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ABSTRACT: Tax competition is modelled as a game played by a finite number of jurisdictions, among which the tax base is perfectly mobile. Jurisdictions play Nash in tax rates. Existence of equilibrium is established, under fairly strong assumptions. Smaller jurisdictions are shown to levy lower taxes. The convergence to a "small open economy" of a jurisdiction as its share of the population shrinks is analyzed.

The difficulties caused by jurisdictions competing for a mobile tax base have long been recognized by economists. Since an increase in the tax rate of a single jurisdiction induces some of the tax base to move elsewhere, one might expect all jurisdictions to levy inefficiently low tax rates in equilibrium. This argument was made in Stigler's 1957 congressional testimony, and probably can be traced much further back. Recently, it has been analyzed formally by Wilson, and by Zodrow and Mieszkowski. Wildasin's survey of local public finance cites many of the other contributions in this literature.

A common assumption in this literature is that each jurisdiction is small. "Small" is usually taken to mean that one jurisdiction's policy has no influence on national variables. For example Zodrow and Mieszkowski assume[1], "the local government in each jurisdiction acts on the assumption[s]... that its actions cannot affect the national net return to capital r". Wilson[2] makes the same assumption. Of course, this is a perfectly natural assumption to make, analogous to the "small open economy" assumption in trade theory, or the assumption that firms cannot influence the price of output in a perfectly competitive industry.

However, it has long been recognized that this sort of assumption rules out the type of strategic interaction that laymen associate with the term "competition". The story told in the opening paragraph of this paper suggests that cooperation among jurisdictions might make all residents better off. If the essence of tax competition is the non-cooperative behaviour of local governments, then a simple non-cooperative game may be a profitable way of modelling such competition.

That is what is done in this paper. A very simple model is presented of a finite number of jurisdictions financing their local public sector by taxing capital, which is assumed mobile among jurisdictions. Each jurisdiction's strategic variable is its tax rate. They behave non-cooperatively; the equilibrium concept is the Nash equilibrium in tax rates.

What distinguishes this paper from most of the literature is that jurisdictions are not assumed "small". At least two other papers deal with tax competition among a finite number of jurisdiction. Mintz and Tulkens present a 2-jurisdiction model, in which a single taxable output may be transported (at some cost) between jurisdictions, and in which residents may commute to work. Kolstad and Wolak discuss two states setting resource taxes to capture rents from distant consumers. They solve numerically for the Nash equilibrium using data from Montana and Wyoming. My paper is (I believe) the first to examine capital tax exporting in a nation with an arbitrary number of jurisdictions. It is also the first to deal analytically with the question of the existence of a Nash equilibrium.

There are two main motives for this paper. First, not all jurisdictions are relatively small. California has 10% of the American population; Ontario 40% of Canada's. Moreover, tax competition may take place among a fairly small subset of the nation's jurisdictions. Even small states may be large relative to the set of states with which they are effectively competing.[3] Further, the issue of tax competition is relevant to local governments in a metropolitan area. Even if there are many jurisdictions in a metropolitan area, the central city usually contains a large fraction of the population.[4]

A second motive for this paper is to provide a foundation for the "small open jurisdiction" literature. In the model presented here, there is a natural measure of a jurisdiction's relative size--its share of the nation's population. I examine what happens to a jurisdiction's behaviour as its relative size shrinks to zero. This exercise is an attempt to do for tax competition among jurisdictions what Novshek & Sonnenschein did for competition among firms.[5]

In particular, two definitions of "small" have been offered: a small share of population and a negligible influence on national variables. Does small in the first sense imply small in the second? The implication seems sensible; it certainly is consistent with the models of Wilson and of Zodrow and Mieszkowski. But earlier work (in a somewhat different model) by Mieszkowski on tax competition supports H.G. Brown's view that "the small size of a taxed sector is not a sufficient reason for ignoring general equilibrium adjustments".[6] Appendix 3 presents an example of a jurisdiction whose relative size declines to zero, but whose influence on national variables does not decline to zero.

The organization of this paper is as follows. Section 1 presents a simple model of the tastes and technology, and people's behaviour. Section 2 shows how the allocation of capital among jurisdictions depends on local tax policy. Section 3 derives each jurisdiction's optimal tax policy, and discusses the existence of Nash equilibrium. Section 4 derives some properties of equilibrium: whether the public sector's output is underprovided, and how tax rates vary with relative size. Section 5 considers equilibria in which all jurisdictions levy the same tax rate, and how that tax rate varies with the number of jurisdictions. Section 6 establishes some limit results; conditions are derived under which a jurisdiction's influence on the national net return to capital approaches zero as its size does.

Section 7 discusses efficiency of equilibrium briefly. Section 8 examines a specific form of asymmetric equilibrium, when there are only 2 jurisdictions which differ in size. Section 9 offers the usual concluding remarks.

I. THE MODEL

It is assumed that one country is divided into I jurisdictions. This division is treated as exogenous. All residents of the country are identical (in ability and in tastes), and are immobile among the jurisdictions. Each resident of the country owns an equal share of the country's capital stock. There are two inputs to production, labour and capital services. It is assumed residents' supply of labour is fixed. Since residents are immobile, it then follows that production in any jurisdiction depends only on the supply of capital services to that jurisdiction.

There are two consumption goods, a private good and a publicly provided good. It is essential to realize from the outset that the publicly provided good has a pure private good technology. That is, if some combination of inputs can provide a level g of consumption of the publicly provided good to each of N people, then the same level g can be provided to each of kN people by using k times the level of each input. More concisely, there are neither economies nor diseconomies of scale in population.

Further, it is assumed that both the private good and the publicly provided good use the same factor proportions. Both are produced under constant returns to scale in the 2 inputs. Essentially then, there is one output which can be transformed into either the private good or the publicly provided good. The production technology in each jurisdiction is the same. Because of the constant returns to scale in population, the output per capita in jurisdiction i can be written as a function $f(k_i)$ of the capital services per capita used in production in that jurisdiction. Each jurisdiction's intensive production function is assumed to obey the Inada conditions

$$f(0) = 0;\ f' \geq 0;\ f'' < 0;\ \lim_{k\to 0} f'(k) = \infty;\ \lim_{k\to\infty} f'(k) = 0$$

and to be twice continuously differentiable.

The assumptions of constant returns to scale serve to emphasize the behavioural asymmetries introduced by tax competition. Even though all jurisdictions have the same technologies their residents will have different consumption bundles in equilibrium if the jurisdictions have different populations.

Although the output can be used for either of the consumption goods, it is taken as an institutional constraint that only a jurisdiction's government can provide the publicly provided good. The cost of this good must be financed by a unit tax on capital employed in the jurisdiction. Thus this paper takes the country's constitution as given, and explores the implications of its provisions. Hence the inefficiency of the equilibrium should not be taken too seriously. A new set of tax rules could yield an efficient equilibrium easily (and might be preferred by all identical residents of the country). The institutional constraint can be written

$$g_i = t_i k_i \qquad (1)$$

where

$g_i \equiv$ per capita consumption of publicly provided good in jurisdiction i.

$t_i \equiv$ unit tax on capital services provided in jurisdiction i

$k_i \equiv$ per capita level of capital services provided in jurisdiction i

It is assumed that perfect competition prevails in each jurisdiction, so

$$R_i = f'(k_i) \tag{2}$$

where R_i is the gross return to capital.

Let

$\lambda_i \equiv$ proportion of the country's population living in jurisdiction i

$k^* \equiv$ capital stock per capita in the country

so that each of the country's residents has an endowment k^* of capital. Capital is perfectly mobile among jurisdictions, so that its net return is the same everywhere. Denoting that return by r, capital mobility then implies

$$r = R_i - t_i \qquad i=1, \ldots I \tag{3}$$

The total capital stock in the country is fixed, and inelastically supplied, except that free disposal is assumed, so that its net return cannot be negative. Thus if the net return is positive all capital will be used; otherwise the net return is zero and there may be an excess supply.

$$r \geq 0; \quad \sum_{i=1}^{I} \lambda_i k_i \leq k^* \text{ with equality if } r > 0 \tag{4}$$

Because of constant returns to scale, per capita labour earnings in jurisdiction i are output minus gross capital earnings, or $f(k_i) - R_i k_i$. Each resident's total income, labour earnings plus the return to capital ownership, are spent on the private good. If x_i denotes per capita consumption of the private good

$$x_i = f(k_i) - f'(k_i)k_i + rk^* \tag{5}$$

Each resident has the same utility function $u(x_i, g_i)$. It is assumed utility is quasi-concave. In fact a further stronger assumption is made, that indifference curves become steeper as one moves vertically up and shallower as one moves to the right. Moreover, they are assumed not to intersect the axes.[7] If the marginal rate of substitution is denoted $m(x_i, g_i)$,

$$m(g_i, x_i) \equiv \frac{\partial u}{\partial g_i} \bigg/ \frac{\partial u}{\partial x_i}$$

then it will be assumed

$$\frac{\partial m}{\partial g_i} < 0 \ , \quad \frac{\partial m}{\partial x_i} > 0 \ , \quad \lim_{g \to 0} m(g, x) = \infty, \ \lim_{x \to 0} m(g, x) = 0$$

Finally, there is the behaviour of each jurisdiction's government. Each government chooses a tax rate t_i to maximize the utility of its identical residents, treating other jurisdictions' tax rates as given. Thus each jurisdiction recognizes its own influence on both its own capital stock and on the national net return to capital. In particular, these variables are determined from the I tax rates by equations (3) and (4). The variables which enter residents' utility then can be determined from equations (1) and (5). Hence each jurisdiction picks t_i to maximize $u(x_i, g_i)$, by solving equations (1) - (5) treating all t_j's $(j \neq i)$ as fixed. A Nash equilibrium set of tax rates is then defined in the usual way, namely that each t_i is jurisdiction i's optimal choice given the other t_j's.

2. THE ALLOCATION OF CAPITAL

In this section the effects of the tax rates t_i on the supply of capital services to the various jurisdictions are derived. Note that these effects can be determined from the system of equations (3) and the inequality (4). Two regimes are possible; when tax rates are high enough the net return to capital is zero and there is excess supply of capital. I will first deal with the case of a positive net return to capital. When $r > 0$, the I k_i's and r are determined by the simultaneous solution to

$$\begin{aligned} r &= f'(k_i) - t_i \quad i=1,\ldots I \\ \Sigma\lambda_i k_i &- k^* = 0 \end{aligned} \tag{6}$$

Differentiating (6),

$$\begin{bmatrix} f''(k_i) & 0 & 0 & 0 & -1 \\ 0 & f''(k_2) & & & \\ & & & 0 & \\ 0 & & 0 & f''(k_I) & -1 \\ \lambda_1 & & \lambda_{I-1} & \lambda_I & 0 \end{bmatrix} \begin{bmatrix} dk_1 \\ \\ \\ dk_I \\ dr \end{bmatrix} = \begin{bmatrix} dt_1 \\ \\ \\ dt_I \\ dk^* \end{bmatrix} \tag{7}$$

The determinant Δ of the matrix in (7) is

$$\Delta = \left[\prod_{i=1}^{I} f''(k_i)\right]\left[\sum_{i=1}^{I} \frac{\lambda_i}{f''(k_i)}\right] \qquad (8)$$

Since sgn Δ = sgn $(-1)^{I+1}$, it can be seen the principal minors of the matrix alternate in sign, so the uniqueness of any allocation of capital can be shown for any $(t_1, \ldots t_I)$.[8] Further computation shows

$$\frac{\partial k_i}{\partial t_i} = \frac{1}{\Delta}\left\{\left[\prod_{\substack{j=1\\j\neq i}}^{I} f''_i\right]\left[\sum_{\substack{j=1\\j\neq i}}^{I} \frac{\lambda_i}{f''_i}\right]\right\} < 0 \qquad (9)$$

$$\frac{\partial k_i}{\partial t_j} = -\frac{\lambda_i}{\Delta}\left\{\prod_{\substack{m=1\\m\neq i\\m\neq j}}^{I} f''_m\right\} > 0 \quad i\neq j \qquad (10)$$

$$\frac{\partial r}{\partial t_i} = -\frac{\lambda_i}{f''_i}\,\frac{1}{\Delta}\left[\prod_{i=1}^{I} f''_i\right] < 0 \qquad (11)$$

where I have used the shorthand $f''_i \equiv f''(k_i)$.

The boundary between regimes is the set of t_i's solving $\Sigma\lambda_i k_i = k^*$ and $f'(k_i) = t_i$, $i=1,\ldots I$. Equation (11) shows that if all tax rates except t_i and t_j are held constant, that $\partial t_i/\partial t_j < 0$ along this boundary. The Inada conditions ($\lim_{k\to\infty} f'(k) = 0$) show the boundary never hits any t_i - axis. Above the boundary, there is excess supply of capital. There

$$f'(k_i) = t_i \qquad i=1,\ldots I \tag{12}$$

provided that $\Sigma\lambda_i k_i \leq k^*$. In this excess supply regime, connections between jurisdictions are broken. One jurisdiction can increase its tax base without lowering any other's. Here

$$\frac{\partial k_i}{\partial t_i} = \frac{1}{f''_i} < 0 \tag{13}$$

$$\frac{\partial k_i}{\partial t_j} = 0 \qquad i \neq j \tag{14}$$

$$\frac{\partial r}{\partial t_i} = 0 \tag{15}$$

Note that in either regime the jurisdiction's supply of capital is strictly decreasing in its own tax rate. Since (9) can be rearranged to

$$\frac{\partial k_i}{\partial t_i} = \frac{1}{f''_i} \left\{ 1 - \frac{\lambda_i}{f''_i} \Big/ \sum_{j=1}^{I} \frac{\lambda_j}{f''_j} \right\} \tag{16}$$

the supply response of capital to the jurisdiction's own tax rate becomes steeper as the locus r=0 is crossed from below.

Some further notational shorthand may prove useful. Let

$$b_i \equiv \frac{\lambda_i}{f''_i(k_i)} \Bigg/ \left(\sum_{j=1}^{I} \frac{\lambda_j}{f''_j(k_j)} \right)$$

so that

$$1 \geq b_i \geq 0 , \quad \sum_i b_i = 1$$

and the earlier equations (9) - (11) can be rewritten

$$\frac{\partial k_i}{\partial t_i} = \frac{1-b_i}{f''_i} \tag{17}$$

$$\frac{\partial k_i}{\partial t_j} = - \frac{b_i}{f''_j} \tag{18}$$

$$\frac{\partial r}{\partial t_i} = -b_i \tag{19}$$

Of course b_i is not a constant unless f is quadratic. In fact, since an increase in t_i lowers k_i, and raises all k_j's$(j \neq i)$, if $f''' > 0$ everywhere then $\frac{\partial b_i}{\partial t_i} > 0$, and if $f''' < 0$ everywhere then $\frac{\partial b_i}{\partial t_i} < 0$. Also $\frac{\partial b_i}{\partial t_j}$ will depend on the fourth derivatives of f.[9] Finally, if $k_1 = k_2 = \quad = k_I = k^*$, then $b_i = \lambda_i$.

3. EQUILIBRIUM

Consider now the choice of tax rate of jurisdiction i. Residents wish to choose a t_i to maximize $u(x_i, g_i)$. Since $x_i = f(k_i) - f'(k_i)k_i + rk^*$, and $g_i = t_i k_i$, then from (17) and (19)

$$\frac{\partial x_i}{\partial t_i} = -(1 - b_i)k_i - b_i k^* \tag{20}$$

$$\frac{\partial g_i}{\partial t_i} = k_i + \frac{t_i(1 - b_i)}{f''_i} \tag{21}$$

if the return to capital is positive, and

$$\frac{\partial x_i}{\partial t_i} = -k_i \tag{22}$$

$$\frac{\partial g_i}{\partial t_i} = k_i + \frac{t_i}{f''_i} \tag{23}$$

if there is excess supply of capital. In either regime $\frac{\partial x_i}{\partial t_i} < 0$. As equation (20) shows, when $r > 0$, a tax rate increase reduces private income in two ways: by driving down the jurisdiction's capital-labour ratio it lowers wage income, and by decreasing the national return to capital it reduces capital income.

The net change in utility (divided by the marginal utility of income) from a tax increase is $\frac{\partial x_i}{\partial t_i} + m(x_i, g_i)\frac{\partial g_i}{\partial t_i}$. If I define

$$t_{-i} \equiv (t_1, \ldots t_{i-1}, t_{i+1}, \ldots t_I)$$

and

$$F_i(t_i; t_{-i}) \equiv \frac{\partial x_i}{\partial t_i} + m(x_i, g_i)\frac{\partial g_i}{\partial t_i}$$

then an optimal choice of tax rate must involve either $F_i = 0$ (if the allocation is strictly within a regime) or $F_i^- \geq 0$; $F_i^+ \leq 0$ (if $(t_1,\ldots t_I)$ implies r=0 and $\sum_{j=1}^{I} \lambda_j k_j = k^*$).

Substituting from (20) - (23)

$$F_i = -(1 - b_i)k_i - b_i k^* + m_i\left[k_i + \frac{t_i(1 - b_i)}{f''_i}\right] \quad \text{if } r > 0 \qquad (24)$$

$$F_i = (m_i - 1)k_i + \frac{m_i f'_i}{f''_i} \quad \text{if } \sum_j \lambda_j k_j < k^* \qquad (25)$$

Since k_i is the solution to $f'(k_i) = t_i$ in the regime of excess supply, (25) shows that each jurisdiction's optimal choice of tax rate in that regime depends neither on the size λ_i of the jurisdiction, nor on the policies of other jurisdictions.

Since $\frac{\partial x_i}{\partial t_i} < 0$, supply of the publicly provided good must be increasing in the tax rate at any optimum. Equations (24) - (25) also show that at any optimum

$$m_i > (1 - b_i) \qquad (26)$$

$$m_i > 1 \quad \text{if} \quad k_i \leq k^* \tag{27}$$

Since m_i is jurisdiction i's marginal rate of substitution between the publicly provided good and the private good, the Samuelson first-best condition for public good provision is $m_i = 1$.[10] Hence (27) shows that at least one jurisdiction must under-provide the publicly provided good (in the sense that Σ MRS > MRT)[11] in any equilibrium.

The reaction correspondence of jurisdiction i is the set of tax rates which maximize residents' utility, for a given t_{-i}. Such a correspondence need not be single-valued. Since F_i is continuous in t_{-i}, the reaction correspondence will be continuous if it is single-valued in some neighbourhood.[12]

If I denote by ϵ_R the elasticity of the gross return to capital with respect to the quantity of capital

$$\epsilon_R \equiv -\frac{\partial R}{\partial k}\frac{k}{R} \equiv -\frac{f''}{f'}k$$

then (25) shows a necessary condition for any optimum to involve an excess supply of capital is $\epsilon_R > 1$. For instance, if technology is Cobb-Douglas, $f \equiv k^{\alpha}$, then $\epsilon_R = (1 - \alpha) < 1$, and jurisdictions will always wish to reduce taxes until there is no longer an excess supply of capital.

There is another circumstance in which an excess supply of capital can be ruled out. Let

$$\alpha(k) \equiv \frac{f'(k)k}{f(k)}$$

denote capital's share of output. Consider now the problem of dividing a fixed output f(k) between the two goods. Suppose this optimum always involves spending more than labour's share on the private good. That is

$$m[(1 - \alpha(k))f(k), \alpha(k)f(k)] < 1 \tag{28}$$

Then the tax base provided for the public good is large enough, in the sense that residents could finance all their desired public expenditure from a unit tax on capital, if capital were fixed.

In appendix 1 it is shown that the capital stock per person can be bounded below by some positive k_o. If condition (28) holds for all $k_o \leq k \leq k^*$, then there is some strictly positive r_o such that $r \geq r_o$ in any equilibrium. To see the result, note first that if r=0, then $g_i = \alpha f(k)$, $x_i = (1-\alpha)f(k)$, so that if (28) holds, then $m_i < 1$ for some i with $k_i < k^*$. From (24) and (25), $m_i < 1$ and $k_i < k^*$ imply $F_i < 0$. So any equilibrium must involve $r > 0$. To find a lower bound r_o, define r_o as

$$r_o = \min\{r | m[(1-\alpha)f(k) + rk^*, \alpha f(k) - rk] \geq 1 \text{ for some } k_o \leq k \leq k^*\}.$$

From assumption (28), the fact that m increases with r, and the fact that $[k_o, k^*]$ is a compact set, such an r_o must exist and be positive. And (24) shows $m \geq 1$ is necessary for any equilibrium.

Thus the only circumstances in which an equilibrium will exist with an excess supply of capital are if all of capital's income is less than residents wish to spend on the publicly provided good.

Consider next the uniqueness of the reaction of jurisdiction i to the given tax vector t_{-i}. A sufficient condition for such uniqueness (at least in the region in which r > 0) is that $F_i' \leq 0$ whenever $F_i = 0$. From (24)

$$F_i' = [2m_i - (1-b_i)] \frac{(1-b_i)}{f_i''} + \frac{\partial m_i}{\partial t_i} \left[k_i + \frac{t_i(1-b_i)}{f_i''} \right] +$$

$$+ \frac{\partial b_i}{\partial t_i} \left[\frac{m_i t_i}{f_i''} + k_i - k^* \right] - m_i t_i f_i''' \frac{(1-b_i)^2}{(f_i'')^3} \qquad (29)$$

where by $\frac{\partial m_i}{\partial t_i}$ I mean $\frac{\partial m}{\partial x_i} \frac{\partial x_i}{\partial t_i} + \frac{\partial m}{\partial g_i} \frac{\partial g_i}{\partial t_i}$.

It has already been shown that $m_i > (1-b_i)$ if $F_i = 0$, so that the first term on the right side of (29) must be negative. Also, at any optimum $\frac{\partial g_i}{\partial t_i} = k_i + \frac{t_i(1-b_i)}{f_i''} > 0$. Since m_i must decrease with an increase in the tax rate, the second term is also negative. It was noted earlier that if f" does not change signs, sgn $(\frac{\partial b_i}{\partial t_i})$ = sgn f"'. Hence $k_i \geq k^*$ and f"' < 0 are sufficient for both the third and fourth terms on the right to be negative. Of course it cannot be true that $k_i \geq k^*$ for all i, unless $k_1 = k_2$ = = $k_I = k^*$.

Actually, it can be shown somewhat tediously, that the third term on the right must be negative if $f''' < 0$, regardless of whether $k_i \geq k^*$. That is what this paragraph shows. Suppose $k_i < k^*$. Then $F_i = b_i(k_i - k^*) + \frac{m_i}{f''_i} t_i(1-b_i) + (m_i-1)k_i$, so that $F_i = 0$ requires $m_i > 1$. Next, note that $f'(k_i) - f'(k_j) = t_i - t_j$. Since $k_j > k^*$ for some j, $t_i > t_i - t_j = f'(k_i) - f'(k_j) > f'(k_i) - f'(k^*)$. If $f''' \leq 0$ then $f'(k_i) - f'(k^*) \geq (-f''(k_i))(k^*-k_i)$, so that $-\frac{m_i t_i}{f''_i} > k^*-k_i$, proving $\frac{\partial b_i}{\partial t_i}[-\frac{m_i t_i}{f''_i} + k_i-k^*] < 0$ if $f''' \leq 0$. Therefore

> the reaction function of each jurisdiction is single valued within the regime in which the return to capital is positive provided that $f''' \leq 0$.

Also, consider the jump in F_i at the locus r=0. From (24) and (25),

$$F_i^- - F_i^+ = b_i\{(k_i-k^*) - \frac{m_i t_i}{f''_i}\},$$ which has just been shown positive.

Therefore, the jump in the reaction function is in the right direction.

> $f''' < 0$ ensures each jurisdiction's reaction function is everywhere single-valued.

Since F_i is continuous in each component of t_{-i}, each jurisdiction's reaction will be a continuous function with a compact range,[13] provided it is single-valued. Hence $f''' \leq 0$ is sufficient to ensure the existence of a

Nash equilibrium in tax rates. However, this condition is fairly strong: a Cobb-Douglas production function certainly violates it.

The above result is the only general existence theorem in this paper. I feel it is probably as good as one can get in this model. Clearly if f''' is sufficiently large, then second-order conditions will fail at some critical points of the jurisdiction's optimization. Since the third derivative is a local property, a perverse perturbation of f''' at some equilibrium could change the sign of F_i' without altering behaviour anywhere else. If reaction correspondences are multiple-valued and discontinuous, as might occur if $f''' > 0$, I know of no applicable general existence result. The results of Novshek (1985), or McManus, for discontinuous reactions require conditions on the sign of the slope of reaction correspondences. From the definition of b_i it can be seen that $\frac{\partial b_i}{\partial t_j}$, and hence $\frac{\partial F_i}{\partial t_j}$, depends in a rather complicated way on fourth derivatives of f. Thus I can see no meaningful restrictions on the primitive functions which would ensure that reaction correspondences are non-increasing, or that jumps in the correspondences are jumps down, which is the sort of requirement Novshek (1985) shows is minimal in a sense for existence of Nash equilibrium.

4. CHARACTERISTICS OF EQUILIBRIUM

Much of the literature involving large numbers of jurisdictions has discussed the under-supply of the publicly provided good induced by tax competition. Such a result seems intuitive; attempting to attract a tax base, jurisdictions lower tax rates more than they otherwise would. In the model presented here, the Samuelson efficiency condition (Σ MRS = MRT) is $m=1$. Thus an under-supply can be said to exist in the "over-pricing" sense, if $m > 1$. Inequality (27), in the previous section, already noted that $m_i > 1$ for any jurisdiction in which $k_i \leq k^*$. Thus at least one jurisdiction (that with the highest tax rate) must undersupply the publicly provided good. It is also clear from equation (25) that all jurisdictions must under-supply the good in this sense if there is an equilibrium with excess supply of capital. But as the subsequent discussion showed, this under-supply is due to a statutory tax base which is too small, not to tax competition.

Must all jurisdictions undersupply the public good? Inequality (26) suggests that if the parameter b_i were small enough m_i might approach a number greater than 1. In fact (24) shows if $b_i=0$, $(m_i-1)k_i = -\dfrac{m_i t_i}{f''_i}$. Note that the size parameter λ_i is in the numerator of the definition of b_i. Hence some support has been given to the notion that all jurisdictions must under-supply if they are all sufficiently small. This notion will be made more precise in subsequent sections; one must of course be careful that other variables don't approach the wrong limits as $\lambda_i \to 0$.

If all jurisdictions are not small enough, they all need not under-supply the public good. This can be seen by substituting a very large value of k_i in (24). However, note for any two jurisdictions, if $t_i < t_j$, then $x_i > x_j$. That result is a simple consequence of the definition of x_i (equation (5)) and the fact that $f'(k_i) - f'(k_j) = t_i - t_j$. Recall that the valuation m of the publicly provided good is increasing in x and decreasing in g. Since $t_i < t_j$ implies $x_i > x_j$, then a necessary condition for $m_i < m_j$ (if

$t_i < t_j$) is $g_i > g_j$. Therefore, if a jurisdiction i sets a relatively low tax rate t_i, so that $k_i > k^*$, then one (or both) of two possibilities must occur. Either $m_i > m_j > 1$, or residents of jurisdiction i consume more of each commodity than residents of jurisdiction j, where j is any jurisdiction in which $k_j < k^*$.

Which jurisdictions set which tax rates? One of the main results of this paper is that there is a tendency for small jurisdictions to set low tax rates.[14] To see this tendency, suppose two jurisdictions i and j were to set $t_i = t_j = t$. Suppose further that jurisdiction i is the smaller of the two, so that $\lambda_i < \lambda_j$. If $t_i = t_j$, then $k_i = k_j$ in any allocation. The definition of b_i implies $\frac{b_i}{b_j} = \frac{\lambda_i}{\lambda_j}$ if $k_i = k_j$. Equation (24) can be written

$$F_i = (m_i - 1)k_i + \frac{m_i t_i}{f''_i} + b_i[k_i - k^* - \frac{m_i t_i}{f''_i}] \tag{30}$$

If $t_i = t_j$, then $x_i = x_j$, $g_i = g_j$ and $m_i = m_j$. Hence $F_i - F_j = (b_i - b_j)$ $[k_i - k^* - \frac{m_i t_i}{f''_i}]$. Clearly, if $k_i = k_j \geq k^*$, then $F_i < F_j$ when $\lambda_i < \lambda_j$ and $t_i = t_j$. Thus if the larger jurisdiction's optimal tax rate is t_j, and $k_j \geq k^*$, the smaller jurisdiction i will want to lower taxes below t_j. Note if I=2 and $t_1=t_2$, then $k_1 = k_2 = k^*$.

An immediate consequence of the above observation is that a symmetric equilibrium [i.e. one in which $t_1=t_2= =t_I$] can occur only if there is excess supply of capital, or if all jurisdictions are the same size [$\lambda_1 = \lambda_2 =$

$= \lambda_I = \frac{1}{I}$]. Despite the constant returns to scale in population, different-sized jurisdictions choose different tax rates in equilibrium.

The above conclusion can be strengthened if it is again assumed that $f''' \leq 0$. For then, if $\lambda_i < \lambda_j$ and $t_i = t_j$, $F_i < F_j$ regardless of whether or not $k_i \geq k^*$. The remainder of the paragraph proves this assertion, and may be ignored by those willing to accept the proof on faith. Suppose $\lambda_i < \lambda_j$, $t_i = t_j$ and $k_i = k_j < k^*$. Earlier results (equation (27)) then imply $m_i = m_j > 1$. From (30) a necessary and sufficient condition for $F_i < F_j$ is $k_i - k^* - \frac{m_i t_i}{f''_i} > 0$. Since $m_i > 1$, a sufficient condition is thus $(-f''_i)(k^* - k_i) < t_i$. Let jurisdiction m set $t_m > 0$ so that $k_m \geq k^*$. Then $t_i \geq t_i - t_m = f'_i - f'_m$. If $f''' \leq 0$, $f'_i - f'_m \geq (-f''_i)(k_m - k_i) > (-f''_i)(k^* - k_i)$, which proves the assertion.

If either (i) I=2 or (ii) $f''' \leq 0$, then when two jurisdictions set the same tax rate, the larger must have a higher value for its "utility gradient" F_i. This higher value indicates a tendency to set a higher tax rate. Appendix 2 shows this tendency can be translated into a characterization of equilibrium tax rates. What that appendix proves is as follows.

Suppose jurisdictions are numbered so that $\lambda_1 \geq \lambda_2 \geq .. \geq \lambda_I$. Suppose further that <u>either</u>

(i) I=2 and both reaction correspondences are single-valued <u>or</u>

(ii) $f''' < 0$. Then there exists an equilibrium with $t_1 \geq t_2 \geq .. \geq t_I$.

Notice there are no asymmetries here in technology. If all jurisdictions set the same tax rates, all residents would consume the same bundles of commodities. But different sizes of jurisdiction induce behavioural asymmetries. Examination of equation (17) shows that if

$t_i = t_j$ and $\lambda_i < \lambda_j$, then $\frac{\partial k_i}{\partial t_i} < \frac{\partial k_j}{\partial t_j} < 0$. Smaller jurisdictions perceive a more elastic reaction of the capital stock per capita to the tax rate. It is this perceived reaction that is the essence of tax competition; jurisdictions are reluctant to raise taxes, since their tax base moves elsewhere. Since smaller jurisdictions face a more elastic reaction, they are more loathe to raise taxes. They will (under circumstances delineated above) set lower tax rates.

However, this bigger tax competition effect does not necessarily mean "small is bad". Earlier in this section the possibility was raised of residents of low tax jurisdictions consuming more of both goods. In Section 8 it will be shown that residents of the small jurisdiction must be better off in equilibrium in a 2-jurisdiction nation if disparities in size are sufficiently wide.

5. SYMMETRIC EQUILIBRIUM

In this section, only identical jurisdictions are considered (i.e. those for which $\lambda_1 = \lambda_2 = .. = \lambda_I = \frac{1}{I}$). The previous section showed that only under such circumstances could a symmetric equilibrium arise. The focus of this section is on symmetric equilibria, and on their behaviour as the number of jurisdictions grows large.

To begin, consider the values t of the tax rate such that $F_1 = F_2 = \ = F_I = 0$ when $t_1 = t_2 = \ = t_I = t$. Any symmetric equilibrium in which $r > 0$ must satisfy such a property. But in addition it must be true that F_i'

≤ 0 for the choice of $t_i = t$ to be optimal. There is at most one value of t satisfying $F_i(t; t_{-i}) = 0$ when $t_{-i} = (t,...t)$. To see this, consider first the value of F_i when all jurisdictions and tax rates are identical. Then $k_i = k^*$, and $b_i = \frac{1}{I}$. Hence

$$F_i = (m - 1)k^* + \frac{mt}{f''(k^*)} \frac{I - 1}{I} \quad (31)$$

where $m = m\,[f(k^*) - tk^*, tk^*]$. Since $m[f(k^*), 0] = +\infty$, F_i will be positive for small values of t. As t increases, m must fall. The derivative of F_i with respect to t is $[k^* + \frac{t}{f''}\frac{I-1}{I}]\frac{\partial m}{\partial t} + \frac{I-1}{I}\frac{m}{f''}$. The expression in square brackets must be positive when $F_i = 0$ and $\frac{\partial m}{\partial t} < 0$, proving that there is at most one value of t such that $F_i = 0$.

For the net return to capital to be positive, $t < f'(k^*)$. Therefore, if $(m - 1)k^* + \frac{mf'(k^*)}{f''(k^*)}\frac{I-1}{I} < 0$ when $m = m\,[(1-\alpha)f(k^*), \alpha f(k^*)]$, exactly one value of t exists satisfying $F_i = 0$, and $r > 0$. If $(m-1)k^* + \frac{mf'(k^*)}{f''(k^*)} > 0$ when $m = m\,[(1-\alpha)f(k^*), \alpha f(k^*)]$ a symmetric equilibrium with excess supply of capital exists. Otherwise a symmetric equilibrium exists with r=0, and no excess supply of capital.

Suppose now that some $t < f'(k^*)$ exists satisfying $F_i = 0$. Does it represent a symmetric equilibrium? From equation (29)

$$F'_i = [2m - \frac{I-1}{I}] \frac{I-1}{I} \frac{1}{f''(k^*)} + \frac{\partial m_i}{\partial t_i} [k^* + \frac{t}{f''(k^*)} \frac{I-1}{I}]$$

$$+ \frac{\partial b_i}{\partial t_i} [\frac{mt}{f''(k^*)}] - mt\, f'''(k^*) (\frac{I-1}{I})^2 (f''(k^*))^{-3} \qquad (32)$$

If the third derivative of f is sufficiently positive, F'_i may still be positive. Moreover, this possibility does not disappear if the number of jurisdictions increases. Although $\frac{\partial b_i}{\partial t_i}$ must then approach zero, the final term in (32) does not shrink as I grows.

Thus the assurance is false that a symmetric equilibrium must exist if all jurisdictions are the same, and small enough to have no influence on the national return to capital. Such an assurance requires restrictions on the third derivative of the production function (namely that $(2m-1) + \frac{\partial m_i}{\partial t_i}$ $[f''(k^*)k^* + t] - mt\, f'''(k^*) (f''(k))^{-2} > 0$ when $(m-1)f''(k^*)k^* + mt = 0$ where $m = m(f(k^*) - tk^*, tk^*)$). If these restrictions are not satisfied, and if $(m-1)k^* + \frac{mf'(k^*)}{f''(k^*)} < 0$, there can be no symmetric equilibrium, no matter how many jurisdictions there are. There may be asymmetric equilibria, there may be no equilibria. But there is a unique potential symmetric equilibrium, and second-order conditions must be satisfied there for it to represent an equilibrium.

What happens to the symmetric equilibrium, if it exists, as the number of jurisdictions increases? Nothing if the equilibrium involves excess supply of capital. If the equilibrium is on the boundary between regimes, it must be true that $(m-1)k^* + \frac{mf'(k^*)}{f''(k^*)} \frac{I-1}{I} \geq (m-1)k^* + \frac{mf'(k^*)}{f''(k^*)}$ (where $m = m((1-\alpha)f(k^*), \alpha f(k^*)))$. Since the two sides of this inequality converge to each other as I grows, the likelihood of such an equilibrium falls. More precisely, if the symmetric equilibrium for some value of I involves a positive net return to capital, so will symmetric equilibria for all larger values of I.

A simple examination of (31) shows what happens to tax rates in symmetric equilibria with $r > 0$ as the number of jurisdictions rises. They fall. For if $F_i = 0$ for some value of I and t, $F_i < 0$ for all larger values of I. Tax rates do not fall to zero. They start (at I=1) with the value yielding m=1, and fall towards the value yielding $m(f(k^*) - tk^*, tk^*)$ $(k^* + \frac{t}{f''(k^*)}) = k^*$ as $I \to \infty$. Since m is a decreasing function of t, this MRS must rise with the number of jurisdictions. Increasing tax competition exacerbates the undersupply of the publicly provided good.

In any symmetric equilibrium, $k_i = k^*$ for all i. At the equilibrium, $b_i = \frac{1}{I}$, so that $b_i \to 0$ as $I \to \infty$. From (19), then, the influence any jurisdiction's policy has on the national net return to capital disappears as the number of identical jurisdictions grows. Thus some support is given to the usual assumption made in "small open economy" models that if the number of identical jurisdictions is large enough, each can take the national return to capital r as unaffected by its own policy.

However, the result established above held only for symmetric equilibrium. It need not hold for arbitrary choice of policy. For instance, imagine a sequence of economies with $\lambda_1 = \ldots = \lambda_I = \frac{1}{I}$, in which $t_2 = t_3 = \ldots = t_I = f'(\frac{k^*}{2})$ and in which $t_1 = \frac{1}{2}\{f'[(\frac{I+1}{2})k^*]\}$. Such policies imply a sequence of k_i's in which k_2 through k_I approach $\frac{k^*}{2}$, r approaches 0 (from above), and k_1 approaches $(\frac{I+1}{2})k^*$, which of course grows infinitely large with I. From the definition of b_1, b_1 approaches zero as $I \rightarrow \infty$ if and only if $I f''[(\frac{I+1}{2})k^*]$ approaches infinity with I. If $f = k^\alpha$, this will will not be the case.

What the above example is intended to show is that, even when all jurisdictions are identical, it is not true that the mere fact that a jurisdiction is small implies so is its influence on national variables. This influence is (in this model) endogenous, and depends on the choice of policies by all jurisdiction.

In the above example, the choice of policies described cannot be an equilibrium. Jurisdictions 2 through I would be better off setting

$t_i = f'(k^*)$, for example.[15] But this sort of peculiarity cannot be ruled out a priori. To establish that small size implies small influence, one must establish either (i) the technology is such that $b_i \to 0$ as $\lambda_i \to 0$ regardless of what are $k_1, .. k_I$ or (ii) rational policy choice by each jurisdiction imply $b_i \to 0$ as $\lambda_i \to 0$. The next section on "limit results" establishes some such restrictions.

6. LIMIT RESULTS

The purpose of this section is to establish when sufficiently small size can be said to imply negligible influence on national variables. This implication is usually assumed in "small open economy" models. Under what circumstances can it be derived from examining the behaviour of large economies as the number of jurisdictions shrinks?

More formally, equation (19) shows that b_i measures jurisdiction i's influence on the net national return to capital r. In this model λ_i measures the relative size of jurisdiction i. For any small positive δ is there some small positive ε such that $\lambda_i < \varepsilon$ implies $b_i < \delta$? Ideally, one would like this limiting function $\varepsilon(\delta)$ to depend only on tastes and technology, not on the number of jurisdictions, or their policies.

Note that attention here is not being restricted to the symmetric case. Any sequence of economies $\sigma=1,\ldots\infty$ will be allowed, provided $i \le I^{(\sigma)}$ for all σ, and $\lambda_i^{(\sigma)} \to 0$. It may be a symmetric sequence, in which $I^{(\sigma)} = \sigma$, and $\lambda_i^{(\sigma)} = \frac{1}{\sigma}$. But one may also consider an asymmetric sequence. For example, one might have $I^{(\sigma)} = 2$, $\lambda_1^{(\sigma)} = 1 - \frac{1}{\sigma}$, $\lambda_2^{(\sigma)} = \frac{1}{\sigma}$ and examine whether $b_2^{(\sigma)} \to 0$.

Most of the section is fairly mechanical. I begin by establishing a lower bound for $(-\sum_j \frac{\lambda_j}{f''(k_j)})$ depending only on f and k^*. Appendix 1 shows that there is some strictly positive k_o such that no jurisdiction would ever choose a per capita capital stock less than k_o. This k_o depends only on f and k^*. Next note that at least half the population must live in jurisdictions in which the capital stock per capita is $2k^*$ or less. Now define

$$A_o = 2 \inf_{k_o \le k \le 2k^*} \left\{ - \frac{1}{f''(k^*)} \right\}$$

Since f is strictly concave, A_o is strictly positive. Since at least half the population must live in jurisdictions in which $k_o \le k \le 2k^*$, then

$$- \sum_j \frac{\lambda_j}{f''(k_j)} \ge A_o > 0,$$ establishing the required lower bound.

From the definition of b_i

$$b_i \equiv \frac{\lambda_i}{f''(k_i)} \bigg/ \left(\sum_{j=1}^{I} \frac{\lambda_j}{f''(k_j)} \right)$$

it can be seen that a sufficient condition for $b_i \to 0$ as $\lambda_i \to 0$ is that $-f''(k_i)$ be bounded below away from zero as $\lambda_i \to 0$. For if $-f''(k_i) \ge A_1 >$

0, then $b_i \le \frac{\lambda_i}{A_o A_1}$ which establishes the result.

If $f''' < 0$, then the job is done. For then appendix 1 establishes $-f'' \ge -f''(k_o) > 0$.

The restriction $\Sigma\, \lambda_j k_j \le k^*$ implies $k_i \le \frac{k^*}{\lambda_i}$. Therefore if

$-kf''(k) \to \infty$ as $k \to \infty$

the result also obtains. The reasoning is straightforward. Suppose a sequence of k_i's exists with $-\frac{\lambda_i}{f''(k_i)} > a_o$. Then $-\frac{f''(k_i)}{\lambda_1} < \frac{1}{a_o}$. Define γ as $k_i \lambda_i$, so $-\frac{1}{\gamma} f''(k_i)k_i < \frac{1}{a_o}$. The restriction $\Sigma\lambda_j k_j \le k^*$ implies each γ is less than or equal to k^*, so that $f''(k_i)k_i < \frac{k^*}{a_o}$, which cannot occur if $\lim_{k\to\infty} kf''(k) = -\infty$. Hence even if $f''' > 0$, if f'' does not decrease too quickly in absolute value, b_i must approach zero with λ_i.

An alternative method of showing $b_i \to 0$ is by providing restrictions on jurisdictions' behaviour which imply k_i must be bounded above for all i, independent of λ_i.

Earlier, it was shown that if the tax base was large enough, then the net return to capital could be bounded below away from zero. Specifically under condition (28)

$$m[(1-\alpha(k))f(k),\ \ \alpha(k)f(k)] < 1 \tag{28}$$

for all $k_o \leq k \leq k^*$ then it was shown there was some r_o such that $r \geq r_o >$ 0 in any equilibrium. Since $f'(k_i) \geq r$ for all i, an upper bound for each k_i has been found, namely the solution to $f'(k) = r_o$.

Again using earlier results, recall the elasticity of capital's marginal product was defined by

$$\varepsilon_R \equiv -\frac{f''}{f'} k$$

Consider now a sequence of economies in which every jurisdiction's size approaches zero. Such a sequence need not be symmetric; one might have $I^{(\sigma)} = \sigma$, $\lambda_i^{(\sigma)} = i\ \phi(\sigma)$ where $\phi(\sigma)$ is defined by $\phi(\sigma) = 2[\sigma(\sigma+1)]^{-1}$, so that $\frac{\lambda_i^{(\sigma)}}{\lambda_{2i}^{(\sigma)}} = 2$ for all $\sigma \geq 2i$. In such a sequence it must always be true that some $k_j \leq k^*$. Thus for large enough σ, b_j will be arbitrarily small for some j. Suppose now that $r^{(\sigma)} \to 0$. Then for large enough σ, $\frac{\partial g_j}{\partial t_j}$ will approach $k_j + \frac{f'_j}{f''_k}$ arbitrarily closely, since $b_j \to 0$ and $t_j = f'_j - r \to f'_j$. If $\varepsilon_R < 1$, then $k_j + \frac{f'_j}{f''_j}$ must be negative. But $\frac{\partial g_j}{\partial t_j}$ cannot be negative in equilibrium for any jurisdiction j. Thus $\varepsilon_R < 1$ (for all k) implies r cannot approach 0 as $\sigma \to \infty$, which again bounds k above. Notice that the Cobb-Douglas technology satisfies $\epsilon_R = (1-\alpha) < 1$. Summarizing the above argument

> if in a sequence of economies, each jurisdiction's relative size approaches 0, then $\epsilon_R < 1$ is sufficient to ensure each $b_i \to 0$ in equilibrium.

The above results indicate some plausible restrictions which imply a jurisdiction's influence must become small as its size does. The example of Appendix 3 shows this need not always be true, however.

Where are the implications of small size? Examination of (24) shows that if $b_i \to 0$, and each k_i is bounded then eventually $m_i > 1$, unless $t_i \to 0$. But if $t_i \to 0$, so does g_i, and $m(x,0) > 1$. So if a jurisdiction's influence is small it must undersupply the public good. The conclusion need not hold if the jurisdiction's size alone (λ_i) is small, without b_i shrinking as well. If $m[(1-\alpha), \alpha f(k)] \geq 1$ as k becomes large, then $m_i > 1$ as $\lambda_i \to 0$ even if b_i does not approach zero.[16]

Another implication of the "small open" economy model is that all residents attain identical utility. For if jurisdiction j made its residents better off than jurisdiction i, jurisdiction i could simply imitate jurisdiction j's policy. If r is affected by a jurisdiction's policy, this equal-utility result need not hold. For by changing its tax rate to t_j, jurisdiction i is altering both r and k_i. It can certainly achieve the same utility as residents of jurisdiction j would get if $t_i = t_j$. But that utility may be lower than what residents of jurisdiction j get currently, with $t_i \neq t_j$. It should be clear that the "equal utility" result will hold in the limit provided that each $b_i \to 0$ and all per capita stocks k_i are bounded above.

In this vein, consider what happens to F_i if $b_i \to 0$. It approaches $(m_i - 1)k_i + \frac{t_i m_i}{f''_t}$. Suppose each $b_i \to 0$. Then in the limit there is a nearly symmetric equilibrium, with $k_i \to k^*$ and $t_i \to$ the solution to $(m-1)k^* + \frac{mt}{f''(k^*)}$, where $m = m[f(k^*) - tk^*, tk^*]$, provided that the second-order conditions discussed in the earlier section on symmetric equilibrium hold. Even if relative size differences persist in the limit (as in the earlier example where $\frac{\lambda_i}{\lambda_j} = \frac{i}{j}$), not only will all residents' utilities converge to the same value, so will each jurisdiction's policy. Thus

if $f''' < 0$, then if $\lambda_i^{(\sigma)} \to 0$, $\forall$ i, there is a sequence of equilibria in which all t_i's converge to the same value.

7. EFFICIENCY

A word should be said about efficiency of equilibrium. That word is that Nash equilibrium is never efficient (unless there is only one jurisdiction). This result is hardly very surprising.

Given the MRT between the two consumption goods, there are 2 efficiency conditions: GNP should be maximized, and output divided efficiently between the goods. Maximization of GNP requires $k_1..k_I$ be chosen to maximize

$\sum_i \lambda_i f(k_i)$ subject to $\Sigma k_i \lambda_i \leq k^*$, implying the obvious condition $k_1 = k_2 =$ $= k_I = k^*$, under which capital's marginal product is equated across jurisdictions. Allocative efficiency requires each person's MRS equal 1. If attention is restricted to allocations in which all residents of a given jurisdiction are treated the same, the set of efficient allocations is the set of $\{k_i, x_i, g_i\}_{i=1,..I}$ such that $k_i = k^*$, $m(x_i, g_i) = 1$ and $\Sigma \lambda_i (x_i + g_i) = f(k^*)$. One such allocation is the one which treats all residents of all jurisdictions the same, so that $(x_i, g_i) = (x^*, g^*)$, the unique solution to $x^* + g^* = f(k^*)$, $m(x^*, g^*) = 1$.

If all the jurisdictions merged, voters would choose the efficient equal-utility solution outlined above, provided that $m[(1-\alpha(k^*))k^*, \alpha(k^*)k^*] < 1$. If this condition does not hold, then the tax base is inadequate. The maximum possible g (for the one-jurisdiction nation) is $\alpha(k^*)k^*$, in which case the net return to capital has been driven down to zero. Provided that $\alpha(k^*)k^*$ is less than the desired expenditure on the publicly provided good, the capital tax is efficient, since aggregate capital supply is inelastic (if $r > 0$), and voters recognize their dual roles as consumers and as owners of capital.

If $I > 1$ the previous sections show equilibrium must be inefficient. The production efficiency condition $k_1 = k_2 = \quad = k_I = k^*$ can hold only if $\lambda_i = \lambda_2 = \ .. = \lambda_I = \frac{1}{I}$. But then all m_i's are greater than one.

Note also that utility attained in symmetric equilibrium declines as the number of jurisdictions increases. In symmetric equilibrium, the problem is that residents of each jurisdiction treat their own capital stock as endogenous, when the aggregate supply is fixed. Increased jurisdictional fragmentation exacerbates this problem further, as the perceived elasticity $\left| \frac{\partial k_i}{\partial t_i} \frac{t_i}{k_i} \right|$ rises with I.

I do not regard the inefficiency of equilibrium in this model as providing any justification for intergovernmental grants, or federal equalization payments. Within the context of the model, there are far simpler ways of achieving efficiency. Merge the jurisdictions. Replace the capital tax by a head tax. Levy the capital tax on its owners, not its users.

Why might residents put up with the inefficient institutional structure hypothesized here? The next section shows that some residents may be better off under inefficient fragmentation than in the efficient equal-utility allocation. Thus the simple solutions listed above do not necessarily represent Pareto improvements on the Nash equilibria of the model.

8. TWO JURISDICTIONS

To examine briefly the effects of asymmetries in jurisdictional population, consider a 2-jurisdiction nation, where by convention 1 will assume

$$\lambda_1 \geq \frac{1}{2} \geq \lambda_2$$

From the previous results, there is a tendency for the smaller jurisdiction 2 to be a tax haven, setting a lower tax rate because of the more elastic response of its per person capital stock to its tax rates.

Of particular interest here is the effect of increasing the asymmetry, by raising λ_1 and lowering λ_2. As $\lambda_1 \to 1$, it might be expected that residents of jurisdiction 1 choose a tax policy approaching the optimum, as their economy becomes less and less open. If $b_2 \to 0$ as $\lambda_2 \to 0$, then $F_1 \to m_1k_1 - k^*$. Also, if $\lambda_2k_2 \to 0$ as $\lambda_2 \to 0$, then k_1 must approach k^* (if there is no excess supply of capital). Hence if $\lambda_2k_2 \to 0$ and $b_2 \to 0$, jurisdiction 1's policy approaches the first-best, $k = k^*$ and $m = 1$.

Appendix 3 presents an illustration (using a Cobb-Douglas production function) to show that it need not be true that $b_2 \to 0$ as $\lambda_2 \to 0$. This example requires that the capital stock be an inadequate tax base, in that $m[(1-\alpha)f(k^*), \alpha f(k^*)] > 1$. Although it is not true that the influence of jurisdiction 2 (measured by b_2) shrinks to zero with the population, it is true in this particular example that $k_1 \to k^*$. However m_1 does not approach 1. In the limit, in this example, both jurisdictions underprovide the publicly provided good, in that $m_2 > m_1 > 1$. Consumption per capita of each good by residents of jurisdiction 2 rises without bound as the population λ_2 shrinks. Thus, in Appendix 3, it certainly pays to live in a very small jurisdiction.

This result holds more generally. Earlier it was shown that if reaction correspondences were continuous, an equilibrium would exist in which $t_1 \geq t_2$. Here it will be shown that if λ_1 is large enough, utility in such an equilibrium must be higher in the smaller jurisdiction.

First, note that since $g_i = t_ik_i$, then $\dfrac{\partial g_1}{\partial t_2} = t_1 \dfrac{\partial k_1}{\partial t_2} > 0$.

Next, consider $\dfrac{\partial x_1}{\partial t_2}$. Since $x_1 = f(k_1) - f'(k_1)k_1 + rk^*$, $\dfrac{\partial x_1}{\partial t_2} =$

$= f''(k_1)k_1 \frac{b_1}{f''_2} - b_2 k^*$. If I denote the denominator $-\left(\frac{\lambda_1}{f''_1} + \frac{\lambda_2}{f''_2}\right)$ of b_i by b, then

$$\frac{\partial x_1}{\partial t_2} = \frac{1}{b}\{-\frac{\lambda_1}{f''_2} k_1 + \frac{\lambda_2}{f''_2} k^*\} \tag{33}$$

Since $k_1 > k_o$, if $\frac{\lambda_1}{\lambda_2} > \frac{k^*}{k_o}$, then $\frac{\partial x_1}{\partial t_2} > 0$.

Consider now an equilibrium set of tax rates (t_1^*, t_2^*) with $t_1^* > t_2^*$, for an economy in which $\frac{\lambda_1}{\lambda_2} > \frac{k^*}{k_o}$. Suppose jurisdiction 2 raised its tax rate to t_1^* from t_2^*. The previous paragraph showed that one byproduct of such an increase would be increased consumption of each good, and thus increased utility for residents of the larger jurisdiction.

Denote by $U_1[t_1, t_2]$ and $U_2[t_1, t_2]$ the utility residents of the two jurisdictions obtain when the tax vector is (t_1, t_2). It has just been shown that $U_1[t_1^*, t_1^*] > U_1[t_1^*, t_2^*]$. Since the tax rate t_2^* is optimal for jurisdiction 2, $U_2[t_1^*, t_2^*] \geq U_2[t_1^*, t_1^*]$. If $t_1 = t_2$, then $u_1 = u_2$, so that $U_2[t_1^*, t_2^*] \geq U_2[t_1^*, t_1^*] = U_1[t_1^*, t_1^*] > U_1[t_1^*, t_2^*]$, proving the result. It pays to be small.

Suppose now that as $\lambda_2 \to 0$, a sequence of Nash equilibria exists, in which k_2 is bounded. As noted above, then jurisdiction 1's policy approaches the "first-best' $k_1 = k^*$, $m_1 = 1$ (provided $m[(1-\alpha(k^*))f(k^*), \alpha(k^*)f(k^*)]$

< 1). But utility in the small jurisdiction is strictly higher. In fact, if λ_2 is small enough, utility there must be be strictly higher than in the first-best, equal treatment solution. Since $\frac{\partial g_1}{\partial t_2} = -t_1 b_1/f''_2$, $u_2 - u_1$ can be strictly bounded below away from zero.[17] Hence as $\lambda_1 \to 1$, u_1 approaches the first-best equal-treatment level, and u_2 approaches something strictly better.

Of course equilibrium is not efficient, no matter how small is λ_2. The big gains from segregation accruing to the few residents of jurisdiction 2 are outweighed by the small losses to the many residents of jurisdiction 1. However, the results of this section give an explanation as to why residents of a small jurisdiction may veto merger, even when it leads to efficiency. If the merged jurisdiction is to treat all its residents equally, former residents of the small jurisdiction are made worse off by the merger. Only if the new merged jurisdiction can commit itself to treating these residents better than others will merger represent a Pareto improvement. Given majority rule, it may be difficult to commit to this sort of favourable treatment.[18]

9. CONCLUDING REMARKS

Some of the results of this paper may seem somewhat disappointing. A very simple stylized model has been presented. All residents were assumed identical. The technology was quite simple. No possibility of commodity trade was allowed. Residents were immobile. Political behaviour was very unsophisticated. Nonetheless, establishing the existence of equilibrium required fairly strong (and somewhat implausible) restrictions on technology.

I have tried to argue that this problem is in the nature of the beast; there is no simple set of more general, or economically meaningful, restrictions which guarantee existence of equilibrium.

Similarly the limiting results can be viewed as somewhat negative. Without stronger assumptions than are usually made, the presumption that a jurisdiction's influence on national variables shrinks as its size does cannot be supported in general. On the other hand, counter-examples to this presumption are likely to be somewhat contrived and implausible.

In defense of these aspects of the paper, a large literature on tax competition does exist. It is worthwhile to establish rigorously the foundations upon which this literature must rest.

In a more positive vein, some specific implications for tax competition are presented when jurisdictions are relatively large. Even if consumption possibilities per capita do not vary with population, and people are identical, heterogeneity in tax rates will arise if jurisdictions differ in population. Large jurisdictions should worry less about tax base erosion, and levy higher taxes. The small tax havens may be able to free ride on larger neighbors; residents should be better off in the smaller jurisdictions. Indeed the small jurisdiction may wind up with a better public sector, despite its lower tax rates. An explanation is provided for smaller jurisdictions holding out credibly for a disproportionate share of the gains from federation with other jurisdictions.

APPENDIX 1

In this appendix, it will be shown that there is a maximal tax rate T any jurisdiction would ever set, and a minimal per capita capital stock $k_o > 0$, and that T and k_o depend only on the technology and tastes (and k^*), and not on the number of jurisdictions or their relative populations.

First, define the function $\phi(\ )$ by

$$f'[\phi(x)] = x \tag{34}$$

so that $\phi' = \frac{1}{f''} < 0$. The Inada conditions imply ϕ maps $[0,\infty]$ onto $[0,\infty]$.

Second, note $\frac{\partial x_i}{\partial t_i} < 0$, whether or not there is excess supply of capital (equations (20) and (22)).

(i) Since $f(0) = 0$, and f is concave, $t_i k_i \leq f'(k_i)k_i < f(k_i)$, so that $t_i k_i < f[\phi(t_i)]$.

(ii) Define $\hat{t}(\varepsilon)$ by $f[\phi(\hat{t}(\varepsilon))] = \varepsilon$. Then if $t > \hat{t}(\varepsilon)$, $g_i = t_i k_i < \varepsilon$.

(iii) If $r > 0$, then $\Sigma\lambda_i k_i = k^*$, so that $k_j \geq k^*$ for some j. Then $r < f'(k_j) \leq f'(k^*)$.

If $r = 0$, clearly $r < f'(k^*)$.

So in any allocation, $r < f'(k^*)$.

(iv) Let $0 < \hat{k} < k^*$ satisfy $f'(\hat{k}) = 2f'(k^*)$.

Suppose jurisdiction i sets $t_i = f'(k^*)$.

Then $f'(k_i) = r + t_i < 2f'(k^*)$ (from (iii))

$= f'(\hat{k})$ (by definition).

Therefore a tax rate of $t_i = f'(k^*)$ guarantees a tax base of at least $\hat{k}$, no matter what other jurisdictions do.

(v) From (ii) if $t_i > \hat{t}[f'(k^*)\hat{k}]$, then $g_i < f'(k^*)\hat{k}$.

(vi) Consider any tax rate higher than $\hat{t}[f'(k^*)\hat{k}]$, and compare it as a policy with $t_i = f'(k^*)$. From (v) the higher tax rate yields consumption of g_i of at most $f'(k^*)\hat{k}$.

From (iv) the lower tax rate yields a level of g_i of at least $f'(k^*)\hat{k}$.

Since $\frac{\partial x_i}{\partial t_i} < 0$, setting $t_i = f'(k^*)$ dominates any tax rate of $T = \hat{t}[f'(k^*)\hat{k}]$, bounding tax rates above.

(vii) From (iii) and (vi), $f'(k_i) = r + t_i < f'(k^*) + T$ in any allocation.

Let $k_o = \phi[f'(k^*) + T]$, so that $k_i > k_o$ in any allocation.

APPENDIX 2

Here two results are proved involving larger jurisdictions levying higher tax rates. The first result requires slightly weaker assumptions about the technology, and may have a more intuitively appealing proof, but is valid only when there are two jurisdictions. In what follows, by convention $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_I$.

THEOREM 1: If I=2, and both reaction correspondences are continuous, then a Nash equilibrium exists with $t_1 \geq t_2$.

PROOF: From equation (30), if $t_1 = t_2$, then $k_1 = k_2 = k^*$, and $F_1 \geq F_2$.

For any i, there can be at most one value of t for which $F_i = 0$ when $t_1 = t_2 = t$ (for a proof see the beginning of section 5).

Consider then the reaction curves of the 2 jurisdictions in the set $\{(t_1, t_2) | t_1 \geq t_2, 0 \leq t_i < T\}$.

Since $F_1 \geq F_2$ along the line $t_1 = t_2$, jurisdiction 1's reaction curve touches this line to the right of jurisdiction 2's (where t_1 is on the horizontal axis, t_2 on the vertical).

Jurisdiction 1's reaction curve must eventually hit the line $t_2 = 0$ (there is some optimal reaction to $t_2 = 0$). It never hits the line $t_1 = T$ (appendix 1 shows $t \geq T$ is never optimal).

Jurisdiction 2's reaction curve cannot hit the line $t_2 = 0$ (the optimal tax is always positive), and must hit the line $t_1 = T$.

Hence figure 1 shows the 2 curves must cross somewhere in the set $\{(t_1, t_2) | t_1 \geq t_2; 0 \leq t_i \leq T\}$ proving the theorem.

<u>THEOREM 2</u>: If $f''' < 0$, there is a Nash equilibrium in which $t_1 \geq t_2 \geq \quad \geq t_I$.

<u>PROOF</u>: If $f''' < 0$ then if $F_i = 0$, $F_i' \leq 0$, and t_i is an optimal reaction for jurisdiction i. Hence it suffices to prove there is some $(t_1, \ldots t_I)$ at which $F_i = 0$ for all i.

A mapping will be constructed whose fixed point is such a tax vector.

Since $F_i > 0$ at $t_i = 0$ for all i, and all t_{-i}, there is some strictly positive t_o such that $F_i > 0$ at $t_i = t_o$ for any $t_{-i} \in [0,T]^{I-1}$ (this construct merely eliminates the potential problem that $F_i = +\infty$ at $t_i = 0$).

Let $\Gamma = \{(t_1, \ldots t_I) | t_1 \geq t_2 \geq \ldots \geq t_I$, and $t_o \leq t_i \leq T\}$.

Note that Γ is a compact convex set.

For any t in Γ, define $\hat{\alpha}(t)$ as the maximum scalar α such that $t + \alpha F(t) \in \Gamma$, where $F(t) = (F_1(t), \ldots F_I(t))$. Since Γ is compact and convex, a unique non-negative such α must always exist if $F \neq 0$.

Pick an arbitrary positive $\bar{\alpha}$, and let $\alpha(t) = \min(\hat{\alpha}(t), \bar{\alpha})$. Define $\Phi(t)$ on Γ by

$$\Phi(t) = t + \alpha(t)F(t) \qquad (35)$$

By construction Φ is continuous, and maps Γ into itself, so must have a fixed point.

At the fixed point, either $\alpha(t) = 0$ or $F(t) = 0$.

Could $\alpha(t) = 0$ at a fixed point?

Then t must be on the boundary of Γ.

If $t_i = t_o$, $F_i > 0$.

If $t_i = T$, $F_i < 0$.

If $t_i = t_{i+1}$, $F_i \geq F_{i+1}$.

Therefore if t is on the boundary of Γ, F points into Γ (or along its boundary). Hence $\hat{\alpha}(t) > 0$.

Therefore at the fixed point of Φ, $\alpha(t) > 0$, so $F(t) = 0$. Hence a tax vector t in Γ has been found in which $F_i(t) = 0$ for all i, proving the theorem.

APPENDIX 3

Here an example is presented of a 2-jurisdiction nation, with Cobb-Douglas technology $f = k^\alpha$, in which as $\lambda_1 \to 1$, $\lambda_2 \to 0$, $k_2 \to \infty$ and $b_2 \to \bar{b}_2 > 0$ in any equilibrium.

A necessary condition for this result is that the tax base be inadequate, i.e. $m[(1-\alpha)f(k^*), \alpha f(k^*)] > 1$, as will be seen subsequently.

With Cobb-Douglas technology there can be no excess supply equilibrium: in such an equilibrium the value of t must satisfy $t = f'(k)$ for a k which maximizes $u[(1-\alpha)f(k), \alpha f(k)]$, and clearly no such finite k exists.

If $r = 0$ in equilibrium, $t_i = f'_i$, so that $\frac{\partial g_i}{\partial t_i} = k_i - (1-b_i)\frac{k_i}{1-\alpha}$.

If $b_2 \to 0$, then $\frac{\partial g_1}{\partial t_1}$ becomes negative, so t_1 cannot be optimal. Hence any equilibrium on the boundary between regimes cannot involve $b_2 \to 0$.

Moreover if $\alpha > \frac{1}{2}$ no such equilibrium can exist. For if $1-\alpha < \frac{1}{2}$, it cannot simultaneously be true that $\frac{1-b_1}{1-\alpha} < 1$ and $\frac{1-b_2}{1-\alpha} < 1$, so either $\frac{\partial g_1}{\partial t_1} < 0$ or $\frac{\partial g_2}{\partial t_2} < 0$.

If a sequence of equilibria were to exist in which $b_2 \to 0$ (with $r > 0$), then (24) becomes $F_1 \to m_1 k_1 - k^*$

Since $m((1-\alpha)f(k^*), \alpha f(k^*)) > 1$, and $tk < \alpha f(k)$, $F_1 = 0$ implies $k_1 < k^*$. In the limit, then, it cannot be true that $k_1 \to k^*$ if $b_2 \to 0$. But if k_2 is bounded, then $k_1 \to k^*$, since $\lambda_1 k_1 + \lambda_2 k_2 = k^*$.

Therefore, it has been proved that in any sequence of equilibria, $k_2 \to \infty$. Suppose $k_2 \to \infty$ and $b_2 \to 0$.

If $b_2 \to 0$ it must be true that $\dfrac{\lambda_2}{f''_2} \to 0$, or $\lambda_2 k_2^{(2-\alpha)} \to 0$.

Since $\lambda_2 k_2^{2-\alpha} = (\lambda_2 k_2) k_2^{1-\alpha}$, then $k_2 \to \infty$, $b_2 \to 0$ implies $\lambda_2 k_2 \to 0$, so that $k_1 \to k^*$, which has just been shown impossible.

Therefore in any sequence of equilibria, $k_2 \to \infty$ <u>and</u> $b_2 \to \bar{b}_2 > 0$.

What does the limit equilibrium look like?

So far it has been shown that in any limit equilibrium $k_2 \to \infty$, $b_2 \to \bar{b}_2 > 0$, and $r > 0$, and $r > 0$ if $\alpha > \frac{1}{2}$. Also $r \to 0$, since $r < f'(k_2)$.

Let ρ be the asymptotic ratio of jurisdiction 2's tax rate to its gross return to capital, $\dfrac{t_2}{f'(k_2)}$.

Then $t_2 \to \rho f'(k_2) = \alpha\, \rho\, k_2^{\alpha-1}$.

Since $k_2 \to \infty$, then

$F_2 \to -b_1k_2 + m_2(k_2 - \frac{\rho b_1}{1-\alpha} k_2)$ so that in the limit

$$m_2(1 - \frac{\rho b_1}{1-\alpha}) = b_1 \qquad (36)$$

$$\text{where } m_2 = m[(1-\alpha)f(k_2),\ \rho\ \alpha\ f(k_2)] \qquad (37)$$

If tastes are homothetic, m_2 is independent of k_2, so that (36), (37) can be solved for ρ as a function of b_1. Let $a_i = -\lambda_i/f''_i$, so that $b_i = \frac{a_1}{a_1 + a_2}$. The limiting b_1 cannot be zero, since $m((1-\alpha), \rho_2) > 1$. (see equation (36)).

Since $a_1 = \lambda_1 k_1^{2-\alpha}$ is bounded, and $b_1 \to \bar{b}_1 > 0$ therefore $a_2 \to \bar{a}_2 < \infty$. Hence $\lambda_2 k_2^{(2-\alpha)} \to \bar{a}_2$, so that $\lambda_2 k_2 \to \bar{a}_2 k_2^{\alpha-1} \to 0$.

Therefore in the limit, $k_1 \to k^*$.

Since $r \to 0$, $t_1 \to f'(k^*)$.

Hence first-order conditions on jurisdiction 1 become

$$F_1 \to -1 + m_1(1 - \frac{b_2}{1-\alpha}) \qquad (38)$$

implying $b_2 = (1 - \alpha)\frac{m_1 - 1}{m_1}$ where $m_1 = m((1-\alpha)f(k^*), \alpha f(k^*))$.

Hence the limiting equilibrium can be derived by solving (38) for b_2, and then (36) and (37) for ρ.

As far as second-order conditions are concerned, lowering ρ will raise m_2 (from (37)), raise b_2 (since k_2 increases) and hence raise F_2. Thus $F_2' < 0$.

For jurisdiction 1, note that since $k_1 \to k^*$, the two positive terms in the equation (29) for F_1' can be bounded. Thus if there is sufficient curvature in the indifference curves to make $\left|\frac{\partial m_1}{\partial t_1}\right|$ very large, $F_1' < 0$.

Some qualitative properties of the limiting equilibrium: Clearly $u_2 \to \infty$ and u_1 is bounded, so it pays to be in the small jurisdiction.

If tastes are homothetic $m_2 \geq m_1$ (since $\rho \leq 1$), so that $m_2 \geq m_1 > 1$. Both jurisdictions undersupply the public good.

If $\alpha > \frac{1}{2}$, then (38) shows $b_2 < (1-\alpha) < \frac{1}{2} < b_1$.

But of course the main point is that $b_2 > 0$, so that even as jurisdiction 2 shrinks, its influence $\frac{\partial r}{\partial t_2}$ on national variables does not become negligible.

FOOTNOTES

[1]Page 359.

[2]Page 198, paragraph 3.

[3]Montana and Wyoming, for example, produce most of the American output of low-sulfur coal. This observation central to Kolstand and Wolak.

[4]The two-jurisdiction, asymmetric model of section 8 below is meant to apply to this situation.

[5]See Novshek (1980), or Novshek & Sonnenschein for example.

[6]Mieszkowski, page 81, footnote 6.

[7]This assumption can be weakened at the cost of some increased tedium in the mathematics.

[8]It is true that if $(t_1, \ldots t_I)$ yields an allocation in which $\Sigma\, \lambda_i k_i = k^*$, it cannot yield an allocation in which $\Sigma\, \lambda_i k_i < k^*$. Also at most one allocation with excess supply exists for a given tax vector $(t_1, \ldots t_I)$, as equation (12) makes clear.

[9]If I=2, then $\frac{\partial b_i}{\partial t_j} > 0$ if $f''' < 0$ everywhere, and $\frac{\partial b_i}{\partial t_j} < 0$ if $f''' > 0$ everywhere $(i \neq j)$, but $I > 2$ implies an increase in t_j increases k_i and decreases k_j but also increases other k_m's, so the effect on b_i cannot be signed from the sign of f''' alone.

[10]Because of the straight-line production possibility frontier, the marginal rate at which 1 unit of the numeraire (x) is transformed into one unit of g to each of N people is N, and Σ MRS = N•m.

[11]Throughout this paper, I will use this dual relationship Σ MRS = MRT to characterize under-provision, although Atkinson & Stern have demonstrated

this characterization is not the same as a lower quantity of g being provided than in some first-best solution.

[12]Since F_i is continuous in F_{-i} the convex hull of jurisdiction i's reaction correspondence is a continuous correspondence from R^{I-1} into R.

[13]Appendix 1 shows that optimal tax rates can be bounded above.

[14]In the remainder of this section, attention is restricted to allocations in which $\Sigma \lambda_i k_i = k^*$. It should be clear that in an allocation with excess supply of capital, all jurisdictions will levy the same tax rate, regardless of size, if there is a unique $k < k^*$ which maximizes $u[(1-\alpha(k))f(k), \alpha(k)f(k)]$.

[15]If $f = k^\alpha$, then $f'(k)k$ increases with k. Setting $t_i = f'(k^*)$ yields $g_i \sim f'(k^*)k^*$ which is better than $f'(\frac{k^*}{2})\frac{k^*}{2}$. Since $r \to 0$, for large I the decrease in rk^* induced by setting $t_i = f'(k^*)$ will be more than offset by the increase in wage income. Hence $t_i = f'(k^*))$ implies more of both commodities than $t = f'(\frac{k^*}{2})$, for sufficiently large I. This observation is a special case of a result obtained in Section 6, that a jurisdiction's capital stock per person must converge to k^* if its influence gets small enough.

[16]If b_i is bounded away from zero even if $\lambda_i \to 0$, then $k_i \to \infty$. Since $m[x, g] > m[(1-\alpha)f(k), \alpha f(k)]$, then $m > 1$ as $\lambda_i \to 0$.

[17]Raising t_2^* to t_i^* must raise g_1 by $-t_i \int^{b} \frac{1}{f''_2}$ where the integral is over values of k_1, k_2 as t_2 rises. This is bounded away from zero. Hence $U_1[t_1^*, t_1^*] - U_2[t_1^*, t_2^*]$ does not approach zero as $\lambda_2 \to 0$, which (along with the earlier paragraph) proves the result

[18]Alternatively, small states merging into a federation may demand some provisions in the new merged federation's constitution as a form of commitment.

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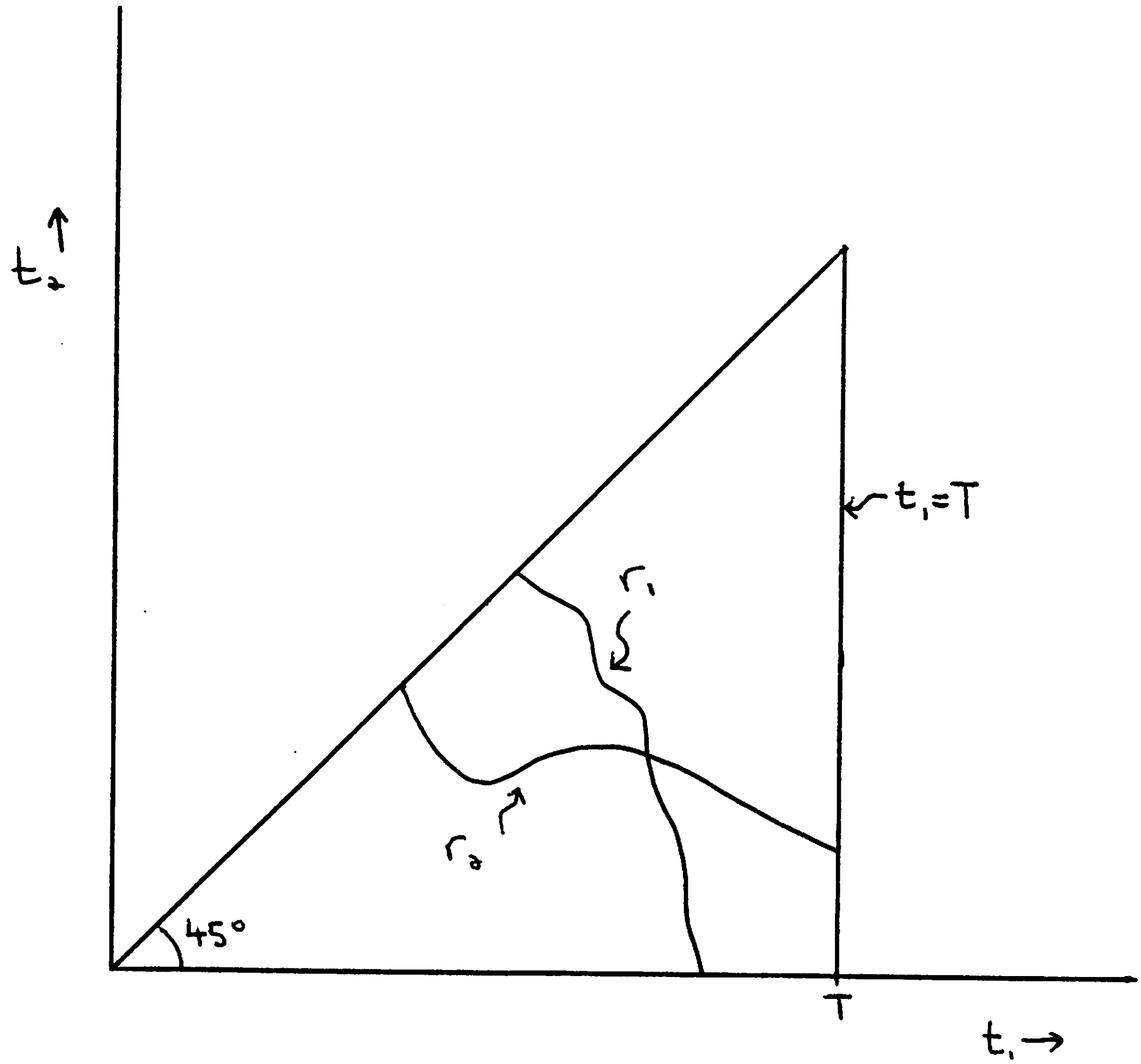

Figure 1