

1986

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Citation of this paper:

Konieczny, Jerzy D.. "Inflation and Costly Price Adjustment: A Macroeconomic Analysis." Department of Economics Research Reports, 8603. London, ON: Department of Economics, University of Western Ontario (1986).

ISSN:0318-725X
ISBN:0-7714-0729-7

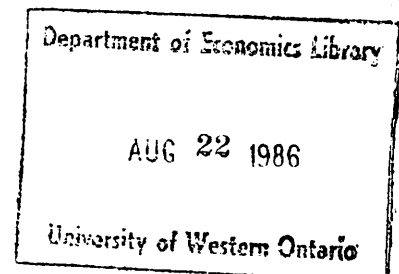
RESEARCH REPORT 8603

INFLATION AND COSTLY PRICE ADJUSTMENT:
A MACROECONOMIC ANALYSIS

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A general equilibrium model in which firms face fixed costs of price adjustment is constructed. The general equilibrium requirement that the average real price be equal to one is shown to be of crucial importance. Relative price dispersion is positively correlated with inflation and the costs of changing prices. Increased relative price variability leads, for a class of profit functions, to smaller output and higher employment. This is because output variability increases and the firms face increasing marginal costs. Real wages rise but profits, money holdings and welfare fall. The effect on the frequency of price changes is ambiguous. The optimal rate of inflation is zero.

I INTRODUCTION

High inflation rates in the 1970s led to increased interest in analysis of the inflationary process. One approach builds on the assumption that changing prices is costly. A predecessor of this literature is a well known paper by Barro (1972)¹; later papers include Sheshinski and Weiss (1977,1979,1983), Mussa (1977,1981), Rotemberg (1982,1983), Danziger (1983, 1984, 1985) and Parkin (1985).

The papers by Sheshinski and Weiss, Mussa and Danziger are set in a microeconomic, partial equilibrium framework.² A monopolistic firm is assumed to face fixed costs of price adjustment and maximize profits which are a function of its real price.³ Due to the costs of price changing it cannot increase prices in step with inflation, which is modelled either as a deterministic or a stochastic process. Under general assumptions about the nature of the profit function and the inflationary process the optimal pricing policy is shown to be of the (S,s) type, known from the inventory literature (Scarf, 1959).⁴ The nominal price is held constant over intervals of time and the real price is allowed to vary between two fixed bounds. Whenever it reaches the lower bound, s, the nominal price is increased so that the new real price is equal to the upper bound, S. The papers derive comparative statics results. It is shown, in general, that real price variability is positively correlated with the rate of inflation and size of the costs of changing prices while frequency of price changes is positively correlated with inflation and negatively with adjustment costs. Variability of inflation appears to have qualitatively similar effects to changes in adjustment costs.⁵

This approach appears to be of interest to the macroeconomist. The (S,s) pricing policy leads to prices which are sticky in an optimal sense, leaving no mutually advantageous trades unexploited.⁶

To make the model suitable for macroeconomic analysis, individual firms' behaviour should be consistently aggregated in a general equilibrium framework. This is the main goal of the paper. It is a difficult task; as Lovell mentioned in the context of inventory models, "the main disadvantage of the (S,s) decision rules is that aggregation is extremely difficult" (Lovell, 1981, p. 511). The difficulty lies in the requirement that the expected and actual rates of inflation coincide. Once the expectations are given, each firm's time path of prices is determined. At each moment of time, aggregation over firms should produce the expected inflation rate.⁷

The problem is solved in the paper by considering steady states of an economy in which firms are uniformly distributed with respect to their last price change. In a recent paper Caplin and Spulber (1985) show, in a stochastic model, that this distribution in fact occurs in steady state.

Under this assumption the expectations of the firms are rational: they follow (sS) pricing policies and, given their expectations of inflation the Divisia-type price index of their prices increases at a rate equal to those expectations.

In this type of framework the main problem with a partial equilibrium approach is that the (single) monopolistic firm can set any real price it wants. Parkin (1985) and Rotemberg (1983) construct macroeconomic models

on the basis of optimal pricing policy derived for a single firm. The average real price in the economy must, however, be equal to one. This condition, called the macroeconomic constraint in the paper, is not redundant. It is easy to show that a change in the inflation rate changes the average real price of a firm which follows the (sS) type policy. This relationship depends on the form of the real profit function and thus the average real price in an economy in which firms follow microeconomic considerations will, in general, be different from one.⁸ It is shown in the paper that the macroeconomic variables: aggregate output and real wage must adjust for this constraint to be met.

Rotemberg (1982) sets similar goals for his analysis. The crucial difference is in the treatment of the price adjustment costs. He assumes that they are convex in the size of price change and, by solving the dynamic programming problem of firms, derives time paths of prices and output. The assumption is crucial for his model; the optimal pricing policy is not of the (S,s) type, but rather leads to a partial adjustment rule. If firms perceive changes in their optimal (in the absence of changing price adjustment costs) price, they adjust prices slowly so as to avoid large, costly changes. It is, however, inherent in the microeconomic literature that the price adjustment costs be bounded away from zero.

The paper generalizes the partial equilibrium, micro models mentioned above into a macroeconomic framework. The model presented is an equilibrium one in a double sense: first, all markets continuously clear and, second, given the costs of price adjustment, no agent can benefit from altering his actions. The model also provides a rationalization for the

price stickiness apparent at the micro level.

It is assumed throughout the paper that the costs of price changing are fixed and independent of the size of price change. This is the simplest assumption given the requirement that they be bounded away from zero. Analysis of those costs is beyond the scope of the paper (for some details see, for example, Alchian (1969), Rotemberg or Sheshinski and Weiss). They are modelled here as real costs which require a fixed amount of labour effort. The results of the paper are not changed if the costs are thought of as subjective, as in Rotemberg (1982).

The analysis shows that the positive relationship of the inflation rate and/or costs of price adjustment with the variability of relative prices (Sheshinski and Weiss, Danziger, Barro) is not altered by macro-economic considerations.

For a class of profit functions which meet a condition derived in part IV, in particular for symmetric (and so also for quadratic) functions inflation is shown to reduce output, real money balances and firms' profits and increase employment and real wages. The reason is simple: as relative price variability increases with inflation, so does variability of quantities produced (there are no inventories). As firms face increasing marginal costs, the rise in variability of production leads, given the average output, to higher demand for labour (which is the only input). Thus, real wages increase and profits of the firms fall. Eventually, a lower average output is produced with higher average quantity of the labour input. Thus the model implies a Philips Curve which is positively sloped with respect to employment but negatively sloped with respect to output.

It should be noted that those relationships do not depend on the effect of inflation on labour used in price changing; in fact, an increase in inflation has ambiguous effect on the frequency of price adjustment.

The model has strong welfare implications: even though inflation increases both employment and real wages the consumers/workers are worse off as they consume less output, work more and hold smaller real balances. Inflation redistributes income from profits to wages but the fall in profits more than offsets the increase in wages.

With general functional forms of the profit functions a rise in inflation reduces output or increases real wages.

Finally, since the effects are identical for the case of deflation, the model implies that the optimal rate of inflation is zero. That is, of course, dependent on the way money enters the economy. Here it is assumed that interest is paid on existing money holdings so that real return on money is not affected by inflation.

The plan of the paper is as follows: the model is presented in the next section. It is solved in section III. In the following section the comparative statics results are derived and analyzed. The final section contains discussion and extensions.

II THE MODEL

The economy consists of a continuum of monopolistic firms, a representative consumer and a government.

Notation:

- y^d, y^s - demand/supply for a firm's product;
 Y - aggregate demand factor (defined below);
 D - goods demand (aggregate);
 ℓ^d, ℓ^s - labour demand/supply (aggregate);
 m^d, m^s - real money demand/supply
 P - the nominal price index;
 $P(x), p(x)$ - firm's x nominal/real price; $p(x) = P(x)/P$;
 c - the cost of changing prices (in units of labour);
 w - the real wage;
 g - the inflation rate;
 $\tau \equiv \ln(S/s)/g$ - the length of interval between price changes.

(a) Firms

Each monopolistic firm produces a single, nonstorable good. Except for the difference in their output, all firms are identical: they face the same demand functions and use the same technology.⁹ It is convenient to index firms by x , $x \in [0,1)$ (one can think of x as indexing goods) and analyze a typical firm. Its demand function is given by:

$$(2.1) \quad y^d = Y \cdot p(x)^{\frac{1}{\gamma-1}} \quad 0 < \gamma < 1$$

Labour is the only factor in this economy. The technology is described by the inverse production function:

$$(2.2) \quad \ell^d = \ell(y^s) \quad \ell', \ell'' > 0$$

Thus the firm faces increasing marginal costs. $\ell(\cdot)$ is such that the firm's real profit function is strictly quasiconcave in p .

The firm satisfies demand at all times ($y^s = y^d \equiv y$). It does so by varying its nominal price and output. Labour input (and thus output) can be varied costlessly; on the other hand, each price adjustment requires labour effort, quantity of which is independent of the size of the change. Thus costs of adjusting prices depend on the real wage.

The firm maximizes real profits per unit of time.¹⁰ Its profit function is thus seen to depend on its real price, aggregate demand factor, real wage, costs of price adjustment and inflation rate. From its point of view the last four variables are exogenously determined. The profit function can thus be written as:

$$(2.3) \quad F = F(p, \hat{Y}, \hat{w}, \hat{c}, \hat{g})$$

where hats have been put over variables exogenous to the firm.

The firm's problem is to maximize F with respect to p , given c , Y , w and g . It is therefore identical to the one solved in a partial equilibrium framework.

The assumptions made above are sufficient for the optimal pricing

policy to be of the s, S type.¹¹ The first-order conditions are:

$$(2.4) \quad G = \frac{1}{\tau} \int_0^{\tau} \{Y(Se^{-gt})^{\frac{Y}{Y-1}} - w\ell[Y(Se^{-gt})^{\frac{1}{Y-1}}]\} dt - \frac{cW}{\tau}$$

$$(2.5) \quad G = Y \cdot S^{\frac{Y}{Y-1}} - w\ell(YS^{\frac{1}{Y-1}})$$

$$(2.6) \quad G = Ys^{\frac{Y}{Y-1}} - w\ell(Ys^{\frac{1}{Y-1}})$$

Here G denotes the maximized profit function. The conditions (2.4)-(2.6) say that average profits per unit of time are maximized when they are equal to profits at terminal prices.

In order for the firm to satisfy demand, with optimal pricing policy its profits must be nonnegative and, in the range of prices charged, price must be higher than marginal cost. If the first condition is not met, the firm will close down; if the second is violated the firm would ration some customers.¹²

From (2.4)-(2.6) the optimal price bounds, s and S , are seen to depend on the aggregate demand variable, real wage, costs of price adjustment and the rate of inflation:

$$(2.7) \quad S = S(\hat{Y}, \hat{w}, \hat{c}, \hat{g})$$

$$(2.8) \quad s = s(\hat{Y}, \hat{w}, \hat{c}, \hat{g})$$

(b) Government

The only role for the government is to provide money to the economy. In order to concentrate on the costs-of-price-adjustment effects of inflation I assume that monetary transfers are provided in proportion to

existing money holdings. Thus inflation does not affect the real return on money and the price adjustment costs are the only source of monetary nonsuperneutrality.

(c) Consumers

The economy is populated by a representative consumer who owns the firms and supplies labour services in a competitive market. His income consists of firms' profits and reward for labour services. For consistency with the firms' problem it is assumed that there is no discounting. He maximizes:

$$(2.9) \quad U = U(D_t, m_t^d, l_t^S) - R(m_t^d) \quad R' > 0$$

where $R(\cdot)$ is the disutility of money holdings (Samuelson, 1969) We assume that $-R''(m)/R'(m) < a$. D_t is given by:

$$(2.10) \quad D_t \equiv \left[\int_0^1 d_t^Y(x) dx \right]^{1/\gamma}$$

D_t will be called aggregate goods demand, to distinguish it from Y .

To determine the representative agent's budget constraint we have to analyze financial flows in the economy. They take place in meta-time. The agent has initial money holdings, obtains current profits from the firms, wages, rewards for changing prices as well as money transfers proportional to his current holdings. Inflation, equal to the rate of monetary creation, depreciates his nominal money holdings. He immediately allocates his real resources optimally between good purchases and real money holdings. That description is valid in stationary states studied here. Thus his budget constraint, given the way money transfers are distributed, is equivalent to:

$$(2.11) \quad \dot{m}_t + D_t + (1 - \lambda_t^S)w_t = G_t + w_t$$

where total labour supply has been normalized at 1.

Define P_t , the price index, as:

$$(2.12) \quad P_t = \left(\int_0^1 P_t(x)^{\frac{\gamma}{\gamma-1}} dx \right)^{\frac{\gamma-1}{\gamma}}$$

This is similar to the Divisia price index, often used in empirical work (Theil, 1967).

The formulation of the consumer problem allows us to solve it in two stages. In stage 1, he maximizes (2.10) subject to:

$$(2.13) \quad \int_0^1 P(x) d(x) dx = DP$$

For convenience, the time subscripts have been dropped. Note that (2.13) is written in nominal terms.

The maximization yields, using (2.12):

$$(2.14) \quad d(x) = D p(x)^{\frac{1}{\gamma-1}}$$

The above procedure thus derives formally demand functions postulated in the literature (where, however, D is treated as constant).

In the second stage, using a Cobb-Douglas utility function, the consumer maximizes:

$$(2.15) \quad U = D_t^a m_t^{d(1-a)} (1-l_t^s)^b - R(m_t^d)$$

In steady state the solution to (2.15) s.t. (2.11) is, dropping time subscripts:

$$(2.16) \quad D = \frac{a}{a+b} (G+w)$$

$$(2.17) \quad (m^d)^a R'(m) = K (G + w)^{a+b} w^{-b} \quad \text{where } K > 0$$

$$(2.18) \quad \ell^S = \frac{a}{a+b} \left(1 - \frac{bG}{aw}\right)$$

Using (2.14) it is seen that all goods, money and leisure are normal while an increase in the real wage increases labour supply.

Thus in this model firm's behaviour is given by demand functions (2.1), technology (2.2) and equations (2.4)-(2.6); consumer behaviour is described by the three first-order conditions (2.16)-(2.18) and demand functions (2.14).

III SOLUTION OF THE MODEL

(a) The Timing of Price Changes

In stationary state the firms must expect a constant inflation rate and their expectations must be consistent in the sense that the rate of change in the price index (2.12) is equal to the expected rate of inflation. This restricts the distribution of price changes over time. It is straightforward to prove the following:

Proposition 1.

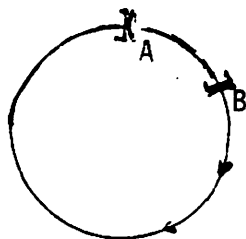
If all firms expect the same inflation rate, their expectations are consistent if and only if they are uniformly staggered across time with respect to the timing of their last price change.^{13,14}

In a recent paper Caplin and Spulber (1985) consider a similar model in a stochastic setting. Inflation is described by a two-state stochastic process. In state 0 there is no inflation, in state 1 there is a constant inflation rate, $g > 0$. Sojourn times in both states are independently

and identically distributed according to different exponential distribution. They proceed to prove that the steady state distribution of firms' prices is uniform in the log of real price. The deterministic model considered here is a limit case of that economy: with the sojourn times going to zero, the price level increases at a constant rate and the uniform distribution in the log of real price becomes equivalent to the uniform distribution in the time of last price change.

Some intuition can be provided by a simple diagram. For clarity of exposition I assume that the high real price is never charged. Until t_0 the rate of inflation was equal to g and the firms expect it to remain unchanged. They are ordered, according to the time of their last price change, on a circle of circumference equal to 1 (Fig. 1), where the arrow points in the direction of passing time.

FIG. 1:



At t_0 the firm located at A is just about to change price; its real price is therefore s and its nominal price equals sP_{t_0} . The real price of a firm located at distance U , $U \in [0,1)$, measured clockwise along the circumference, is se^{gU} ; its nominal price is $P_{t_0} se^{gU}$. The distance from A to B is Δt ; the firm located at B has real(nominal)price equal to $se^{g\Delta t}(P_{t_0} se^{g\Delta t})$.

Consider now the situation at $t = t_0 + \Delta t$. The firm located at B is about to change its price; so its real price is s ; its nominal price is $P_t s = P_{t_0} se^{g\Delta t}$. The nominal price of a firm located at a clockwise

distance U from B ($U \in [0,1)$) is therefore $P_{t_0} se^{g\Delta t} \cdot e^{gU}$. If we now turn the circle counterclockwise so that B moves to A , we obtain a similar situation to that at time t_0 ; however at each point of the circle we have a firm with a price increased by a factor $e^{g\Delta t}$. Thus the price level has increased by the same factor and the inflation rate between t_0 and t was equal to g .

The assumption of uniform staggering makes it possible to determine prices charged by all firms at a given moment of time. Let x denote the firm which changed its price τx ago; its real price is $p(x) = Se^{-g\tau x}$. Let the index of output be of the form (2.10). Then, using (2.1), the aggregate output supplied at any moment of time, denoted by \tilde{Y} , is:

$$(3.1) \quad \tilde{Y} = \left\{ \int_0^1 \gamma^\gamma (Se^{-g\tau x})^{\frac{\gamma}{\gamma-1}} dx \right\}^{1/\gamma}$$

Aggregate labour demand is:

$$(3.2) \quad \lambda^d \equiv \int_0^1 \lambda \left[\gamma \cdot (Se^{-g\tau x})^{\frac{1}{\gamma-1}} \right] dx + \frac{c\gamma}{\lambda nS/s}$$

It will be useful to note that the values of output, employment and profits in the economy at each moment of time are equal to average output, employment and profits of a single firm over a pricing cycle. Consider, for example, profits. Average profit per unit of time is equal to:

$$(3.3) \quad \frac{1}{\tau} \int_0^\tau \tilde{F}(Se^{-gt}) dt - \frac{cW}{\tau}$$

where \tilde{F} is the real profit function written as a function of the real price only. Aggregate profits at any moment of time are:

$$(3.4) \quad \int_0^1 \tilde{F}(Se^{-g\tau x}) dx - \frac{CW}{\tau}$$

Substituting $\tau x = t$ in (3.4) we obtain (3.3). Thus G given by, say, (2.4) gives aggregate profits forthcoming at any moment of time.

Thus we have proved:

Proposition 2 Aggregate profits are constant over time.

The equality between average and aggregate variables makes it possible to analyze a single firm across time rather than the economy at a given moment of time.

(b) Equilibrium

There are three markets in this economy: for labour, money and goods. In equilibrium they must clear. Thus:

$$(3.5) \quad m^d = m^S \equiv m$$

$$(3.6) \quad \ell^S = \ell^d$$

$$(3.7) \quad d = y \Leftrightarrow \text{from (2.1), (2.14): } D = Y$$

The last equation makes the consumer's demand functions (2.14) identical to those faced by firms (2.1). This implies that the aggregate goods market also clears.

In equilibrium there is also a macroeconomic constraint in operation. While each firm is free to choose (s, S) given Y and w , the average real price must be equal to one. Dividing both sides of (2.12) by P_t we obtain

$$(3.8) \quad \int_0^1 p(x) \frac{Y}{Y-1} dx = 1 .$$

As things stand now, there are too many equations. One is eliminated by:

(c) Walras' Law

From the consumer budget constraint (2.11) using the market clearing conditions (3.5) and (3.7), the definition of G in (2.4) and the macroeconomic constraint (3.8) and (3.2) we get

$$\begin{aligned}
 -\lambda^S &= \frac{1}{w} \left\{ \underbrace{\int_0^1 [Y(Se^{-g\tau x})^{\frac{Y}{Y-1}} - w\lambda(Y(Se^{-g\tau x})^{\frac{1}{Y-1}})] dx}_{= Y} - \frac{wcg}{\lambda nS/s} - Y \right\} = \\
 &= - \int_0^1 \lambda(Y(Se^{-g\tau x})^{\frac{1}{Y-1}}) dx - \frac{cg}{\lambda nS/s} = -\lambda^d
 \end{aligned}$$

In equilibrium, by Walras' law, one market clearing equation is dependent. Equilibrium in goods markets and in the money market implies labour market clearing.

As (3.1) implies that aggregate demand, D , given by (2.10) is equal to total output, \tilde{Y} , we have $D = Y$ so also $Y = \tilde{Y}$ and thus (3.1) is equivalent to the macroeconomic constraint (3.8).

(d) The Summary of the Model

The remaining system consists of three first-order conditions for the consumer problem, (2.16)-(2.18); three first-order conditions for the firm (2.4)-(2.6); two market clearing conditions (3.5) and (3.7); the labour demand equation (3.2), the macroeconomic constraint (3.8) and the equation $g\tau = \lambda \ln S/s$, which is used for notational convenience.

There are 12 endogenous variables: supply and demand: for goods - D and Y ; for labour - ℓ^d and ℓ^s ; for money - m^d and m^s ; the nominal price level P , real profits G , real wage w and the parameters of the firm's optimizing process: S, s and τ . The model is closed by specifying the money supply equation:

$$(3.9) \quad \frac{M^S}{P} = m^S$$

We can eliminate D , ℓ^s , m^d and m^s from the system. Replacing M^S/P with m we finally obtain the following system of 8 equations in the remaining 8 endogenous variables:

$$(3.10) \quad \lambda = \frac{1}{a+b} \left(a - \frac{bG}{w} \right)$$

$$(3.11) \quad (m)^a R'(m) = K (G + w^{a+b} w^{-b})$$

$$(3.12) \quad g\tau = \lambda nS/s$$

$$(3.13) \quad Y = \frac{a}{a+b} (G+w)$$

$$(3.14) \quad G = \int_0^1 \left\{ Y (Se^{-g\tau x})^{\frac{Y}{Y-1}} - wL(Y(Se^{-g\tau x})^{\frac{1}{Y-1}}) \right\} dx - \frac{w c g}{\lambda n S / s}$$

$$(3.15) \quad G = YS^{\frac{Y}{Y-1}} - wL(YS^{\frac{1}{Y-1}})$$

$$(3.16) \quad G = Ys^{\frac{Y}{Y-1}} - wL(YS^{\frac{1}{Y-1}})$$

$$(3.17) \quad \int_0^1 (Se^{-g\tau x})^{\frac{Y}{Y-1}} dx = 1$$

The system is dichotomized. To solve it one has to solve the block (3.13)-(3.17), using (3.12), and use the solutions for Y , G , S , s and w to obtain equilibrium values for the labour supply and money holdings.

IV COMPARATIVE STATICS

I assume that the model described by equations (3.10)-(3.17) has an equilibrium. Recall that marginal costs are increasing. The analysis of this section is conducted in terms of average profits of a single firm; it has been shown in section III (see (3.3) and (3.4)) that under the assumption of uniform staggering this is equivalent to analyzing the behaviour of firms at a point of time.

(a) Preliminaries

In contrast with the production function of the firms, an investment in notation seems to yield increasing returns in this model. It will allow to write the first-order expansion of (3.12)-(3.17) in a compact form. Let:

$$f(x) \equiv \frac{1}{Y-1} Yx^{\frac{2-Y}{Y-1}} (Yx - wL'(Yx^{\frac{1}{Y-1}}))$$

$$h(x) \equiv x^{\frac{1}{Y-1}} (x - wL'(Yx^{\frac{1}{Y-1}}))$$

$$L(x) \equiv L(Yx^{\frac{1}{Y-1}})$$

$$\bar{h} \equiv \int_s^S \frac{h(x)}{x} dx$$

$$\bar{L} \equiv \int_s^S \frac{L(x)}{x} dx ; \quad \hat{L} \equiv L + \frac{cg}{\ln S/s}$$

$$A \equiv (S^{\gamma-1} - 1/S) / (s^{\gamma-1} - 1/s)$$

$$k \equiv \frac{a}{a+b}, \quad k < 1$$

The description of the notation follows. The flow of real profits to a firm charging real price x is:

$$(4.1) \quad \tilde{F}(x) = Yx^{\frac{\gamma}{\gamma-1}} - wL(Yx^{\frac{1}{\gamma-1}})$$

Thus the profit function can be written as a function of aggregate output, real wage and real price:

$$(4.2) \quad \tilde{F} = \tilde{F}(Y, w, x)$$

Taking the total derivative of \tilde{F} we obtain

$$(4.3) \quad d\tilde{F} = h(x)dY - L(x)dw + f(x)dx$$

Thus $h(\cdot)$, $f(\cdot)$ and $L(\cdot)$ are partial derivatives of the firm's profit function with respect to Y , x and w . The latter is equal to the productive labour demand of the firm. \bar{L} and \bar{h} are averages of the corresponding functions over the pricing cycle¹⁵ and \hat{L} is average total labour demand, inclusive of labour used in the price changing process. The variable A is obtained by differentiating (3.17) totally.

Certain properties of the functions $f(\cdot)$, $h(\cdot)$ and $L(\cdot)$ will be useful when the comparative statics results are analyzed:

(a) Let x^* denote the profit maximizing real price in the absence of price adjustment costs, given Y and w . Then $f(x^*) = 0$, so:

$$(4.4) \quad \gamma x^* - w \ell'(\gamma x^* \frac{1}{Y^{\gamma-1}}) = 0; \quad \text{also} \quad f(S) < 0 < f(s).$$

(b) We will be interested in the sign of $h'(\cdot)$. Evaluating it at x^* we obtain:

$$(4.5) \quad h'(x^*) = - \frac{1}{\gamma-1} \gamma x^* \frac{3-\gamma}{Y^{\gamma-1}} w \ell''(\gamma x^* \frac{1}{Y^{\gamma-1}}) > 0 .$$

Since $h'(\cdot)$ is continuous, it is positive within some neighbourhood of x^* . So we have:

$$(4.6) \quad h(S) > \bar{h} > h(s)$$

$$(c) \quad \text{As } 0 < \gamma < 1, \text{ we have } h(S) = S^{\frac{1}{\gamma-1}} (S - wL'(yS^{\frac{1}{\gamma-1}})) < 1 .$$

As long as $h'(\cdot)$ is positive, therefore,

$$(4.7) \quad \bar{h} < 1$$

(d) As $L(\cdot)$ is decreasing in x we have

$$(4.8) \quad L(S) < \bar{L} < L(s)$$

Eliminating τ from (3.13)-(3.17) and linearizing the system around its equilibrium point, we obtain, using the above definitions:

$$(4.9) \quad dY = kdG + kdw$$

$$(4.10) \quad dG = \bar{h}dY - \hat{L}dw - \frac{wC}{2nS/s} dg$$

$$(4.11) \quad dG = h(S)dY - L(S)dw + f(S)dS$$

$$(4.12) \quad dG = h(s)dY - L(s)dw + f(s)ds$$

$$(4.13) \quad ds = AdS$$

In this system, Y , G , w , s and S are endogenous while g is treated as exogenous. In fact, the rate of change of money supply is exogenous, but in steady state equilibrium analyzed here $g = \frac{\dot{M}^S}{M^S}$. Note that in obtaining (4.10) use has been made of the fact that G was maximized with respect to s and S .

Equation (4.9) represents equilibrium in the goods market. The consumer is seen to spend a constant fraction of his gross income on goods, with the remaining part being allocated to leisure. It incorporates the assumption that all markets clear. Equations (4.10)-(4.12) represent equilibrium for a typical firm. They require that the price bound (S, s) be chosen so as to make profits at extremal prices, $G(S)$ and $G(s)$, equal to the average profits per unit of time net of the costs of price adjustment.

Equation (4.13) deserves some discussion. It represents the macroeconomic, general equilibrium constraint and was absent from the partial equilibrium version of the model. Since $s < 1 < S$ we have $A < 0$ and s and S must change in the opposite direction for (4.13) to hold.

Its form is based on the assumption that the firms are uniformly staggered with respect to the timing of their last price change. That is implied by the requirements of steady state equilibrium (see Proposition 1). The actual functional form is dictated by the price index (2.12). It is not as restrictive as it seems. In general, a price-quantity index (sum of prices weighted by values positively related to quantities) implies that $S-1 > 1-s$, as long as quantities are negatively related to prices. That is also the case here. To prove, define $b(x) \equiv \frac{\gamma-1}{\gamma} x^{\gamma-1} - \ln x$.

Then (3.17) requires that $b(S) = b(s)$. We have $b'(x) = 0$ for $x = 1$; $b''(x) < 0$, $b'''(x) > 0$ in some neighbourhood of $x = 1$. The function $b(x)$ is concave, has a unique maximum at $x = 1$ and its slope decreases with x . Therefore it is strongly skewed to the right¹⁶ and for s, S such that $b(s) = b(S)$ we have $S-1 > 1-s$. It also implies that

$$(4.14) \quad 0 > A > 1$$

(b) Superneutrality of Money

From (4.9) we obtain that Y is a function of G and w only. From (4.10), (4.11) or (4.12) it can be seen that G depends on Y, w, c, g, s and S . Substituting G away in (4.9) we obtain a formula for aggregate output which depends on real wage, w , costs of price adjustment, c , the price bounds s, S and the rate of inflation, g :

$$(4.15) \quad Y = Y(w, c, s, S, g)$$

Equation (4.15) determines the value of output which, given the real wage rate, costs of price adjustment, the price bounds and the inflation rate, clears the goods market (and, since the system is dichotomized, by Walras' Law clears the labour market as well). Thus inflation is not neutral in this model, barring specific forms of the demand functions (2.1), production function (2.2) and the indices (2.10) and (2.12).

Rotemberg (1983) argues that output is not affected by inflation in this model. This is clearly a special result, due to the functional forms and the partial equilibrium framework he uses.¹⁷

(c) The Effect of Inflation and Price Adjustment Costs on Price Dispersion

Equation (4.13) is an implicit relationship between the price bounds. It can be used throughout to eliminate the lower bound, s .

As (4.10) gives the profits forthcoming to the consumers from the ownership titles, combining it with (4.9) we obtain an equation of the form:

$$(4.16) \quad \gamma^1 = \gamma^1(w, c, g)$$

It can be understood as determining the wage rate which, given the costs of price adjustment, inflation rate and the optimal policy of firms, leads utility maximizing consumers to buy the quantities produced.

Combining (4.10) with (4.11) and (4.10) with (4.11) and (4.12) we obtain two relationships of the form:

$$(4.17a) \quad \gamma^2 = \gamma^2(w, S, c, g)$$

$$(4.17b) \quad \gamma^3 = \gamma^3(w, S, c, g)$$

They may be understood as determining the values of aggregate demand and real wage which lead profit maximizing firms to set price bounds so that the macroeconomic constraint holds. Recall that profit maximization requires that average real profits be equal to profits at the low and at the high prices; hence 2 equations.

We can use (4.16) to eliminate w from (4.17), thus obtaining the two symmetric equations:

$$(4.18a) \quad \{h(S) - \bar{h} - (1 - k\bar{h}) \frac{L(S) - \hat{L}}{k(1 - \hat{L})}\} dY + f(S) dS = \frac{cW}{\lambda n S / s} \left\{ \frac{L(S) - 1}{1 - \hat{L}} \right\} dg$$

$$(4.18b) \quad \{h(s) - \bar{h} - (1 - k\bar{h}) \frac{L(s) - \hat{L}}{k(1 - \hat{L})}\} dY + Af(s) dS = \frac{cW}{\lambda n S / s} \left\{ \frac{L(s) - 1}{1 - \hat{L}} \right\} dg$$

To sign the coefficients I need to assume that $L(s) > \hat{L}$. That would be true if the variations in productive employment during a pricing cycle were, in a typical firm, large relative to employment in price changing. That seems to be a reasonable assumption.

Equations (4.18) are equivalent to:

$$(4.19a) \quad D_1 dY + D_2 dS = D_3 dg$$

$$(4.19b) \quad D_4 dY + D_5 dS = D_6 dg$$

The properties (a)-(d) of the functions $f(\cdot)$, $h(\cdot)$ and $L(\cdot)$ make it possible to sign the coefficients D_1 - D_6 . As $h(S) > \bar{h} > h(s)$ ¹⁸; $1 - k\bar{f} > 0$, $L(S) < \hat{L} < 1$; $h(S) < 0$; $h(s)A < 0$, D_1 is positive while the remaining coefficients are negative.

Performing the required comparative statics exercise we get $dS/dg > 0$. As, from (4.13), s and S are negatively related, so $ds/dg < 0$. Thus:

Proposition 3

Dispersion of relative prices is positively correlated with the inflation rate.

This result has been obtained in partial equilibrium framework by Sheshinski and Weiss, Barro and Danziger. Proposition 3 shows that

macroeconomic considerations do not alter it.

There is extensive empirical evidence on the relationship between inflation rate and relative price dispersion. The early classics are Mills (1927) and Graham (1930); more recent studies include Glejser (1965), Vinning and Elwertowski (1976), Fisher (1981) and Blejer (1983).¹⁹ Of the six explanatory theories discussed by Fisher (1981) only the cost-of-price adjustment model can account for dispersion effects of anticipated inflation. It is thus significant that the positive correlation remains valid in a macroeconomic model.

It is evident from (4.18) that:

Proposition 4

Dispersion of relative prices is positively correlated with the size of price adjustment costs.

The same result has been shown to hold in a partial equilibrium model by Sheshinski and Weiss (1977) and Danziger (1984). As expected, with the friction of costly price adjustment removed ($c=0$) inflation is superneutral in this model.

To interpret the results, consider the equations (4.18). They produce two schedules in the S, Y space: (4.18a) yields a positively sloped schedule AA while (4.18b) a negatively sloped schedule BB (Fig. 2 on page 26). Given the optimizing behaviour of firms, a rise in Y is required for them to set higher values for the price bound S and s . Here, AA represents the relationship for the upper bound, S ; BB for the lower bound s (recall that S is negatively correlated with s). A detailed discussion is in the next section.

An increase in g or c shifts both schedules up, thus increasing S and reducing s .

(d) The Effect of Inflation and Price Adjustment Costs on Equilibrium Output, Employment, Real Wages, Profits, Money Holdings and Welfare

Equation (4.16) is a relationship in two endogenous variables, Y and w . Another relationship is obtained by eliminating G , s and S from (4.10)-(4.13). Both are written below in explicit form:

$$(4.20a) \quad (k\bar{h}-1)dY + k(1-\hat{L})dw = k \frac{CW}{\ln S/s} dg$$

$$(4.20b) \quad \{\bar{h}-h(s)-B(\bar{h}-h(S))\}dY + \{L(s)-\hat{L}-B(L(S)-\hat{L})\}dw = (1-B)\frac{CW}{\ln S/s} dg$$

where $B \equiv \frac{f(s)A}{f(S)}$, $B > 0$.

An interpretation follows. Equation (4.20a) encompasses the clearing condition for the goods markets and the consumer maximizing behaviour. It gives, for each value of Y , the real wage workers must receive so that, as consumers, they buy the output produced. The implied relationship is positively sloped in (w, Y) space: to lead consumers to buy more output they must, as workers, obtain a higher real wage. That seems to be a reasonable relationship. It will be called the consumer equilibrium schedule and be denoted CC (Fig. 3).

Equation (4.20b) uses the first-order conditions for the firm and the macroeconomic constraint. It represents the relationship between Y and w which leads profit maximizing firms to set the price bounds so as

to meet the constraint. It is negatively sloped in the (w, Y) space, as $h(s) < \bar{h} < h(S)$, $L(s) > \hat{L} > L(S)$. It says that when the firms are faced with a higher real wage, in order to keep average real prices equal to 1 they must reduce output. That, again, seems to be a reasonable relationship. The implied schedule, PP (Fig. 3) is called the producer equilibrium schedule.

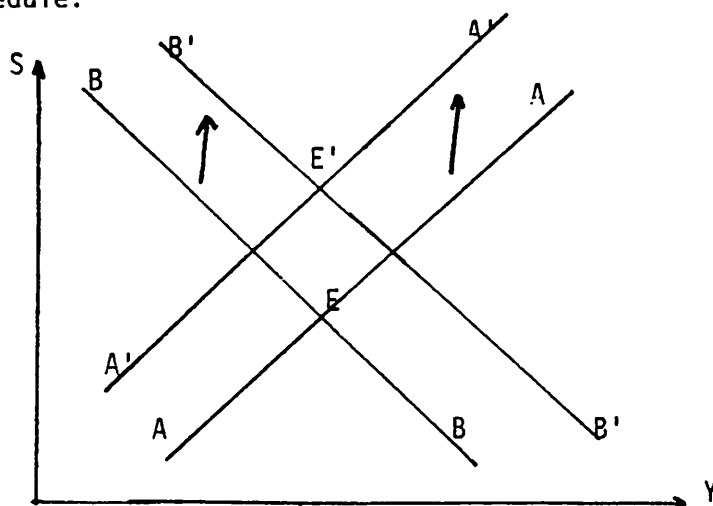


Fig. 2

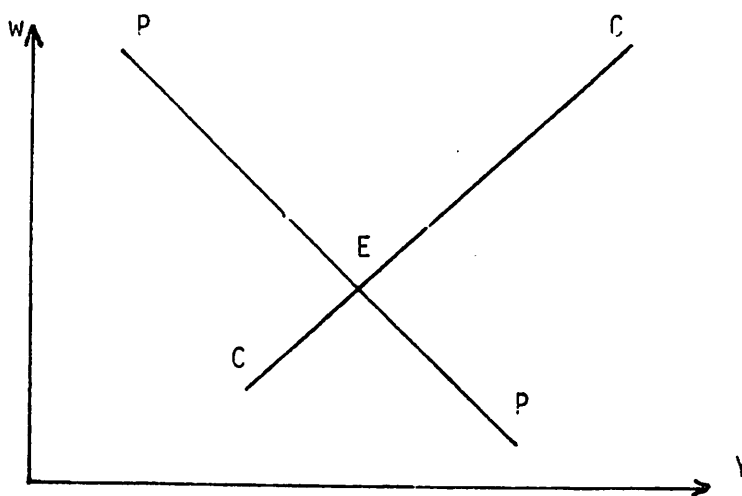


Fig. 3

To understand that relationship, consider optimal partial equilibrium response of a typical firm to an exogenous (for it) increase in real wage, w . It will treat Y as fixed and adjust its price bounds s and S so as to maintain the equalities $G(s) = G_{AVE} = G(S)$. The first-order conditions are given by (4.10)-(4.13) where $dY = dg = 0$:

$$\left. \frac{dS}{dw} \right|_{Y, g \text{ const.}} = \frac{L(S) - \hat{L}}{f(S)} > 0$$

(4.21)

$$\left. \frac{ds}{dw} \right|_{Y, g \text{ const.}} = \frac{L(s) - \hat{L}}{f(s)} > 0$$

as $f(S) < 0 < f(s)$, $L(S) < \hat{L} < L(s)$.

The typical firm attempts to increase both s and S and thus to set its new average real price over the cycle above 1. The explanation of such behaviour is simple. Consider the necessary equality $G(S) = G_{AVE}$. A rise in real wage reduces $G(S)$ less than G_{AVE} since, when the firm charges high price S its employment is lower than average. To restore the equality the firm must reduce $G(S)$. This is accomplished by increasing S , since $f(S) < 0$. Similar explanation holds for the low price, s .

It was shown in part III that the average real price of a typical firm over the price cycle is equal to the economy average real price. Therefore, if the firm succeeded in setting it above 1, the macroeconomic constraint would be violated. It is easy to see that, in order to induce it to maintain average real price equal to 1, an (exogenous for the firm) drop in Y is required. Again, optimal response of the firm is given by (4.10)-(4.12) with $dw = dg = 0$. Thus:

$$\left. \frac{dS}{dY} \right|_{w,g \text{ const.}} = \frac{\bar{h}-h(S)}{f(S)} > 0$$

(4.22)

$$\left. \frac{ds}{dY} \right|_{w,g \text{ const.}} = \frac{\bar{h}-h(s)}{f(s)} > 0$$

since $h(S) > \bar{h} > h(s)$.

A drop in Y reduces $G(S)$ more than G_{AVE} . To restore their equality $G(S)$ must be increased. That is done by reducing S . Similar explanation holds for the low real price.

It is worth noting that optimal response of a monopolistic firm facing costs of price adjustment to changes in demand and cost of inputs turns out to be similar to the reaction of a standard monopolistic firm which can change prices costlessly.

The economy is in equilibrium at the intersection of the two equilibrium schedules. Equations (4.20) can be written as:

$$\begin{array}{r} E_1 dY + E_2 dw = E_3 dg \\ - \quad + \quad + \end{array}$$

(4.23)

$$\begin{array}{r} E_4 dY + E_5 dw = E_6 dg \\ + \quad + \quad \text{sign}(1-B) \end{array}$$

Here $E_1 < 0$; $E_2 - E_5 > 0$; $\text{sign}(E_6) = \text{sign}(1-B)$. Simple algebra establishes that:

$$(4.24a) \quad \frac{dY}{dg} \begin{matrix} \geq \\ < \end{matrix} 0 \text{ as } \frac{1-L(s)}{1-L(S)} \begin{matrix} \geq \\ < \end{matrix} \frac{Af(s)}{f(S)} (= B)$$

$$(4.24b) \quad \frac{dw}{dg} \begin{matrix} \geq \\ < \end{matrix} 0 \text{ as } \frac{1-kh(s)}{1-kh(S)} \begin{matrix} \geq \\ < \end{matrix} \frac{Af(s)}{f(S)} (= B)$$

It can be shown that the sign of the derivatives depends on the relationship between the ratios of shift to slope of the two schedules. Unambiguous results are obtained when

$$(4.25) \quad \frac{1-kh(s)}{1-kh(S)} > B > \frac{1-L(s)}{1-L(S)}$$

This condition is met for a class of real profit functions, in particular for symmetric (and thus quadratic) functions. Those functional forms have been used by Mussa (1977, 1981), Parkin (1985) and Rotemberg (1982, 1983) as well as in numerous papers not related to costly price adjustment (see also discussion below and note 20).

When (4.25) is met we obtain immediately from (4.24) that a rise in inflation leads to a fall in output and an increase in real wage. From (3.11) real profits and money holdings fall (recall that $\frac{R''(m)m}{R'(m)} < a$) while equilibrium employment increases. Finally, as output, real money balances and leisure decrease consumers, even though they obtain higher real wages, are unambiguously worse off.

Before discussing the adjustment process we shall analyze the results. Recall that, when inflation increases, relative prices vary more. Therefore, variability of output over each pricing cycle also rises; immediately after changing price output is smaller and just preceding a price change it is larger than before. What is true for a firm over a pricing cycle holds for

the economy at any moment of time. Since there are diminishing returns to scale, more variable production, at unchanged output, requires a bigger labour input. To hire the additional workers firms must offer higher real wages. Condition (4.25) ensures that output actually falls, but not enough to reduce labour demand below previous level.

In the economy analyzed here, there is high and instantaneous labour mobility. Most firms reduce employment in a continuous fashion while the firms changing prices sharply increase their work force. No firm has any trouble hiring people who would change the price for its product. In the real world with less than instantaneous labour turnover one can expect that frictions in the labour market would lead to a smaller rise in employment and even greater reduction of output.

In this model, high employment and real wages are not necessarily a good thing. Consumers are unambiguously worse off as they work more and consume less (as well as hold smaller real balances). As firms have lower revenues and higher production costs their profits fall and this fall more than offsets the rise in real wages. Inflation therefore has a strong redistributive effect,

The welfare costs of price dispersion are usually analyzed in a stochastic framework, where it leads to misallocation of resources and search. Those problems are absent in the deterministic model, so the result presented here offers another argument: dispersed prices are harmful in an economy in which firms operate in the range of diminishing returns.

It should be stressed that all of the above effects do not depend on what happens to the average cost of price changing. The effect of inflation on the frequency of price changes is ambiguous. The appropriate derivative is impossible to sign so the argument is indirect. Sheshinski and Weiss (1977) showed, in a microeconomic model, that frequency of price changes may fall with inflation. Comparison of (4.18) with similar formulas derived in a micro model shows that it is feasible that relative price dispersion increases more in the present model.

To analyze the adjustment process in detail it is convenient to consider two cases: $B \geq 1$ and $B < 1$.

Consider $B \geq 1$ first. If $B > 1$ then $1-B < 0$ and an increase in the inflation rate shifts the consumer equilibrium schedule up and the producer equilibrium schedule down (Fig. 4). (If $B = 1$ then the producer schedule is unaffected by inflation rate.)

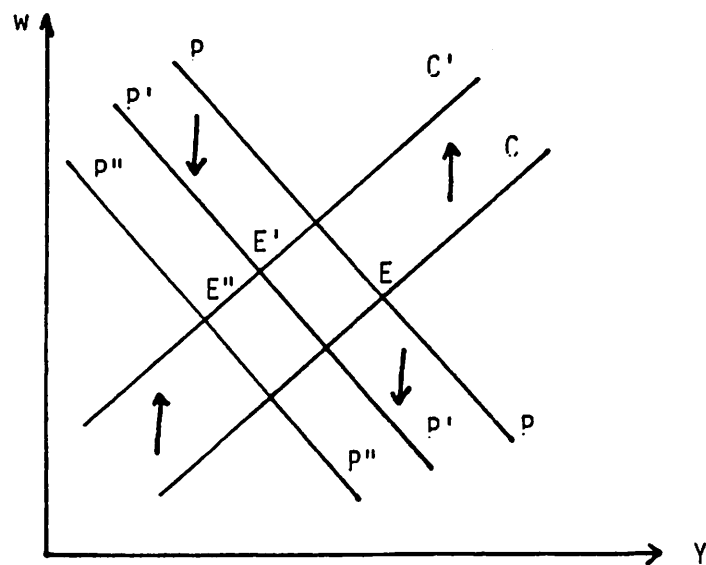


Figure 4

Thus Y must fall while the direction of the change in w depends on the ratios of shifts to slopes of both schedules. The flatter is the PP schedule (the steeper CC), or the bigger is its shift (the smaller the shift in CC), the more likely is w to fall. From (4.20a) the position

of CC is unaffected by changes in B while from (4.20b) an increase in B increases the shift in PP.²¹

A rise in the inflation rate reduces average profits per unit of time (see (2.4)). Given Y, w consumer demand, due to drop in profit income, falls; thus at the initial equilibrium there is excess supply of goods. The firms find that G_{AVE} have decreased; they therefore adjust s, S so as to maintain $G(S) = G_{AVE} = G(s)$, while treating Y and w as constant. But, given dS ,^{it/} implies that s decreases too little for the macroeconomic constraint to be met.²² Thus the average real price charged by the firm is greater than 1.

Restoration of equilibrium requires, first of all, a reduction in output produced. That moves both the economy and the producers closer to (macroeconomic) equilibrium, since both aggregate supply and average real price fall. If B is high (precisely if $B > \frac{1-kh(s)}{1-kh(S)}$ - see (4.24b)) then at the original values of Y and w the average real price would have been much higher than 1. In this case restoration of equilibrium requires that the drop in output be accompanied by a drop in real wage with average real price decreasing on both counts. Had only output fallen (with w increased), at the values of Y, w, G consistent with consumer equilibrium the firms would have still charged average real price greater than 1. That can be seen in Fig. 4, where P"P" corresponds to a large value for B . A drop in wage which maintains consumer equilibrium (i.e. a movement along the CC schedule) implies further drop in Y . Both reduce average real price, thus restoring producer equilibrium.

If B is low ($1 < B < \frac{1-kh(s)}{1-kh(S)}$) then, at the original values the

difference between the average real price and unity is small. A fall in Y is accompanied by a rise in w . Any drop in both Y , w which restores consumer equilibrium results in average real price smaller than 1.

If $B < 1$ the analysis is similar. A rise in the rate of inflation shifts both curves up (Fig. 5).

Therefore, real wages must increase and the behaviour of output depends on the familiar relationship between shift/slope ratio, as expressed by (4.24). A rise in inflation rate results, at constant Y , w , in excess supply of goods and in average real price, given optimal partial equilibrium response of the firms, smaller than 1. A rise in w increases average real price and demand. The behaviour of Y depends on the relationship given in (4.24b).

From (4.20) it is easy to see that the effects of changes in price adjustment costs on output and real wage are similar to those of inflation.

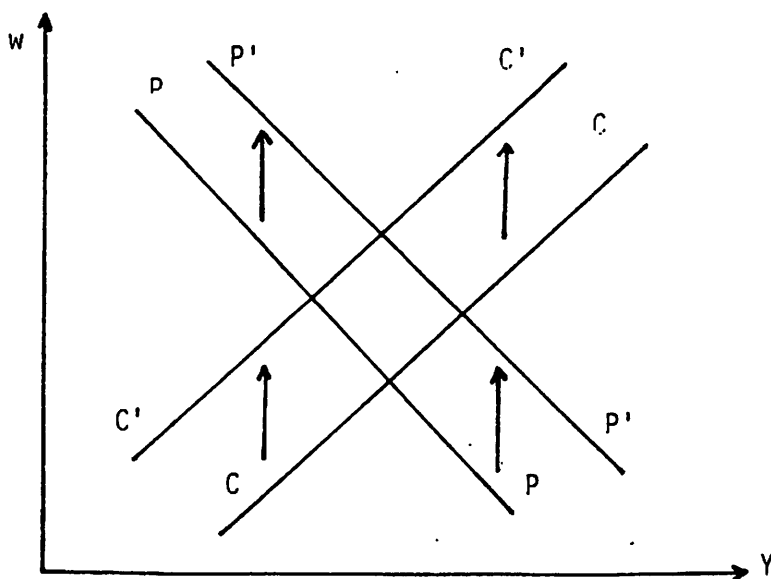


Fig. 5

V DISCUSSION, CONCLUSIONS AND EXTENSIONS

(a) Partial versus General Equilibrium

As pointed out in the introduction, models set in partial equilibrium framework fail to take account of the requirement that the average real price be equal to one (called the macroeconomic constraint here). The analysis shows that the requirement is crucial and thus results obtained in the microeconomic models may be misleading. It is thus reassuring that they generalize to the more general framework.

The effects of inflation on output in a partial equilibrium model have been analyzed in my (1985b) paper. It was shown there that they depend on the forms of the demand and profit functions. Faced with increased inflation firms raise S and reduce s so as to make the first order conditions hold. The size of those changes depend on the slopes of the profit function at the extremal prices. The effect on output, in turn, is determined by demand at those prices. It must be stressed that those considerations are absent here. Due to the macroeconomic constraint and the form of the quantity index (2.10) movements in the price bounds do not affect output. All effects come from changes in the aggregate demand variable, Y , which is treated as constant in microeconomic analysis. Thus the model concentrates on general equilibrium effects of inflation. It is interesting to note, however, that those effects depend on similar considerations as before. The parameter B , which is crucial for the results, is affected by slopes of the real profit function at the extremal prices (ie. on $f(s)$ and $f(S)$). That is because, faced by a change in inflation rate, the firms adjust their price bounds in a manner identical to that in a partial equilibrium model. Given that adjustment, the role of aggregate variables, Y and w , is to lead firms to set the price bounds so that the macro constraint is met.

(b) Generality of the Results

In this type of framework the forms of demand and profit functions have very definite effects on the results. It is a widespread practice in

macroeconomics to chose convenient functional forms. The "innocent" second order approximation of the loss function from charging suboptimal prices is equivalent to a profit function quadratic in the real price. When combined with log-linear demand functions it results in output being independent of inflation; a result that does not hold with other functional forms.

In the model I tried, therefore, to avoid using specific functional forms. The freedom, however, is limited; to obtain a tractable model the profit function must depend on the relative price only. This requires that CES utility functions as well as the Divisia-type price index be used.²³ The CES demand functions affect the forms of the functions $f(\cdot)$ and $h(\cdot)$ only; the derivations in section IV are general with the above functions being the appropriate derivatives of the momentary profit function. The Cobb-Douglas utility functions was introduced for simplicity; the only assumption on the technology is that marginal costs are increasing. The way money is injected into the economy has been chosen so as to make the results independent on the real-return-on-money considerations. Finally note that the assumption about the costs of price adjustment using real resources did not play any essential role. All results remain unchanged if those costs were subjective (as in Rotemberg, 1982). That is due to the assumption about the relative size of labour employed in price changing (see p.23).

(c) Welfare Costs of Inflation

The simple setting of the model does not allow deriving many welfare implications formally. One result is when (4.25) holds; then welfare is shown to be negatively affected by inflation. At the same time employment and real wages increase; thus high employment and real wages are not necessarily good things in this model. In the case considered,

the rise in real wage is more than offset by a drop in output, real money holdings (both due to the fall in profit income) and leisure. Those effects capture some of the welfare costs of price dispersion. As costs are convex, increased amount of labour is needed to produce reduced output. Not surprisingly welfare falls. This result confirms the widespread opinion that relative price variability reduces welfare (for example Laidler, 1977, Fisher, 1981b). That effect is not captured in most analyses of steady state inflation (see, for example, the discussion in Jaffe and Kleiman).²⁴

If the positive correlation of inflation and price dispersion holds in a general equilibrium, stochastic model, the usual argument about inefficiency brought about by search (Alchian, 1970) and mistaken, suboptimal decisions would apply.

The model can be viewed as a version of a classical, general equilibrium model with a friction on the producer side. Without costs of price adjustment it reduces to a model in which inflation is superneutral. The friction of price adjustment represents an inefficiency in a monetary economy. It has been shown that the effects of increases in inflation are similar to those of price adjustment costs hikes. Thus perfectly anticipated inflation can be seen to increase the friction and take the economy away from an efficient equilibrium. Monetary authority can eliminate the friction by reducing inflation to zero.

(d) Extensions

An obvious drawback of the model is its steady-state, deterministic setting. A generalization to a model in which inflation is stochastic is thus desired, perhaps by combining the inflationary process, modelled as in Sheshinski and Weiss (1983), with the present framework.

A stochastic model would allow to capture effects of unanticipated inflation. There is extensive evidence on positive correlation between unanticipated inflation and price dispersion (apart from some of previously mentioned references, Parks, 1978; Herzowitz, 1981 and Taylor, 1981, are good examples). It appears that it is stronger than with anticipated inflation. The usual explanation is in terms of the

equilibrium "surprise" model developed by Lucas (1973) and Barro (1976) (for example Cukierman, 1982, 1983). That framework cannot explain the apparent effect of anticipated inflation. The present approach has the potential of accounting for both effects. In fact Sheshinski and Weiss (1983) show, in a partial equilibrium model, that dispersion of relative prices is ^{more} strongly correlated with the variance of inflation than with its level. This seems therefore to be a promising research strategy.

The friction of costly price adjustment is exogenous in the present model. A formal modelling of the costs is an important task. In the deterministic model they are seen as real, actually incurred costs. In a stochastic framework they may be understood as search and reputation costs and the periodically fixed price may be a form of an implicit contract between firms and their customers.

The assumption adopted here is that the costs of price adjustment are fixed and independent of inflation. As they imply that profits fall with inflation, it may be expected that, with rising inflation, firms would undertake measures to reduce them. This can be modelled by altering the form of the profit function so as to allow a tradeoff between costs of producing output and costs of price changes.

Finally, the costly price adjustment framework has been often criticized for the asymmetric assumptions about changing prices and output. While the standard technology the firm uses implies no fixed costs of varying production and seems to be a reasonable assumption, the criticism may be avoided by considering durable goods. The firms would respond to changes in demand by varying output and inventories. Inventories allow to smooth the production process and thus avoid, to some extent, increased costs of bigger price dispersion.

NOTES

I am grateful to Joel Fried, David Laidler, Michael Parkin, Eytan Sheshinski and especially to Peter Howitt for helpful comments and suggestions. I am responsible for any remaining errors.

1. In his model the expected rate of change in the price level. The model analyzed in section III of my 1985b paper is a simple (and, it seems, the only possible) extension of the Barro model to inflationary environment.
2. The Mussa (1981) paper addresses some macroeconomic questions; however, Rotemberg (1983) has shown that the assumptions of the model are not consistent.
3. Sheshinski and Weiss (1979) assume that profits depend also on the quantity of fixed or quasi-fixed factor.
4. The direction of price changes is not important; the optimal pricing policy is similar in the case of deflation. To obtain two parameter characterization in stochastic models, however, it is required that the changes in the general price level be unidirectional.
5. Exceptions are noted in my 1985b paper.
6. It thus avoids the criticism which is, perhaps, best expressed in Barro (1977) as well as accounts for nominal stickiness. For a review of those issues, see Gordon (1981); for evidence on nominal stickiness see Gordon (1981, 1983); I have informally discussed related issues in my 1985a paper.
7. This problem has been discussed by Sheshinski and Weiss (1983).
8. Parkin and Rotemberg avoid this problem due to the specific form of the profit function they use.

9. In particular, all firms are of the same size. Generalization which would allow firms to differ in size is straightforward.
10. This is the usual criterion when there is no discounting. Introduction of positive interest would complicate the model greatly without altering the results.
11. For a proof - see Sheshinski and Weiss (1977). The first-order conditions that follow were derived in my (1985b) paper. It was shown there that the second-order conditions are met.
12. The second assumption is sufficient for absence of rationing. It will be useful when comparative statics results are derived.
13. i.e. for any interval of time of length t , $t \leq \tau$, t/τ firms change their price.
14. I have not been able to derive necessary or sufficient conditions for the case when expectations about inflation differ across firms. There are several obvious arguments for assuming identical expectations, especially in the setting of this model (steady state, identical firms, constant money supply growth rate).
15. More precisely \bar{h} and \bar{c} are averages with respect to time. In the definitions integration is with respect to prices, hence x appears in the denominators.
16. A function $m(z)$ with a maximum at z^* is strongly skewed to the right iff (a) it is strictly quasiconcave (b) for any $z_1 < z^* < z_2$ such that $m(z_1) = m(z_2)$ we have $-b'(z_2) < b'(z_1)$.

17. Linear demand and quadratic profits in the log of real price. His treatment is essentially microeconomic and so in his model the macroconstraint is not likely to hold.
18. This holds in the neighbourhood of x^* where $h'(\cdot) > 0$. For $\ell(\cdot) = q^2/2$, various values of γ yield neighbourhood extending to $1.2x^*$ to $1.3x^*$. Moreover, I require only that $h(S) > \bar{h} > h(s)$ which holds also in some region in which $h'(\cdot) < 0$, so the admissible range is wider. The limitation does not seem restrictive.
19. Blejer and Leiderman (1980) did not find this relationship significant.
20. More precisely, this is true at low inflation rates when A is approximately equal to -1 and $S-1 \approx 1-s$. At higher inflation rates for (4.25) to hold the real profit function must be strongly right skewed to offset the fact that $|A| < 1$ and $S-1 > 1-s$.
21. The effect of B on the slope of PP depends on the second derivatives of $h(\cdot)$ and $L(\cdot)$ as well as on the proportion of active labour employed in price changing. It is seen from the analysis that the effect of B on the shift of PP dominates.
22. From the partial equilibrium considerations the typical firm, treating Y as constant, adjusts s according to $ds|_p = \frac{f(S)}{f(s)} dS|_p$ (where the subscript p denotes partial equilibrium). However, for macro equilibrium we must have $ds = AdS$. Since $B = \frac{f(s)A}{f(S)} > 1$ by assumption, so $A < \frac{f(S)}{f(s)} < 0$ and thus $ds|_p > AdS|_p$, i.e. the average real price is greater than 1.
23. I am grateful to Peter Howitt for pointing it out.

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