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PUBLIC EDUCATION, INCOME DISTRIBUTION,
AND VOTING

Arthur J. Robson

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PUBLIC EDUCATION, INCOME DISTRIBUTION, AND VOTING

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Abstract: What is the income distributional impact of public education? A dynamic model of investment in human capital is considered. When individuals differ in ability, initial human capital, interest rate faced, or working lifetime, the impact is almost certainly regressive. If, however, the length of schooling is used as a proxy for the first two parameters, and differing interest rates or working lifetimes are considered, education apparently gives the greatest relative boost to the disadvantaged. Finally, individuals with high percentage gains from increments in public education vote for high levels of its provision, when it is financed by a proportional income tax.

Footnote: Steve Shavell read an earlier draft and made a number of suggestions for which I am grateful. Errors and omissions are mine.

1. Introduction

This paper addresses the basic question: What is the income distributional impact of the subsidy of public education? A second question is related to the first: Who will vote for high levels of education?

Empirical literature bearing on the first question indicates no marked equalizing effect of public education (see Hansen and Weisbrod (1967), for example). The theoretical question is: Is this surprising? The answer suggested by the present paper is: It is not.

A dynamic lifetime model of investment in human capital is used. The maximand is taken to be the present discounted value of earnings. Although unearned income is neglected, it is likely that this is highly correlated with earned income, and the conclusions of the paper stand. Leisure, too, is omitted from consideration, for the sake of simplicity. Education, which is public in the sense of being equally provided to all, is an input to the production of human capital. At each point in time an individual can reinvest his entire stock of human capital or can rent it all in a competitive market. This simplification to a binary choice is made because government subsidy of education is substantial in reality only during specialization in education. The production function for human capital is taken to be Cobb-Douglas and depreciation of human capital is ignored.

The simplifications made facilitate the analysis. However, the model might also be taken as a counterexample to the hypothesis that the subsidy of public education has a progressive impact on income distribution.

Individuals are allowed to differ in the values assigned the following parameters.

- (1) The initial endowment of human capital. The theory suggests this as a cause of inequality, although direct measurement seems to be impossible.
- (2) Ability, represented as a constant in the production function for human capital. This constant reflects the differential rate at which individuals can increase their stock of human capital. Although measurement of this parameter is possible in principle, it is ambiguous in practice.
- (3) The rate of interest facing an individual. Plausibly poor individuals face a higher rate of interest than do rich individuals. (This is more general than assuming that certain individuals cannot borrow at all.) Such imperfections in the market for capital might appear to form the basis of an argument in favor of the subsidy of education. However, for political and informational reasons, such subsidy is largely across-the-board in practice. Hence the distributional impact of the subsidy of public education must be ascertained.
- (4) The working lifetime planned. Women, for example, may anticipate a shorter period of participation in the workforce. (A more general formulation would include the possibility of anticipating a shorter workweek.) Planning a shorter working lifetime does not imply being worse off, since there are non-pecuniary benefits such as leisure. All the same, the strictly financial consequences of

educational subsidy are of interest for such individuals. All individuals will be assumed to have the same biological lifetime for consumption.

How do individuals who differ as above differ in the percentage gains they derive from public education?

In answering this question, it is shown that treating the length of formal education as exogenous might lead to incorrect results. This yields a caveat for empirical investigations of the impact of public education upon income distribution.

The question of the voting for public education is taken up next. When education is financed by a proportional income tax, the individuals who gain most as a percentage from increments in public education will vote for the highest levels.

Related theoretical papers

Stiglitz (1974) uses a static context where education is equally provided to all to relate the majority-voting equilibrium to the efficient allocation. When median income (wealth) is less than mean income (wealth), the preferences of the poor tend to dominate the voting outcome. Differing wealth then leads to an overprovision of education, and differing ability to an underprovision. The result for ability is consistent with the result in the present paper, although the framework is different.

The dynamic model of investment in human capital is due to Ben-Porath (1967). In his paper, individuals are permitted to reinvest a fraction of their human capital stock, and he allows human capital to depreciate. Also there is a private input to the production of human capital, which is replaced in the present paper by public education.

Wallace and Ihnen (1975) modify Ben-Porath's model to consider capital market imperfections and they investigate the dependence of the length of schooling upon the parameters of the model. They assume loans to finance education are not allowed, although loans for consumption must be permitted. The present paper investigates how capital market imperfections affect the distribution of the benefits of education, by varying the rate of interest facing an individual.

2. The Basic Model

Consider first the production of human capital. At time t , take the inputs to be public (that is, equally-provided) education, E , and an amount of human capital, $R(t)$, selected by the individual. It seems reasonable to allow the individual to select $R(t)$ freely between zero and his total current stock of human capital, $H(t)$, say. If $R(t) = H(t)$ corresponds to formal schooling, then $0 < R(t) < H(t)$ corresponds to on-the-job training. However, a simple model would imply that the government subsidize on-the-job training as much as formal schooling. This is not realistic. A more complicated model might permit different levels of subsidy or allow the addition of private inputs. For simplicity, the individual is constrained here to select $R(t) = 0$ or $R(t) = H(t)$. As further simplifications, depreciation of human capital is ignored and the production function is taken to be Cobb-Douglas. Hence

$$\dot{H}(t) = \alpha E^\beta (R(t))^\gamma = \delta (R(t))^\gamma \quad (1)$$

where α , β , γ are positive constants, $\beta + \gamma < 1$, and $\delta = E^\beta$. The parameter α is taken to represent ability and to differ between individuals. Suppose that the initial stock of human capital, $H(0)$, is I , say. This will also differ between individuals.

The maximand for an individual is the present discounted value of lifetime earnings,

$$Y = \epsilon \int_0^T (H(t) - R(t))e^{-rt} dt \quad (2)$$

where ϵ is the competitive rate of return to human capital, r is the rate of interest at which an individual can borrow or lend, and T is the working lifetime of the individual. Individuals differ in r and T again.

The problem of maximizing (2) subject to the differential equation (1), the control constraints $R(t) = 0$ or $R(t) = H(t)$, and the initial condition $H(0) = I$, is soluble by the Maximum Principle. Define the Hamiltonian

$$\mathcal{H} = \epsilon(H(t) - R(t))e^{-rt} + \Psi(t)\delta R(t) \quad (3)$$

where $\Psi(t)$ is a costate variable with adjoint equation

$$\Psi'(t) = - \frac{\partial \mathcal{H}}{\partial H} = - \epsilon e^{-rt}$$

The transversality condition for unconstrained choice of $H(T)$ is $\Psi(T) = 0$.

Hence

$$\Psi(t) = \frac{\epsilon}{r} (e^{-rt} - e^{-rT}) \quad (4)$$

The Hamiltonian, (3), is to be maximized over the two choices for $R(t)$.

However,

$$\mathcal{H} \Big|_{R(t)=0} = \epsilon H(t)e^{-rt} \quad (5)$$

$$\mathcal{H} \Big|_{R(t)=H(t)} = \frac{\epsilon}{r} \delta (e^{-rt} - e^{-rT}) H(t) \quad (6)$$

Consider the relative magnitudes of (5) and (6). Since $H(t)$ is non-decreasing and $(1 - e^{-r(T-t)})$ is decreasing, the general form of the solution is¹

$$R(t) = \begin{cases} H(t) & 0 \leq t \leq S \\ 0 & S < t \leq T \end{cases} \quad (7)$$

where S is thus the length of an initial period of schooling, $0 \leq S < T$.

In fact, if

$$I^{1-\gamma} < \frac{\alpha E^\beta}{r} (1 - e^{-rT}) \quad (8)$$

then $S > 0$. If (8) does not hold $S = 0$. Consider the case for $S > 0$.

Then S is the unique solution to

$$H(S)^{1-\gamma} = \frac{\delta}{r} (1 - e^{-r(T-S)}) \quad (9)$$

However,

$$\dot{H}(t) = \delta(H(t))^\gamma \quad 0 \leq t \leq S \quad (10)$$

since all human capital is reinvested during this initial period. Hence

$$H(S)^{1-\gamma} = I^{1-\gamma} + (1 - \gamma)\alpha E^\beta S \quad (11)$$

and (9) can be rewritten as

$$\frac{I^{1-\gamma}}{\alpha E^\beta} + (1 - \gamma)S = \frac{1}{r}(1 - e^{-r(T-S)}) \quad (12)$$

which cannot be solved explicitly.

Determinants of the length of schooling

How does the length of schooling, S , depend upon the level of public education, E , and the individual parameters I , α , r , and T ? Firstly, from (12),

$$\frac{\partial S}{\partial E} = \frac{\beta I^{1-\gamma}}{\alpha E^{1+\beta} [(1-\gamma) + e^{-r(T-S)}]} > 0 \quad (13)$$

showing that the length of schooling increases as the level of public education increases. Also from (12),

$$\frac{\partial S}{\partial (I^{1-\gamma}/\alpha)} = \frac{-1}{E^\beta [(1-\gamma) + e^{-r(T-S)}]} < 0 \quad (14)$$

so that the length of schooling decreases for an increased initial endowment of human capital, I , but increases for a greater level of ability, α . Here, as in future results, these two causes of inequality have opposite effects. Again, from (12),

$$\frac{\partial S}{\partial r} = - \frac{[1 - e^{-r(T-S)}(1 - r(T-S))]}{r^2 [(1-\gamma) + e^{-r(T-S)}]} < 0 \quad (15)$$

so that a higher rate of interest leads to a decrease in the length of schooling. Finally, from (12),

$$0 < \frac{\partial S}{\partial T} = \frac{1}{(1-\gamma)e^{r(T-S)} + 1} < 1 \quad (16)$$

Hence both the length of schooling, S , and the length of time spent working, $T-S$, increase with the total working lifetime, T .

These results for the determinants of S will be used in Section 3.

3. Public Education and Income Distribution

Consider the present discounted value of lifetime earnings, V . After an individual finishes schooling his wage rate is constant and

$$V = \epsilon H(S) \int_0^T e^{-rt} dt \quad (17)$$

If (8) holds, so that $S > 0$, this becomes

$$V = \epsilon \alpha^{\frac{1}{1-\gamma}} E^{\frac{\beta}{1-\gamma}} e^{-rS} \left\{ \frac{1 - e^{-r(T-S)}}{r} \right\}^{\frac{2-\gamma}{1-\gamma}} \quad (18)$$

using (9). If (8) does not hold and $S = 0$,

$$V = \epsilon I \left(\frac{1 - e^{-rT}}{r} \right) \quad (19)$$

Consider now the impact of public education E upon V . This is not straightforward due to the implicit dependence of S on E . However, it may be analyzed as follows. If $E = 0$, (8) cannot hold, $S = 0$, and V is given by (19). As E is increased, V remains constant until the value E^* is reached, where

$$I^{1-\gamma} = \frac{\alpha E^*{}^\beta}{r} (1 - e^{-rT}) \quad (20)$$

At this level the individual begins to undertake schooling. For $E > E^*$,

$$\frac{dV}{dE} = \frac{\beta}{(1-\gamma)E} - r \left\{ \frac{1 + \frac{2-\gamma}{1-\gamma} e^{-r(T-S)}}{1 - e^{-r(T-S)}} \right\} \frac{\partial S}{\partial E}$$

so that, using (13), and simplifying

$$\frac{dV}{dE} = \frac{\beta r S}{E[1 - e^{-r(T-S)}]} = \bar{g}(E, S, r, T) = g(E, I^{1-\gamma}/\alpha, r, T) \quad (21)$$

say. The first function, \bar{g} , is the explicit form given. The second, g , accounts for the dependence of S upon the original parameters, as in (12). Equation (21) describes how the percentage gains from an increment in public education are distributed, given that individuals engage in schooling. In all cases here, it turns out that the individuals who have the lowest values of E^* also have the highest values of (21). Unambiguously, then, these individuals benefit the most as a percentage from public education.

Consider in turn the different individual parameters.

Different initial human capital and ability

Suppose individuals differ in I and α . From (20) it follows that a low value of $I^{1-\gamma}/\alpha$ implies a low value of E^* . Also, from (21),

$$\begin{aligned} \frac{\partial g}{\partial (I^{1-\gamma}/\alpha)} &= \frac{\partial \bar{g}}{\partial S} \cdot \frac{\partial S}{\partial (I^{1-\gamma}/\alpha)} \\ &= \frac{-\beta r [1 - e^{-r(T-S)} + r S e^{-r(T-S)}]}{E^{1+\beta} [1 - e^{-r(T-S)}]^2 [(1-\gamma) + e^{-r(T-S)}]} < 0 \end{aligned} \quad (22)$$

Hence a low value of $I^{1-\gamma}/\alpha$ also implies a high percentage gain from an increment in E .

Altogether, then, individuals with initial human capital, I , and ability, α , such that $I^{1-\gamma}/\alpha$ is low, have long periods of schooling, S , and benefit the most from public education, E , as a percentage.

Are such individuals rich or poor? If differing discounted earnings arise from differing abilities, the richer individuals spend more time in school and derive a greater percentage benefit from public education. On the other hand, if the reason for differing discounted earnings is differing initial human capital, the richer individuals spend less time in school and benefit less as a percentage. It seems in reality likely that ability and initial human capital are positively correlated but that better endowed individuals spend more time in school. Then there is a regressive impact to the subsidy of public education.

Consider now a possible empirical investigation of the determinants of the percentage gains from an increment in the level of public education. Since it is reasonable to assume that all individuals undertake some schooling, it is the form of (21) that is being investigated. The only way that initial human capital, I , and ability, α , enter (21) is as $I^{1-\gamma}/\alpha$ and then because of the effect of this ratio on the length of schooling, S . Since measurement of I and α is difficult, it might be tempting to use S as a proxy for both. Such treatment of S as exogenous will lead to incorrect conclusions

as to the effect of different interest rates, r , and working lifetime, T , upon the percentage gain from education. This will be demonstrated in the following subsections.

Different interest rates

What effect do capital market imperfections have upon the distributional impact of public education? The imperfections apparently favor educational subsidy, but since this is actually mostly non-discriminating, its distributional impact should be examined.

With an income profile as in the model, individuals are likely to wish to borrow for present consumption against future income. A factor contributing to inequality is that certain disadvantaged individuals face higher rates of interest than others. How are the percentage gains from public education now distributed?

Varying the interest rate apparently causes difficulties in the use of the present discounted value of earnings as a criterion of welfare. However, Appendix I considers the simple case where utility is logarithmic and discounted over time. Then the change in discounted utility is proportional to the percentage change in the present discounted value of income. The constant of proportionality is independent of the interest rate. Hence individuals with the same percentage gain in discounted income have the same gain in discounted utility regardless of differing interest rates. If utility is not logarithmic, there is no compelling reason for considering percentage changes in income. The criterion seems to mean as much (or as little) as usual.

Additionally, it is shown in Section 4 that voting behavior can be predicted by considering the percentage gain in discounted income arising

from increments in public education. This is independent of the possibility of different interest rates for different individuals.

Consider first the use of the length of schooling, S , as an exogenous variable in an investigation of the determinants of the percentage gain from an increment in public education. From (21),

$$\frac{\partial \bar{g}}{\partial r} = \frac{\beta S [1 - (1 - r(T-S))e^{-r(T-S)}]}{E[1 - e^{-r(T-S)}]^2} > 0 \quad (23)$$

Hence individuals facing a higher rate of interest, but nevertheless having the same length of schooling, benefit more as a percentage from an increment in public education. Then it would appear that education helps the most in relative terms those disadvantaged by facing a high interest rate.

However, this is false. Individuals who have the same length of schooling despite an increase in the interest rate, must differ in initial human capital, ability, or both. Consider individuals who are identical apart from interest rate faced. From (20), an increase in r raises the value of E^* , the minimum level of public education needed for an individual to engage in schooling. Moreover, from (21),

$$\frac{\partial g}{\partial r} = \frac{\partial \bar{g}}{\partial r} + \frac{\partial \bar{g}}{\partial S} \frac{\partial S}{\partial r} = - \frac{\beta I^{1-\gamma} [1 - r(T-S)] e^{-r(T-S)}}{\alpha E^{1+\beta} [(1-\gamma) + e^{-r(T-S)}] [1 - e^{-r(T-S)}]} < 0 \quad (24)$$

Hence an increase in the interest rate faced decreases the relative gain from public education. Public education has a regressive impact in this respect also.

Different working lifetimes

Consider now the impact of the subsidy of public education on individuals who plan to stop working at different points in their lives. Although

varying T is not as important as varying the rate of interest, r , it still holds some interest for a comparison between men and women, for example. Since everyone lives for the same total lifetime, varying the working lifetime does not cause a problem with the use of the present discounted value of earnings as a criterion of welfare.

Consider again the use of the length of schooling, S , as an exogenous variable in an empirical investigation. From (21),

$$\frac{\partial \bar{g}}{\partial T} = - \frac{\beta r^2 S e^{-r(T-S)}}{E[1 - e^{-r(T-S)}]} < 0 \quad (25)$$

Hence it will seem that education helps the most in relative terms those disadvantaged by having a shorter working life.²

Again, this conclusion is invalid for identical individuals. In fact, from (20), a longer working lifetime, T , decreases E^* , the minimum level of public education needed so that an individual will engage in schooling. Furthermore, from (21),

$$\frac{\partial g}{\partial T} = \frac{\partial \bar{g}}{\partial T} + \frac{\partial \bar{g}}{\partial S} \frac{\partial S}{\partial T} = \frac{\beta I^{1-\gamma} r^2}{\alpha E^{1+\beta} [(1-\gamma) + e^{-r(T-S)}] [1 - e^{-r(T-S)}]^2} > 0 \quad (26)$$

Hence individuals with longer working lifetimes benefit relatively more from increments in public education. This is another regressive aspect to the subsidy of public education.

4. Voting for Public Education

Suppose now that the level of public education, E , is to be decided upon. Which individuals will vote for high levels of E ? With a proportional income tax, the answer is: those with high percentage gains from increments in E .

Suppose that expenditure on education is given by

$$M = \theta K(E) \quad K'(E) > 0 \quad (27)$$

where $0 \leq \theta \leq 1$ is the tax rate, and $K(E)$ is the tax base which depends on the level of education, E . The level of expenditure, M , translates into a level of provision of public education, E , as

$$E = F(M) \quad F'(M) > 0 \quad (28)$$

where the function $F(\cdot)$ represents any diminishing effectiveness of expenditure on education as crowding increases.

Consider first the function $K(\cdot)$. The assumption used is that

$$\frac{dK}{dE} < \frac{K}{E} \quad (29)$$

or equivalently that the average tax base, K , per unit of public education, E , declines as E increases. The assumption (29) can be derived when the tax base, $K(E)$, is the earned incomes of individuals composing the society. The crucial point is that, from (18), using (13), it can be shown that

$$\frac{dV}{dE} < \frac{\beta}{1-\gamma} \frac{V}{E} < \frac{V}{E} \quad (30)$$

since $\beta + \gamma < 1$ by assumption. The construction of the tax base, $K(E)$, which depends on the detailed structure of society, is messy and seems peripheral.

Consider now the question of the congestion of the education facilities, that is, the function $F(\cdot)$. The assumption here is that

$$\frac{dF}{dM} < \frac{F}{M} \quad (31)$$

or that the average number of units of public education provided per dollar of expenditure declines as expenditure rises. This seems a natural interpretation of congestion of educational facilities.

Equations (27) and (28) simultaneously determine a required tax rate, θ , for each level of provision of public education, E . In fact, θ should satisfy

$$E = F[\theta \cdot K(E)] \quad , \quad 0 \leq \theta \leq 1 \quad (32)$$

Assume there exists an $\tilde{E} > 0$ such that

$$F[K(\tilde{E})] = \tilde{E} \quad (33)$$

Then, if $E \leq \tilde{E}$, a θ satisfying (32) exists. If also

$$\theta'(E) > 0 \quad (34)$$

the value of $\theta(E)$ is unique as written. Differentiating (32), and rearranging,

$$\theta'(E) = \frac{M}{KE} \left\{ \frac{E}{M} \frac{dM}{dE} - \frac{E}{K} \frac{dK}{dE} \right\} \quad (35)$$

Then (34) follows immediately from (29) and (31). Also, (34) holds at $E = 0$. Hence the expansion of the tax base can never allow the tax rate to decline, as the level of expenditure on education is increased.

An individual, 1, say will vote for the level of public education which maximizes his after-tax discounted earnings

$$\bar{V}_1(E) = (1 - \theta(E)) \cdot V_1(E) \quad (36)$$

where $V_1(E)$ is the before-tax discounted earnings as in Section 3, emphasizing its dependence on E . The presence of the tax $\theta(E)$ does not itself affect $V_1(E)$ in this simple model. Suppose that individual 1 votes for $\bar{E}_1 > 0$.³ A necessary condition for an interior maximum can be written as

$$f(E) = g_1(E) \quad (37)$$

where

$$f(E) = \frac{\theta'(E)}{1-\theta(E)} > 0$$

and

$$g_1(E) = \frac{V_1'(E)}{V_1(E)} = \frac{\beta r_1 S_1}{E[1 - e^{-r_1(T_1 - S_1)}]} > 0$$

as in (21), where the parameters have been subscripted "1". The length of schooling, S_1 , depends on E as in (12). Consider the functions $f(\cdot)$ and $g_1(\cdot)$. Suppose $E \leq E_1^*$, where E_1^* is the minimum level of public education needed for 1 to attend school at all. Then $S_1 = 0$, and $g_1(E) = 0$. However $f(E) > 0$. As E tends to \bar{E} , as in (33), then $\theta(E)$ tends to 1, and $f(E)$ tends to infinity. However, $g_1(E)$ remains bounded.

Since existence of an interior optimum is assumed, there are an even non-zero number of values of E satisfying (37). Second-order considerations rule out odd-numbered candidates. A global comparison must still be made to find the optimum, \bar{E}_1 .

Consider now an individual 2, such that $g_2(E) > g_1(E)$ for all E greater than E_1^* (which is greater than E_2^*). How does the level of public education voted for by 2, \bar{E}_2 , compare with \bar{E}_1 ? In fact, regardless of multiple local maxima, $\bar{E}_2 > \bar{E}_1$. The proof is elementary but tedious, and is sketched here. At $E = \bar{E}_1$, individual 2 is better off than at any $E < \bar{E}_1$. For individual 1 was at the optimum here, and 2 gains on 1, in relative terms as E increases, by assumption. At \bar{E}_1 , there remains a marginal net gain to 2 of increasing E . A positive optimum exists for 2 since it did for 1; hence $\bar{E}_2 > \bar{E}_1$.

Section 3 analyzed the factors that might cause $g_2(E) > g_1(E)$, for $E > E_1^*$. To recapitulate, if individual 2 spends more time in school than 1, due to different ability, α , initial human capital, I , or both, then $g_2(E) > g_1(E)$, $E > E_1^*$. Individual 2 will vote for more education than 1. If individual 2 instead faces a lower interest, r , or has a longer working

lifetime, T , then also $g_2(E) > g_1(E)$, $E > E_1^*$. Again individual 2 will vote for more education than 1.

5. Conclusions

There seems to be little support for advocacy of public education as a means of reducing inequality to be derived from this model.

The only possible progressive impact of the subsidy of public education would arise if the predominant reason for inequality were differing initial human capital. Then it would have to be that richer people would choose to spend less time in school than poorer people. Even allowing for the omission of consumption benefits of education, this seems improbable. Otherwise, the impact of the subsidy of public education upon persons of differing initial human capital or ability is regressive.

Two other factors leading to inequality were investigated--differing interest rates and working lifetimes. An individual who faces a higher interest rate or a shorter working lifetime is, of course, worse off. In both of these cases, also, there was found to be a regressive impact to the subsidy of public education.

A common practice in empirical work is to use length of schooling as an exogenous variable. Since ability and initial human capital are hard to measure, length of schooling will then act as a proxy for these two variables. Then the model predicts that public education will apparently benefit relatively the most those disadvantaged by a high interest rate or a short working lifetime.

Finally, it is shown that, under a proportional tax, individuals who gain the most in percentage terms from increments in public education will vote for the highest levels of its provision.

APPENDIX I

When interest rates differ, how well does the criterion of the percentage gain in discounted income work to predict changes in intertemporal utility? As well as it ever does, that is, in the case for logarithmic utility, discounted over time. Then intertemporal utility is

$$W = \int_0^L e^{-\rho t} \log C(t) dt \quad (1')$$

where ρ is the rate of discount for utility, L is the total lifetime, and $C(t)$ is the rate of consumption at time t . W is to be maximized subject to a budget constraint on consumption, say

$$V = \int_0^L C(t) e^{-rt} dt \quad (2')$$

where V is the present discounted value of income, and r is the rate of interest.

The Lagrangian is

$$\mathcal{L} = \int_0^L e^{-\rho t} \log C(t) - \lambda \int_0^L C(t) e^{-rt} dt \quad (3')$$

for a Lagrange multiplier, λ . The first-order condition implies

$$C(t) = \frac{1}{\lambda} e^{(r-\rho)t} \quad (4')$$

If, as seems likely, $\rho > r$, $C(t)$ decreases over time. In any case, using (2')

$$\lambda = \frac{1}{\rho V} (1 - e^{-\rho L}) \quad (5')$$

and then, from (1')

$$W = \int_0^L e^{-\rho t} \left\{ \log \left(\frac{\rho V}{1 - e^{-\rho L}} \right) + (r - \rho)t \right\} dt \quad (6')$$

Hence, if V increases,

$$\frac{dW}{dV} = \frac{1}{\rho V} [1 - e^{-\rho L}] \quad (7')$$

Hence equal percentage changes in V cause the same increment in W , regardless of r .

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FOOTNOTES

¹Existence is readily established here, so that the solution presented is the unique solution.

²In fact, individuals who spend the same time in school despite a higher interest rate or a shorter working lifetime, may be either better off or worse off than their fellows. Although they are worse off if only initial human capital differs, either possibility exists if ability differs. Hence it is not true that the impact of public education will necessarily appear progressive. Details of these assertions are omitted.

³If $g_1(E)$ is small everywhere, individual 1 will vote for $\bar{E}_1 = 0$. This might be true even when $g_1(E)$ and $f(E)$ intersect. This corner solution $\bar{E}_1 = 0$ should strictly be considered a candidate too. This causes no particular problem, but does not seem to be an important possibility.