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ABSTRACT

Recent literature on the phenomenon of sustained growth has emphasized the role of increasing returns to scale technologies. We suggest in this paper a microeconomic foundation for the existence of increasing returns technologies. Production is assumed to require a combination of capital and labor in a standard, constant returns to scale technology. However, this technology is affected multiplicatively by a productivity factor. This factor is assumed to be a result of a research and development process. The R&D process is modeled as a search problem. Firms face a known fixed distribution of productivity factors from which they can sample. Sampling is costly in terms of capital, and therefore firms, which possess a certain amount of capital, have to decide when to stop sampling, hire labor in a competitive labor market, and invest the remainder of their capital in the CRS technology multiplied by the productivity factor they have uncovered. The paper analyzes the search problem faced by the firms, and shows that under certain assumptions about the probability distribution function which governs the behavior of the productivity factor, output is likely to display increasing returns to scale with respect to capital (in a probabilistic sense). This result is embedded in a Diamond-like growth model. It is argued that the model can possess sustained growth paths with interesting stochstaic features. In particular, growth rates of the poorer economies are likely be lower than those of the richer economies, but less variable. This result seems to be corroborated in the data.

1. INTRODUCTION

This work describes an environment in which optimizing behavior implies that incentives to capital accumulation do not diminish as the capital stock grows, thus allowing for sustained growth in output per worker asymptotically. The main idea is that improvements in production technologies, broadly interpreted, occur as random outcomes of search process over potential "untried" technologies, where the search investment is financed from savings. Countries with higher output per worker can - and under the optimal search strategy will choose to - invest more in search, thus enjoying higher expected profits, which in turn will allow them to further increase their investment in search. What is novel about our approach is that we demonstrate how this process can generate sustained growth even though new output is produced via neo-classical constant returns to scale functions of capital and labor, and the distribution of technologies searched over is *time invariant*, so that better technologies become harder to discover.

Recent literature on growth, for instance, Lucas (1988), Romer (1986), Rebelo (1987), and Jones and Manuelli (1988), has been successful in modeling sustained growth *endogenously*, by relying on some notion of increasing returns to scale, or on some lower bound on capital productivity, thus avoiding the diminishing returns to the factor of production that can be accumulated.[1] Little emphasis is put on the micro foundation for such a technology, although its micro underpinning might be crucial for designing

[1] See Sala-i-Martin (1990) for a comprehensive review of this literature.

policies aimed at promoting growth. A different mechanism for generating endogenous growth is provided by explicit models of financial intermediation, which exploit increasing returns in information processing, or from economizing on monitoring costs of funded investment projects, (Greenwood and Jovanovic (1990), Williamson (1987)). Yet, in all the aforementioned works the underlying production technology is taken as given, while relatively few works recognize the importance of the costs and the structure of returns to the process of *finding* these technologies, (Aghion and Howitt (1990), and Jovanovic and Rob (1990) being notable exceptions). We suggest that production technologies, broadly interpreted, have to be *discovered*, that the discovery process is *random*, and that there is a *fixed cost* element involved in searching for better technologies. These features will be shown to imply, for certain environments, a random growth process which can display increasing, maintained or decreasing conditional first moments of growth rates, depending on the beginning of period capital stock. The model is based on a costly search process over potential technologies with unknown productivity rates. For sufficiently high per capita beginning of period capital stock, optimal search generates output process exhibiting increasing returns to "gross" capital, while exhibiting constant or decreasing returns to "productive" capital. Search costs account for the difference between "gross" and "productive" capitals.

Our approach to modeling growth through search for better technologies differs from some recent works on the same subject. In particular, we assume that within a period, each firm is precluded from observing the search results of its rivals. It simply operates as a price taker with respect to its factors of production, doing the best it can in searching for a production technology.

Thus, the only advantage in finding a "good" technology is the economic value of that technology itself, without any additional monopoly rents. This, to be contrasted with Aghion and Howitt (1990), where in the context of R&D races, the finder of the best technology enjoys monopoly rights, until a better technology is found. Moreover, we further reduce the incentive to invest in R&D by assuming a time-invariant probability distribution of *levels* of technologies, as opposed to Aghion and Howitt (1990), or Jovanovic and Rob (1990), where *increments* to the technologies have time-invariant distributions. Thus, in our environments it becomes monotonically harder over time to find a better technology, and yet we show that incentives to invest in search can be large enough to induce ever increasing investment, which generates sustained growth per capita.

The increasing returns to capital in this search-investment model create a natural role for *intermediation*, interpreted simply as the process of pooling individual savings to exploit these increasing returns. In a deeper analysis, the equilibrium structure and extent of intermediation of this kind is determined endogenously and jointly with the growth process itself, (as in Greenwood and Jovanovic (1990)). Here, however, we simply impose a pooled investment process, which amounts to exploiting in full the benefits from conducting joint search. As in Boyd and Prescott (1986), pooled investment will economize on project evaluation costs.

In addition to generating equilibrium growth process with (conditional) increasing first moments, our search theoretic approach has several additional attractive features. First, it allows us to examine the implications of alternative government policies aimed at promoting growth, such as subsidies to R&D. In previous analyses, such policies were modeled as exogenous shifts in the productivity of R&D investment. Since our approach derives the optimal search strategy as a function of the (fixed) sampling cost from a given distribution of technologies, subsidies to that cost can be studied without assuming any "implicit" change in the productivity of R&D[2].

Second, our search theoretic approach has testable implications which depend on the assumed distribution of technologies. Given such a distribution, our approach can use observable differences among searchers to estimate and test implications to their objectives and production functions. For instance, we can view different sectors in the economy as conducting search from the same distribution but with different initial capital stocks, and use the differential sectors growth rates to test the model.

Third, the resulting stochastic growth process has implications to relations among higher moments of time series of growth processes which have not received due attention before. For instance, Kormendi and Meguire (1985) found that post World War II sample means of annual growth rates of real GNP in a sample of 47 countries are positively and significantly correlated with the variability of each country's growth, (as measured by sample standard deviations of annual growth rates in each country).

Finally, a search theoretic setup can explain apparent differences between aggregate and firm specific data of output and R&D outlays. For instance,

[2]This is analogous to the examination of unemployment insurance in search theoretic models of unemployment, for instance, Lippman and McCall (1976).

Pakes and Schankerman (1984) found hardly any relationship between the growth rates of individual firms and their intensity of R&D activity, while the growth in sector output aggregates accounts for over 50% of the variance in sector aggregates of R&D investment.

This paper outlines a model which can address some of the issues mentioned above. In section 2, we analyze a one-period problem of sequentially searching over investment opportunities, and characterize the optimal search strategy. In section 3 we embed the sequential search process in an otherwise standard overlapping generations growth model, a la Diamond (1965). The sense in which the solution to the search problem can give rise to an aggregate production technology that exhibits increasing returns to scale is illustrated in section 4. In particular, we provide in that section an example economy which will have, for sufficiently high initial capital stock, a random equilibrium growth path with *increasing* expected growth rates, conditioned on beginning of period stock of capital. For sufficiently low initial capital, that same economy can have an equilibrium with no search and negative growth. Further implications of the model on the stochastic growth process are reviewed in the concluding section.

2. SEQUENTIAL SEARCH FOR INVESTMENT OPPORTUNITIES

2.1 *A single period problem*

Consider the following search problem. An investor has Q units of capital good, (hereafter, investment capital), available for investment at the current time. For a fixed price of α units of capital the investor can sample one investment project at a time, from an infinitely large population of projects, indexed by their (constant) productivity rate, θ. The cumulative probability distribution of the productivity rates is denoted by $H : \mathbb{R}^1 \rightarrow [0, 1]$, $H(\theta) = 0$ for $\theta < \underline{\theta}$, $H(\theta) = 1$ for $\theta \geq \bar{\theta}$, $0 \leq \underline{\theta} < \bar{\theta} \leq \infty$.[3] This distribution is assumed to be invariant under successive sampling, (i.e., sampling with replacement). After each sampling the investor has the option of accepting or rejecting the project just examined. Rejecting the project means sampling at least one more time. Accepting means activating the project with all the remaining capital and possibly additional factors of production which can be hired at that point. Thus, the search is done "without recall" during the search process, so that a project rejected cannot be adopted later on, (it will be shown below how to modify the analysis to allow for recall within the search period).

A project with a particular θ in which k units of capital and optimal levels of other factors are utilized will generate a payoff to the searcher, denoted $\varphi(k,\theta)$, where $\varphi(\cdot,\cdot)$ increases monotonically in both arguments. The index φ will be interpreted in the next section as the utility from the profit generated by operating the project θ with k units of capital. It is assumed that the only cost of search is the direct sampling cost, and in particular, the search activity does not take any time. The investor's goal is to maximize the expectations of the index φ by specifying an optimal stopping strategy for the sequential search process. Denote by σ a stopping strategy, mapping histories of sampled projects and remaining capital into the binary space,

[3]Note that θ can be infinity, a feature that will be critical for sustained growth.

{"accept" , "reject"}. Then each σ induces a probability distribution over the final values of θ and k that will determine that searcher's payoff, denoted $\tilde{\theta}_\sigma$ and $\tilde{k}_\sigma$. The searcher seeks to find a stopping rule σ such that

$$E\ \varphi(\tilde{k}_\sigma,\tilde{\theta}_\sigma) \geq E\ \varphi(\tilde{k}_{\sigma'},\tilde{\theta}_{\sigma'})$$

for every feasible stopping rule σ'.

Since resources are limited, the search will be finite in the number of observations sampled. Let $V(k,\theta)$ be the value of the objective searching optimally, k units of capital remain for investment, and the available productivity rate is θ. The function $V(\cdot,\cdot)$ must then satisfy the following functional equation:

$$(2.1)\qquad V(k,\theta) = \text{Max}\ \{\ \varphi(k,\theta)\ ,\ E\ V(k-\alpha,\tilde{\theta})\ \},\qquad \alpha \leq k$$
$$V(k,\theta) = \varphi(k,\theta),\qquad 0 \leq k \leq \alpha,$$

where E is the expectations operator.

Note that if the searcher can always go back to any project sampled earlier, then equation (2.1) changes to:

$$(2.2)\qquad V(k,\theta) = \text{Max}\left\{\ \varphi(k,\theta)\ ,\ E\ \text{Max}\{V(k-\alpha,\theta)\ ,\ V(k-\alpha,\tilde{\theta})\}\ \right\},\qquad k \geq \alpha;$$
$$V(k,\theta) = \varphi(k,\theta),\qquad k < \alpha.$$

Since $\varphi(k,\cdot)$ is monotonically increasing in θ, the optimal search strategy takes the form of a reservation productivity rates. Let $\theta^*(k)$ denote the minimal productivity rate which is acceptable when the remaining investment capital is k. Then, from (2.1), we have:

$$(2.3) \qquad V(k,\theta) = \begin{cases} \varphi(k,\theta), & \text{if } \theta > \theta^*(k) \\ E\, V(k-\alpha,\tilde{\theta}), & \text{otherwise.} \end{cases}$$

If $\theta^*(k) > \underline{\theta}$, then it must equate the two terms in the maximand of (2.1), in which case we have:

$$(2.4) \qquad \varphi(k,\theta^*(k)) = E\, V(k-\alpha,\tilde{\theta}) =$$

$$= H[\theta^*(k-\alpha)]\cdot\varphi(k-\alpha,\theta^*(k-\alpha)) + \int_{\theta^*(k-\alpha)}^{\bar{\theta}} \varphi(k-\alpha,\theta)\, dH(\theta)$$

while $\theta^*(k) = \underline{\theta}$ for $k \in (0,\alpha)$.

Equation (2.4) is a recursive relation from which $\theta^*(k)$ can be found for any k > 0. Note also that the first equality in (2.4) allows us to state that if σ^* denotes the optimal search strategy (without recall) from initial capital stock of Q then

$$(2.5) \qquad E\, \varphi(\tilde{k}_{\sigma^*},\tilde{\theta}_{\sigma^*}) = \varphi(Q,\theta^*(Q)).$$

Finally, note that the analog of (2.4) for the case *with* recall can be found from (2.2) in the same way, to yield the (non-recursive) equation in $\theta^*(k)$:

(2.4')

$$V(k,\theta^*(k)) =$$

$$\varphi(k,\theta^*(k)) = E \text{ Max} \left\{ V(k-\alpha,\theta^*(k)) \ , \ V(k-\alpha,\tilde{\theta}) \right\}$$

$$= H(\theta^*(k))\cdot\varphi(k-\alpha,\theta^*(k)) + \int_{\theta^*(k)}^{\bar{\theta}} \varphi(k-\alpha,\theta')dH(\theta'),$$

where the last equality in (2.4') follows from assuming that $\theta^*(\cdot)$ is non-decreasing, to be proven below.

2.2 *Properties of* $\theta^*(k)$

Without additional assumptions on the index $\varphi(\cdot,\cdot)$ it is difficult to characterize the optimal search strategy. We assume, hereafter, that:

(2.6)
$$\varphi(k,\theta) = f(k)\cdot g(\theta), \qquad k \geq 0, \ \ \theta \in [\underline{\theta} \ , \ \bar{\theta}],$$

where f and g are non-negative, monotonically increasing, f is concave and f(0)=0. These assumptions will be motivated in the context of a dynamic growth model in the next section. With assumption (2.6), equation (2.4) becomes:

(2.7)
$$g(\theta^*(k)) = \text{Max} \left\{ \frac{f(k-\alpha)}{f(k)} E \text{ Max}\{g(\theta^*(k-\alpha)),g(\tilde{\theta})\} \ , \ g(\underline{\theta}) \right\}$$

For sufficiently large k relative to α, the RHS of (2.7) equals its first argument, and this recursive relation can be examined to establish some important properties of the function $\theta^*(\cdot)$. In particular, we have:

CLAIM 1: (i) $\theta^*(k)$ increases monotonically in k for all $k > 0$;

(ii) $\theta^*(k) \longrightarrow \bar{\theta}$ as $k \longrightarrow \infty$;

(iii) $g(\theta^*(k)) - g(\theta^*(k-\alpha)) \longrightarrow 0$ as $k \longrightarrow \infty$.

Proof:

For sufficiently low k, at least $k \in (0,\alpha)$, we have from (2.7) that $\theta^*(k) = \underline{\theta}$, so that (i) holds weakly. Assume then that $d\theta^*(k-\alpha)/dk > 0$, and that $\theta^*(k)$ is given implicitly by:

$$(2.8) \qquad g(\theta^*(k)) = \frac{f(k-\alpha)}{f(k)} \cdot E \text{ Max } \{ g(\theta^*(k-\alpha)) , g(\tilde{\theta}) \}$$

Differentiating both sides of (2.8) w.r.t. k, we get

$$(2.9) \qquad g'(\theta^*(k))\frac{d\theta^*}{dk}(k) = \frac{d\left(\frac{f(k-\alpha)}{f(k)}\right)}{dk} \cdot E \text{ Max } \{g(\theta^*(k-\alpha), g(\tilde{\theta})\}$$

$$+ \frac{f(k-\alpha)}{f(k)}\left(H(\theta^*(k-\alpha))g'(\theta^*(k-\alpha))\frac{d\theta^*}{dk}(k-\alpha)\right).$$

Since $f(\cdot)$ is a concave increasing function, $f(k-\alpha)/f(k)$ also increases in k. It follows, since $g'(\cdot) > 0$ by assumption, that $\frac{d\theta^*}{dk}(k) > 0$ whenever $\frac{d\theta^*}{dk}(k-\alpha) \geq 0$.

To prove (ii), note that $\theta^*(\cdot)$ is strictly increasing in k for sufficiently large k, and is bounded from above by $\bar{\theta}$, since the RHS of (2.8) is bounded from above by $g(\bar{\theta})$. However, for any $x < \bar{\theta}$, $E_{\theta} \text{ Max } \{ x , \theta \} > x$, while $f(k-\alpha)/f(k)$ converges to 1. Hence we cannot have $\lim_{k \to \infty} \theta^*(k) < \bar{\theta}$.

Finally, to prove (iii), use (2.8) again to write

$$
(2.10) \qquad g(\theta^*(k)) - g(\theta^*(k-\alpha)) =
$$

$$
\frac{f(k-\alpha)}{f(k)} \left(E \text{ Max } \{ g(\theta^*(k-\alpha)) , g(\tilde{\theta}) \} - g(\theta^*(k-\alpha)) \right)
$$

$$
- \frac{f(k) - f(k-\alpha)}{f(k)} g(\theta^*(k-\alpha)).
$$

As $k \longrightarrow \infty$ and $\theta^*(k-\alpha) \longrightarrow \bar{\theta}$, the first term on the RHS of (2.10) goes to zero, while the second term is positive. Since $g(\theta^*(k))$ increases monotonically in k, it must be that $g(\theta^*(k)) - g(\theta^*(k-\alpha))$ converges to zero.∎

Note that the fact that first α-differences of $g(\theta^*(k))$ go to zero does not necessarily imply that first α-differences of $\theta^*(k)$ go to zero, unless g is a convex increasing function. Hence, without further restrictions on g we cannot establish in general a property like concavity of $\theta^*(\cdot)$. Nevertheless, in the next section we examine example economies in which this is the case.

Finally, note that the properties of the threshold productivity levels, $\theta^*(\cdot)$, for the case of searching *with recall* within the period, are identical with those established for the case *without* recall. The functional equation which defines the optimal strategy for this case, equation (2.2), can be shown to yield the following equation in θ^*:

$$(2.11) \qquad g(\theta^*(k)) = \text{Max}\left\{\frac{f(k-\alpha)}{f(k)}\,\text{E Max}\{\, g(\theta^*(k)), g(\tilde{\theta})\,\}\ ,\ g(\underline{\theta})\right\}$$

One can establish the same properties listed in claim 1 for the solution to (2.11) along much the same lines specified in the proof. While for any particular k, the threshold productivities will differ in the two cases, it is easy to establish that the one corresponding to the case *with* recall is always higher, and that the difference between the two vanishes as $k \to \infty$.

3. SEARCH IN A DYNAMIC MODEL

Here we embed the one-period search-investment problem in an overlapping generations model. Each period N identical two period lived agents appear. Agents supply labor services (inelastically) when young, and allocate the wage income received between first period consumption and savings, where the latter are invested in a competitive firms given a known distribution of the rate of return on saving. The capital income on savings provides the sole source of second period consumption, and accordingly the amount saved is determined by an expected utility maximization problem. Formally, we assume a time separable utility function, so that an agent born at period t maximizes

$$(3.1) \qquad v(c_{1t}) + E\ u(\tilde{c}_{2t})$$

subject to

(3.2) $$c_{1t} = w_t - s_{t+1}$$

(3.3) $$\tilde{c}_{2t} = \tilde{R}_{t+1} s_{t+1}$$

Here $v(\cdot)$ and $u(\cdot)$ are strictly concave, twice differentiable increasing functions, E is the expectations operator, c_{it} is the consumption at period i of the agent's life, $(i = 1,2)$, w_t is the wage rate the agent obtains when young, s_{t+1} is the agent's saving and $\tilde{R}_{t+1}$ is the random gross rate of return the agent faces.

The firm attracts savings by issuing equities, or equivalently, by offering a cumulative probability distribution of rates of return, $\mathcal{R}(\cdot)$. The firm generates the distribution $\mathcal{R}(\cdot)$ by searching for an acceptable production technology and, when one is found, utilizing it with its remaining investment capital. The firm's profits are then distributed to share holders as returns on their savings. Here, we assume that the firm can, at each period t, adopt the technology that was employed at t-1, θ_{t-1}, at no (search) cost.

Specifically, denote the underlying technology of the firm by $\tilde{\theta}F(k,\ell)$, $F(\cdot,\cdot)$ strictly concave, twice differentiable and increasing and homogeneous of degree one in both arguments. At the beginning of the period, the firm views the wage rate it will ultimately have to pay its workers, when it will decide on its optimal employment level, as a random variable, $\tilde{w}$, with an exogenously known cdf $\mathcal{W}(\cdot)$. The firm has a given initial amount of investment capital, Q, and has to search for an acceptable productivity rate, θ, (which might be θ_{t-1}), so as to maximize the expected utility of its share owners. When an acceptable project is found, the firm decides on optimal labor input, $\ell^*(\theta,k,w)$, where k is the residual investment capital net of search costs and w is the realized wage rate. It will then produce the profits

$$\pi(k,\theta \mid w) = \theta F(k,\ell^*(\theta,k,w)) - w\ell^*(\theta,k,w),$$

which, by the constant returns to scale assumption on F, is a linear function of k. The return on the firm's initial capital is then given by $R = \pi(k,\theta \mid w)/Q$. Note that R will be random at the beginning of the period, being a function of both k and θ -whose final realizations depend (stochastically) on the firm's strategy- and on w, which will be determined by the aggregate demand for labor in the economy, thus potentially reflecting the search results of all the firms in the market. In choosing its own search strategy and labor hiring, the firm treats its own initial investment capital Q as given. However, with only one firm, Q is given by Ns, where N denotes the number of savers, and s is the per capita savings. Therefore, when the firm chooses an optimal search strategy which maximizes the expected utility of a representative saver, $u(\tilde{R}s)$, in effect it maximizes the expected utility from the per-saver profit, $\pi(k,\theta \mid w)/N$. We also have assumed that although there is only one firm in the industry, it treats the probability distribution of the ensuing wage rate as independent of its own search behavior, (see footnote 3 below).

Thus, the optimal search policy of a firm which has access to productivity θ, (which might be θ_{t-1} or a realization drawn during the current search), and owns k units of capital must satisfy the following functional equation:

$$(3.4) \qquad V(k,\theta) = \text{Max}\left\{E\ \pi(k,\theta,\tilde{w})/N\ ,\ E\ \text{Max}\{\ V(k\text{-}\alpha,\theta),V(k\text{-}\alpha,\tilde{\theta})\ \}\right\}^4$$

In equilibrium, the realization of the wage rate w must satisfy

$$(3.5) \qquad \ell^*(\theta,k,w) = N$$

where k and θ are, respectively, the final investment capital and the productivity level of the project that were accepted by the searching firm. Thus, w, k and θ will all be realizations of random variables, whose distributions are interdependent: given the distribution on w, the search strategy determines the joint distribution of k and θ, which, in turn, determines via (3.5) the distribution of w.

The intertemporal problem involves another (related) fixed point argument. The

[4]In formulating the firm's optimal search strategy in (3.4) we have chosen to ignore the impact of that strategy on the wage probability distribution. This assumption is warranted in an environment with a large number of competing firms, but may be questioned under our assumption of a single firm in the market. Pursuing a fixed-number-of-firms version, with labor market clearing wage rate, will generate an equilibrium growth path with essentially the same dynamics and similar stochastic structure to the 1-firm case. The main difference being that with many firms conducting *statistically independent* searches from the same distribution of productivity rates - the variance of the resulting output, (and possibly of growth rates too), will be lower. However, this just begs the question of the evolution of the market structure along the growth path, bringing into the picture additional considerations such as the desired diversification in agents portfolios. These issues are left for future work.

Alternatively, we could model the single firm at a point in time as facing a regular upwards sloping labor supply curve. The firm will then take into account the impact of its search strategy on the monopoly wage it will have to pay, thus considerably complicating the characterization of the optimal search strategy without, as we see it, comparable gain in the implications of the model.

distribution of the rate of return $\mathcal{R}(\cdot)$ determines (in general) the saving of the young, s. That amount, in turn, determines the initial amount of capital the firm has, thus affecting its search strategy and hence, the distribution of k and θ (and of w). In equilibrium, the rate of return distribution $\mathcal{R}(\cdot)$, which is taken as given by the young, should match the one which is generated by the firm's behavior.However, as noted above, this fixed point is determined jointly with the equilibrium relationships governing the behavior of k, θ, and w.

Finally, note that equations (3.4) and (2.2) agree with each other when $\varphi(k,\theta) = E\ u[\pi(k,\theta,\tilde{w})/N]$. As assumed in the analysis of the search problem in section 2, $\varphi(\cdot,\cdot)$ is strictly monotone, and is a concave function of k whenever $u(\cdot)$ is a concave function, by the linearity of $\pi(k,\cdot,|\cdot)$. In the next section we describe an environment in which $\varphi(\cdot,\cdot)$ can be further decomposed in accordance with (2.6).

4. AN EXAMPLE

In this section we provide an example which considerably simplifies the fixed point problems discussed above, and displays the sense in which costly searching for a productive technology from a distribution with an unbounded support can give rise to sustained growth. First we show that if the objective function takes a multiplicative form of a particular type, the optimal strategy is independent of the distribution of the wage rate, so that that distribution can be calculated after the optimal search strategy has been determined. Then we establish the fact that we can also sever the link between the distribution of the rate of return and the saving behavior, while maintaining the structure that unlinks the optimal search strategy from the distribution of the wage rate. Finally we calculate some lower bounds on the expected rate of growth of the economy, and show that for a particular class of probability distributions on productivity rates - the economy may have a sustained growth path (in expectation).

Claim 2: Suppose that $u[\pi(k,\theta \mid w)/N]$ can be written as $a(N)h(w)\xi(k,\theta)$. Then, if the random variable $\tilde{w}$ is treated as though it is independent of the random variables $\tilde{k}$ and $\tilde{\theta}$, the optimal strategy which satisfies (3.4) depends only on $\xi(k,\theta)$, and is independent of the distribution of w.

Proof: The objective of the search is to maximize the expected utility of the representative owner:

$$(4.1) \qquad E\ u[\pi(\tilde{k},\tilde{\theta},\tilde{w})/N] = a(N)\cdot E \left\{ h(\tilde{w})\cdot\xi(\tilde{k},\tilde{\theta}) \right\} = a(N)\cdot E\ h(\tilde{w})\cdot E\ \xi(\tilde{k},\tilde{\theta}),$$

where the last equality follows from the independence assumption in the claim. Let $\varphi(k,\theta)$ be given by $a(N)\cdot E\ h(\tilde{w})\cdot\xi(k,\theta)$ and substitute this specification of $\varphi(\cdot,\cdot)$ into (2.4'), to get:

$$(4.2) \qquad a(N)\cdot E\ h(\tilde{w})\cdot\xi(k,\theta^*(k)) = H(\theta^*(k))\cdot a(N)\cdot E\ h(\tilde{w})\cdot\xi(k-\alpha,\theta^*(k)) +$$

$$+ \int_{\theta^*(k)}^{\bar{\theta}} a(N)\cdot E\ h(\tilde{w})\cdot\xi(k-\alpha,\theta')\cdot dH(\theta').$$

Since the factor $a(N)\cdot E\ h(\tilde{w})$ cancels out, the function $\theta^*(\cdot)$ which solves

(4.2) for any k is independent of the distribution of w. ■

Remark: Obviously, the result of Claim 2 holds also for the case in which $u[\pi(k,\theta|w)/N)$ is *additively* separable in N, w, and (k,θ).

Suppose now that the production function is Cobb-Douglas, so that

$$F(k,\ell) = Ak^{\gamma}\ell^{(1-\gamma)}, \quad A > 0, \quad 0 < \gamma < 1. \tag{4.3}$$

Then we obtain that given a wage rate w and productivity θ, the profit corresponding to optimal labor hiring is given by:

$$\pi(k,\theta|w) = A\gamma(1-\gamma)^{(1-\gamma)/\gamma}\theta^{1/\gamma}(1/w)^{(1-\gamma)/\gamma}k. \tag{4.4}$$

Suppose in addition that the utility function $u(\cdot)$ is CRRA, so that $u(c) =(1/\delta)c^{\delta}$, $\delta \le 1$. Then $u[\pi(k,\theta|w)/N]$ clearly satisfies the conditions of the claim, so that if the firm takes the distribution of w as given, its optimal search strategy is independent of that distribution and of N. Hence the equilibrium distribution of the wage rate is induced by the optimal search, but does not affect it.

If, in addition, we assume that preferences are logarithmic in both periods, the saving behavior is independent of the distribution of the rates of return, (as long as agents have no independent source of second period income). Thus, the beginning-of-next-period capital stock is unaffected by the search strategy to be employed in that period, and depends only on the current wage rate the young obtain. As the logarithmic utility function satisfies CRRA as well as the additive separable version of Claim 2 - we have eliminated all the intertemporal fixed point links between the search strategy and the wage and rates of return distributions.

For the remainder of this section we assume that the production function is Cobb-Douglas, and preferences are logarithmic. For this case, we obtain

$$(4.5) \qquad \xi(k,\theta)) = \ln(\theta^{1/\gamma}k).$$

We can now establish the relationship between the objective of the firm and output. In particular, we show that what the firm is maximizing is related to output in a way that places a lower bound on expected output in terms of the threshold productivity associated with the optimal search strategy.

Consider a firm which starts the search at period t with Q units of capital, and has free (of search cost) access to θ_{t-1} at each point during the search. The sequential search within the period, however, is done *without recall.* That is, the firm cannot go back to any previously sampled but rejected project, but can choose θ_{t-1} at any point. Proceeding along the optimal search strategy will generate a value function which satisfies:

$$(4.6) \qquad V(k,\theta) = \text{Max}\left\{\varphi(k,\theta)\ ,\ \text{E Max}\{V(k-\alpha,\theta_{t-1})\ ,\ V(k-\alpha,\tilde{\theta})\}\right\}$$

where k is the remaining investment capital and θ is the largest of the most recently sampled productivity and θ_{t-1}.

The optimal search strategy that attains the solution to (4.6), σ^*, can be characterized again by a threshold productivity function, $\theta^*(k,\theta_{t-1})$, such that the search continues as long as $\theta^*(k,\theta_{t-1}) > \mathrm{Max}\{\ \theta,\theta_{t-1}\ \}$, and stops the first time the last inequality is reversed. This version of the search environment is a convex combination of the versions described in section 2: it has some recall feature, captured by θ_{t-1} being always a fall back option, but it does not allow the firm to return to any project rejected *during* the current search. Obviously, the ability to choose θ_{t-1} at any point in time will work like a shift factor, raising the entire θ^* schedule above the schedule associated with the no-recall search. Since the search strategy corresponding to no-recall is a feasible strategy, it follows from (2.5) that the value of the search problem with recall across periods, starting with capital stock of Q, and no prior productivity to fall back on, satisfies:

$$E\ \xi(\tilde{k}_{\sigma^*},\tilde{\theta}_{\sigma^*}) \geq \xi(Q,\theta^*(Q)),$$

where $\theta^*(\cdot)$ characterizes the optimal search strategy for the *no-recall* case.For the log utility and Cobb Douglas production function, we get

$$\xi(k,\theta) = \ln\ \{\theta^{(1/\gamma)}k\},$$

so that when searching optimally, we have:

$$(4.7)\qquad E\ \xi(\tilde{k}_{\sigma^*},\tilde{\theta}_{\sigma^*}) = E\ \ln\ (\tilde{\theta}_{\sigma^*}{}^{(1/\gamma)}.\tilde{k}_{\sigma^*}) \geq \ln\ ([\theta^*(Q)]^{(1/\gamma)}.Q).$$

Actual output resulting from the optimal search, denoted $\tilde{y}_{\sigma^*}$, is given by:

(4.8) $\tilde{y}_{\sigma*} = \tilde{\theta}_{\sigma*} \cdot \tilde{k}_{\sigma*}{}^{\gamma} \cdot N^{(1-\gamma)}$.

Therefore,

$$(4.9) \quad E\{\tilde{y}_{\sigma*}\} = E\{\tilde{\theta}_{\sigma*}\tilde{k}_{\sigma*}{}^{\gamma}N^{(1-\gamma)}\} = N^{(1-\gamma)} \cdot E\{(\tilde{\theta}_{\sigma*}{}^{1/\gamma} \cdot \tilde{k}_{\sigma*})^{\gamma}\} =$$

$$= N^{(1-\gamma)} \cdot E\left\{\left[e^{\ln(\tilde{\theta}_{\sigma*}^{1/\gamma} \cdot \tilde{k}_{\sigma*})}\right]^{\gamma}\right\} >$$

$$> N^{(1-\gamma)} \cdot \left[e^{E\{\ln(\tilde{\theta}_{\sigma*}^{1/\gamma} \cdot \tilde{k}_{\sigma*})\}}\right]^{\gamma} \geq$$

$$\geq N^{(1-\gamma)} \cdot \left[e^{\ln([\theta^{*}(Q)]^{1/\gamma} \cdot Q)}\right]^{\gamma} = N^{(1-\gamma)} \cdot \theta^{*}(Q) \cdot Q^{\gamma}$$

where the first inequality in (4.9) follows from Jensen Inequality, and the second from (4.7). Thus we have established a lower bound on the expected output (conditional on Q) in terms of Q and $\theta^{*}(Q)$. It is evident from (4.9) that in order for the lower bound on output, given beginning of period capital stock Q, to be a convex function of Q, $\theta^{*}(Q)$ has to increase at least as fast as $Q^{1-\gamma}$. As we show next, the convexity of the lower bound on output in beginning of period capital implies sustained growth in the dynamic model.

Consider the dynamics of savings in the logarithmic utility case, where saving rates do not depend on future returns. Specifically, let aggregate saving S_{t+1} be given by

(4.10) $S_{t+1} = \beta(1-\gamma)y_t$

where β is the saving rate, y_t is the realized output at period t, and $(1-\gamma)$ is the labor share of output. Letting beginning of period t+1 capital stock be given by S_{t+1}, and using (4.9), we get a lower bound on the *expected conditional rate of growth*:

$$(4.11) \qquad E\left\{ \frac{\tilde{y}_{t+1}}{y_t} \,\middle|\, y_t \right\} \geq \theta^*(\beta(1-\gamma)y_t)(\beta(1-\gamma)y_t)^{\gamma}N^{(1-\gamma)}/y_t - 1.$$

Next we show that for an appropriate choice of the pdf of θ, the expected output can be bounded below by a convex function of Q, and hence (4.11) will imply that the economy may grow at a sustained rate in expectation. Let the cdf $H(\cdot)$ of θ be the Pareto distribution given by:

$$(4.12) \qquad H(\theta) = 1 - \theta^{-\lambda}, \qquad \theta \geq 1, \ \ \lambda > 0,$$

implying $E(\theta) = 1/\lambda$. Then, equation (2.4) becomes:

$$(4.13) \qquad \ln(\theta^*(k)) = \gamma\ln[(k-\alpha)/k] + [1-\theta^*(k-\alpha)]\ln[\theta^*(k-\alpha)] + \int_{\theta^*(k-\alpha)}^{\bar{\theta}} \ln(\theta)\, \lambda\theta^{-(\lambda+1)} d\theta.$$

We approximate the solution to (4.13) by the solution to the problem of search with recall, in which $\theta^*(k-\alpha)$ on the RHS is replaced by $\theta^*(k)$, thus avoiding the recursive nature of the problem. It can be shown that the approximation improves as k increases. This implies:

$$(4.14)\qquad [\theta^*(k)]^{-\lambda}\cdot\ln[\theta^*(k)] = \gamma\ln[(k-\alpha)/k] +$$

$$+ [\theta^*(k)]^{-\lambda}[\ln[\theta^*(k) + 1/\lambda],$$

and the solution of (4.14) for θ(k) satisfies:

$$(4.15)\qquad [\theta^*(k)]^{-\lambda} = -\lambda\gamma\ln(1-\alpha/k).$$

For sufficiently large values of k relative to α, we use the approximation $\ln(1-\alpha/k) \cong -\alpha/k$, so that (4.15) implies

$$(4.16)\qquad \theta^*(k) \cong (k/\alpha\gamma\lambda)^{1/\lambda}.$$

Accordingly, we get from (4.11):

$$(4.17)\qquad E\{\frac{\tilde{y}_{t+1}}{y_t}\ |y_t\} > \Lambda\cdot N^{(1-\gamma)}\cdot {y_t}^{\gamma+(1/\lambda)-1}$$

where Λ is a constant which depends on α, β, γ and λ. Thus, a sufficient condition for the economy to have a sustained growth path (in expectation) is $\gamma+1/\lambda > 1$.

Notice, however, that the probability distribution under consideration in this example has no more than the first λ moments. Larger λ implies the existence of additional moments, a faster declining upper tail of the density function, and a lower expected value for θ. Consequently, larger λ must be matched by a larger capital share of output, γ, to satisfy the aforementioned sufficient condition for sustained expected growth. For instance, when $\lambda=2$, (so that both $E(\theta)$ and $Var(\theta)$ exist), γ has to exceed 0.5 to meet that sufficient condition.

The stochastic behavior of this economy may display several patterns. In particular, the economy may have the property that it can reach an absorbing state in which it "collapses", while other configurations of the parameters imply that the economy can never collapse. Specifically, let $\hat{k}$ be the smallest sustainable capital stock, assuming that each period at least one sample has to be taken, at a cost α. Then, if such $\hat{k}$ exists, it is the smallest root of:

$$(4.18) \qquad \hat{k} + \alpha = (1-\gamma)\cdot\beta\cdot\underline{\theta}\cdot F(\hat{k},N),$$

and $k_t \geq \hat{k}$ implies $k_{t+1} \geq k_t$, for all possible θ_t. Next, consider the highest stock of capital at which the optimal search strategy still implies that *any* productivity rate is acceptable, $\underline{k} = \text{Sup}\ \{k \mid \theta(k) = \underline{\theta}\}$.

Suppose first that $\hat{k} < \underline{k}$. Then, if the economy starts with any $Q > \underline{k} + \alpha$, the worst that can happen is that the search program depletes capital to $\underline{k}$, which will be utilized with $\underline{\theta}$. But even then, according to (4.18), next period capital stock will again exceed $\underline{k}$ by at least α, so that the economy will never have a capital stock below $\underline{k}+\alpha$, if it ever exceeds this level. And if sufficiently high value of θ is drawn, the economy may start growing.

On the other hand, if $\hat{k} > \underline{k}$, the economy may collapse. Even if the economy starts with an amount of capital which exceeds $\hat{k}$, the search process may deplete capital to a level k_t which is below $\hat{k}$, and draw a sufficiently low value of θ so that $k_{t+1} < k_t \leq \hat{k}$. In this case, the economy may collapse totally, in the sense that there will not be enough capital for even one draw. Clearly, if $\hat{k}$ does not exist, in that the sense that $k + \alpha \geq (1-\gamma)\beta\underline{\theta}F(k,N)$ for all $k \geq 0$, then the probability of collapse is always positive.

Finally, we describe how the option of adopting the previous period utilized technology without investing in search affects the resulting growth path. Suppose that at some period t, a relatively high θ is found and adopted, and denote it by θ_0. It then follows that $\theta_0 \geq \theta^*(k_t,\theta_{t-1})$. By virtue of the fall back option, $\theta^*(\cdot,\theta_{t-1}) \geq \theta^*(\cdot)$ for any θ_{t-1}, where $\theta^*(\cdot)$ is the threshold productivity for the no-recall case. Using (4.16) we then have:

$$\theta_0 \geq k_t^{1/\lambda}(\alpha\gamma\lambda)^{-1/\lambda}.$$

Likewise, for all subsequent periods t+s in which θ_0 is adopted without search, $\theta_0 \geq k_{t+s}^{1/\lambda}(\alpha\gamma\lambda)^{-1/\lambda}$, so that as long as no new search is undertaken the capital stock evolves according to:

$$k_{t+s+1} = \beta(1-\gamma)\theta_0 k_{t+s}^{\gamma} N^{(1-\gamma)} \geq \beta(1-\gamma)N^{(1-\gamma)}k_{t+s}^{\gamma+(1/\lambda)}.$$

It follows, that if $\gamma + (1/\lambda) > 1$, the capital stock will grow at a positive rate bounded away from zero, and will eventually reach a sufficiently high level, say K for which $\theta^*(K,\theta_0) > \theta_0$. At that period search will resume. The growth path, for such an economy, will have spells of no search, during which a previously discovered technology is adopted by successive generations. Eventually, search will resume, but only at a low level, so that the previous technology might still be the one adopted, and the capital stock will continue to grow at the previous rate. As search efforts will intensify, the likelihood of a "break through" productivity will increase, and when one occurs, the economy will enter the next spell of no search, which will be longer, but finite nonetheless. Notice that the condition that suffices for this pattern of growth, $\gamma + (1/\lambda) > 1$, is the same condition that implies sustained asymptotic growth, depending only on the productivity of search, and the capital elasticity of output

5. CONCLUSION

The framework developed in this paper was chosen not so much for its realistic features, as for its interesting behavior concerning the returns to scale. These returns emerge as a combination of the returns to sequential search for investment opportunities, and a standard decreasing returns production technology. It is instructive to examine the impact of relaxing some assumptions that were used in our model.

In particular, it is obvious that higher degree of risk aversion will reduce the selectivity of the optimal search, and hence, will make sustained growth harder to obtain. The assumptions on the transferability of technological know how over time also have important implications. Our results demonstrate that even in the case where each period firms start the search without the option to fall back on previously discovered technologies - the growth process in per capita terms may be sustained at positive rates. The specification "with recall between periods" has the additional attractive feature in that it allows for spells of "no search" following a particular "good" discovery by any generation. It would also make the possibility of a negative growth spell, and ultimate total collapse of the economy, less likely.

On the other hand, by abstracting from the structure of investment intermediation, we have also affected the stochastic growth path in a particular way. Consider a competitive intermediation sector, a typical firm in which conducts search for investment projects each period, financed by savings which it manages to attract.Such multi firm sector might exist since risk averse agents will take the opportunity to split their savings among firms that offer identically distributed returns. However, splitting the search between many investment firms will reduce the initial capital of each firm, and will result in a dominated distribution of rates of return on savings, (optimal search induces a rate of return distribution which is ordered by first degree stochastic dominance according to the initial capital). Accordingly, allowing for endogenously determined intermediation structure will likely reduce the conditional first *two* moments of growth rates, in addition to having implications on such things as the length and severity of downturns along the growth path.

The simplicity of the model described in this work allows one to simulate the growth path for variety of specifications, and calculate simulation moments of growth rates, search cost shares of total output, probabilities of collapse, duration and intensity of periods with negative growth, etc. These simulation results will be used to confront observations like the positive (and significant) correlation between mean growth rates over time and mean standard deviations of growth rates exhibited by industrialized countries over long periods, both before and after the two World Wars. Moreover, since we get that sustained (expected) growth rates depend on the level of employment, in addition to the level of capital stock, we may have another way to test the model's implications.

Moreover, as we noted in the introduction, this search theoretic framework accords well with Pakes and Schankerman (1984) observations on the differences between individual firms data, and sector aggregates. If each firm within a sector conducts independent search from its own sector-specific distribution of technologies, and individual firm's search outcomes are highly random, then individual firms data would display low correlation between firm's search costs and output, both at levels and in growth rates. When aggregating over firms in a given sector, however, individually random search outcomes would wash out, leaving the positive effect of "better" sector distributions on sector aggregate search investment.

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