1978

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RESEARCH REPORT 7806

THE EFFECTS OF MINIMUM WAGES LEGISLATION

IN TWO SECTOR FIXED COEFFICIENT MODELS

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I.   <u>INTRODUCTION</u>

Some work has been carried out on analysing the impact of unions
and minimum wage laws on the distribution of income.[1] The consequences
and implications of such minimum wage laws for economic growth, however,
have not been explored in any rigorous way. From the redistribution of
income effects caused by the minimum wage laws, it follows that there will
be, in general, effects on growth patterns.

This paper firstly analyses the static and comparative static
effects of minimum wages, when these minima are not necessarily equal be-
tween sectors. This is followed by an analysis of the implications for
growth of such laws. Minimum wage laws are only of interest in a context
of growth where some factor, other than labour, can be accumulated out of
the output of an industry. Two sector fixed coefficient, growth models
such as those studies by Corden [3] and Ocker [10], are therefore the
simplest relevant types.

One of the conclusions of the paper is that in general a full em-
ployment balanced growth path does not exist. If we define a balanced
growth path as allowing the possibility of unemployment, less than full
employment growth paths may exist involving varying proportions of unemployed
to employed labour. In the cases where unemployed labour is characteristic
of the balanced state, the capital to employed labour ratio will naturally
be higher than the capital to total (employed plus unemployed) labour ratio.

11. DEFINITION OF MINIMUM WAGE

Johnson [6], Johnson and Mieszkowski [7] have carried out some work on the impact of unionization and minimum wage laws on the distribution of income. In [6] Johnson assumes that "the minimum wage is effective in raising the real wage of labour in one industry". The major shortcoming of this definition is that since there is a lack of knowledge of final prices, it is not known what value in x_1 goods the minimum wage in x_2 goods really is. Minimum wage laws are based on maintaining <u>present consumption levels</u> so that the earnings of a capital asset would generally not be regarded as a satisfactory component of minimum wages. Hence we will use a more general definition where the minimum wage consists of a certain value bundle of goods. The workers in sector 2 receive sufficient x_2 to enable them to exchange, at the current competitive market price, these goods for at least $\hat{x}_1$ goods. There is some flexibility in the way producers decide to pay the real wage; any mixture of x_1 and x_2 yielding a given value in x_1, $\hat{x}_1$, will do.

The minimum wage is set by the government or one of its authorities who instructs employers that if any labour is employed, it must be paid this minimum. The owners of the other factor, capital, compete for what is left of the total product so that the rental rate, r, on capital is calculated as a residual. The minimum wage has a maximum $\hat{w}_{max}$ (the minimum wage times the number of people employed) which must be less than or equal to the total product (national income). If $\hat{w}_{max}$ exceeds the total product firms go out of business. Thus there are maximum wages which could be paid if an industry is to remain viable in the short run.[2]

We examine the effects of a minimum wage on (i) the production

possibility locus and the corresponding relative supply curve and

(ii) the conditions for the existence of static equilibrium and longrun

growth.

III. STATIC ANALYSIS WITHOUT MINIMUM WAGES

We firstly set up the model without a minimum wage and then introduce the minimum wage. This will enable us to isolate and examine more clearly the impact of the minimum wage on the model.

Formally the model is as follows:

Goods x_1 and x_2 are produced using capital (K) and labour (L). At any instant outputs x_1 and x_2 must satisfy the inequalities

$$a_{K_1} x_1 + a_{K_2} x_2 \leq K \tag{1}$$

$$a_{L_1} x_1 + a_{L_2} x_2 \leq L \tag{2}$$

where a_{ij} is the input factor i required for one unit of output of good j, $i=K,L$ $j=1,2$. Constraints (1) and (2) define a production possibility set like $OABD$ in Figure 1 where KK' and LL' represent the capital and labour constraints respectively. The diagram is drawn on the assumption that x_2 is relatively labour intensive i.e. $a_{K_2}/a_{L_2} < a_{K_1}/a_{L_1}$. Our analysis proceeds on this assumption.[3]

Figure 1
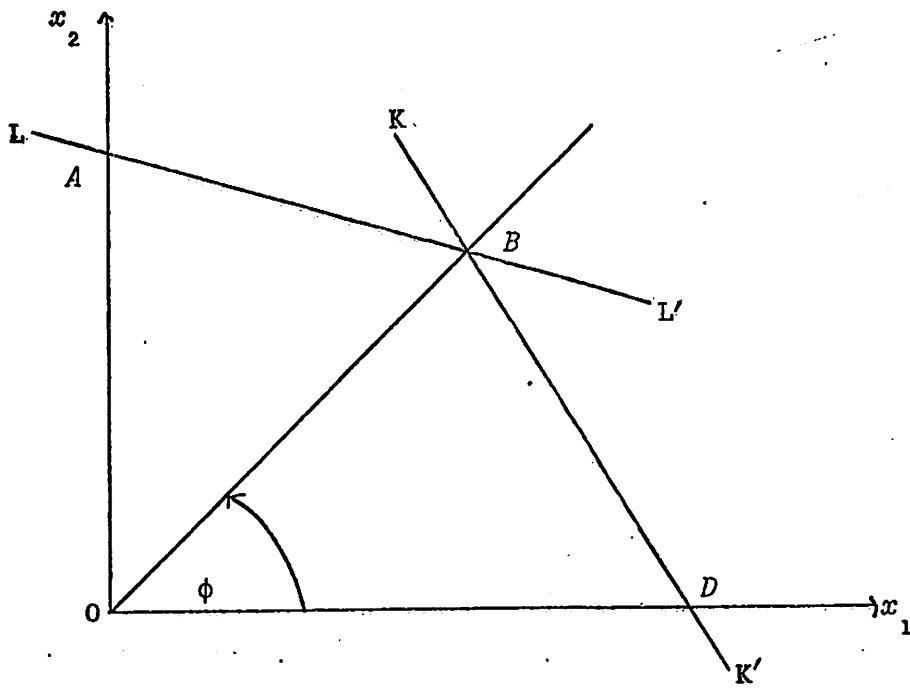
In terms of Figure 1, along AB labour is the only binding constraint, so only the wage rate is non-zero; along BD labour is redundant so only the rental rate is non-zero.[4]

Thus from the "price equals cost of production" rule, the price ratio along AB is

$$\frac{P_2}{P_1} = a_{L2}/a_{L1} \tag{3}$$

while the price ratio along BD is

$$\frac{P_2}{P_1} = a_{K2}/a_{K1} \tag{4}$$

At the corner B, we have a range of price ratios depending on the value of r (the rental rate) and w (the wage rate). Thus at the full employment point B,

$$P_2/P_1 = \frac{a_{K2} + a_{L2}\, w}{a_{K1} r + a_{L1} w} \tag{5}$$

Assume that P_1 is the numeraire. Let the full employment price ratio be $P \equiv P_2/P_1$. The rental and wage rate must be non-negative. Thus

$$P = a_{K2} r + a_{L2} w \tag{6}$$

$$1 = a_{K1} r + a_{L1} w \tag{7}$$

This system has a unique solution for r and w if and only if the matrix

$$X = \begin{bmatrix} a_{K2} & a_{L2} \\ a_{K1} & a_{L1} \end{bmatrix}$$

is non-singular i.e. if and only if Det $(X) \neq 0$.[5] This determinant is positive if $a_{K_2}/a_{L_2} > a_{K_1}/a_{L_1}$, zero if $a_{K_2}/a_{L_2} = a_{K_1}/a_{L_1}$ and negative if $a_{K_2}/a_{L_2} < a_{K_1}/a_{L_1}$. At B, the full employment point there is a range of price ratios with the price ratios defined by (3) and (4) forming the limits.

The relative supply curve describes the relationship between the output ratio x_2/x_1 and the price ratio P; x_2/x_1 = S(P). The relative supply curve for x_2 labour intensive is shown in Figure 2. We assume that capital stock is such that the full employment of both factors point has both outputs positive. The relative supply curve is upper semi-continuous.[6]

Instantaneous equilibrium is now discussed. If the preferences of all individuals are identical, convex, and homothetic-to-the-origin, then the relative demand curve is well defined and is, in particular, independent of the level of income.[7] We may write the relative demand x_2/x_1 = D(P), where D'(·) < 0.[8] A particularly important case is the neoclassical savings model in which commodity 2 is an investment good, commodity 1 is a consumption good and preferences are of the Cobb-Douglas type. This implies the relative demand curve x_2/x_1 = S/(1 - S)P where S is the rate of savings (= investment) and $P \equiv P_2/P_1$. Relative demand and relative supply can be equalized, instantaneous equilibria are guaranteed to exist. For any relative factor intensity, the downward sloping demand curve will only have one intersection with the relative supply curve thus equilibrium is unique. In the case shown in Figure 2, equilibrium occurs at the point E. Point E is along the full employment of both factors portion of the relative supply curve.

FIGURE 2
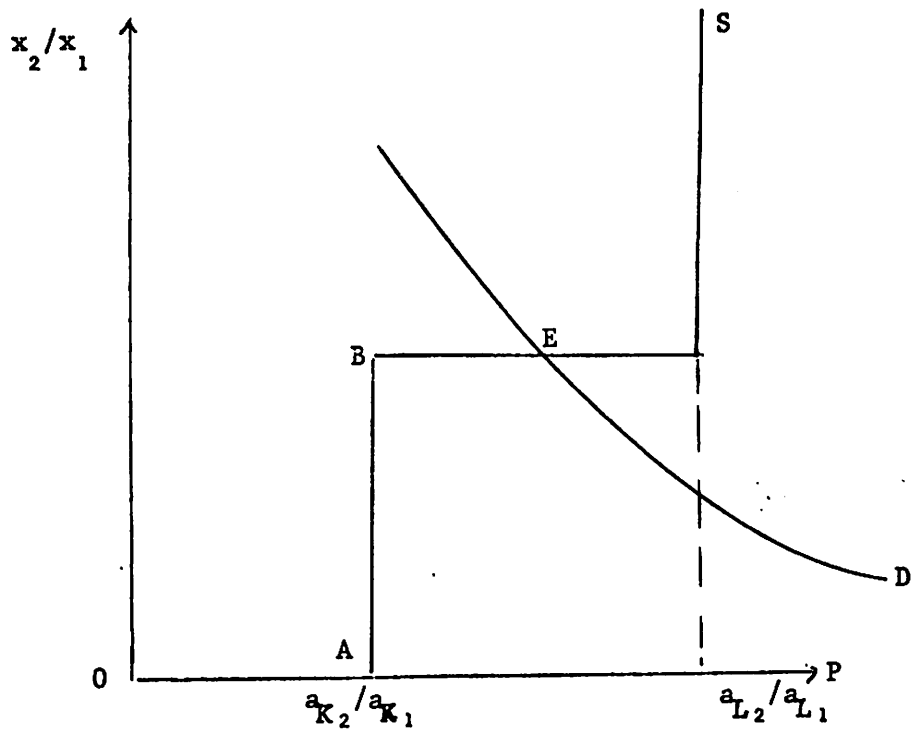
By using relative demand and supply functions, we have implicitly assumed that the only equilibrium points we are considering are those where both outputs are positive. Corner solutions, where one of the outputs are zero, are feasible under different demand conditions from those specified in this section. The possibility of corner solutions has important implications for the minimum wage case and these are examined in the next section.

IV. STATIC ANALYSIS WITH MINIMUM WAGES

We now introduce the minimum wage (defined as in section II) into the model presented in section III and analyze the implications for equilibrium.

The minimum wage is paid in terms of good x_1.[9] Formally, using P_1 as the numeraire, the minimum wage is

$$\hat{w} \equiv \hat{x}_1 \equiv Px_2 \tag{8}$$

where $\hat{x}_1$ represents the minimum wage bundle in value terms, Px_2 represents the equivalent value of x_2 goods measured at the competitive market price ratio.

Consider the case where the uniform minimum wage is imposed in both sectors. Consider the full employment point B. Equations (6) and (7) now become

$$P = a_{K2}r + a_{L2}\hat{w} \tag{9}$$

$$1 = a_{K1}r + a_{L2}\hat{w} \tag{10}$$

from which

$$r = (1 - a_{L1}\hat{w})/a_{K1} \tag{11}$$

$$P = \frac{a_{K2}}{a_{K1}} - \hat{w}. \quad (B) \tag{12}$$

where $B = (a_{L1}a_{K2} - a_{L2}a_{K1})/a_{K1} \gtreqless 0$ as $k_2 \gtreqless k_1$,

where $k_i = a_{Ki}/a_{Li}$, $i = 1 ,2$.

Thus the effect of introducing the minimum wage is to decrease r and change the value of relative prices. The effect on r is always a reduction whereas the effect on relative price depends on the relative factor intensity of the two sectors.

In the case of a different minimum wage in each sector, and assuming $\hat{w}_2 = u\hat{w}_1$, u(constant) $\neq$ 1, (9) and (10 become

$$P = a_{K_2} r + a_{L_2} \hat{w}_2 \qquad (13)$$

$$1 = a_{K_1} r + a_{L_1} \hat{w}_1 \qquad (14)$$

from which

$$r = (1 - a_{L_1} \hat{w}_1)/a_{K_1} \qquad (15)$$

$$P = a_{K_2}/a_{K_1} - \hat{w}_1 (B') \qquad (16)$$

where $B' = (a_{L_1} a_{K_2} - u a_{L_2} a_{K_1})/a_{K_1} \gtreqless 0$ as $k_2 \gtreqless u k_1$. k_1 and k_2 are defined as in (12).

In the variable coefficients case the presence of factor reward diffemetials generally leads to a shrinking in of the production possibility locus (with the end points common to both the non-distorted and distorted frontiers) and a non-tangency between the price ratio and the production possibility locus.[10] In the fixed coefficient case, the frontier does not change since it is determined by purely technological conditions so that only the non-tangency results.

In the case where the minimum wage differs in each sector, we are in effect, introducing two distortions. The imposition of the minimum wage is one while the other comes about by assuming the minimum wage differs between the sectors.

Thus from (16) the sign of B' in the presence of two distortions depends on the factor value intensity rankings of the two industries.

Consider that part of the production possibility function, BD, where labour is redundant and capital is the tight constraint. If the same minimum applies in each sector, equations (9) and (10) apply and again r decreases and relative prices increase or decrease as k_1 is greater than or less than k_2. In the case of a different minimum wage in each sector, (13) and (14) apply so that in this case the movements in prices depends on the relative factor value intensity rankings of the two sectors. Along BD, the perfectly competitive wage rate is zero since capital is the tight constraint. Hence if $\hat{w}_1 = 0$ (implying of course that $\hat{w}_2 = 0$), (12 and (16) reduce to

$$P = a_{K2}/a_{K1} \tag{17}$$

which is the "normal" result in the fixed coefficient model. In the cases where r = 0 and a minimum wage is imposed, production must be zero as no redistribution can take place.

In discussing the effect on r and P along the production possibility frontier, we have so far assumed that outputs of both sectors are positive.

The defining of supply and demand in terms of the relative quantities of the two goods restricts the discussion of equilibrium positions to the cases

where both goods are produced in positive amounts. It is quite possible, of course, that equilibrium can occur at the corner points (of the production possibility frontier) A or D of Figure 1. There are a wide class of demand conditions which would permit such an equilibrium.

Allowing demand to enter the model in this way means, of course, that the property that price equals cost of production does not then always hold. This possibility of a specialization equilibrium has important implications.

Firstly, the notion of an upper bound on $\hat{w}$, $\hat{w}_{max}$, alters. If $\hat{w} = Px_2$, a corner solution allows the possibility that P becomes sufficiently large to accommodate any finite $\hat{w}_{max}$. The demand conditions under which this occurs would be where the wage increases shift the demand for x_2 well to the right (with P on the vertical axis).

Secondly, referring to Figure 3, the broken lines represent price ratios and indifference curves could be tangent to price lines at points A, B and D. Interior "one-sided" tangents are permitted though only one point in each diagram would apply. Thus in diagrams (a) and (b) in Figure 3, equilibrium could be at the point A, when $x_1 = 0$, or the point D, where $x_2 = 0$. Thus depending on demand conditions, more than one price ratio may be consistent with a given minimum wage.

Our main concern is in examining the growth implications so we will assume that both outputs are produced in positive amounts. This is not to distract attention from the fact that corner solutions may be of some interest.

Assuming that the physical and value factor intensity rankings are the same, we can examine the effect on the relative supply curve of enforcing the minimum wage. Figure 4a is for the x_2 capital intensive case while Figure 4b is for the x_2 labour intensive case. The relative supply curve for x_2 capital intensive alters from ABDS to BD′S′ while for x_2 labour intensive it alters
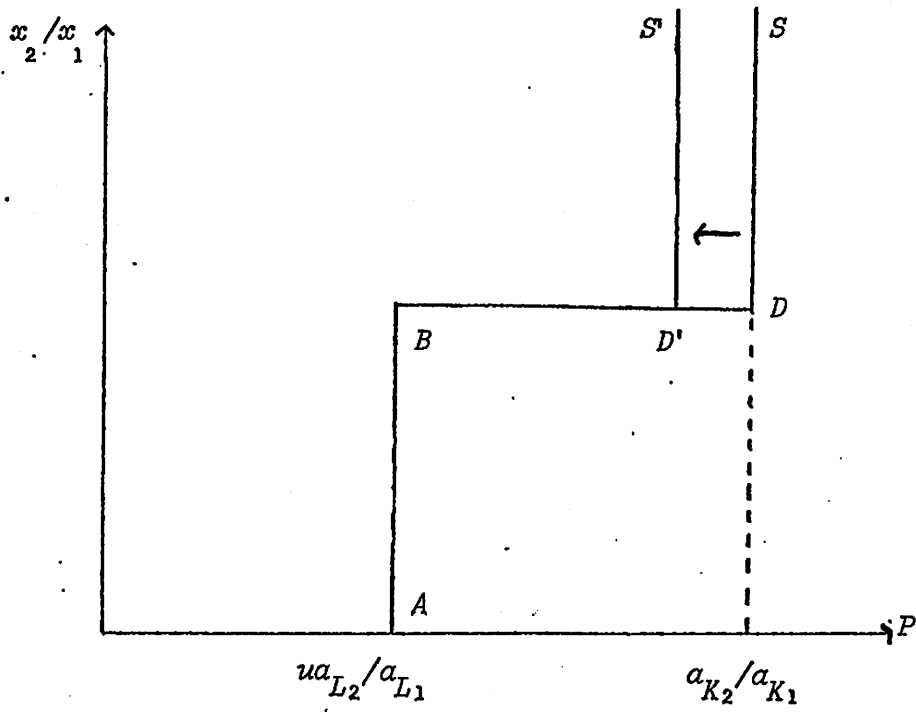
FIGURE 3
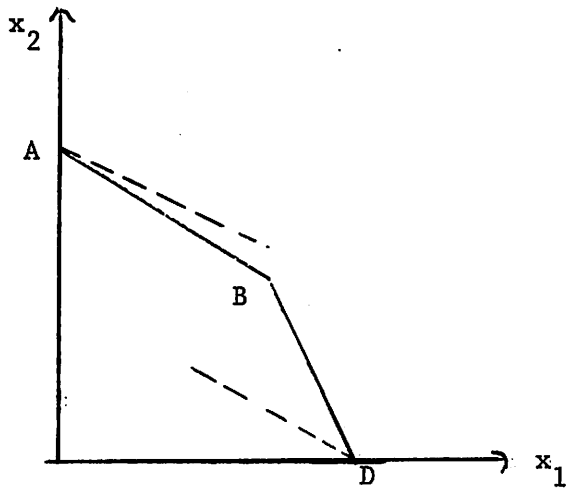
(a)
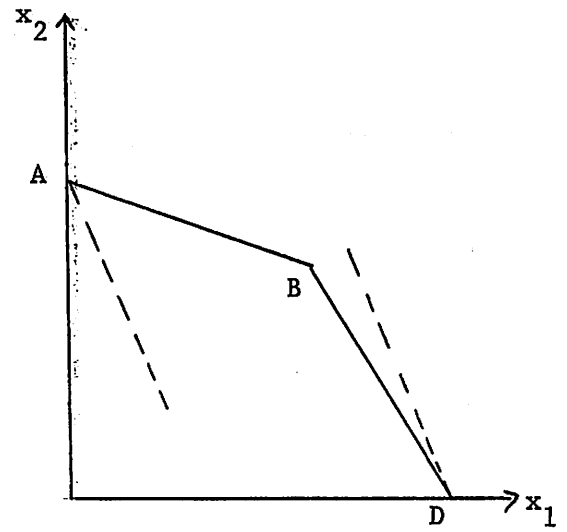
(b)
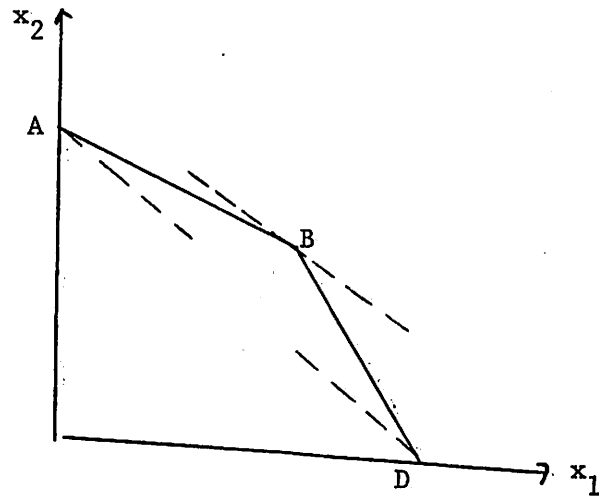
(c)

FIGURE 4a

from ABDS to $A'B'D$. Note that the section of the pre-minimum wage relative

supply curve where labour is the tight constraint (AB in Figure 4a and DS in

Figure 4b) is deleted from the post-minimum wage relative supply curve.

If the physical and value rankings differ, then the effect of the

minimum wage on P, and hence the relative supply curve results in perverse

output responses. For x_2 capital intensive, the relative supply curve shifts

from ABDS to $B'DS$, (Figure 5a) while for x_2 labour intensive, the relative

supply curve shifts from ABDS to $A'B'D$ (Figure 5b). Thus if the effect of

minimum wages is severe enough to cause factor value intensity reversals,

then perverse supply responses to price movements result . This particular

result also occurs in fixed coefficient models with factor-price differentials.[11]

V.   COMPARATIVE STATICS

The effects of minimum wages results in a redistribution of

income effect as well as a shift in the relative supply curve.

Formally

$$(r - r_m) \leq 0 \text{ if and only if } (\hat{\omega} - w) \leq 0 \tag{18}$$

where r and r_m are pre- and post-minimum wage rental rates respectively,

w is the competitive wage while $\hat{\omega}$ the minimum wage.

In the case of the same minimum wage in each sector, from (11)

$$(r - r_m) = (\hat{\omega} - w)a_{L_1}/a_{K_1} = (\hat{\omega} - w)/k_1 \tag{19}$$

where k_1 is defined as in (12). Thus the income redistribution effect

depends on the magnitude of the minimum wage itself as well as on the

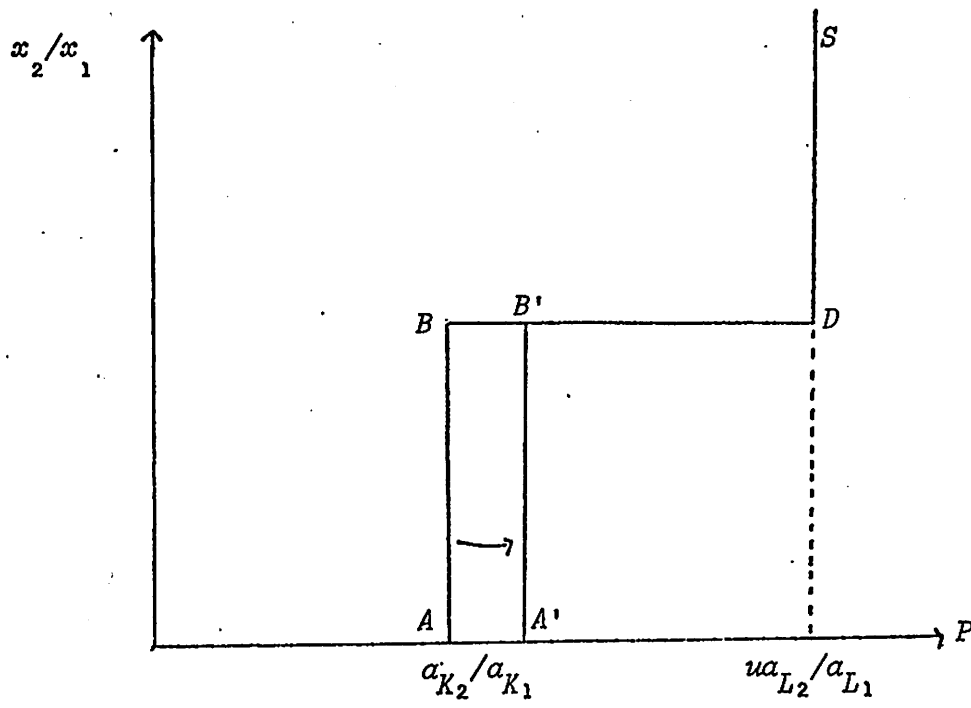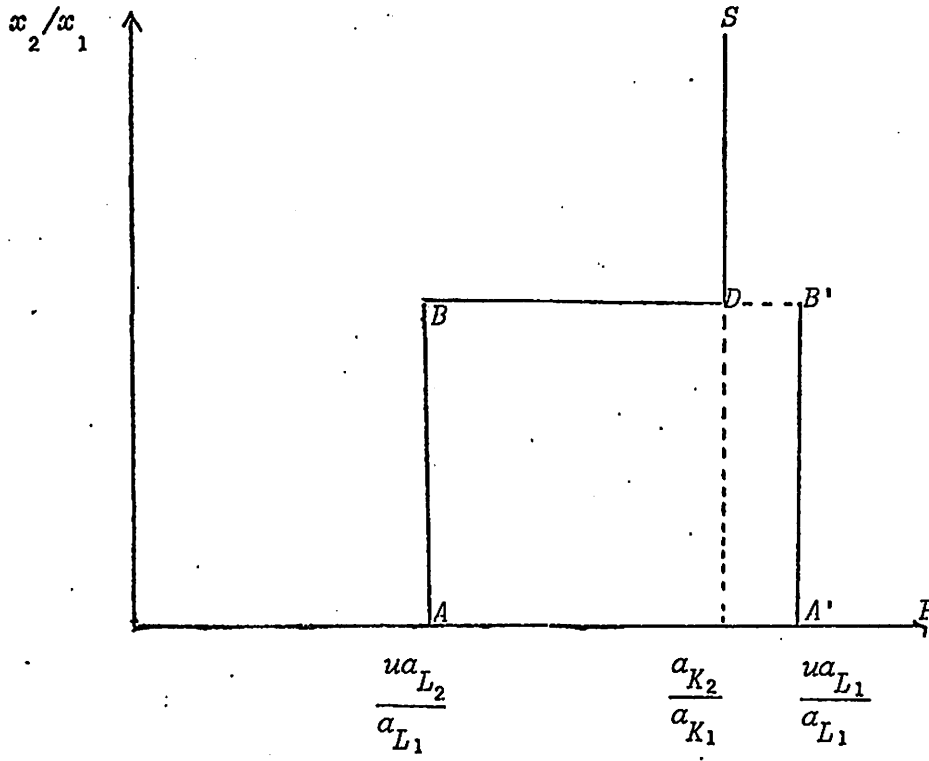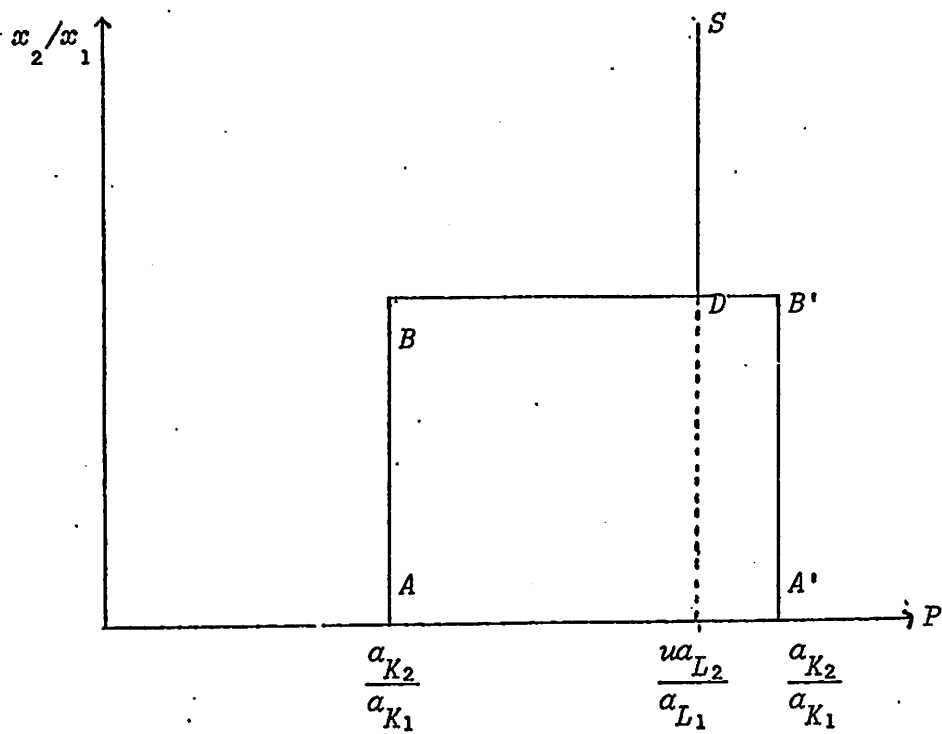capital intensity of sector 1.

FIGURE 4b

FIGURE 5a

FIGURE 5b

In the case of different minimum wages in each sector,

$$(r - r_m) = (\hat{\omega}_1 - w) a_{L_1}/a_{K_1} = (\hat{\omega}_1 - w)/k_1 \tag{20}$$

Recall that $\hat{\omega}_2 = u\hat{\omega}_1$. In this case the income redistribution effect depends on the magnitude of $\hat{\omega}_1$ and the capital intensity in sector 1.

Concerning the effects on relative prices in the case of different minimum wage rates to each sector,

$$P_m = a_{K_2}/a_{K_1} - \hat{\omega}_1 (B') \tag{21}$$

when B' is defined as in (16) and P_m is the post-minimum wage price ratio. From (6) and (7)

$$P = a_{K_2}/a_{K_1} - w(B) \tag{22}$$

where B is defined as in (12). Thus combining (21) and (22)

$$(P - P_m) = \hat{\omega}_1 B' - w B \tag{23}$$

In the case of the same minimum wage rate in each sector, (23) simplifies to

$$(P - P_m) = B(\hat{\omega} - w) \tag{24}$$

Thus from (24) the effect on the change in relative prices due to the minimum wage depends on the physical and value factor intensity of both sectors as well as on the magnitude of the minimum wage. In the case of the same minimum wage in each sector, (24) the change depends on the physical factor intensities as well as the magnitude of $\hat{\omega}$.

In the case where $r = 0$, and a minimum wage is imposed, the effect on relative prices can be easily derived but is of little interest since production is zero.

VI. STATIC EQUILIBRIUM WITH MINIMUM WAGE

Define the x_2/x_1 output ratio at the full employment point

as ϕ. Define the level of capital stock at which full employment

occurs as K^{**}. If the minimum wage applies

and assuming no physical or value factor intensity reversals, the

relative supply curve is $BD'S'$ for x_2 capital intensive (Figure 4a)

and $A'B'D$ for x_2 labour intensive (Figure 4b). In both cases

there are discontinuities in the relative supply curve. Equilibria

exists only if the graph of the relative demand curve defined in section III

does not pass through any discontinuity in the relative supply curve. The

discontinuity arises when $P_2/P_1 = ua_{L2}/a_{L1}$. In the case x_2 capital

intensive existence (and uniqueness) of equilibrium is guaranteed

if, and only if, $D(a_{K_2}/a_{K_1}) \gtrless \phi\, a_{L_2}/a_{L_1}$. In the case x_2 labour intensive,

the condition becomes $D(a_{K_2}/a_{K_1}) \leq \phi\, a_{L_2}/a_{L_1}$. Thus a necessary and

sufficient condition for the existence of a momentary equilibria is

that, for x_2 labour intensive.

$$S \geq \frac{\phi\, a_{L_2} a_{K_2}}{\phi\, a_{L_2} a_{K_2} + a_{K_1} a_{L_1}} \tag{25}$$

where ϕ is the full employment output ratio S is the fixed savings rate.

Further assume that labour consumes all its income while capital

owners perform the savings function. For x_2 capital intensive, the

inequality in (25) is reversed.[12]

In the case where factor value intensities reverse, then condition

(25) is not sufficient to ensure a unique equilibrium. In general

multiple equilibria will occur. For x_2 capital intensive (in the

physical sense) and a factor value intensity reversal, any downward

sloping curve may cut the relative supply curve $B'DS$ twice (Figure 6).

Similarly for x_2 labour intensive.

Figure 6
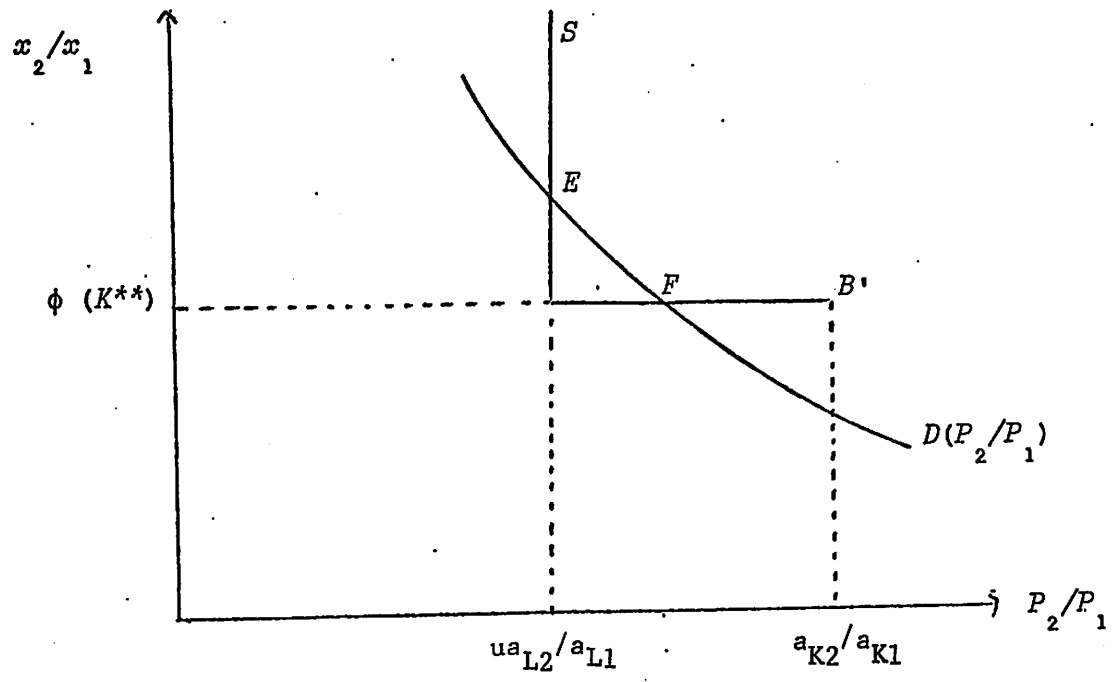
VII. GROWTH EXCLUDING MINIMUM WAGES

The previous results apply to any two-sector fixed coefficient model. A model of economic growth is obtained by letting commodity 2 be the investment good (I), and commodity 1 be a consumption good (C). Exponential depreciation of capital at a rate m, and labour force growth rate at rate n are assumed when $m + n > 0$. Growth involves changes in investment, consumption and capital stock per worker. Interpreting the analysis in section III in per capita terms, K is the capital labour ratio, and $L = 1$. The accumulation of capital satisfies

$$\dot{K} = I - (m + n)K \qquad (26)$$

where $\dot{K} = dK/dt$ and I is a function of K if there is a unique equilibrium at every instant. We now analyse this relationship for I goods labour, intensive.

Define K_1 as the capital labour ratio below which capital is the only binding constraint. Define K_2 as the capital labour ratio above which labour is the only binding constraint. Between K_1 and K_2 both capital and labour are binding constraints. For $K \in [0, K_1]$, I is proportional to K; it is increasing if the savings rate, $S > 0$. For $K > K_2$, I is constant. For K between K_1 and K_2 both outputs vary linearly with K_1 just because of the requirement that both factors are fully employed in this range. The quantity of good 1, the consumption good, increases with K possibily at a faster rate than it did for $K < K_1$, while the quantity of good 2, the investment good declines. Figure 7 shows the relationship between I and K just described, as well as the investment necessary to overcome population growth and depreciation.

Figure 7
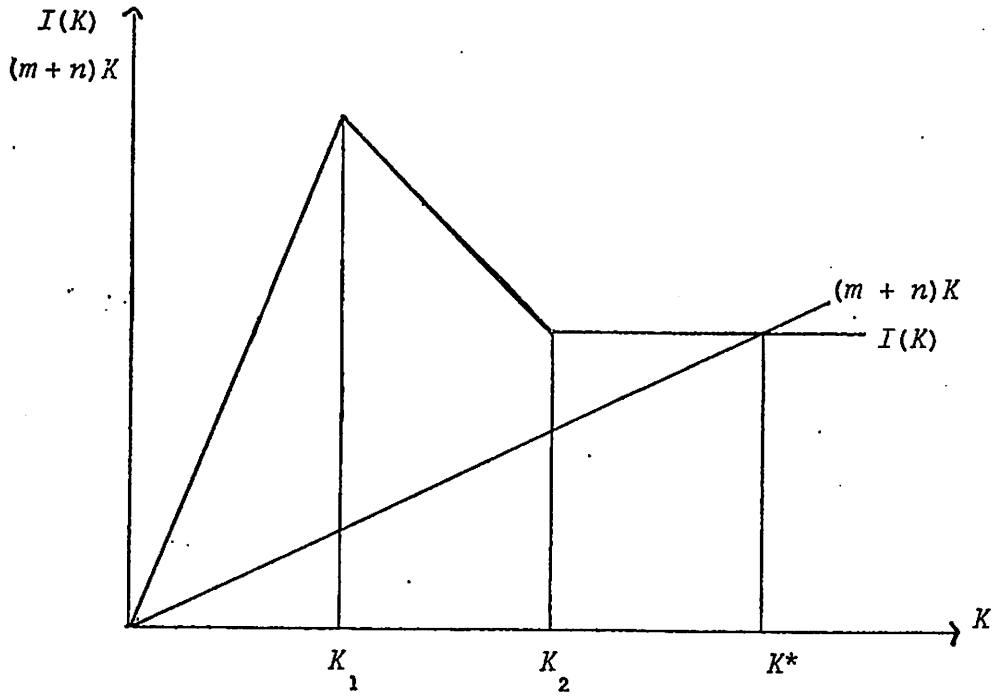
Using (26) if the slope at OA exceeds $(m + n)$, for $K < K_1$, there is a unique, globally stable, balanced state capital/labour ratio K^* such that $I(K^*) = (m + n)K^*$. Labour is never redundant in this state. For $K \gtrless K^*$ $I(K) \lessgtr (m + n)K$. Within the range $K < K_1$ capital is the tight constraint, so that income in terms of I goods is

$$Y_I = I + (a_{K_1}/a_{K_2})C = K/a_{K_2} \tag{27}$$

on using the capital constraint (1). But since the savings rate, , is a fixed proportion of income (total output).

$$I = S\,Y_I = S\,K/a_{K_2} \tag{28}$$

on using (27). Therefore over the range $K < K_1$,

$$dI/dK = S/a_{K_2} \tag{29}$$

Unique non-trivial globally stable balanced states exist if and only if,

$$S/a_{K_2} > (m + n) \tag{30}$$

Condition (30) has an easy interpretation. This requires that the rate of savings be such that the savings generated by one unit of capital stock exceed population growth and depreciation.[13]

VIII. GROWTH WITH MINIMUM WAGES

Assuming no factor value intensity reversals and considering the x_2 labour intensive (in the physical sense) case, the analysis in sector VII, in the presence of minimum wages, is modified as follows. From Figure 4b, for capital stocks greater than the full employment capital stock (K^{**}), equilibrium does not exist. Consequently in deriving a relationship between I and I(K) as in Figure 8, for $K > K_2$, labour is the binding constraint in this case, there is a discontinuity in the I(K) function. For $K > K_2$, labour is the binding constraint; therefore capital is not fully employed and receives a zero return. Thus producers who are forced to pay the workers the minimum wage (by definition this must be greater than the competitive wage) will just stop producing and the industry is no longer viable.

If the slope of OA is equal to (m + n), there are an infinite number of balanced state capital-labour ratios, up to and including K_1. All these balanced states are stable but involve, excluding the point A in Figure 8, unemployed labour. In all these states, however, the employed labour receives the minimum wage.

If the slope of OA is less than (m + n), the only balanced state capital-labour ratio is the trivial case of the origin. The balanced state at the origin is stable. Thus even if the rate of savings, S, is such that the savings generated by one unit of capital stock exceed population growth and depreciation, a full employment balanced growth path may not exist. Specifically, the balanced growth capital labour ratio K^* must be between the origin and K_2. For $K_1 < K^* < K_2$, labour is fully employed.[14]
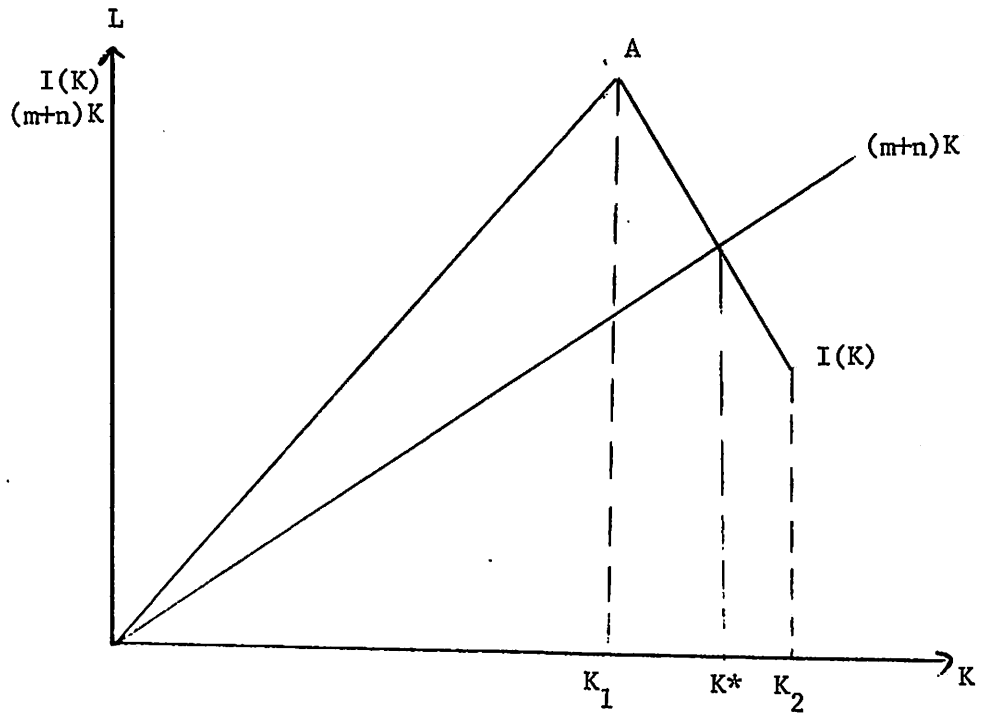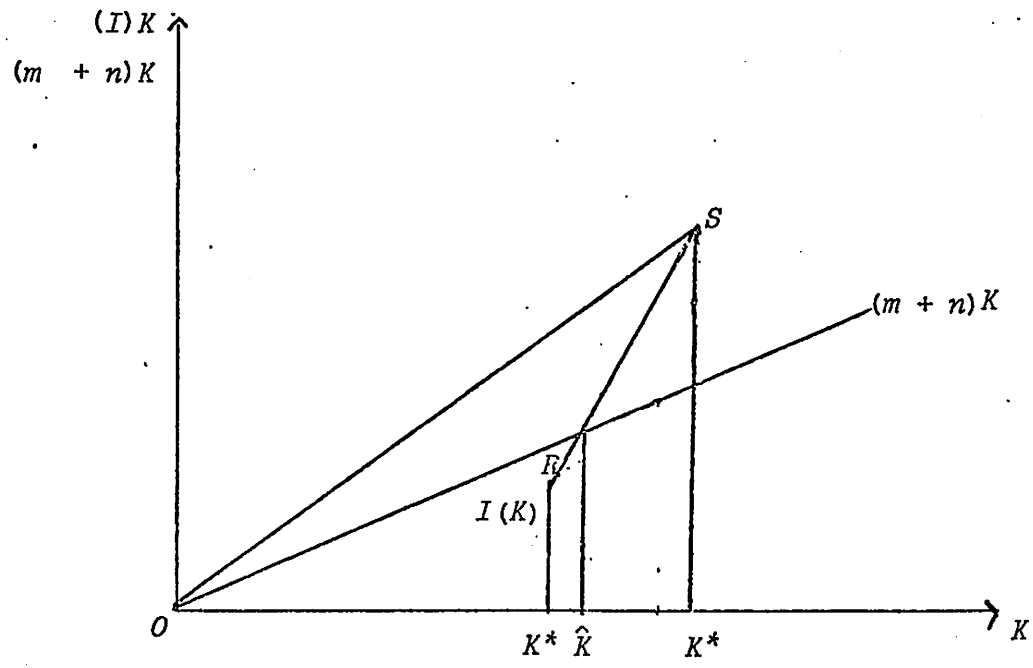
Figure 8

Figure 9

In the case of factor value intensity reversals, multiple equilibria can occur, at most two (Figure 6). Figure 9 shows the consequent relationship of I and K, as well as the investment $(m+n)K$ needed to overcome population growth and depreciation. The segment OS of the graph of $I(K)$ is linear stops at $K = K^{**}$, and corresponds to the equilibrium at E. The segment RS corresponds to the full employment equilibria (at F of Figure 6) and varies linearly with K. Again the condition that the slope of OS exceeds $m + n$ is necessary but not sufficient to ensure a non-trivial balanced growth path. One may exist, say, $\hat{K}$, between K^* and K^{**}, i.e., the range where both factors are fully employed. In this range there is an equilibrium at the point of full employment of both factors and there is one along the part of the relative supply curve where capital is the tight constraint (E in Figure 6). That equilibrium which give the highest level of investment also gives the highest level of satisfaction according to the preferences underlying demand. Unlike the case in [9], workers receive some income at <u>both</u> equilibrium points so that utility maximization will not guarantee that one of these equilibria will always be selected. Hence the indeterminacy remains.

Referring to Figure 9 if the slope of $(m+n)K$ exceeds the slope of OS, the natural rate of growth (including depreciation) exceeds the warranted

growth rate. If the slope of OS exceeds (m+n) to such a degree that

there is no intersection along SR, then the warranted growth rate exceeds

the natural rate of growth, labour shortages increase, but there is

no balanced growth path. In both cases a trivial balanced growth

path exists at the origin. If the slopes coincide, there are multiple

balanced growth paths, all of which, except the one at the corner S, involve

labour unemployment. This implies that the imposition of a uniform (binding)

minimum wage would result in the following: (1) the growth rate of employed

labour would naturally fall below the growth rate of total (employed plus un-

employed) labour and (2) fix the capital/employment ratio at a higher level.

In all cases the employed labour would be receiving the minimum wage.

Comparing Figure 7 and Figure 8, it is easy to see

that (with minimum wages) if a full employment balanced growth state

exists, the long-run K* is the lower in the presence of minimum wages.[15]

This K is the ratio of total capital of total labour, i.e., the aggregate

capital/labour ratio. This is different from the capital/employed labour

ratio if unemployment exists. A possible trade-off exists, between short

term gains in the form of minimum wages and a higher long-run balanced

growth capital/labour ratio K*, only if a full employment balanced growth

path exists.

IX.   TRADE-OFF BETWEEN MINIMUM WAGES AND GROWTH

The presence of minimum wages will in general necessitate

extra conditions to ensure static equilibrium as well as long-run

balanced growth. In the model, labour consumes all its income while

capital owners perform the savings function. Since there is a

redistribution of income from the capitalists to the wokers, total

savings as a proportion of total income will fall. Consequently

investment will also fall. Hence if a full employment balanced growth capital/
labour ratio exists, there is a trade-off between adopting
(and enforcing) minimum wage laws and achieving an "optimum"
rate of investment and growth.

In general, a full employment balanced growth path may not exist. This
may occur for mainly two reasons: Assuming condition (30)   is
satisfied, (a) there may not be any intersection of the I(K) and
the (m + n)K functions because of the discontinuity in the
I(K) function, and (b) an intersection may occur but if factor
value intensities have reversed, uniqueness cannot be guaranteed
and hence indeterminacy results.

If a balanced growth path is defined as allowing the
possibility of unemployment, the other possibilities arise.
Even if condition (30)   does not hold, less – than full
employment balanced growth paths may exist involving varying
proportions of unemployed to employed labour. In the cases where
unemployed labour is a characteristic of the balanced state, the
capital to employed labour ratio will naturally be higher than the
capital to total labour (employed plus unemployed) ratio.

FOOTNOTES

[1]Work on this has been carried out by Johnson [6], Johnson and Mieszkowski [7].

[2]If we allow the possibility of specialization in production, i.e., a corner point; then it is possible that relative prices can become sufficiently large to allow any finite $\hat{w}_{max}$.

[3]The case of x_2 capital intensive can be dealt with in a similar manner.

[4]These are familiar conditions in fixed coefficient models. See Dorfman, Samuelson and Solow [5 pp. 106-203, 346-89.]

[5]If Det (X) = 0, the production frontier is a straight line; in general there is no full employment of both factors.

[6]Debreu [4, p. 17].

[7]If the tastes of capitalists and workers differ, the method of payment of the minimum wage will affect the supplies of the factors and markets and hence raise problems of uniqueness.

[8]The notation D' denotes the first derivative of D with respect to its argument, in this case, relative prices.

[9]Lefeber [8] uses a similar definition while Brecher [1] uses Johnson's definition to examine some implications for international trade.

[10]See for example Bhagwati and Srinivasan [2].

[11]See for example Manning and Sgro [9].

[12]The stability properties for the x_2 capital intensive case are more complex for reasons suggested in Solow [12] is his comment on the two-sector variable coefficients growth model.

[13]Solow [1, p. 75] derives the following condition for a stable equilibrium for the Harod Domar model:

$$\frac{\sigma}{\alpha} > \eta$$

When σ is the constant savings rate, α is the output/capital ratio while η is the labour force growth rate. The LHS can be interpreted as the warranted rate of growth while the RHS is the natural rate of growth. In our case we also have a parameter for capital depreciation.

[14]See also Manning and Sgro [9] for a discussion of this result.

[15]This K is the ratio of total capital to total labour, i.e., the aggregate capital/labour ratio. This is different from the capital/employed labour ratio if unemployment exists.

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