

1987

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Citation of this paper:

Harrison, Glenn W., Jack Hirshleifer. "An Experimental Evaluation of Weakest-Link/Best-Shot Models of Public Goods." Department of Economics Research Reports, 8707. London, ON: Department of Economics, University of Western Ontario (1987).

10388

ISSN:0318-725X
ISBN:0-7714-0858-7

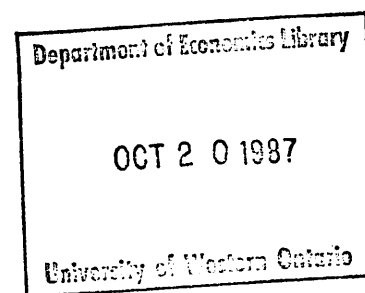
RESEARCH REPORT 8707
AN EXPERIMENTAL EVALUATION OF WEAKEST-LINK/
BEST-SHOT MODELS OF PUBLIC GOODS

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October 1987



**AN EXPERIMENTAL EVALUATION OF WEAKEST-LINK/BEST-SHOT
MODELS OF PUBLIC GOODS**

by

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* University of Western Ontario and University of California, Los Angeles, respectively. We are grateful to the Foundation for Research in Economics and Education, the Social Sciences and Humanities Research Council of Canada, and the UCLA Center for Managerial Economics and Public Policy for financial assistance. Helpful comments have been received from Joel Guttman, Mark Isaac, Charles Plott, Stan Reynolds, E.E. Rutström, and Lynne Zucker, and extremely valuable suggestions were provided by two referees.

ABSTRACT

Experiments are designed to test whether voluntary private provision of public goods meets theoretical expectations under the assumption of rational self-interested behavior. We go beyond the previous experimental literature in examining individual and group choices under alternative social composition functions, making explicit use of alternative decision protocols.

Three social composition functions were studied: (1) STANDARD SUMMATION--where, as in the usual textbook case, the social aggregate of the public good is the sum of the amounts privately provided; (2) WEAKEST LINK--where the aggregate is the minimum of the amounts individually provided; and (3) BEST SHOT--where the aggregate is the maximum of the individual provisions. Theoretical considerations indicated that the traditional result as to "underprovision" of public goods (in comparison with socially efficient totals) under STANDARD SUMMATION would be substantially mitigated under WEAKEST LINK, but aggravated under BEST SHOT.

Under our Sequential protocol we conducted a trio of experiments, one for each of the social composition functions. In each group (pair) of subjects the second-mover had enough information to make an explicit optimizing choice, but the first-mover had to act in ignorance of his or her partner's likely later behavior. All the Marginal Benefit and Marginal Cost schedules were identical although this fact was not revealed to the subjects. However, subjects had some opportunity to learn about partners' likely behavior in the course of repeated periods of play. Even though the informational conditions did not meet the requirements for this solution concept, we "predicted" that the subjects would be able to attain the perfect equilibrium. As it turned out, the actual results averaged over periods and replications squared remarkably with those predicted.

Under our second protocol, Sealed bid, both group members were in the dark as to their partners' likely behavior. The conditions for perfect equilibrium being inapplicable here, for purposes of prediction we used the weaker Nash equilibrium concept. To overcome non-uniqueness of this concept, two supplementary principles were appealed to: (i) only symmetrical solutions were considered, and (ii) among symmetrical solutions the Pareto-superior one was chosen. Since the learning problem was notably more difficult under Sealed-bid, and just what would constitute rational behavior subject to some question, we anticipated a poorer fit between observed and predicted results. While this indeed occurred, in some respects the subjects did manage to go a surprising distance toward theoretical anticipations.

For the Sequential experiments, our results may be regarded as strongly confirming a compound hypothesis that the subjects: (a) acted in a rational, self-interested way, (b) believed that their partners would behave similarly, and (c) could learn the correspondence between their own and their partner's payoff functions. For the experiments conducted under the Sealed-bid protocol, this compound hypothesis was less adequately confirmed. Further study will be necessary to specify which portions failed, and to what degree.

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CANADA

Standard economic theory predicts that self-interested agents will undersupply public goods, in comparison with efficient social totals. This inference derives from modelling the provision of public goods as a single-period Prisoners' Dilemma in a continuous-strategy space. However, people in both natural and designed experimental settings appear to solve the underprovision problem to a surprising degree, so that far less "free-riding" occurs than anticipated. As an important historical example, it has been observed that disasters tend to elicit an extraordinary degree of seemingly unselfish behavior.¹ Outsiders rush in to contribute relief to the stricken community, while community members themselves pitch in to rescue neighbors in peril and to engage in other forms of voluntary mutual aid.² In experimental settings as well, the evidence is at least mixed with regard to the observed extent of free-riding behavior.³

While humanly gratifying, these results are somewhat disturbing for standard theory. Among the possible explanations are: (i) individuals may not be as selfish, or as rational, as standard theory assumes; (ii) public goods provision often corresponds not to a single-period but to a repeated-interaction game, for which free riding is not uniquely predicted by the theory; and (iii) even in a single-period situation, public goods provision may not always be a Prisoners' Dilemma. In this study we will be making use of laboratory conditions that render the repeated-game explanation inapplicable. But, employing a generalization of the theory of public goods along the lines suggested in Hirshleifer [1983], we will show that single-period public-goods provision is indeed not always a Prisoners' Dilemma. With due attention paid to the alternative decision situations involved -- so that underprovision is predicted in some cases, but not in

others -- our results (subject to certain important qualifications) tend to confirm theoretical expectations under the maintained assumption of rational self-interested behavior.

In predicting the decisions of rational self-interested parties as to the voluntary provision of public goods, it is essential to attend to two different aspects of the game setting: the Social Composition Function (SCF) and the protocol of play. The SCF refers to the way in which individual contributions are amalgamated into an available social aggregate of the public good. The protocol corresponds to the extensive form of the game, which specifies (among other things) the sequence of moves and the informational conditions under which play takes place. In designing our experiments we have been careful to specify both the Social Composition Function and the protocol, so that comparisons could accurately be drawn between what theory predicts and what experimental subjects actually do.

Alternative Social Composition Functions

Public goods are defined in terms of a peculiar feature on the demand side: the amount produced is equally available for non-rivalrous consumption by all members of the community. But our standard models do not assume anything special on the supply side, specifically about the SCF which converts individual provisions into a socially available aggregate amount of the public good. On the standard assumption, this aggregate is always the simple sum of the individual contributions.

But the observation that people often overcome the free-rider problem in disaster situations provides a clue that the standard assumption about the form of the SCF may not be applicable. Such situations often correspond to "weakest link" environments, which are characterized by the fact that failure of any unit may be fatal to the whole. In desperate circumstances where each

person must do his duty (and even more) if the community is to survive, what appears to be self-sacrificing behavior may actually be selfishly optimal in swinging the balance between community viability and social collapse.

Table 1 displays the three simple social composition functions--relations between the individual contributions q_i and the socially available total Q of the public good--that our experiments cover. The SCF implicit in the usual theory, which of course leads to the usual prediction of free-riding, is called STANDARD SUMMATION. But this is only one point along a spectrum of possibilities. At one extreme of the spectrum is WEAKEST LINK where the available aggregate is the minimum of the individual provisions. Here, as suggested above, the generalized theory predicts that free-riding on the part of rational self-interested individuals will fall to low levels. (In fact, it should disappear entirely if the population is homogeneous.) At the other extreme in Table 1 is BEST SHOT, where the socially available amount is the maximum of the individual provisions. Under BEST SHOT, as will be shown, free-riding is predicted to be even more predominant and intractable than under STANDARD SUMMATION.⁴

The WEAKEST LINK model describes a variety of situations where each member of a social group has a kind of veto power over the extent of collective achievement. Examples might include: (i) townspeople manning sectors of a levee when the river is in flood (where any person's failure means that the water will break through and inundate the entire community), (ii) military units defending segments of the front against an enemy offensive, or (iii) a group of agents responsible for dredging successive stretches of a navigation channel (since the minimum depth dredged determines how much traffic can flow). All of these cases are parallel to the disaster situation. Once the usual protections and redundancies supporting the social

TABLE 1

Alternative Social Composition Functions

Social composition function	Formula	Predicted extent of free-riding
STANDARD SUMMATION	$Q = \sum_i q_i$	Intermediate
WEAKEST LINK	$Q = \min_i q_i$	Least
BEST SHOT	$Q = \max_i q_i$	Most

TABLE 2

Summary of Experiments

Experiment	Protocol	Social Composition Function	Periods	Replications per Period
SQ-1	Sequential	STANDARD SUMMATION	6	3
SQ-2	Sequential	WEAKEST LINK	6	3
SQ-3	Sequential	BEST SHOT	6	3
SB-1	Sealed Bid	STANDARD SUMMATION	10	5
SB-2	Sealed Bid	WEAKEST LINK	10	5
SB-3	Sealed Bid	BEST SHOT	10	3

division of labor can no longer be relied upon, breakdown is in prospect unless everyone cooperates. Turning to the opposite extreme, the BEST SHOT model typically applies when different teams engage in a contest where "victory" benefits the entire team as a public good, while the scoring rule depends solely upon the best individual performance. Examples might include: (i) anti-missile batteries firing under local control at a single incoming ICBM, (ii) gang wars in which each "family's" gunmen aim solely to assassinate the rival don, or (iii) the mice in the fable attempting to bell the cat.⁵

Of course, many intermediate and variant SCF's are also possible. Table 1 could be generalized to place arbitrary ascending or descending weights upon individual contributions (Hirshleifer [1984]). Or the SCF could be made to depend upon various parameters -- e.g., the mean, or the median, or the average of the upper or lower quartiles -- of the distribution of individual contributions. Or, under either WEAKEST LINK or BEST SHOT, there might be various provisions for "refunding" non-decisive contributions (see Bohm [1972, 1984], Van de Kragt et. al. [1983], and Palfrey and Rosenthal [1984, 1985]). But we limit ourselves here only to the simplest polar alternatives to the STANDARD SUMMATION composition function.

Alternative Protocols

A number of imaginative protocols have been designed by theorists and experimentalists to mitigate the free-riding problem in STANDARD SUMMATION environments. Influential examples include Bohm [1972][1984], Ferejohn, Forsythe and Noll [1979], Ferejohn et. al. [1982], Green and Laffont [1977], Groves and Ledyard [1977] and Smith [1977] [1979a] [1979b] [1980]. These protocols demonstrate the important normative possibility of designing decentralized mechanisms that will provide optimal or near-optimal public good provision levels.

Figure 1

MARGINAL BENEFIT (MB) AND MARGINAL COST (MC) SCHEDULES

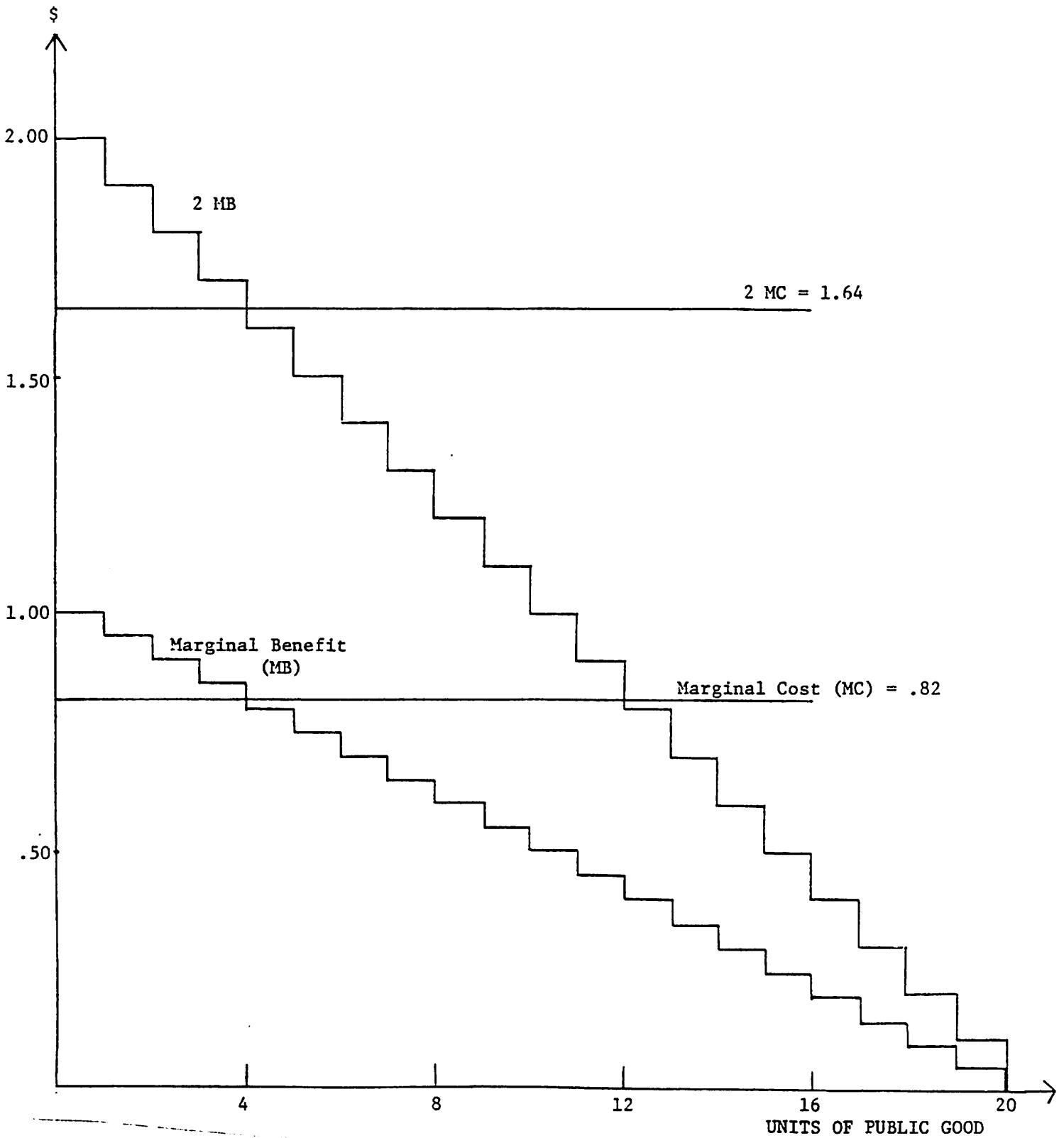


TABLE 3

Redemption Value Sheet

<u>Project Level (Units)</u>	<u>Redemption Value of Specific Units</u>	<u>Total Redemption Value of All Units</u>
1	\$1.00	\$1.00
2	0.95	1.95
3	0.90	2.85
4	0.85	3.70
5	0.80	4.50
6	0.75	5.25
7	0.70	5.95
8	0.65	6.60
9	0.60	7.20
10	0.55	7.75
11	0.50	8.25
12	0.45	8.70
13	0.40	9.10
14	0.35	9.45
15	0.30	9.75
16	0.25	10.00
17	0.20	10.20
18	0.15	10.35
19	0.10	10.45
20	0.05	10.50
21	0.00	10.50

Our experiments involved two-person groups only. We employed two basic protocols: Sequential and Sealed-Bid. The Sealed-Bid protocol is a simultaneous-move arrangement, following the direct contribution procedure of Smith [1979b], Isaac, McCue and Plott [1985] and Banks, Plott and Porter [1986]. Here in each two-person group the agents privately and concurrently specify their individual levels of provision for the public good. In contrast, the Sequential protocol is an alternating-move arrangement. In this case one agent in each group declares his or her provision level first, that decision is then made public, and then the other makes a choice in response. As we shall see, the protocol employed importantly affects both the theoretical predictions and the behavioral outcomes.

1. EXPERIMENTAL DESIGN

A summary of our experiments appears in Table 2. All subjects were economics undergraduates at the University of Western Ontario.⁶ In the Sequential protocol session there were 18 subjects segregated randomly into 9 groups of 2. Three of these groups were then randomly assigned to each of the social composition functions in each period. In the Sealed-bid protocol session we had 26 subjects in 13 groups of 2. The two sessions were run "back-to-back", with subjects from a larger pool of volunteers randomly assigned to a particular session.

Nobody ever knew the identity of his or her partner within the larger population of subjects. Furthermore, the partners were changed each period. This feature was designed to prevent possible extraneous influences, such as the desire to make friends or build reputations, from contaminating the

experiment. All subjects were given the same fixed valuation schedule for the public good, valid for each experimental period, as shown in Table 3 and pictured in Figure 1.

No subject was informed of the payoffs of any other subject in our experiments, and in particular the fact that all valuation schedules were the same was not revealed. Our theoretical analysis, in contrast, presumes that the payoffs are common public knowledge. This informational discrepancy made our experiments quite a severe test of the underlying theory. We were in effect "predicting" on the basis of a compound hypothesis that: (i) each subject was a rational self-interested economic agent, and believed that his partner was also, and (ii) the subjects correctly conjectured that their payoffs were identical. Owing to the possibility of learning, we would expect condition (ii) to more accurately describe the situation in later than in earlier experimental replications, and this in fact occurred (as will be explained in more detail below). However, we should point out that learning was far from trivially easy, since subjects were told (as in fact occurred) that the partnership assignments were to be reshuffled each period. Thus there was no chance of mutual education and accommodation as between any given pair.⁷

Under each of the two different experimental protocols, a trio of experiments was conducted -- one member of the trio corresponding to each of the social composition functions of Table 1.

In the Sequential protocol (experiments SQ-1, SQ-2, and SQ-3) one subject in each pair was randomly selected to have the first move. The first-mover was required to declare his or her own irrevocable contribution to the pair's joint provision of the public good. The other member of the pair

could then use that information in choosing a best response, in the form of a second-move decision as to how much to contribute in turn. Although the last-mover has an informational advantage over the first-mover, he is also at a severe disadvantage with respect to his ability to commit to a certain strategy. We will see that the commitment asymmetry dominates any informational asymmetry.

In the second trio of experiments under the Sealed-bid protocol (experiments SB-1, SB-2, and SB-3), each subject had to select a level of voluntary contribution in ignorance of the simultaneous choice being made by his or her partner. Here it might be thought that the poorer information (about the other side's moves) available to the pair as a group would make it more difficult for the subjects to arrive at their self-interested optimal choices. This inference was in fact supported, to a marked extent, by our experimental results.

All subjects in the Sealed-bid experiments SB-1, SB-2 and SB-3 were made familiar with the following instructions:

You are about to participate in a decision process in which one of numerous competing alternatives will be chosen. This is part of a study intended to provide insight into certain features of decision processes. The instructions are simple. If you follow them carefully and make good decisions, you might earn a considerable amount of money. You will be paid in cash.

This decision process will proceed as a series of ten periods. In each period the level of a project will be determined and financed. The "level" can be at zero "units" or more. Attached to the instructions you will find a sheet called the Redemption Value Sheet. It describes the value to you of decisions made in each period. You are not to reveal this information to anyone. It is your own private information.

During each period a level of the project will be determined. For the first unit provided during a period you will receive the amount listed in row 1 of the Redemption Value Sheet. If a second unit is also provided during the period, you will receive the additional amount listed in row 2 of the Redemption Value Sheet. If a third unit is provided, you will receive, in addition to the two previous amounts, the amount listed in row 3, etc. As you can see, your individual total payment in each period is computed as a sum of the redemption values of specific units. These totals of redemption values are tabulated for your convenience on the right-hand side of the page.

The earnings per period, which are yours to keep, are the differences between the total of redemption values of units of the project provided and your individual expenditures on the project. Suppose, for example, your Redemption Value Sheet was as below and two units were provided.

REDEMPTION VALUE SHEET (EXAMPLE)

<u>Provided Level (Units)</u>	<u>Redemption Value of of Specific Units</u>	<u>Total Redemption Value of All Units</u>
1	600	600
2	500	1100
3	400	1500

Your redemption value for the two units would be 1100 and your earnings would be computed by subtracting your individual expenditures from this amount. If 2.5 units were provided, the redemption value would be determined by the redemption values of the first and second unit plus half of the third unit, that is, $600 + 500 + (0.5)400 = 1300$.

The process by which the level of the project is decided will proceed as follows. Each unit of the project costs \$0.82. At the beginning of each period you are to write on the Expenditure Form the amount you will spend individually. This number should also be recorded on row 2 of your Individual Record of Earnings. These individual Expenditure Forms will be collected. The number of units of the project is then determined by applying one of the following three Rules:

Rule I: The number of units provided is the total of the individual expenditures divided by the cost per unit.

Rule II: The number of units provided is the smallest of the individual expenditures divided by the cost per unit.

Rule III: The number of units provided is the largest of the individual expenditures divided by the cost per unit.

You will be told at the beginning of each period which of these Rules applies to you in that period. After the level of the project has been determined it will be announced. Your individual expenditures will not be made public. Note that your individual expenditures are binding on you, irrespective of the Rule used to determine the level of the project.

When the level of the project is announced, you should enter the Total Redemption Value of all units obtained from the Redemption Value Sheet on row 1 of your Individual Record of Earnings. You should then subtract row 2 from row 1 on this record to determine your earnings for this period. Row 4 provides a place for you to record the number of units of the project provided in each period.

During this process you are not to speak to anyone or otherwise attempt to communicate. There may be several groups making decisions at once. You will be told which group you are participating with in each period and how many members are in your group. Your Individual Record of Earnings identifies your Individual Number, and has a row for you to note your Group Number in each period. The group you are assigned to in making the decision in each period will be dissolved immediately thereafter, and your new group assignment will be different each period. Furthermore, in no event will you ever be told who else is in the group with you.

Are there any questions?

The Redemption Value Sheet referred to is shown in Table 3, and was common to each participant in all the experiments. Our instructions closely follow those used by Isaac, McCue and Plott [1985, pp. 70-73] except for the references to alternative Rules and the shuffling of subjects from group to group.

The instructions to subjects in the Sequential experiments SQ-1, SQ-2, and SQ-3 were simple modifications of those reproduced above, in accordance with the changed protocol.

3. EFFICIENT OUTCOMES UNDER ALTERNATIVE SOCIAL COMPOSITION FUNCTIONS

To clarify the nature of the social composition functions, and to illustrate the basis for our theoretical predictions as to the outcomes, the three pairs of matrices shown in Table 4 represent simplified versions of the

TABLE 4

Simplified Illustration of the
Three Social Composition Functions

WEAKEST LINK

<u>Algebraic</u>			<u>Numerical</u> (b = 2, c = 1)		
	P	N		P	N
P	b-c, b-c	-c, 0	P	1, 1	-1, 0
N	0, -c	0, 0	N	0, -1	0, 0

BEST SHOT

<u>Algebraic</u>			<u>Numerical</u> (b = 2, c = 1)		
	P	N		P	N
P	b-c, b-c	b-c, b	P	1, 1	1, 2
N	b, b-c	0, 0	N	2, 1	0, 0

STANDARD SUMMATION

<u>Algebraic</u>			<u>Numerical</u> (B = 4, b = 2, c = 3)		
	P	N		P	N
P	B-c, B-c	b-c, b	P	1, 1	-1, 2
N	b, b-c	0, 0	N	2, -1	0, 0

decision processes involved. In this illustration (but not in the actual experiments) cooperation, in the form of a decision to contribute to the provision of the public good, is simply a yes/no affair: the individual is either a provider (P) or a non-provider (N).

Starting with the WEAKEST LINK case, the matrix on the left shows the respective payoffs to the four possible combinations of P and N strategies, where b is the benefit received by each player should both contribute, and c is the cost to either of contributing. If the WEAKEST LINK model is to apply, it is necessary that $b > c$. The matrix on the right is a numerical illustration for $b = 2$ and $c = 1$.

For the BEST SHOT case, b is the benefit received by each player should either contribute, while c remains the cost to either of contributing. Again, a necessary condition is $b > c$. As before, the matrix on the right is a numerical illustration assuming $b = 2$ and $c = 1$.

In the STANDARD SUMMATION case, matters are somewhat more complex since we must now distinguish two possible levels of benefits obtained. Specifically, let B signify the benefit to each player when both contribute, and b the benefit to each when only one contributes. The necessary conditions here can be expressed as $B > c > b$, but $B - c < b$. The numerical illustration on the right assumes $B = 4$, $b = 2$, and $c = 3$. As is well known, this STANDARD SUMMATION situation for the private provision of public goods is a Prisoners' Dilemma.

The efficient levels of provision of the public good are easily visualized in the simplified numerical illustrations of Table 4. For WEAKEST LINK the maximum joint payoff is $1+1 = 2$, achieved if the parties adopt the

strategy-pair [P,P]. Since, under the social composition function represented by WEAKEST LINK, when each party chooses P only 1 unit of the public good becomes available, the efficient quantity of public good provided is 1. For BEST SHOT the maximum joint payoff of $2+1 = 3$ is achieved at either of the off-diagonal cells. Here one player chooses P and the other N, which under BEST SHOT suffices to generate 1 unit of the public good. For STANDARD SUMMATION, finally, the maximum joint payoff is $1+1 = 2$, achieved if the parties both choose P. Under this social composition function, the efficient quantity of the public good is 2.

Turning from these simplified illustrations to the actual experimental situation, with benefits and costs as pictured in Figure 1, the efficiency conditions and corresponding numerical results are shown in Table 5. Notice that the efficient outcomes depend only upon the social composition functions, and not at all upon the protocols. It is also worth noting that the efficient WEAKEST LINK outcome happens to be the unique Pareto-optimal allocation. Under the two other SCF's, however, there are multiple Pareto-optimal allocations (under our assumption that the players' Marginal Cost functions are level and identical).

The interpretation of Table 5 is as follows. Under STANDARD SUMMATION each individual's contribution goes toward purchasing units of the public good to be enjoyed by both. As shown in Figure 1, each individual can always purchase a unit of the public good at a constant individual Marginal Cost $MC = \$0.82$. The individual Marginal Benefit schedule MB shown in the diagram corresponds of course to the benefits tabulated in Table 3 as "Redemption

TABLE 5

Efficient Outcomes Under Experimental Conditions

Social Composition Function	Efficiency Condition	Numerical		
		q ₁	q ₂	Q
STANDARD SUMMATION:	$MC_1 = MC_2 = MB_1 + MB_2$	6	6	12
WEAKEST LINK:	$MC_1 + MC_2 = MB_1 + MB_2$	4	4	4
BEST SHOT:	$MC_1 = MB_1 + MB_2$ and $q_2 = 0$	12	0	12
	<u>or</u> $MC_2 = MB_1 + MB_2$ and $q_1 = 0$	0	12	12

Values of Specific Units." Since the social Marginal Benefit is simply twice the individual MB, inspection of the diagram reveals that the efficiency condition under STANDARD SUMMATION, to wit, $MC_1 = MC_2 = MB_1 + MB_2$ (where the subscripts identify members of the participating pairs) is met when each individual provides 6 units of the public good. Thus $q_1 = q_2 = 6$, so that the efficient aggregate quantity is $Q = 12$.

Under WEAKEST LINK both members must contribute if a unit of the public good is to be generated that both can enjoy. Here the efficiency condition is $MC_1 + MC_2 = MB_1 = MB_2$, which is met when $q_1 = q_2 = 4$. Since under WEAKEST LINK the social amount provided is the lesser of q_1 and q_2 , the efficient social aggregate is $Q = 4$. Finally, under BEST SHOT a unit of the public good is provided when either contributes. For efficiency here one party should contribute zero while the other should set his or her $MC_1 = MB_1 + MB_2$. Numerically, the member contributing should provide $q_1 = 12$ so that the social amount generated, the greater of q_1 and q_2 , is $Q = 12$.

3. PREDICTED VERSUS ACTUAL OUTCOMES

We now come to the crucial point, comparing the experimental outcomes with those predicted under the assumption of rational self-interested behavior.

The Sequential Experiments

In these experiments the second-mover, knowing his own benefit and cost schedule and having seen his partner's prior choice, could in principle always calculate his privately optimal contribution toward purchase of the public good. However, the rational choice for the first-mover would depend upon his

partner's anticipated response. As already indicated, he does not know his partner's payoffs, nor can he be sure that the latter is a rational self-interested player. But our prediction is that the first-mover will correctly conjecture that the payoffs are identical, and will presume that the partner will choose in accordance with rational self-interest when it is his turn to play. In game theory terms, this mutual rationality condition means that we are making use of the "sub-game perfect equilibrium" concept due to Selten [1975].

On these assumptions, our predicted outcomes and profits for the Sequential group of experiments can be read from Table 6, and are pictured in Figures 2a through 2c. From Figure 1, an individual's profit π_i is the sum of his Marginal Benefits for the number of units socially provided by both partners together, less the cost of whatever units he provides himself. The subscripts 1 and 2 here identify the first-mover and second-mover of each trial pair.

Under the STANDARD SUMMATION social composition function the predicted rational choice on the part of first-mover is to contribute nothing (i.e., to choose $q_1 = 0$). Should he do so, second-mover is then forced in his own self-interest to provide $q_2 = 4$ units, making the social aggregate also $Q = 4$. It would be foolish for first-mover to "generously" choose any positive q_1 . If, for example, first-mover set $q_1 = 1$ then a self-interested second-mover would rationally respond by cutting his own provision back to $q_2 = 3$, leaving the total $Q = 4$ as before.⁸

Turning to the WEAKEST LINK social composition function, here the public good will be provided only to the extent that both contribute. The first-mover, therefore, can be confident that a rational partner would exactly match his contribution, up to $q_1 = 4$. Our consequent prediction is

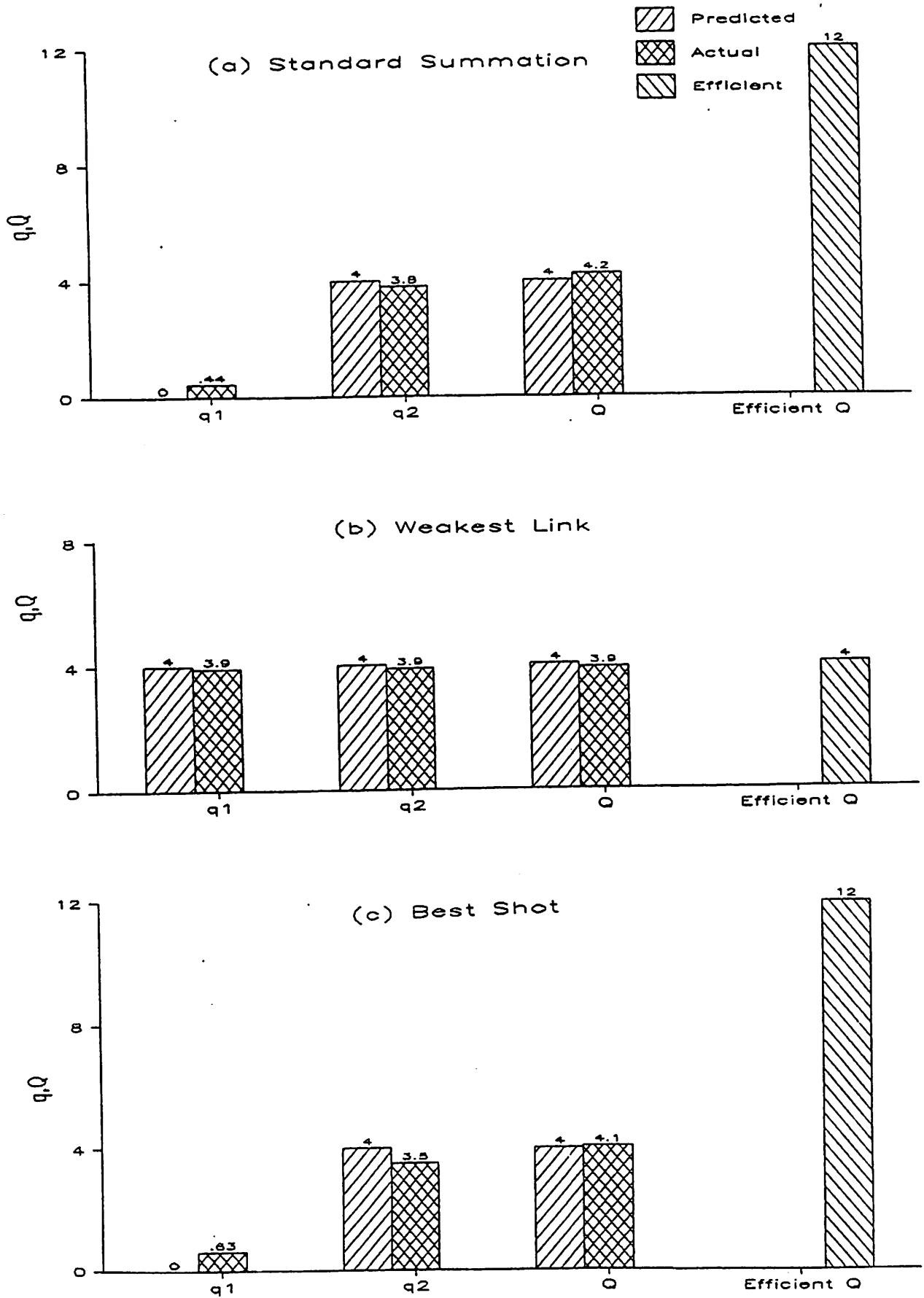
TABLE 6

Predicted and Actual Outcomes: Sequential Experiments

Social Composition Function	q_1	q_2	Q	% of efficient Q	Π_1	Π_2	Π	% of efficient Π
STANDARD SUMMATION (SQ-1)								
Efficient	Indeterminate	12	100%	Indeterminate	7.56	100%		
Predicted	0	4	4	33.3	3.70	.42	4.12	54.5
Observed	.436	3.778	4.204	35.0	3.493	.744	4.237	56.0
WEAKEST LINK (SQ-2)								
Efficient	4	4	4	100%	.42	.42	.84	100%
Predicted	4	4	4	100	.42	.42	.84	100
Observed	3.938	3.889	3.889	97.2	.377	.417	.794	94.5
BEST SHOT (SQ-3)								
Efficient	12 <u>or</u> 0	0 12	12	100%	8.70 <u>or</u> -1.24	-1.24 8.70	7.56	100%
Predicted	0	4	4	33.3	3.70	.42	4.12	54.5
Observed	.629	3.401	4.062	33.8	3.227	.873	4.100	54.2

Figure 2

OUTCOMES OF SEQUENTIAL EXPERIMENTS



$q_1 = q_2 = 4$, which means that the available social total is $Q = 4$ as well. Finally, BEST SHOT is like STANDARD SUMMATION in that a rational first-mover will contribute nothing ($q_1 = 0$), realizing that his partner would once again be left holding the bag and forced in his own self-interest to set $q_2 = 4$. Since Q under BEST SHOT is the larger of q_1 and q_2 , the social aggregate is once again $Q = 4$.

The predicted payoffs for each subject vary with the SCF and the assignment to first-mover or second-mover. They range from a high of \$3.70 (to the first-mover in SQ-1 and SQ-3) to a low of \$0.42 (to the second-mover). Despite this variation across treatments, we assume that our experimental rewards always dominate the subjective costs to agents of computing the optimal solution.

Summarizing, $Q = 4$ is the predicted social aggregate in all three cases. Under WEAKEST LINK this corresponds to efficient provision of the public good, but in the other two instances it is only one-third of the efficient amount. The predicted distributions of the individual contributions differ drastically over the three cases, as indicated in Table 6. It is also of interest to notice that in the two cases where there is an advantage of one player over another (STANDARD SUMMATION and BEST SHOT) the benefit goes to the first-mover, despite the informational asymmetry in favor of the second-mover.

The "actual" figures reported in Table 6 and shown in Figure 2 are the experimental results averaged over 6 periods and 3 replications, or 18 trial-pairs for each of the three social composition functions. As can be seen, the observed results square remarkably with the theoretical predictions.

Furthermore, detailed inspection of the trial-by-trial data reveals that essentially all of such discrepancies as appear in the pooled averages were due to mistaken choices of subjects in their very first or second decision periods. These discrepancies almost always took the form of an "excessive" contribution by the first-mover in the STANDARD SUMMATION and BEST SHOT cases, or a "deficient" contribution in the WEAKEST LINK case. Thus, the parties were able to learn rapidly, despite the informational handicap to rationally optimal choice-making. Hence our predicted results were satisfied quite remarkably under what we regarded as a somewhat severe test.

One possibly puzzling aspect of the data is why, since under BEST SHOT the relation $Q = \max(q_1, q_2)$ applies on any given trial, the average observed social aggregate $Q = 4.062$ was not identical with the average observed $q_2 = 3.501$ -- which is the larger of the averaged q_1 and q_2 . The reason is that although (as predicted) under BEST SHOT the second-mover's q_2 was in fact almost always larger than his partner's q_1 , there were a few instances in early decision periods in which q_1 was larger than q_2 . Thus, the overall averaged Q ended up higher than the averaged q_2 . A corresponding discrepancy in the other direction could have occurred under WEAKEST LINK, where $Q = \min(q_1, q_2)$ for any given trial. But in fact it never did; in the WEAKEST LINK experiments second-mover's contribution never exceeded first-mover's, and so the average of the Q provided was the same as the averaged q_2 . Of course, given this informational situation it would never be rational under WEAKEST LINK for second-mover to exceed first-mover's contribution. This difference between the BEST SHOT and WEAKEST LINK outcomes is therefore another subsidiary confirmation of our rationality prediction.

The Sealed-bid Experiments

The informational obstacles to rational decision-making, already rather severe under the Sequential protocol, are considerably more onerous under the Sealed-bid protocol. In the Sequential experiments one of the players (the second-mover) could always make his or her decision with all relevant information in the open. Under Sealed-bid, in contrast, each of the players had to choose in the dark as to his partner's move. Not only did this ignorance make the decision at any moment more difficult, it also limited what could be learned from experience. So in this group of experiments we anticipated a considerably less perfect match between theoretical and actual results. (Indeed, as will be shown shortly, the theoretical "predictions" themselves become somewhat problematic.) Because of the greater anticipated variability of results, in this group of experiments we generally allowed for more periods of learning and more replications as indicated in Table 2. Table 7 and Figures 3a through 3c summarize the predictions and actual observations under the Sealed-bid protocol.

In the Sealed-bid experiments we employ the Nash equilibrium (NE) solution concept. The Nash equilibrium, for present purposes, may be defined as a strategy-pair such that neither player would find it advantageous to revise his choice given the other's selected strategy. But it turns out that the NE is not unique in any of the cases considered, hence a supplementary principle or principles had to be appealed to. We called upon two such principles. The first is symmetry. Given the completely parallel situations of the two players in the Sealed-bid experiments, we selected as our predicted solution only among those NE's such that the members of each pair make equal contributions to the public good.

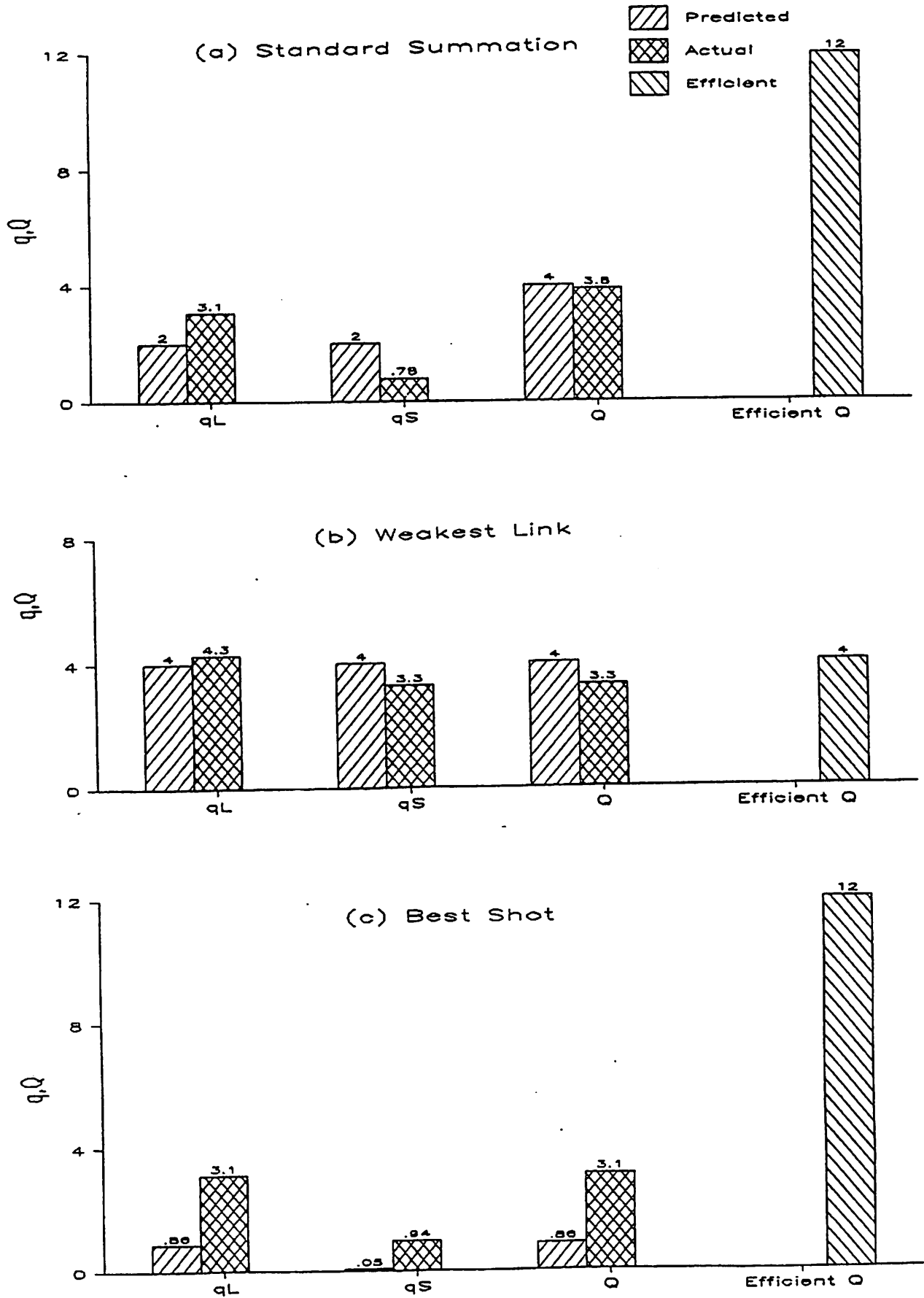
TABLE 7

Predicted and Actual Outcomes: Sealed-Bid Experiments

Social Composition Function	q_L	q_S	\bar{q}	Q	% of efficient Q	Π_L	Π_S	$\bar{\Pi}$	Π	% of efficient Π
STANDARD SUMMATION (SB-1)										
Efficient	Indeterminate	Indeterminate	6	12	100%	Indeterminate	Indeterminate	3.78	7.56	100%
Predicted	2	2	2	4	33.3	2.06	2.06	2.06	4.12	54.5
Observed	3.063	.782	1.922	3.845	32.0	.828	2.640	1.734	3.468	45.9
WEAKEST LINK (SB-2)										
Efficient	4	4	4	4	100%	.42	.42	.42	.84	100%
Predicted	4	4	4	4	100	.42	.42	.42	.84	100
Observed	4.287	3.290	3.659	3.290	82.3	-.386	.381	-.0025	-.005	-0.6
BEST SHOT (SB-3)										
Efficient	12	0	6			8.70	-1.24	3.78		
	<u>or</u>		12		100%	<u>or</u>			7.56	100%
	0	12	6			-1.24	8.70	3.78		
Predicted	.821	.061	.441	.821	6.8	.109	.732	.421	.842	11.1
Observed	3.129	.944	2.036	3.129	26.1	.288	2.080	1.184	2.368	31.3

Figure 3

OUTCOMES OF SEALED-BID EXPERIMENTS



(In the BEST SHOT setting, however, we must also examine asymmetric solutions, for reasons to be made clear below). We also had to employ one other supplementary principle, Pareto-dominance, to be discussed shortly when the WEAKEST LINK case is taken up below.

Under STANDARD SUMMATION, the NE's constitute an infinite class of outcomes, to wit, the continuum of paired non-negative public-good provisions that sum to 4. Among the possibilities are $(q_1, q_2) = (0, 4), (3, 1), (2.5, 1.5)$, etc. If, for example, the parties had chosen the respective provisions $(q_1, q_2) = (3, 1)$, neither would be able to profit by a unilateral revision of his choice. The sole symmetrical member of this class of solutions is obviously $(2, 2)$. Hence our predicted provisions of the public good are 2 for each player. In summarizing the actual data here, Table 7 reports separately the average of the larger provisions in each pair, denoted q_L , and of the smaller, denoted q_S . In this symbolism, our prediction under STANDARD SUMMATION is $q_L = q_S = 2$, so that the social aggregate is $Q = 4$. Notice that while this predicted aggregate is the same as for STANDARD SUMMATION under the previous Sequential protocol, the predicted distribution within pairs has changed drastically from $(4, 0)$ to $(2, 2)$.

Turning now to WEAKEST LINK, here the Nash equilibria once again comprise a continuum of outcomes -- to wit, all the pairs of the form (x, x) such that $0 \leq x \leq 4$. Possible instances include $(0, 0), (1.5, 1.5), (3, 3)$, and $(4, 4)$. Reference to Figure 1 will indicate that, for example, if the parties had each chosen to provide 3 units then neither member of the pair would want to unilaterally revise his choice. Here, since all the NE's are symmetrical, we must call upon our second supplementary principle -- Pareto-dominance. The justification is that since all the NE's pay off equally to the parties,

the most attractive and only reasonable NE should be a "meeting of the minds" such that the mutual profit is as favorable as possible. Using this supplementary principle in addition to symmetry, the predicted provisions are $q_L = q_S = 4$. Under the WEAKEST LINK social composition function, the aggregate quantity of the public good would then be $Q = 4$.

Finally, under BEST SHOT matters are somewhat complicated. There are only two deterministic NE's, in each of which one player provides 4 units of the public good and the other none. Both of these solutions evidently fail to satisfy the symmetry principle. There is, however, a symmetric mixed-strategy NE. Although actually finding it posed quite a daunting problem for our experimental subjects, that was nevertheless the "prediction" we adopted. By varying the contribution level one cent at a time, subjects could effectively provide units of the public good in increments of $q = 0.01219$. The equilibrium mixed strategy involved providing zero units of the public good with probability 0.80012, the remaining probability being distributed almost evenly over every nonzero quantity from $q = 0.01219$ to $q = 4$.⁹

As summarized in Table 7, the results in the Sealed-bid experiments fall considerably short of the excellent matches between predicted and actual results achieved under the Sequential protocol. The one exception is the WEAKEST LINK case, where the observed results here do track the theoretical prediction reasonably well -- though still not nearly as closely as under the previous protocol.

For STANDARD SUMMATION, the predicted equal distribution of the public-good provision, $q_L = q_S = 2$, was not borne out. It looks as if the partners were groping in the dark, trying out all kinds of possibilities, as evidenced by the huge spread between the average of the larger provisions ($q_L = 3.063$) and of the smaller ($q_S = 0.782$). Surprisingly, the average social aggregate $Q = 3.845$ was quite close to the theoretical prediction

$Q = 4$. But this average is misleading, as it hides the serious undershooting and overshooting that occurred in many cases and caused a loss of profit to the players. We discuss this further in the next sub-section.

Finally, for BEST SHOT the shoe is somewhat on the other foot. Here the larger vs. smaller relative provisions are heavily disproportionate as predicted. But, in aggregate, far more units are being provided than predicted. In consequence, however, the parties are getting substantially closer to the efficient solution -- in fact, they are generating an average 26.1% of the efficient number of units, rather than the mere 11.7% that the theory indicated.

Provision Versus Profit

Up to now our evaluation of the experimental results has run entirely in terms of individual and social provisions of the public good: the efficient, predicted, and experimentally observed magnitudes q_i and Q . For some purposes, particularly with regard to degree of efficiency achieved, it is more accurate to think in terms of individual and group "profit" -- a term used here in place of what the textbooks would call consumer surplus.

Among the points of interest, in comparing the results in terms of public-good provisions versus profits, are the following:

1. Quite commonly the partner contributing the smaller provision reaps the larger profit, a result stemming from the nature of public goods and the benefit of free-riding.

2. In terms of efficiency achieved, the results tend to "look better" when scaled in terms of aggregate profit Π rather than in terms of aggregate social provision Q . The reason is that an efficiency failure essentially always takes the form of a shortfall in the social provision of the public good; given the fact of diminishing returns, the shortfall involves units of lower Marginal Benefit than the units actually provided. This argument also indicates why efficiency is more correctly measured in terms of aggregate profit rather than number of units of the public good provided.

3. One notable exception to the preceding generalization is the result for WEAKEST LINK under the Sealed-bid protocol. Here the observed individual and aggregate provisions q_S , q_L , and Q all are not too far from the 100% efficiency predictions, but the π_L and Π profit observations are way off the mark -- in fact, π_L is so heavily negative as to just tilt Π into the negative region. But these anomalies are somewhat "accidental". It so happened that in WEAKEST LINK the theoretical aggregate profits are very small in magnitude compared to the other two social composition functions -- 0.84 versus 7.56. Since each single unit provided costs 0.82, any substantial error made by any individual -- particularly an overshooting -- even on a single trial was liable to seriously affect the overall average. What occurred here, specifically, is that in the very first period when the subjects were still operating entirely in the dark, one player in each of two experimental pairs overshoot by enough to generate a relatively huge negative profit. These two instances were numerically heavy enough to dominate

the average calculated over 50 trials, since in all the other cases the observed profits were (as predicted) quite close to zero in numerical magnitude.

4. In one case (STANDARD SUMMATION under the Sealed-bid protocol), the aggregate Q observed squares nicely with prediction whereas the aggregate Π observed does not. The reason is, as indicated earlier, that the average aggregate Q represented a cancelling-out of some instances of serious undershooting and overshooting. Hence in this case the efficiency achieved as measured by profit looks worse (and actually is worse) than the efficiency indicated by the Q measure.

5. SUMMARY

The experiments reported on here were designed to test whether voluntary private provision of public goods met theoretical expectations under the assumption of rational self-interested behavior. We go beyond the previous experimental literature in examining individual and group choices under alternative social composition functions, making explicit use of alternative decision protocols.

Three social composition functions were studied: STANDARD SUMMATION, WEAKEST LINK, and BEST SHOT. Theoretical considerations indicated that the traditional result as to "underprovision" of public goods under STANDARD SUMMATION would be substantially mitigated under WEAKEST LINK, but aggravated under BEST SHOT.

Under our Sequential protocol we conducted a trio of experiments, one for each of the social composition functions. In each group (pair) of subjects the second-mover had enough information to make an explicit

optimizing choice, but the first-mover had to act in ignorance of his or her partner's likely later behavior. The actual results averaged over periods and replications squared remarkably with those predicted.

Under our Sealed-bid protocol both group members were in the dark as to partners' likely behavior. To overcome non-uniqueness of the NE here, two supplementary principles were appealed to: (i) only symmetrical solutions were considered, and (ii) among symmetrical solutions the Pareto-dominant one was chosen. Since the informational problem was notably more difficult under Sealed-bid, and just what would constitute rational behavior subject to some question, we anticipated a poorer fit between observed and predicted results. While this indeed occurred, in some respects the subjects did manage to go a surprising distance toward theoretical anticipations.

For the Sequential experiments, our results may be regarded as strongly confirming a compound hypothesis that the subjects: (a) acted in a rational, self-interested way, (b) believed that their partners would behave similarly, and (c) correctly conjectured that their payoffs were symmetric with those of their partners. For the experiments conducted under the Sealed-bid protocol, this compound hypothesis was less adequately confirmed. Further study will be necessary to specify which portions failed, and to what degree.

FOOTNOTES

- 1 Even assuming a degree of benevolence, so that improvements to the well-being of disaster victims enter positively into other people's utility functions, there is a free-rider problem. Each potential donor is motivated to contribute less, in the expectation that others will also be providing the desired "good" -- assistance to those in need.
- 2 This "disaster syndrome" has been described and analyzed by a number of economists including Hirshleifer [1963], Dacy and Kunreuther [1969], and De Alessi [1975].
- 3 The experimental evidence ranges from Chamberlin [1978], Marwell and Ames [1979] [1980] [1981] and Schneider and Pommerehne [1981] whose results tend to refute versions of the free-rider hypothesis, to Dawes [1980], Dawes et. al. [1986] and Isaac, McCue, and Plott [1984] who tend to confirm it. Kim and M. Walker [1984] and Isaac, J. Walker and Thomas [1983] have explored various combinations of design features that might explain these seemingly divergent results.
- 4 A number of other studies have dealt, implicitly at least, with social composition functions bearing a family resemblance to WEAKEST LINK or BEST SHOT. Mueller [1979, pp. 13-14] uses the term "jointness of supply" to describe a weakest-link type of situation, where a public good will not be provided at all unless everyone contributes, and correctly indicates that free-riding should then be minimal. Bliss and Nalebuff [1984] discuss "dragon slaying", which is a best-shot type of public good. Lipnowski and Maital [1983] also discuss a best-shot situation which is even closer to the way in which we operationalize it (viz., in terms of the treatment of contributions by agents who do not provide the "best" shot).
- 5 This is essentially the same as the "dragon slaying" problem in Bliss and Nalebuff [1984].
- 6 The dollar payoffs to the subjects were adjusted using the exchange rate (C \$1.32 to US \$1.00) prevailing at the time of the experiments.
- 7 We have subsequently conducted a "complete information" series of experiments, the results being virtually identical with those reported here (and somewhat closer to the BEST SHOT predictions).
- 8 This is of course a standard proposition in public-good theory. As a slight qualification, there will in general be some "wealth effect" owing to the fact that each party's contribution enriches the other, thus making each of them reciprocally willing to purchase somewhat more

of the public good. No wealth effect is allowed for in the experimental Marginal Benefit Schedule given to the subjects. It has been shown, however, that in the provision of public goods any such wealth effect will essentially always be of negligible magnitude (McGuire [1974], Margolis [1982, pp. 19-21].)

- 9 The successive probability weights over the nonzero provisions are remarkably level, though rising very slightly as q increases. Each separate discrete probability is very small ($<.008$). To a good approximation, then, the predicted distribution involves a .8 probability of providing exactly $q = 0$, around a .05 probability of providing between $q = 1$ and $q = 2$ units, and further .05 probabilities for each of the successive intervals between 1 and 2 units, between 2 and 3 units, and between 3 and 4 units. (We would like to thank an anonymous referee for improving our understanding of the equilibrium mixed strategy.)

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