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Dynamic Consistency and Reference Points

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Dynamic Consistency and Reference Points*

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Abstract

This note aims to answer the following question. Is it possible to define a set of preference relations, one for each node of a decision tree, such that these preferences satisfy the reduction of compound lotteries axiom, they are dynamically consistent in the sense that if the decision maker plans to use a certain strategy at a future node N , then once he reaches this node, his current preferences will be to use the planned strategy, and such that they do not converge to expected utility. The key idea is that as uncertainty unfolds, preferences evolve so that the indifference curve through the planned choice at each choice node agrees with the induced preferences from the past. In particular, It follows that this indifference curve is affine. It is argued that the updated preferences are relevant whenever the decision maker has to depart from his original plan. The quadratic model is consistent with one affine indifference curve, thus admitting a natural concept of reference points. It implies randomization aversion for lotteries that are worse than the planned holding X^* but randomization seeking for lotteries that are better than X^* .

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1 Introduction

One normative appeal of expected utility theory is based on the notion that if decision makers have preferences that violate the independence axiom, then they can be manipulated into accepting a sequence of trades that leave them in a position which is stochastically dominated by their initial position. Such a manipulation is called a Dutch book.

Suppose that a decision maker prefers X to Y but $pY + (1 - p)Z$ to $pX + (1 - p)Z$. He begins holding the compound lottery $(X, p; Z, 1 - p)$.¹ For a small $\varepsilon > 0$ he is willing to exchange it for the lottery $(Y, p; Z - \varepsilon, 1 - p)$. At the second stage of the compound lottery, in case the Z event occurs, he has lost ε . If the Y event occurs, the decision maker (who prefers X to Y) is willing to trade Y for $X - \varepsilon$. Thus ex post the decision maker ends up trading $(X, p; Z, 1 - p)$ for $(X - \varepsilon, p; Z - \varepsilon, 1 - p)$, something he should wish to avoid (see Green [10] and Machina [17]).

This manipulation does not force one to obey the expected utility hypothesis, unless one makes the additional assumption that at the second stage of the lottery the decision maker still prefers X to Y . It seems that the essence of avoiding this Dutch book lies in dynamic consistency, implying that preferences change in a fashion consistent with the original plan. This position is advocated by Machina [17] and McClennen [18, 19], suggesting that updated preferences agree with the induced order from past preferences.

Border and Segal [2] show that as uncertainty unfolds, this updating procedure implies that preferences must converge to expected utility. The question the present paper aims to answer is therefore this. Is it possible to define a set of preference relations, one for each node of the decision tree, satisfying the following requirements. (1) Each of them is continuous, monotonic, and transitive, and satisfies the reduction of compound lotteries axiom; (2) The preferences are dynamically consistent in the sense that if the decision maker plans to use a certain strategy at a future node N , then once he reaches this node, his current preferences will be to use the planned strategy; and (3) The preference relations do not converge to expected utility.

This paper offers conditions for such preferences. The key idea is that as

¹Assume that preferences satisfy the reduction of compound lotteries axiom. By this axiom, the decision maker is interested only in the probability of reaching a certain outcome and not in the probabilities of the sequence of stages leading to it.

uncertainty unfolds, preferences evolve so that the indifference curve through the planned choice at each choice node, *and only this indifference curve*, agrees with the induced preferences from the past. In particular, for reasons that are explained in Section 3, It follows that this indifference curve is affine, that is, if X and Y are in this indifference curve then so is $\alpha X + (1 - \alpha)Y$ for all $\alpha \in [0, 1]$. Several non-expected utility theories are consistent with one affine indifference curve, but they may take different shapes from those discussed in the literature. Some may imply a natural concepts of reference points. For example, the quadratic model [5, 6] implies randomization aversion for lotteries that are worse than the planned holding X^* but randomization seeking for lotteries that are better than X^* .

The paper is organized as follows. Section 2 introduces the framework for defining consistency conditions. Section 3 presents results on limits of preferences and discusses the notion of unplanned situations. Section 4 describes the behavior of some special cases of non-expected utility preferences in this framework. Section 5 concludes with a discussion of reference points.

2 Consistent Optimization

Denote the set of lotteries over the interval $[0, M]$ by L . Let \succeq be a complete and transitive preference relation on L . A utility representing \succeq is a function $V: L \rightarrow \mathbb{R}$ such that $X \succeq Y$ iff $V(X) \geq V(Y)$. If V is differentiable, then the derivative of V at Z acts as a continuous linear functional on L , which means that it has a representation in terms of a continuous real function on $[0, M]$. This real function is called the local utility of V at Z , and is denoted $U(\cdot; Z)$. The preferences \succeq can be locally approximated around Z by the expected utility functional using the vN-M utility function $U(\cdot; Z)$ (see Machina [15]).

Decision trees have three kinds of nodes. Terminal nodes specify a single outcome. At a chance node, Nature chooses a branch according to a known probability distribution. At a choice node N , the decision maker is given a choice set L_N of branches, each leading to another node, which may be any of the three kinds. The root node of the tree is denoted N_0 .

The decision maker's initial preferences over L are represented by a utility functional V_0 . At N_0 he makes a plan π of the choices he will make at each choice node of the tree. In general, the decision maker may use a randomized plan and π for node N may be a probability measure on L_N . For each plan

π the decision maker computes its reduced form $Z_0(\pi)$ by computing the probability of reaching each terminal node (this is called the reduction of compound lotteries axiom). Since the decision maker may randomize, it follows that the set of reduced form lotteries is a convex subset of L . He then chooses the plan π_0 whose reduced form $Z_0(\pi_0)$ maximizes V_0 .

Once a decision node is reached the decision maker is free to reevaluate the plan. At choice node N the decision maker uses the utility V_N to evaluate the subplans for the subtrees originating at node N . Again he computes the reduced form Z_N of each subplan at N , and chooses a subplan π_N that maximizes $V_N(Z_N)$. Dynamic consistency can now be defined as follows.

Axiom 1 *If node N' comes after node N , then the optimal choice at N' under $V_{N'}$ was also optimal for N' under π_N .*

If we require that for every choice node N , V_N and V_0 represent the same preference ordering over lotteries, then together with Axiom 1 it implies that V_0 is an expected utility functional (see, for example, Green [10] or Karni and Schmeidler [13]). Machina [17] has argued that the experience of undergoing the risk at earlier nodes may well change the decision maker's utility. The potential problem that can arise from non-expected utility preferences that change when moving down a tree is that the decision maker may "make a book against himself." To solve this problem, Machina offered the following axiom (see also McClennen [18, 19]).

Axiom 2 *Suppose that the V_N -optimal subplan π_N leads to node N' with probability p and to a reduced-form lottery Z_N with probability $1 - p$. Then, as a function of X , $V_{N'}(X)$ and $V_N(pX + (1 - p)Z_N)$ are ordinally equivalent.*

This condition says that at node N' , the preference relation will be the one induced by the preference relation the decision maker used at node N leading to N' . This axiom puts no restrictions on V_0 . However, once V_0 is specified, so are all the functions V_N (up to equivalence). Nevertheless, Border and Segal [2] show that for every differentiable V_0 , as the decision maker moves down the tree, his preference will converge to expected utility.

Although Axiom 1 is sufficient to prevent the Dutch book of the Introduction, it is not restrictive enough to imply any interesting models. I therefore suggest a stronger axiom that is still weaker than Axiom 2.

Axiom 3 *Let the V_N -optimal subplan π_N lead to N' with probability p and to a reduced-form lottery Z_N with probability $1 - p$. Moreover, let X^* be the reduced form lottery resulting from the optimal plan π_N for node N' . Then $\forall X \in L$, $V_{N'}(X^*) \geq V_{N'}(X)$ iff $V_N(pX^* + (1 - p)Z_N) \geq V_N(pX + (1 - p)Z_N)$.*

In other words, if X^* is the π_N -planned optimal choice for decision node N' , then the preference relation between X^* and any other lottery X at N' is the one induced from the preference relation the decision maker had at node N leading to N' . This should hold for all $X \in L$, and not just for $\{X : pX + (1 - p)Z_N \in L_N\}$, as is required by Axiom 1. According to Axiom 3, the utility function V_N determines one indifference curve of $V_{N'}$, the one going through X^* . Note that this is also an indifference curve of $V_{N'}$ according to Axiom 2. Like Axiom 2, the preference relation at node N' depends on the tree. Indeed, changing the outcomes at the terminal nodes may change X^* , and hence it may change $V_{N'}$. To that extent, Axiom 3 agrees with Machina's Axiom 2, that the preferences at each node depend on the tree itself, and the notion of dynamic consistency is with respect to a given choice set. I consider this to be an advantage of the model, as it permits a natural notion of reference points, which must of course be with respect to a given choice set (see below).

3 Zero Probability Events

The analysis of the last section, based upon $V_N(pX + (1 - p)Z)$, is not especially useful when $p = 0$. If the probability of reaching node N' is zero, then under Axioms 2 and 3, $V_{N'}$ is indifferent among all possible lotteries, because ex ante, V_0 (and (V_N)) are indifferent between them. But zero probability events are relevant, because if the original lottery at node N_0 is continuous, then *all* branches of the tree occur with probability zero.

Problems arise even if all outcomes are to be received with positive probabilities as it is somewhat arbitrary to define the risk to which the decision maker has been exposed. Suppose that at N , the decision maker had faced the (optimal) lottery $(X, p; X, q; Z, 1 - p - q)$. At the next stage he holds a ticket for lottery X . What risk did he bear? Is it $(\cdot; Z, 1 - p - q)$, $(\cdot; X, q; Z, 1 - p - q)$, $(X, p; \cdot; Z, 1 - p - q)$, or maybe something else? Border and Segal [2] therefore suggest considering all lotteries as being continuous, so

whatever lottery the decision maker holds, he behaves as though he reached it with probability zero. To solve the problem of updating preferences with respect to zero-probability events, they suggest to take the limit of the induced preferences of Axiom 2, as the probability goes to zero. I adopt a similar solution here to extend Axiom 3 to the case $p = 0$.

Axiom 4 (Zero-Probability Consistency) *Let the V_N -optimal subplan π_N lead to node N' with probability $p = 0$. Also, let X^* be the V_N -maximal choice at N' and let Z_N be the reduced form of the optimal plan π_N for node N . Then $V_{N'}(X^*) \geq V_{N'}(X)$ if and only if there exists $p^* > 0$ such that for every $p \in (0, p^*)$, $V_N(pX^* + (1 - p)Z_N) \geq V_N(pX + (1 - p)Z_N)$.*

Note that since N' is reached with probability zero, Z_N does not depend on X^* , and is the same regardless of what the decision maker receives at N' .

Axioms 4 and 3 only restrict the preference relation between X^* and other lotteries, but they differ in what they assume to be known to the decision maker about past uncertainty. Although Axiom 3 may seem to be more natural, the above discussion shows that the probability p in its definition may not be well defined. Axiom 4 therefore seems to be the best way to capture the intuitive appeal of Axiom 3. Similarly of [2, Theorem 4], we get:

Theorem 1 *Let Z_0 be the reduced form of the V_0 -optimal plan π_0 for the tree. Assume that V_0 is differentiable at Z_0 and that $U_0(\cdot; Z_0)$, the local utility of V_0 at Z_0 , is strictly increasing. Let node N lead to N' , and let X^* be the V_N -best choice for N' . If the decision maker updates preferences according to Axiom 4, then the indifference curve of $V_{N'}$ through X^* is affine and is derived from the expected utility preferences with the utility function $U_0(\cdot; Z_0)$.*

Proof: Suppose node N leads to N' with probability p and to reduced form Z with probability $1-p$. Let $W_{N'}^p$ be the utility function defined by Axiom 2 with indifference curve $I_{N'}^p(X^*)$ through X^* . Let $\tilde{W}_{N'}^p$ satisfy Axiom 3. Obviously, $I_{N'}^p(X^*)$ is an indifference curve of $\tilde{W}_{N'}^p$, as well. By [2, Theorem 4], the preferences represented by $W_{N'}^p$ converge (as $p \rightarrow 0$) to an expected utility relation with the vN-M utility $U_0(\cdot; Z_0)$. In particular, $I_{N'}^p(X^*)$ converge to an affine set which is derived from these expected utility preferences. ■

Figure 1 explains the affinity of the indifference curve through X^* . By construction of the limiting preferences, $V_{N'}(X^*) > V_{N'}(Y)$ if there exists

$p^* > 0$ such that for every $p \in (0, p^*)$, $V_N(X^*, p; Z, 1-p) > V_N(Y, p; Z, 1-p)$. For this, the slope of the chord connecting $pX^* + (1-p)Z$ and $pY + (1-p)Z$ must be steeper than the slope at Z of the indifference curve of V_N . But in that case it follows by the same reasoning that for every $\alpha \in (0, 1)$, $V_{N'}(X^*) > V_{N'}(\alpha X^* + (1-\alpha)Y)$. By taking Y to a limit we get that if $V_{N'}(X^*) = V_{N'}(Y)$, then $\forall \alpha \in [0, 1]$, $V_{N'}(X^*) = V_{N'}(\alpha X^* + (1-\alpha)Y)$.

Theorem 1 implies that if all branches have zero probability, then every V_N -indifference curve through the contingent plan coincides with an indifference curve of the expected utility preferences with the utility function U_0 , which is the local utility of the functional V_0 evaluated at the optimal plan with reduced form Z_0 for the whole tree. Therefore the choices are exactly the same as those an expected utility maximizer would make. This seems to argue that all the above refinements of dynamic consistency are a waste of time, since decision makers must behave as if they maximize expected utility.

But note that a similar problem exists in all models of multi-stage decision making. Since the original choice set L_0 is convex, choosing any undominated lottery out of it can be supported by maximizing an expected utility functional V . As long as the decision maker commits to this plan in subsequent nodes, it is impossible to tell that he does not actually maximize, at each node, the same expected utility functional V . So the inability to distinguish between expected utility and non-expected utility maximizers has nothing to do with Axiom 4 and Theorem 1. But there is a difference between expected and non-expected utility maximizers, as is explained below.

A decision maker may use many different ways to model decision problems he faces. These models depend on his beliefs and on what he finds to be important and salient. For example, he may decide to ignore some of the possible nodes of the decision tree to make his choice problem simpler to handle. This may be due to computation cost that in many cases exceed the expected potential benefit from a better planning. (See Gul and Lantto [12] for a related discussion). Or, the decision maker may make a mistake in figuring out the exact form of the tree. Suppose he reaches node N and realizes that the choice set he faces differs from the one he planned for. Instead of L_N , he has to choose from L'_N . Machina [17] calls such nodes hidden nodes, and argues, correctly, that as long as the decision maker chooses an undominated lottery, whatever he does at such nodes is dynamically consistent. The question nevertheless remains, what should the decision maker do now? I claim that this is where the utility functional V_N matters. The decision

maker should now simply choose the option from L'_N that maximizes V_N . It is therefore precisely in the case of hidden nodes that we may be able to distinguish an expected utility maximizer from others.

Suppose, for example, that for node N , the decision maker planned for the choice set $\{(5, p; 1, 1 - p - q; 0, q) : p \leq 10q - 8.9\}$. His planned choice for this set according to V_0 was $(5, 0.1; 0, 0.9)$. In particular, he preferred this lottery to the lottery $(1, 0.11; 0, 0.89)$. Reaching node N , he realizes that the actual choice set is $\{(5, p; 1, 1 - p - q; 0, q) : p \leq \max\{10q, 0.1\}\}$. Being an expected utility maximizer, he will now have to choose $(5, 0.1; 1, 0.89; 0, 0.01)$. However, if V_N is not an expected utility functional, he may now well choose the sure gain of 1, as is predicted by the Allais paradox.

Does non-expected utility offer any interesting models that retain enough structure to be useful, and yet differentiate itself from expected utility? The next section discusses this question.

4 Some Functional Forms

According to Axioms 2-4, the utility functional V_N over lotteries depends on the specific node of the tree. One can, nevertheless, impose some restrictions on all these preferences. For example, we may wish to require that at each node, the function V_N satisfies some normative assumptions like the independence axiom or betweenness ($X \sim Y \Rightarrow \forall \alpha \in [0, 1], X \sim \alpha X + (1 - \alpha)Y$, see [4], [9], [7]). Since Theorem 1 implies one affine indifference curve, it is obvious that betweenness-satisfying models are consistent with Axiom 4.

Alternatively, suppose that at all nodes the decision maker's utility is quadratic, that is, $V_N(F) = \int \int \varphi_N(x, y) dF(x) dF(y)$ for some symmetric and monotonic function φ_N , although φ_N may vary from one node to another. The quadratic representation is implied by several sets of axioms (Chew, Epstein, and Segal [5, 6]), which have some normative appeal. Moreover, these axioms are meaningful for each node independently of the relations at other nodes. It turns out that under Axiom 4, the quadratic model has strong behavioral implications.

A quadratic functional has the following properties. If it is not ordinally equivalent to expected utility, and has one affine indifference curve, then all indifference curves above it are quasiconcave ($X \sim Y \Rightarrow \alpha X + (1 - \alpha)Y \succ X$, $\forall \alpha \in (0, 1)$) and all indifference curves below it are quasiconvex ($X \sim Y \Rightarrow$

$X \succ \alpha X + (1 - \alpha)Y, \forall \alpha \in (0, 1)$) [5, 6]. According to the above analysis, the indifference curve through X_N^* , the decision maker's current holding, is affine. Therefore, the utility is quasiconcave above X_N^* and quasiconvex below. This is depicted in the left panel of Figure 2. Note that although one non-affine indifference curve of a quadratic functional determines the whole indifference map, many different quadratic functionals can share the same affine indifference curve.

Thus, the quadratic functional implies preferences for randomization in case the choice set turns out to be better than planned. However, if the choice set turns out to include only lotteries that are inferior to his planned holding, the decision maker will exhibit randomization aversion. Consider the following example. Let $X = (1000, 0.9; 0, 0.1)$, $Y = (7500, 0.3; 0, 0.7)$, and let $Z = \frac{2}{3}X + \frac{1}{3}Y = (7500, 0.1; 1000, 0.6; 0, 0.3)$. Suppose that the decision maker's plan was to choose the degenerate lottery $W = (500, 1)$, when he realizes that X and Y (and all lotteries of the form $\alpha X + (1 - \alpha)Y$) are also available. He is indifferent between X and Y , but prefers both to W . The quadratic model now predicts that Z is even better. One possible explanation is that Z combines the best of the two lotteries X and Y . It has a substantial probability of winning \$1000, and a positive (though small) probability of winning the high prize of \$7500. The probability of winning nothing is still substantially less than the probability of winning something.

Suppose, however, that the decision maker planned for the degenerate lottery $W' = (1500, 1)$, when he learns that this option is no longer available. Instead, he has to choose out of the set $\{\alpha X + (1 - \alpha)Y : \alpha \in [0, 1]\}$. According to the quadratic model, if he is indifferent between X and Y , then he is not going to choose Z . Unlike X , it has a significant probability of winning zero, and unlike Y , the probability of winning the high prize of \$7500 is very small. Certainly, a decision maker may adopt the optimistic approach when things turn up better than planned for, but the pessimistic viewpoint in case he is doing worse than planned.

An immediate generalization of the simple quadratic model is obtained by dividing the domain of V into three regions. The function V is quadratic on the lower and upper regions (but not necessarily with respect to the same function φ), and satisfies betweenness on the intermediate region (see [5]). This is depicted in the right panel of Figure 2. If in that case the current holding X_N^* is in the interior of the middle region, then around this point the decision maker behaves as though his preferences satisfy betweenness.

However, for sufficiently big changes, his preferences become quadratic.

Some models are inconsistent with the analysis of Theorem 1. The rank dependent model, first suggested by Quiggin [20], is given by $V(F) = \int u(x)df(F(x))$, where $f: [0, 1] \rightarrow [0, 1]$ is strictly increasing and onto and u is unique up to positive affine transformations. Suppose f is differentiable and that its derivative is positive. This functional is consistent with one non-trivial affine indifference curve if and only if it is reduced to expected utility, hence its strict form is inconsistent with our analysis. Formally:

Lemma 1 *Suppose that V is a rank dependent functional and that there exists $X \notin \{(0, 1), (M, 1)\}$ such that $I(X)$, the indifference curve through X , is affine. Then $f(p) = p$ for every $p \in [0, 1]$.*

Proof: Let x be the certainty equivalent of X . For $\alpha \in (0, \infty)$, let $a_\alpha < b_\alpha$ such that $V(a_\alpha, 1/(\alpha + 1); b_\alpha, \alpha/(\alpha + 1)) = V(x, 1)$. By the affinity of the indifference curve through $(x, 1)$ it follows that for every $p \in [0, 1/(\alpha + 1)]$, $V(x, 1) = V(a_\alpha, p; x, 1 - (\alpha + 1)p; b_\alpha, \alpha p)$. Transform the function u such that $u(x) = 0$ and $u(a_\alpha) = -1$, denote $u(b_\alpha) = B_\alpha$ and obtain by the last equation that $-f(p) + B_\alpha[1 - f(1 - \alpha p)] = 0$. Substitute $p = 1/(\alpha + 1)$ to obtain $B_\alpha = f(1/(\alpha + 1))/[1 - f(1/(\alpha + 1))]$. Therefore, $f(p) = [1 - f(1 - \alpha p)]f(1/(\alpha + 1))/[1 - f(1/(\alpha + 1))]$. Differentiate with respect to p and set $p = 1/(\alpha + 1)$ to obtain $1 = \alpha f(1/(\alpha + 1))/[1 - f(1/(\alpha + 1))]$. Hence $f(1/(\alpha + 1)) = 1/(\alpha + 1)$. ■

This result is parallel to that of Epstein and LeBreton [8], where it is shown that the rank dependent model is inconsistent with Machina's [17] notion of dynamic consistency. (In their analysis, preferences are not updated with respect to zero probability events, hence their proof cannot use the fact that the rank dependent model is inconsistent with affine indifference curves).

5 Reference with Respect to Utility

Standard expected utility, and the standard versions of most of its recent generalizations, deal with preferences over distributions over final level of wealth. So the lottery $(x_1, p_1; \dots; x_n, p_n)$ represents a p_i probability that the decision maker ends with wealth level x_i , $i = 1, \dots, n$. An alternative analysis suggests that lotteries represent changes from the current wealth level. If his

wealth is w and he faces the lottery $(x_1, p_1; \dots; x_n, p_n)$, then the decision maker faces the lottery $(w + x_1, p_1; \dots; w + x_n, p_n)$ over final wealth levels. So two individuals having the same utility function but different wealth levels, will have different preferences over distributions over gains and losses.

Taking this framework one step further, one can claim that decision makers react differently to positive and negative changes from their current wealth level. They may be risk averse with respect to gains and risk loving with respect to losses (Kahneman and Tversky [21, 22]). In that case, the current wealth level serves as a natural reference point. Even more interesting is the case where the lottery itself defines the decision maker's reference point. Such is Gul's [11] disappointment aversion theory. Another possibility is that past lotteries define the reference point (see Bowman, Minehart, and Rabin [3]). In all these cases, each possible outcome of the lottery is evaluated with respect to the reference point, which is by itself a monetary payoff.

Alternatively, one may compare the outcomes of a certain lottery to an alternative lottery that is available to the decision maker. To this group belong Fishburn's [9] non-transitive analysis and the different versions of regret theory (Bell [1] and Loomes and Sugden [14]). Here the reference point is a lottery, but it depends on the specific decision problem the decision maker faces. In these models, there is no unique reference lottery. Each lottery serves as a reference point for the other one.

The analysis of the present paper leads to preferences that depend on the decision maker's current holding, where this holding is a lottery. The reference point is not used to evaluate a single lottery, but to determine the whole preference relation. This reference point is not fixed (as is the case with cumulative prospect theory [22]), nor does it depend on the current choice set (as is the case with regret theory [1, 14] or disappointment aversion [11]). Rather, it depends on the resolution of past uncertainty. Note that in most other models of reference point, the same choice set has always the same best points, while in our analysis the optimal point depends on how the decision maker reached this choice set. (See Section 4 for a numerical example. See also [3]). Although the possibility of the reference point analysis does not depend on the assumption that the reference indifference curve is affine (this affinity follows from Axiom 4), it becomes nevertheless more elegant as some functionals take very specific forms given one affine indifference curve.

I end the paper with a brief discussion of the "weak consequentialism" axiom, suggested by Gul and Lantto [12]. Suppose that at node N_0 the

decision maker considers a certain available lottery X to be optimal. This does not mean that he must choose this lottery, as there may be other optimal lotteries. But suppose that later on in the tree he faces a subset of his original choice set and X is part of it. Weak consequentialism requires that at that point, the lottery X must still be optimal. Gul and Lantto show that this requirement is equivalent to the betweenness axiom.

To a certain extent, weak consequentialism is similar to Axiom 3, which requires that if at decision node N the lottery X is chosen over Y , then later on the decision maker will not choose Y over X . But this 'small' difference between preferences and choice is at the core of the present paper. In order to prevent a Dutch book, behavior, not preferences, should be consistent. Indeed, if preferences are initially strictly quasi convex and both X and Y are optimal, then choosing X implies that in the next period X is strictly preferred to Y (see Section 4). Even though X and Y were initially indifferent, holding X makes it more attractive than Y .

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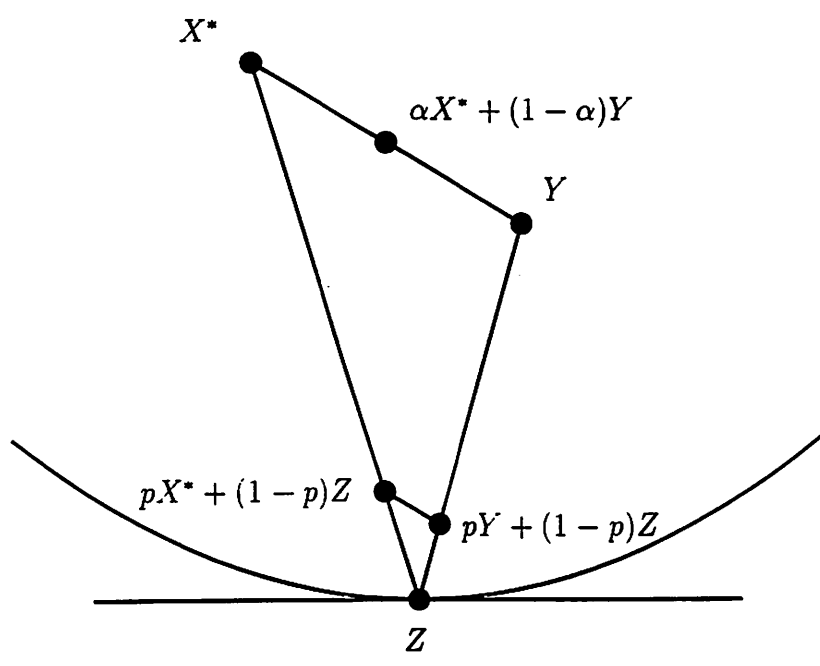


Figure 1: Showing why the indifference curve through X^* is affine.

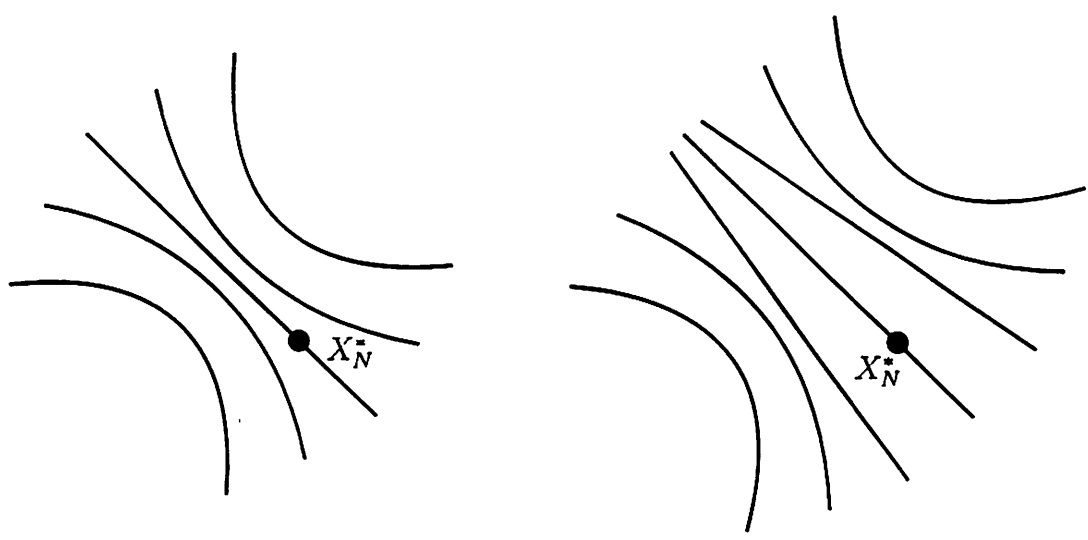


Figure 2: Quadratic and betweenness-quadratic representations