 31.0)	(27.2, 27.2)	1683	1.330	1.332	0.15	0.0026	0.0015	1.72
VI-120 - VI-120	43	(41.6, 25.0)	(33.3, 33.3)	2065	1.346	1.352	0.41	0.0009	0.0023	0.38
$w_1 = 31$	44	(73.6, 49.4)	(61.5, 61.5)	3813	1.390	1.392	0.14	0.0003	0.0006	0.46
$w_2 = 31$	45	(71.8, 22.1)	(47.0, 47.0)	2911	1.354	1.377	1.69	0.0003	0.0028	0.10
	46	(69.1, 41.5)	(55.3, 55.3)	3429	1.383	1.387	0.25	0.0003	0.0009	0.37
No. 42	42	(28.3, 35.7)	(32.0, 32.0)	1408	1.347	1.348	0.09	0.0018	0.0011	1.56
VI-120 - VI-120	43	(62.8, 38.7)	(50.8, 50.8)	2233	1.379	1.382	0.25	0.0004	0.0010	0.39
$w_1 = 22$	44	(87.0, 60.7)	(73.9, 73.9)	3249	1.399	1.401	0.09	0.0002	0.0004	0.49
$w_2 = 22$	45	(97.1, 41.9)	(69.5, 69.5)	3058	1.390	1.398	0.57	0.0002	0.0008	0.19
	46	(59.7, 52.5)	(56.1, 56.1)	2468	1.387	1.388	0.02	0.0004	0.0005	0.78
No. 43	42	(30.5, 52.8)	(41.7, 41.7)	1083	1.364	1.369	0.38	0.0015	0.0005	2.88
VI-120 - VI-120	43	(55.9, 45.2)	(50.6, 50.6)	1314	1.381	1.382	0.05	0.0005	0.0007	0.66
$w_1 = 13$	44	(91.5, 56.9)	(74.2, 74.2)	1929	1.398	1.401	0.16	0.0002	0.0005	0.39
$w_2 = 13$	45	(73.2, 34.5)	(53.9, 53.9)	1400	1.377	1.385	0.58	0.0003	0.0012	0.23
	46	(79.8, 64.7)	(72.3, 72.3)	1879	1.399	1.400	0.03	0.0002	0.0004	0.66

TABLE 4

Subject Number		42	43	44	45	46	Sub-totals
	0.00-0.09	18	12	18	12	12	72
	0.10-0.29	5	7	6	7	7	32
d-values	0.30-0.69	7	7	6	10	3	33
	0.70-1.29	7	7	5	6	1	26
	≥ 1.30	2	9	7	7	19	44
Sub-totals		39	42	42	42	42	207

FIGURE 4.1

VI-15 schedule estimates and data.

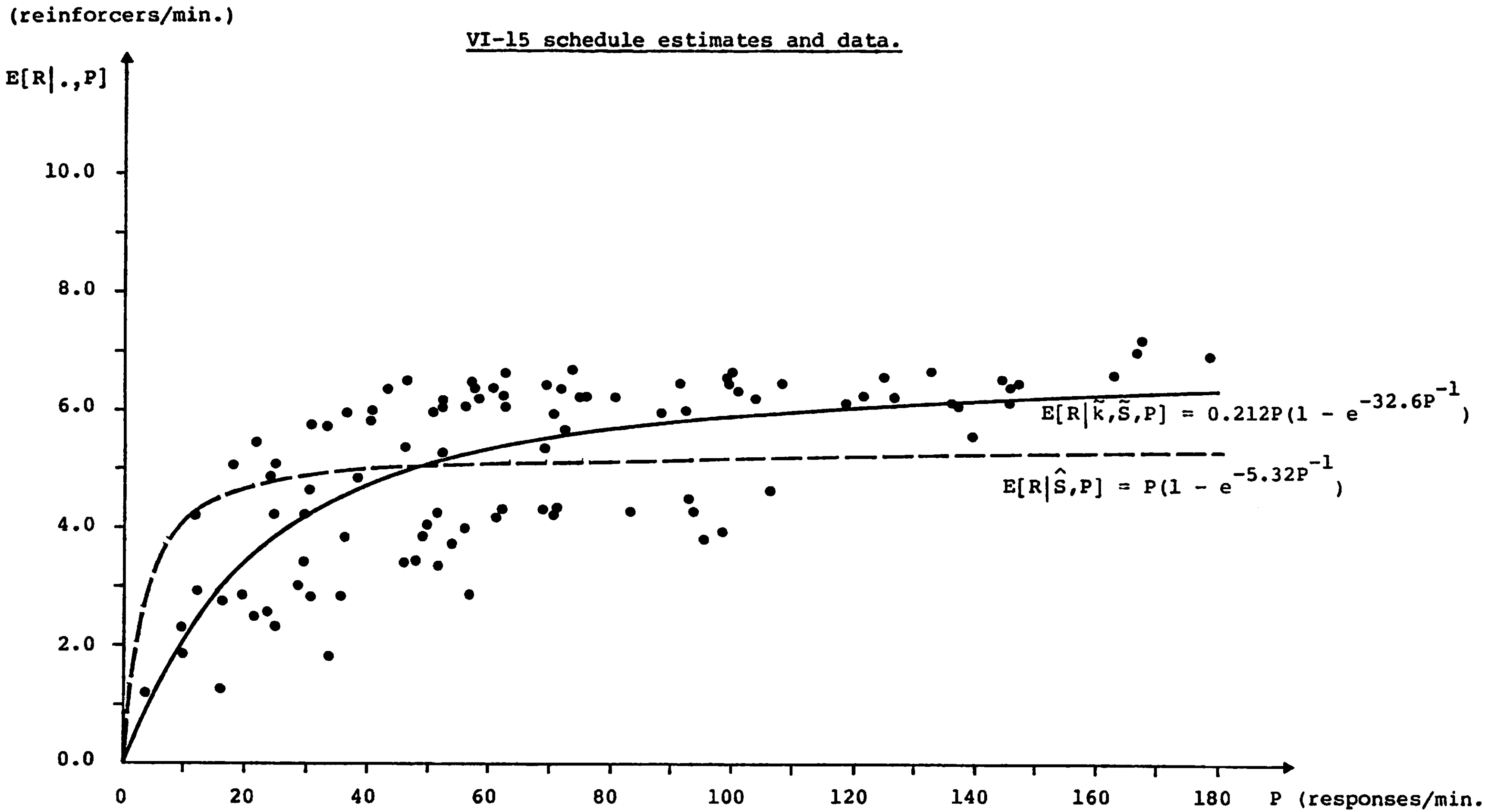

FIGURE 4.2

VI-60 schedule estimates and data.

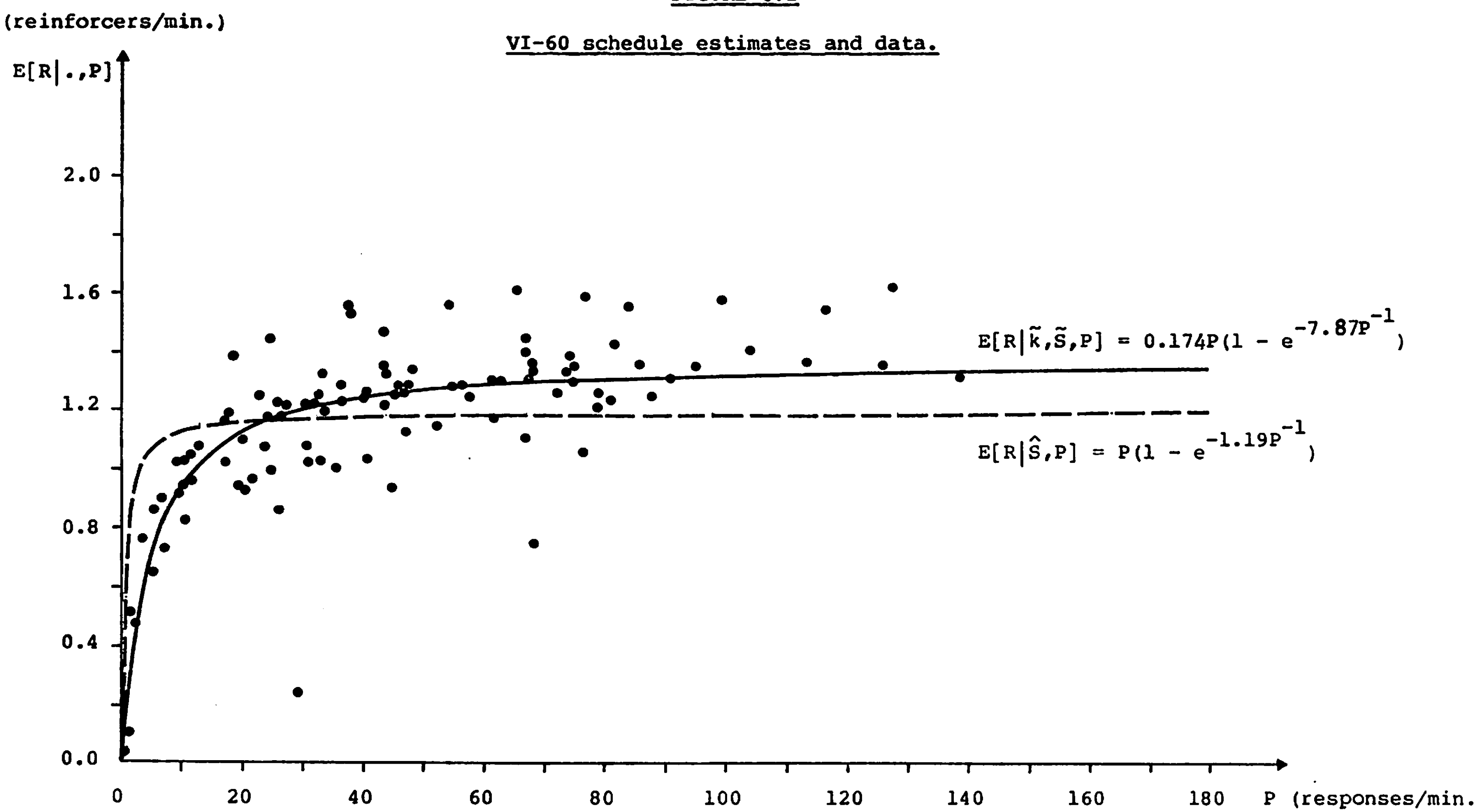

FIGURE 7.1

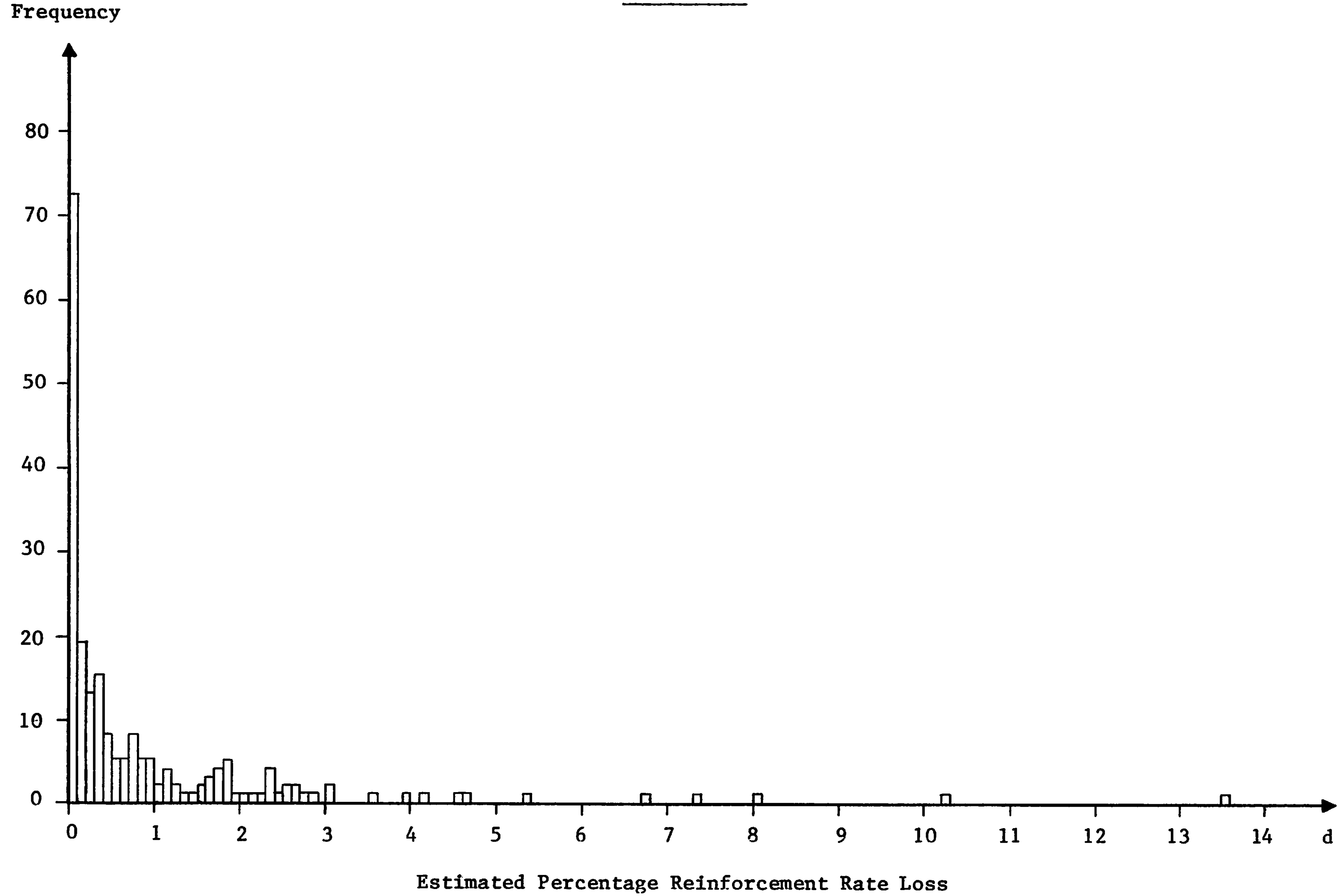
Frequency
80
70
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
d
Estimated Percentage Reinforcement Rate Loss

FIGURE 7.2

Condition 2: VI60-VI60, $w_1 = 40$, $w_2 = 13$.

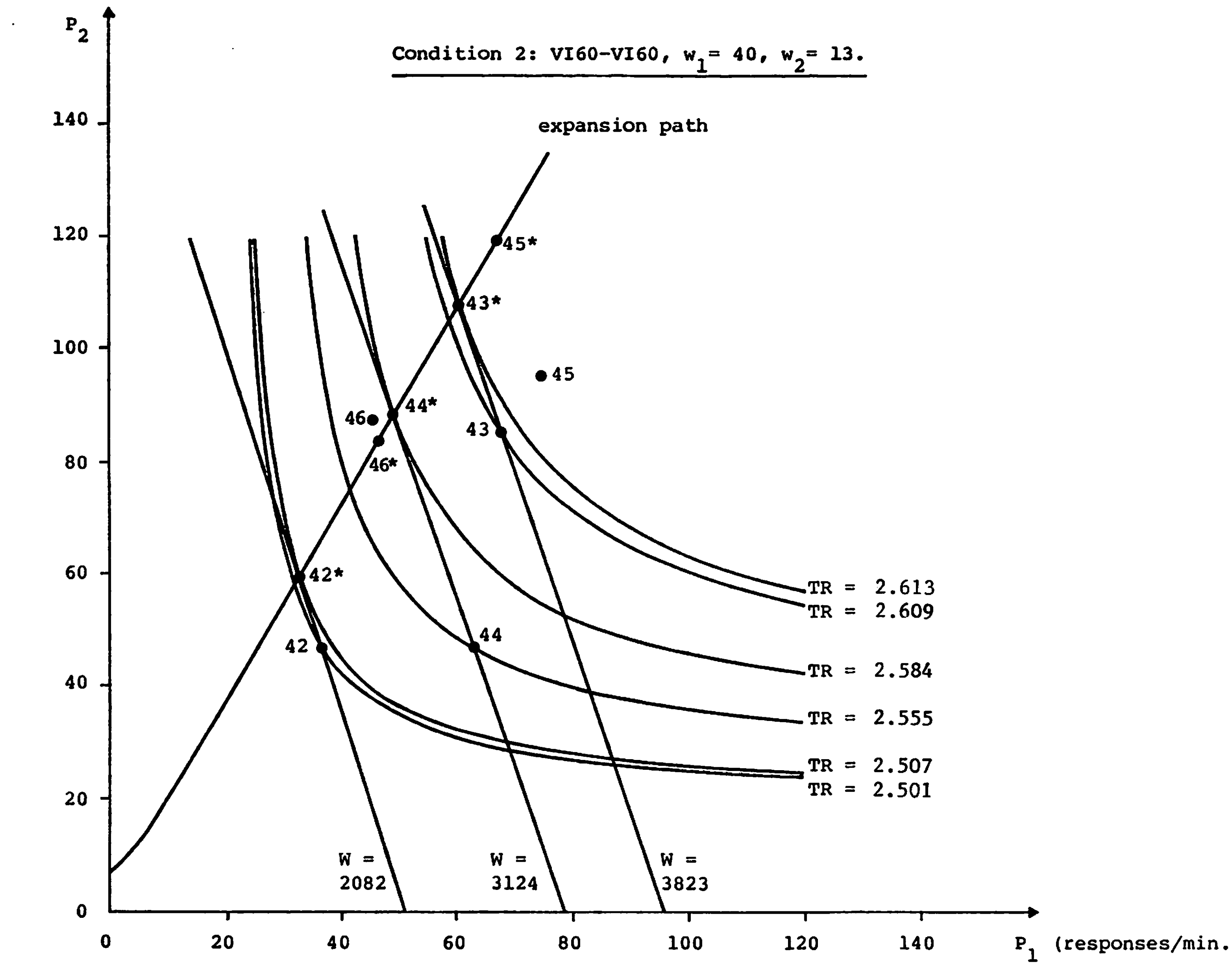

FIGURE 7.3

Condition 27: VI60-VI480, $w_1 = w_2 = 13$.

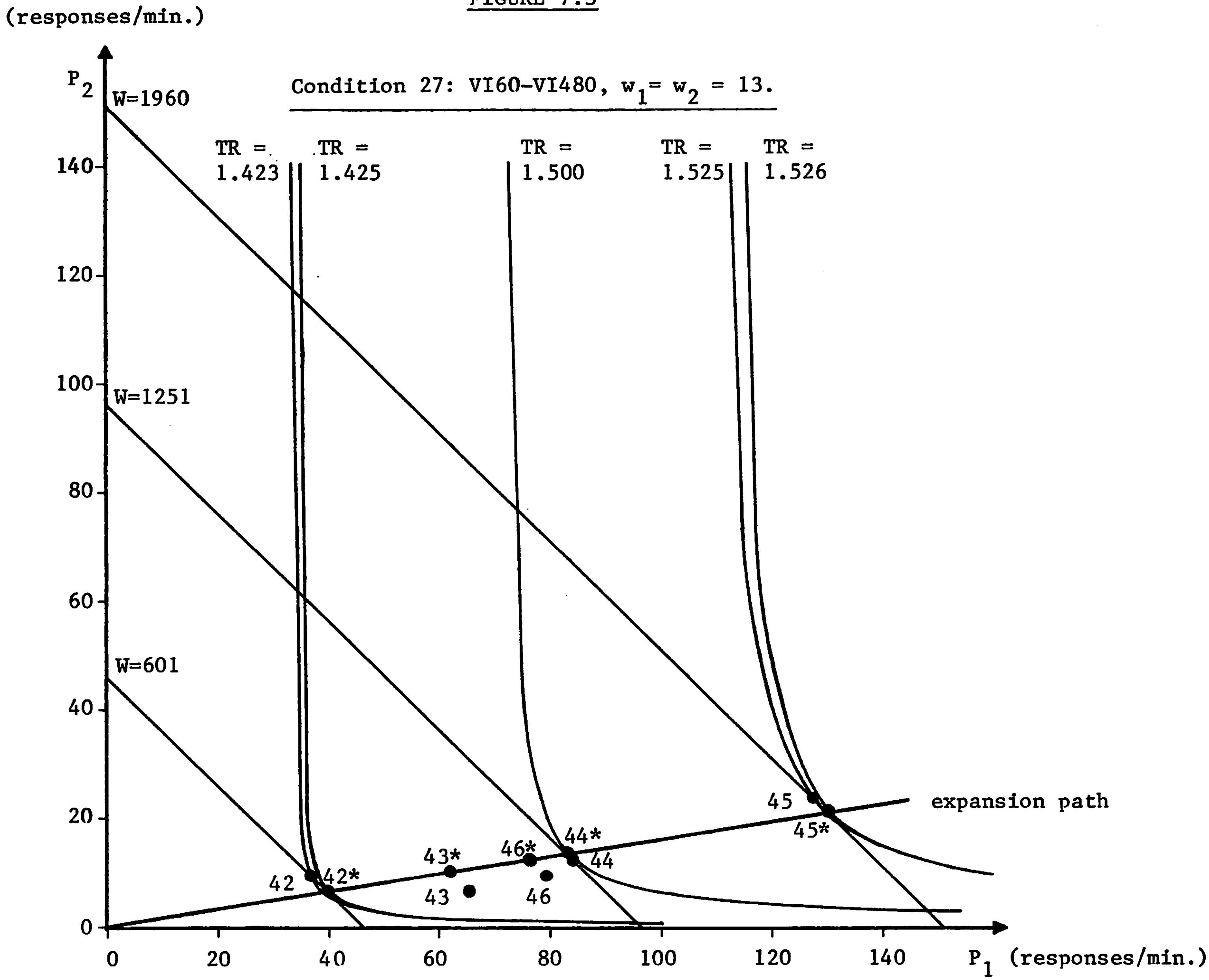

FIGURE 7.4

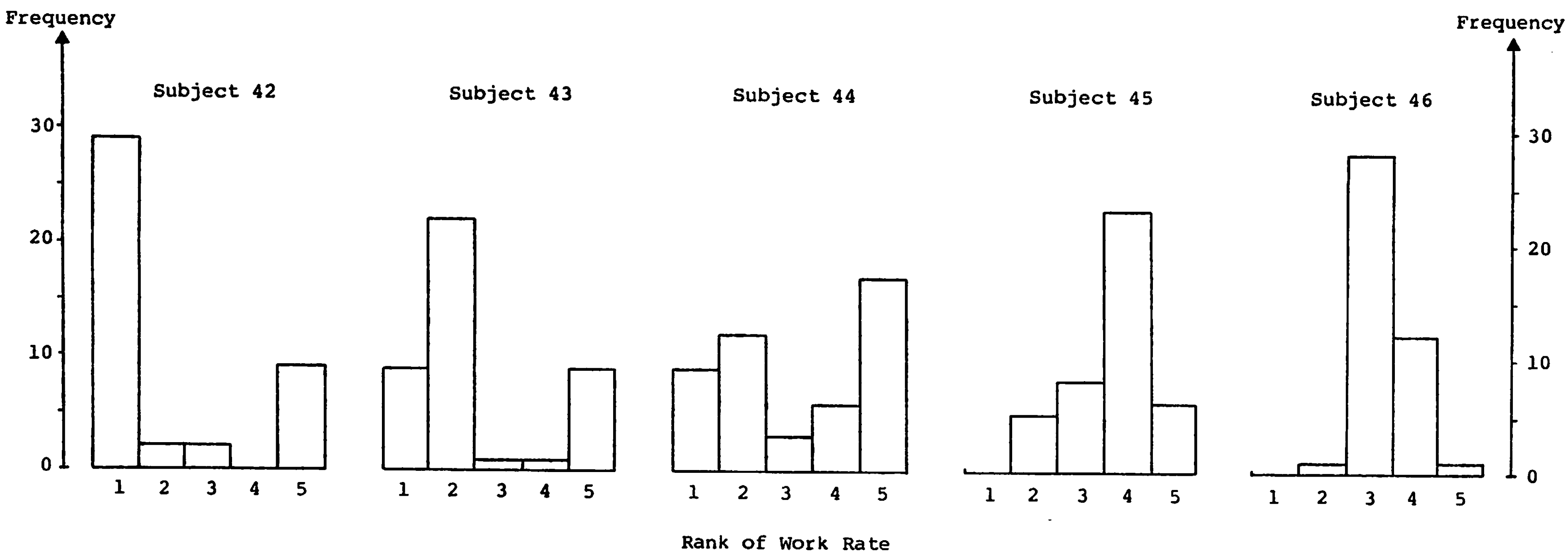
Frequency
Subject 42
Subject 43
Subject 44
Subject 45
Subject 46
30
20
10
0
1 2 3 4 5
Rank of Work Rate
Frequency

FIGURE 7.5

TR',TR* (reinf./min.)