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I. Introduction

Deaton and Paxson (1994) have recently drawn attention to the age profiles of income and consumption inequality. They showed that these profiles are both upward sloping over the lifetime in Taiwan, the U.K., and the U.S. Arguing that only models of consumption behavior which can reproduce such positive slopes are admissible, they examined the predictions for consumption and income inequality coming from the life-cycle and other models of consumption.[1]

Interestingly, while age profiles of inequality for consumption, total household income, and labor earnings do all generally slope up, the opposite is true for non-human wealth up until retirement. A wide variety of studies have found negatively sloped age profiles of inequality for household net worth over the working lifetime, with a rise in inequality in the retirement period. It is interesting to ask what models of consumption behavior are capable of generating this negatively sloped profile, in addition to the positive sloped profiles of consumption and income inequality. This paper asks whether applying this additional requirement puts restrictions on the set of admissible models of consumption behavior which go beyond those found by Deaton and Paxson.

Section II of this paper discusses the stylized facts on the age profile of wealth inequality. The ability of the simple life-cycle model (LCM) to explain the various age profiles of inequality is then studied in Section III. Section IV repeats the exercise for the Keynesian model, and Section V looks at target wealth (buffer stock) models. The possible influence of factors omitted from the analysis is discussed in Section VI. Section VII concludes.

II. Stylized Facts

The stylized facts about the age profile of dispersion and inequality in non-human wealth holdings are simple. First, absolute dispersion increases monotonically with age. Second, relative inequality falls sharply over the first 10-15 years of the working lifetime, continues

falling or at least does not trend upward over the remainder of the working period, and increases in retirement. These stylized facts are found, for example, in studies using either pure survey or synthetic datasets for Canada (e.g. Davies. 1979, and Magee et al., 1991), Italy (Jappelli, 1995), the Netherlands (Alessie et al., forthcoming), the U.K. (Shorrocks, 1975a)[2], and the U.S. (Projector and Weiss, 1964, Table 8, p. 30; Wolff, 1980; and Greenwood, 1987).

Minor exceptions to the above stylized facts are found in two cases. The crucial stylized fact, from the viewpoint of this paper, is that wealth inequality declines, or at least does not trend upwards, over the working lifetime. One possible exception to this may be the case of Germany, where Borsch-Supan (1994, p. 216) reports that inequality, as measured both by the coefficient of variation and the ratio of mean to median financial wealth, increases mildly starting at age 45. The only other exception of which I am aware is for the elderly in Canada, where wealth inequality continues to decline in retirement.

While the pattern of a downward trend in wealth inequality during the working lifetime and increasing wealth in retirement are fairly general, there are some data problems which should be considered. First, most of the studies mentioned above use cross-section data. Panel studies on wealth are beginning to become available, and will allow this deficiency to be corrected. Second, in most cases the published results present grouped data. In the earliest studies families or individuals were typically grouped in 10 year age ranges, but 5 year ranges are now used in some cases. Grouping introduces potentially serious errors in estimated inequality where wealth is trending up or down strongly with age. In a model like the simple LCM, which has a strongly peaked age-wealth profile, striking but spurious falling inequality during the working lifetime and rising inequality in retirement can be generated simply by this grouping error.

In practice, the rise and fall of wealth with age is much less sharp than in the simple LCM, reducing the possible importance of grouping error. And, fortunately, there are also studies which do not suffer from grouping error. An example is provided by Alessie et al. (forthcoming) which looks at single-year age groups in the 1989 Dutch SEP data. Figure 1 shows the Alessie et al. kernel-smoothed age profile for the coefficient of variation of household

net worth in the Netherlands. While this profile shows a small hump in the age range from 30 to 45, overall it shows a downward trend from age 25 to 50. There is then a plateau from 50 to 60, and only after age 60 (by which time most Dutch workers are retired) does an increase in wealth inequality begin.

A further concern about the wealth data is that it generally excludes social security and pension wealth. One approach would be to attempt adjustments to the data to include imputations for these missing forms of wealth. Another, which may be more promising, would be to incorporate in the analysis plausible allowance for state and occupational pensions as a form of retirement income independent of individuals' saving behavior. The form of wealth accumulation studied would then correspond better with the form observed in the data. This latter strategy is on the agenda for future research.

The repeated observation in several countries that wealth inequality declines with age requires some explanation. Friedman (1957) suggested that the ability to reproduce the stylized facts of wealth inequality would be an important test of models of consumption. However, with the important exception of Shorrocks (1975a), who explains how a queuing model of wealth accumulation could generate declining wealth inequality during the working lifetime and rising inequality thereafter, I do not know of any attempts to explain the age profile of wealth inequality.[3]

III. Life-Cycle Model

Our attention is confined here to the simplest version of the "stripped down" LCM. The units considered are families, of constant (adult) composition from "birth" at age 22 (a typical age of labor force entry) to death T years later. All families receive labor income, E_t, from age 22 until retirement after R years of employment. Consumers display certainty equivalence behavior. I assume that the interest rate equals the rate of time preference, and that there is a perfect capital market. Thus, all families desire constant consumption over the lifetime. I will

refer to this specification as the simple LCM.

Carroll (1992) and Hubbard et al. (1993) find that a realistic approximation to the earnings process is provided by a combination of white noise and a random walk. Throughout this paper the following process is assumed for the earnings of an individual family, E_t:

$$(1) \qquad E_t = \begin{cases} E_{t-1} + \varepsilon_t - \beta\varepsilon_{t-1} & \varepsilon_t \sim (0,\sigma^2) \qquad t \leq R \\ 0 \qquad t > R \end{cases}$$

where $\beta = 1$ corresponds to white noise and $\beta = 0$ to a random walk, and I omit any family subscript or superscript, as I will do throughout the paper. Equation (1) assumes perfect correlation of permanent and transitory shocks, but this could be relaxed without altering the qualitative results. Assuming perfect correlation of the two kinds of shock leads to closed form analytical results. Independence of transitory and permanent shocks is equally tractable; how the results would differ with independence is noted as we proceed.

The earnings process assumed implies constant mean earnings, E_t. The following analysis can readily be generalized to allow the age profile of mean earnings to have a realistic hump-shape, but this comes at the cost of tedious algebra, and yields little further insight.[4] I am also ignoring state and occupational pensions. Their inclusion would reduce the strength of the saving motive, producing effects on wealth accumulation in the simple LCM over the working life similar to those of shortening the retirement period. One impact of using additive rather than multiplicative shocks here which should be kept in mind is slightly less of a tendency for wealth inequality to decline over the working lifetime in the models considered.[5]

I have specified an earnings process with additive shocks in order to be able to get analytical results on the behavior of inequality. Multiplicative shocks fit the data better, and conveniently accommodate a lognormal earnings distribution (which is fairly realistic). The formulation (1) might be taken to suggest that, in contrast, I will work with symmetric, or

possibly even normal, distributions. This is not the case. The shapes of the distributions of E_0 and the ε_i's do not have to be unduly restricted. and may be strongly positively skewed.

Notice that in the white noise version of (1), absolute dispersion of E_t, as measured by the variance. $V(E_t)$, or standard deviation will be constant over the lifetime. If we have $\beta < 1$, however, the variance of earnings will rise with age. Since (1) features constant mean earnings, *relative* earnings inequality - - which can be readily measured here by dividing the standard deviation by the mean to get the coefficient of variation, $CV(E_t)$ - - moves in exactly the same way as absolute dispersion. So, if $\beta = 1$ relative as well as absolute dispersion are constant, and with $\beta < 1$ both relative and absolute dispersion are increasing.

In what follows, we find that the behavior of relative and absolute consumption dispersion are also the same, since C_t is constant over the lifetime in the simple LCM. However, the situation is very different for wealth, which is not constant. This illustrates the general point that we must distinguish carefully between the behavior of relative and absolute measures of dispersion. For simplicity, the term "dispersion" will be used as synonymous with "absolute dispersion". Dispersion may be measured using the variance or standard deviation. The term "inequality" will be reserved for relative dispersion, and may be measured with such scale independent measures as the coefficient of variation or the Gini coefficient.

A family of working age has the budget constraint:

$$(2) \qquad \sum_{i=1}^{T} \frac{C_t}{(1+r)^{i-t}} \leq (1+r)W_{t-1} + \sum_{i=t}^{T} \frac{E(E_i)}{(1+r)^{i-t}} \qquad t \leq R$$

where expected future earnings equal the permanent component of current earnings. This gives rise to the consumption rule:

$$(3) \qquad C_t = \frac{[(1+r)W_{t-1} + \beta\varepsilon_t] + (E_t - \beta\varepsilon_t)D_t^{R}}{D_t^{T}}; \qquad D_\ell^{K} = \sum_{i=\ell}^{K} \frac{1}{(1+r)^{i-\ell}}$$

This rule provides much of the intuition behind the positively sloped age profile of consumption inequality, when taken together with:

$$(4) \qquad \bar{C}_t = \bar{E}\frac{-D_1^R}{D_1^T}; \qquad \bar{E} = \bar{E}_0 = \bar{E}_1 = \ldots = \bar{E}_R$$

which indicates that mean consumption is constant. Whereas mean consumption is constant over the lifetime, the variance of W_{t-1}, $V(W_{t-1})$, grows with age (even if wealth inequality is trending downwards), and so does $V(E_t)$ unless we have the pure white noise model, in which case it is constant. Since the dispersion in these determinants of consumption is rising, it is not surprising that the dispersion in C_t is increasing with age, as demonstrated by Deaton and Paxson (1994). But, given that mean consumption is constant this also means that consumption *inequality* is rising with age.[6]

In contrast to consumption, mean wealth rises very quickly up to the retirement age in the simple LCM, and falls quite sharply afterwards. We have:

$$(5) \qquad \bar{W}_t = \begin{cases} i) & \bar{E}\left(\dfrac{D_1^T - D_1^R}{D_1^T}\right)\sum\limits_{i=1}^{t}(1+r)^{t-i} & t \le R \\[3em] ii) & \left(\dfrac{D_{t+1}^T}{D_{R+1}^T}\bar{W}_R\right) & R < t < T \\[3em] iii) & 0 & t = T \end{cases}$$

The sharply peaked mean age-wealth profile is not echoed in real-world data. While hump-shaped age profiles of wealth are observed in cross-section, the rise and fall of wealth is much less sharp than in the simple LCM, and, as has been widely commented, (non-pension) wealth appears to be decumulated fairly slowly in retirement.

Through successive substitution the following expressions are obtained for current wealth in the working and retirement periods:

$$(6) \quad W_t = \begin{cases} i) \quad \sum_{i=1}^{t} \left(\frac{D_t^T - 1}{D_i^T} \right) \beta \varepsilon_i + (1-\beta) \left(\frac{D_1^T - D_1^R}{D_1^T} \right) \sum_{i=1}^{t} \sum_{j=1}^{t} (1+r)^{t-i} \varepsilon_j \\ \qquad\qquad + E_0 \left(\frac{D_1^T - D_1^R}{D_1^T} \right) \sum_{i=1}^{t} (1+r)^{t-i} \qquad t \le R \\ ii) \quad \left(\frac{D_{t+1}^T}{D_{R+1}^T} \right) \bar{W}_R \qquad\qquad R < t < T \\ iii) \quad 0 \qquad\qquad\qquad t = T \end{cases}$$

Noting that consumption in retirement is proportional to wealth, and wealth is in turn proportional to net worth at the retirement date, we have:

Result 1: During the retirement period in the simple LCM, consumption and wealth inequality are equal and constant.

Proof: The result follows directly from (5) and (6), and from noting that consumption is proportional to non-human wealth when earnings are equal to zero for the remainder of the lifespan. ∎

Since the ε_i's are independent of each other, and of E_0, we have :

$$(7) \quad V(W_t) = \sum_{i=1}^{t} \left[\beta \left(\frac{D_t^T - 1}{D_i^T} \right) + (1-\beta) \sum_{j=i}^{t} \left(\frac{D_1^T - D_1^R}{D_1^T} \right) (1+r)^{t-j} \right]^2 \sigma^2 + \left[\left(\frac{D_1^T - D_1^R}{D_1^T} \right) \sum_{i=1}^{t} (1+r)^{t-i} \right]^2 V(E_0)$$

By considering the components of (7) in turn, and comparing their rates of increase with that of the square of mean wealth implied by (5), it is possible to predict the behavior of the squared coefficient of variation of wealth, and therefore $CV(W_t)$, as reflected in the following results.

Result 2: In the absence of earnings shocks inequality in W_t would be constant over the entire lifetime, and would equal inequality in earnings and consumption.

Proof: From (6.i), if the ε_i's are all zero, W_t is proportional to E_0 over the working life, and this result extends to the retirement period through the proportionality of wealth in retirement to wealth at the end of the working lifetime. W_R, shown in (6.ii). Substituting both parts of (6) into (3) one finds that consumption is also proportional to E_0. ■

A sketch proof only is provided for the following result. Details required to complete the proof are in the appendix.

Result 3: If the earnings shocks are purely white noise ($\beta = 1$), $CV(W_t)$ declines over the working lifetime.

Sketch Proof: From Result 2 we know that the $V(E_0)$ term in (7) grows at the same proportional rate as $\overline{W}_t^2$. To establish Result 3, therefore, we need to show that the other term in (7), involving σ^2, will not grow as fast as $\overline{W}_t^2$. Note first that:

$$
(8) \qquad \left(\frac{\overline{W}_t}{\overline{W}_{t-1}}\right)^2 = \left(\frac{\displaystyle\sum_{i=1}^{t}(1+r)^{t-i}}{\displaystyle\sum_{i=1}^{t-1}(1+r)^{t-1-i}}\right)^2 > \left(\frac{t}{t-1}\right)^2
$$

The inequality holds because the last t-1 of the terms of the sum in the numerator are the same as those in the denominator, but the numerator includes an additional t'th term, $(1+r)^{t-1}$, which exceeds any of the terms in the denominator.

Now, let $V^{WN}(W_t)$ be the component of $V(W_t)$ involving σ^2 when $\beta = 1$, and look at its growth rate:

9

$$(9) \qquad \frac{V^{WN}(W_t)}{V^{WN}(W_{t-1})} = \frac{\sum\limits_{i=1}^{t}\left(\frac{D_t^T - 1}{D_i^T}\right)^2}{\sum\limits_{i=1}^{t-1}\left(\frac{D_{t-1}^T - 1}{D_i^T}\right)^2}$$

It is possible to show that:

$$(10) \qquad \frac{V^{WN}(W_t)}{V^{WN}(W_{t-1})} < \left(\frac{t}{t-1}\right) < \left(\frac{t}{t-1}\right)^2$$

This inequality holds because the effect of $D^T_t < D^T_{t-1}$ is sufficiently strong to make the average term in the numerator sum less than that in the denominator sum. With pure white noise we thus have $V(W_t)$ growing less quickly than $\overline{W}_t^2$ and $CV(W_t)$ declines for $t = 1, ..., R$. ∎

Result 4: If the earnings process is a pure random walk, $CV(W_t)$ increases over the working lifetime.

Proof: In this case we again isolate the $V(W_t)$ components involving σ^2. They now represent a random walk rather than white noise. We can form the ratio:

$$(11) \qquad \frac{V^{RW}(W_t)}{V^{RW}(W_{t-1})} = \frac{1 + \sum\limits_{i=1}^{t-1}\left(\sum\limits_{j=i}^{t}(1+r)^{t-j}\right)^2}{\sum\limits_{i=1}^{t-1}\left(\sum\limits_{j=i}^{t-1}(1+r)^{t-1-j}\right)^2}$$

If the numerator consisted only of the term in square brackets, this ratio would exceed that of $(\overline{W}_t/\overline{W}_{t-1})^2$, , i.e. as shown in (8). (The $i = 1$ term in the numerator summation equals $(\overline{W}_t/\overline{W}_{t-1})^2$ times the $i = 1$ term in the denominator, but the corresponding ratio exceeds

$(W_t/W_{t-1})^2$ for higher i's. Adding 1 to the numerator simply increases the overall ratio, and strengthens the result that:

$$(12) \qquad \frac{V^{RW}(W_t)}{V^{RW}(W_{t-1})} > \left(\frac{\bar{W}_t}{\bar{W}_{t-1}} \right)^2$$

Thus, $CV(W_t)$ rises with t when the earnings process is a random walk. ∎

To provide some intuition for these results, think first of what happens in the first period in the white noise model. Families save a very large portion of positive shocks, and dissave the same proportion of negative shocks. In the first period, when mean wealth is low, this produces large relative inequality of wealth. As we move to the second and subsequent periods two things happen to moderate this inequality. The first is that the wealth (or debt) incurred in response to first-period shocks is consumed (paid off). The second is that there are new shocks, uncorrelated with previous ones. Thus, wealth inequality declines.

Adding a random walk element to earnings creates permanent earnings shocks. These shocks tend to make wealth inequality rise with age for the following reason. While the *immediate* impact of these permanent shocks on saving is smaller than that of transitory shocks, the changes they produce in wealth do not fall off with time, as in the white noise model. A permanent negative earnings shock, for example, means lower expected lifetime wealth, lower consumption, and lower wealth throughout the rest of the lifetime. Permanent earnings shocks thus not only produce rising earnings inequality, they also generate rising wealth inequality.

So, the main thing we learn from this analysis is that transitory and permanent shocks have opposite influences on the evolution of wealth inequality over the life cycle. The white noise aspect of the earnings process is tending to make wealth inequality fall, even while earnings inequality is rising. The random walk aspect is tending to make wealth inequality rise. The

remainder of this section asks which effect will dominate when plausible parameter values are assigned to the simple LCM.

Table 1 shows $CV(W_t)$ and $CV(C_t)$ at five year intervals through the lives of a population of simple life cycle savers who work for 35 years and retire for 20 years. With labor force entry this gives retirement at the beginning of the 57th year, and death on the 77th birthday. In view of declining retirement ages, and increasing longevity, it would appear that these figures are more realistic than the 40 year working lifetime and 10 year retirement that featured, e.g., in Atkinson (1971) and some other earlier illustrative calculations. Against this, recall that a longer retirement period implies a greater need for life cycle saving, which has the opposite influence to including state and occupational pensions. Thus, it is important to check the sensitivity of the results to alternative assumptions on the length of the retirement period. This is done in Table 2, which increases the length of the working period, and reduces longevity, alternatively. A further assumption in the calculations reported here is a zero interest rate.

The earnings process in Table 1 is parameterized to mimic the age profile of earnings inequality in the Netherlands. It is appropriate to look at family income, since it is best to think of the decision unit as a married couple, or family. Also, it is best to include all forms of non-capital income, given the considerable importance of transfer payments in the real world. Both these requirements are met with the help of cross-section data from the Dutch SEP survey of 1987, 88, and 89. Aldershof (1994, Figure 29) shows the age profile of the coefficient of variation of non-capital income for Dutch families. The pooled data indicate $CV(E_t)$ of about 0.4 in the age range 25 - 30, and a figure of about 0.6 for the age range 55-60.[7] Accordingly, I have set up the earnings process to make $CV(E_t)$ rise from 0.4 at the start of the working lifetime to 0.6 at the end when β takes on its "best guess" value of 0.5. (The implied value of σ^2 is held constant when β is varied from 0.0 to 1.0 in Table 1. This means that in the current version of the paper the age profile of earnings inequality departs from its benchmark slope when β ≠ 0.5.)

Given the empirical finding that transitory and permanent shocks to earnings are of about equal importance, the central case in Table 1 would appear to be where $\beta = 0.5$. In this case we find that not only does consumption inequality increase with age throughout, as Deaton and Paxson (1994) emphasize is required for realism, but so also does wealth inequality after a very brief decline at the youngest ages. The weight on the white noise component of the earnings process has to be raised to 0.75 before what might be described as realistic behavior of $CV(W_t)$ is obtained. With $\beta = 0.75$, $CV(W_t)$ declines from age 22 to 37, and thereafter rises mildly through to the retirement age.[8]

Table 2 investigates sensitivity to R and T, holding β at 0.5 throughout. The first column repeats the results we have already seen. The second column raises the retirement age to 62, holding the age of death at 77. The last column keeps the retirement age at 62, and reduces the age of death to 72. Interestingly, as the length of retirement is reduced in turn to 15 and then to 10 years from the original 20, the performance of the model improves. Wealth inequality at age 52 is little affected - - rising slightly. But $CV(W_t)$ becomes substantially higher for those aged 22, and also becomes quite a bit higher for those in retirement. The result is a longer period of declining $CV(W_t)$ - - up to age 30 in the third column - - followed by gently rising inequality up until retirement.

Table 2 is of interest not only from the viewpoint of testing sensitivity to R and T, but, as mentioned earlier, because a reduction in the strength of the life-cycle saving motive caused by shortening retirement has a roughly analogous effect on saving to introducing state and occupational pensions. The table therefore suggests that the result of modelling the latter could be to improve the performance of the model significantly.

IV. Keynesian Model

This section repeats the exercise of the previous section for the case where consumption is linear in current income, often referred to as the Keynesian model of consumption:

$$(13) \quad C_t = \alpha_0 + \alpha_1(E_t + rW_{t-1})$$

In this model mean wealth grows according to:

$$(14) \quad \bar{W}_t = \sum_{i=1}^{t}(1+\rho)^{t-i}[(1-\alpha_1)\bar{E}-\alpha_0] = \left[\frac{(1+\rho)^t-1}{\rho}\right][(1-\alpha_1)\bar{E}-\alpha_0]$$

where $\rho = r(1-\alpha_1)$. The following expression for W_t is obtained:

$$(15) \quad W_t = \sum_{i=1}^{t}(1+\rho)^{t-i}[(1-\alpha_1)(\bar{E}+\varepsilon_i)-\alpha_0] + \sum_{i=1}^{t-1}\sum_{j=i+1}^{t}(1+\rho)^{t-j}(1-\alpha_1)(1-\beta)\varepsilon_i$$

and this in turn provides:

$$(16) \quad V(W_t) = \left(\frac{1-\alpha_1}{\rho}\right)^2 \left\{[(1+\rho)^t - 1]^2 V(E_0) + \sum_{i=1}^{t}[\rho(1+\rho)^{t-i} + ((1+\rho)^{t-i}-1)(1-\beta)]^2\sigma^2\right\}$$

The results on the impact of the initial earnings, white noise, and random walk components of earnings on $CV(W_t)$ are qualitatively the same as in the simple LCM, as reported in:

Result 5: Over the working lifetime in the Keynesian model $CV(W_t)$

    (i) is constant if $\sigma = 0$,

    (ii) declines if earnings shocks are white noise ($\beta = 1$),

    (iii) increases if earnings follow a random walk ($\beta = 0$).

Proof: (i) From (14) note that:

$$(17) \quad \left(\frac{\bar{W}_t}{\bar{W}_{t-1}}\right) = \left[\frac{(1+\rho)^t - 1}{(1+\rho)^{t-1} - 1}\right]^2$$

From (16), if $\sigma = 0$, $V(W_t)/V(W_{t-1})$ is also equal to the RHS of (17), establishing part (i) of the result.

(ii) The white noise component of the variance grows according to:

$$(18) \quad \frac{V^{WN}(W_t)}{V^{WN}(W_{t-1})} = \frac{\sum\limits_{i=1}^{t}\left[\rho(1+\rho)^{t-i}\right]^2}{\sum\limits_{i=1}^{t-1}\left[\rho(1+\rho)^{t-1-i}\right]^2}$$

Letting $R = (1 + \rho)$, and using (17) and (18), we need to show that:

$$(19) \quad \frac{V^{WN}(W_t)}{V^{WN}(W_{t-1})} = \frac{R^{2t}-1}{R^{2(t-1)}-1} < \left(\frac{R^t-1}{R^{t-1}-1}\right)^2 = \left(\frac{\bar{W}_t}{\bar{W}_{t-1}}\right)^2$$

As shown in the appendix, the required inequality holds for $R = (1 + \rho) > 1$.

(iii) Finally, if $\beta = 0$, the growth of the random walk component of the variance can be examined:

$$(20) \quad \frac{V^{RW}(W_t)}{V^{RW}(W_{t-1})} = \frac{\sum\limits_{i=1}^{t}\left[(1+\rho)^{t+1-i} - 1\right]^2}{\sum\limits_{i=1}^{t-1}[(1+\rho)^{t-i} - 1]^2}$$

Here the ratio of the first terms in the numerator and denominator sums equals $(\bar{W}_t/\bar{W}_{t-1})$ and the ratio of all the subsequent i-th terms up to $i = t-1$ is greater than $(\bar{W}_t/\bar{W}_{t-1})^2$. The addition of a final term to the numerator strengthens the conclusion that $VRW(W_t)/V^{RW}(W_{t-1}) > (\bar{W}_t/\bar{W}_{t-1})^2$, giving the third part of Result 5. ∎

Table 3 provides results on $CV(W_t)$ and $CV(C_t)$ in the Keynesian model analogous to those in Table 1 for the simple LCM. The model specification remains the same, except that a 5% interest rate is incorporated. Table 4 shows sensitivity to the interest rate. In these calculations $\alpha_0 = 0$, and $\alpha_1 = 0.8$. giving a 20% marginal and average propensity to consume out of current income. While this may seem high, once again we have not included state or occupational pensions, so that the savings modelled are the only provision which families have for retirement.

Notice, first in Table 3 that both $CV(W_1)$ and $CV(C_1) = CV(E_0) = 0.4$. This is a result of making consumption proportional to income, which consists entirely of non-capital income in the first period. Also note that $CV(W_t)$ and $CV(C_t)$ in retirement are the same as $CV(W_R)$, once again due to proportional savings.

The proportional savings model clearly does not do as well as the simple LCM of Table 1. The difference is perhaps most marked for consumption inequality. When $\beta = 0.5$, for example, $CV(C_t)$ rises only from 0.40 to 0.55 over the working lifetime. which is much weaker than the increases found by Deaton and Paxson (1994) in data for the U.S., U.K., and Taiwan. and a much smaller rise than shown here in Table 1 (0.38 to 1.54).[9] (The first column of Table 4, which uses a zero interest rate, indicates that the reason for the poorer performance does not lie in the use of a positive interest rate in Table 3.) Note also that the model predicts a decline in consumption inequality on entry into retirement. The data reported by Deaton and Paxson (1994), in contrast, indicate a levelling-out of consumption inequality in retirement, rather than a decline.

On the wealth side, the proportional consumption model also does less well than the simple LCM, giving rising wealth inequality starting at age 27 even when $\beta = 0.75$. (It is only

at the pure white noise extreme in Table 3 that the model gives declining wealth inequality over a significant part of the working lifetime.) Table 4 shows that this poorer performance is again robust to the interest rate.

Why does the proportional consumption model do less well than the simple LCM? The answer is that it is the transitory shocks which create the tendency for wealth inequality to decline, as in the LCM. But, in the Keynesian model the propensity to consume out of transitory shocks is much higher than in the LCM (and, in fact, equal to the propensity to consume out of permanent shocks). Thus, transitory shocks produce much less wealth accumulation.

V. Target Wealth Model

Recently there has been quite a bit of interest in buffer stock models of consumption. These have a "target wealth" feature. In an attempt to get some idea of how such models might perform in predicting the age profile of wealth inequality, I examine a simple target wealth model. I refer to this ad hoc formulation as a target wealth, rather than buffer stock, model, since the buffer stock models which have drawn attention in recent literature have been derived in an optimizing framework.

Suppose that each family has target wealth, W^*_t, proportional to the permanent component of earnings:

$$(21) \qquad W^*_t = \gamma(E_t - \beta\varepsilon_t)$$

Families are assumed to consume E_t each period, plus an additional amount if their wealth is above its target level:

$$(22) \qquad C_t = E_t + \delta[(1+r)W_{t-1} - W^*_t]$$

This gives:

(23) $C_t = (1-\theta)E_t + \delta(1+r)W_{t-1} - \theta\beta\varepsilon_t$

where $\theta = \delta\gamma$. Wealth grows from period to period according to:

(24) $W_t = \pi W_{t-1} - \theta E_t - \theta\beta\varepsilon_t$

where $\pi = (1 - \delta)(1 + r)$. Wealth is determined by initial earnings and earnings shocks by:

(25) $W_t = \theta\left[\dfrac{\pi^t-1}{\pi-1}\right]E_0 + \theta(1-\beta)\sum_{i=1}^{t}\left(\dfrac{\pi^t-\pi^{i-1}}{\pi-1}\right)\varepsilon_i$

Mean wealth grows according to:

(26) $\bar{W}_t = \theta\left[\dfrac{\pi^t-1}{\pi-1}\right]\bar{E}$

And, finally, the variance is given by:

(27) $V(W_t) = \left[\theta\left(\dfrac{\pi^t-1}{\pi-1}\right)\right]^2 V(E_0) + \left[\dfrac{\theta(1-\beta)}{\pi-1}\right]^2 \sum_{i=1}^{t}(\pi^t-\pi^{i-1})^2)\sigma^2$

The behavior of $CV(W_t)$ over the working lifetime is summarized in Result 6. There are no results for a retirement period, since target wealth is then not defined. This is perhaps not a concern, since buffer stock or target wealth models are thought to capture the behavior of younger families best. It has been suggested, e.g., that young families may save according to a buffer stock model, while those nearing retirement behave move like life-cycle savers.

Result 6: Over the working lifetime in the target wealth model $CV(W_t)$:

(i) is constant if $\sigma = 0$, or if earnings shocks are white noise ($\beta = 1$),

(ii) increases if earnings follow a random walk ($\beta = 0$).

Proof: Comparison of (26) and (27) immediately indicates that $V(W_t)$ and W_t^2 grow at the same rate if $\sigma = 0$. For $\beta = 0$, we examine $V^{RW}(W_t)/V^{RW}(W_{t-1})$ as previously:

$$(28) \quad \frac{V^{RW}(W_t)}{V^{RW}(W_{t-1})} = \frac{\sum\limits_{i=1}^{t}(\pi^t - \pi^{i-1})^2}{\sum\limits_{i=1}^{t-1}(\pi^{t-1} - \pi^{i-1})^2}$$

The $i = 1$ terms in the numerator and denominator sums have the same ratio as $(\overline{W}_t/\overline{W}_{t-1})^2$. The $i = 2, ..., t-1$ terms have a higher ratio: adding a t-th term in the numerator merely exaggerates the inequality. Thus, once again we find $V^{RW}(W_t)/V^{RW}(W_{t-1}) > (\overline{W}_t/\overline{W}_{t-1})^2$, and $CV(W_t)$ grows over time if earnings follow a pure random walk. ∎

The intuition behind these results is straightforward. A family's actual wealth tends to converge on its target wealth. In the absence of earnings shocks, this would mean that all families would save at the same rate until they reached their steady-state. Wealth for families of the same age would be proportional to earnings, and wealth inequality would therefore not vary with age. Transitory earnings shocks would have no effect, since the model says that convergence toward target wealth is independent of such shocks. Permanent shocks, on the other hand, cause rising wealth inequality since they make the target wealths disperse strongly with age, causing actual wealth of families to move off on increasingly divergent trajectories.

Note that with any mixture of white noise and random walk the target wealth model, unlike the simple LCM and Keynesian models, will give wealth inequality rising with age. Hence, the particular form of target wealth model considered here can be regarded as rejected by the data, in the sense that it is inconsistent with the stylized fact that wealth inequality does not rise over the bulk of the working lifetime.

Deaton and Paxson (1994) discussed briefly the simplications of the general class of buffer stock models for age profiles of inequality. They indicate that such models tie the

behavior of consumption and assets closely to that of earnings, and tend to produce the result that inequality in both consumption and wealth will follow the same pattern as that of earnings inequality. This is the same type of behavior as displayed in the ad hoc target wealth model analyzed in this section. Deaton and Paxson do not go so far as to reject buffer stock models since they concentrate on consumption inequality evidence. Given that earnings inequality rises over the lifetime, buffer stock models are clearly capable of producing rising consumption inequality. A stronger test is provided by the age profile of wealth inequality. The failure of the simple target wealth model considered here, and the more general class of buffer stock models analyzed by Deaton and Paxson, to allow falling wealth inequality over the working lifetime would appear to represent a rejection of these models.

VI. Other Factors

The above discussion has looked at very simple models. Many factors which influence the age profile of wealth inequality are beyond the scope of this single paper, but it would be desirable to incorporate others in subsequent versions. Illustrative simple LCM calculations using a positive interest rate; relaxation of the perfect correlation between transitory and permanent earnings shocks; the effect of realistic state and occupational pension plans; and analysis with other preferences (e.g. CRRA) should all be studied. Intuition suggests that these generalizations would alter the main conclusions of the paper relatively little, which would parallel Deaton and Paxson's conclusions in the analysis of consumption inequality.

If a full-blown simulation model of saving by a representative sample of households were to be undertaken, it would be desirable to model plausible differences between families in time preference, rates of return, and ages of labor force entry, retirement, and death. All of these factors tend to increase wealth inequality, and with the exception of differences in the age of labor force entry, are likely to do so increasingly with age. (It is interesting, nonetheless, that differences in rates of time preference and interest rates were incorporated in the simulations of Davies [1979] without removing the downward trend in wealth inequality over the working lifetime.) Importantly, this implies a fortiori that the basic consumption model one begins with

must be able to generate declining wealth inequality when these complicating factors are omitted. Otherwise it will have no chance of reproducing the stylized facts when these additional factors are also incorporated.

Other factors which should, in principle, be studied would include the impact of uncertain lifetime, differential mortality, and intergenerational links, e.g. via bequests. It seems likely that, on balance, these factors would also increase the tendency for wealth inequality to rise with age.[10] Over the working lifetime this again reinforces the need to have a basic consumption model which can generate declining wealth inequality when the complications are not taken into account. And it may be just such factors which one must rely on to explain one of the major stylized facts of the age profile of wealth inequality which has not been addressed in this paper - - the rising trend of wealth inequality in retirement observed in most countries.

VII. Conclusion

This paper has examined simple versions of life-cycle, Keynesian. and target wealth models of consumption. It has drawn out their implications for the age profile of wealth inequality. The latter is observed to decline over the working lifetime and to increase in retirement in a range of advanced industrial countries. Performance of the models in generating a realistic age profile of wealth inequality has been studied, and predictions for consumption inequality have also been noted.

While many factors have been excluded from the analysis in this paper, as discussed in the previous section, a fairly clear ranking of the three kinds of simple model considered here emerges. Not only can the simple life-cycle model (LCM) generate a realistic age profile of consumption inequality, it appears capable of doing the same for wealth inequality, provided that the need for private retirement saving is not too great, and that permanent shocks to earnings do not have a higher variance than transitory shocks. In contrast, a proportional consumption

version of the Keynesian model has been shown to generate too little growth of consumption inequality over the lifetime, and too much of a tendency for wealth inequality to rise in the working lifetime. Finally, a simple target wealth model falls down in predicting that wealth inequality must rise with age, a result which Deaton and Paxson's comments indicate may generalize to a broad class of buffer stock models.
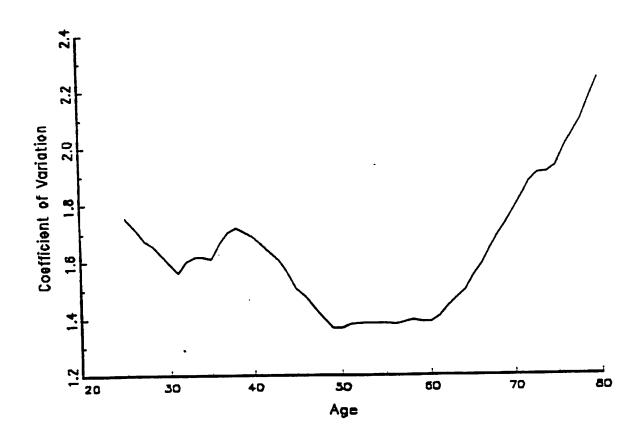
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Figure 1

Coefficient of Variation of Net Worth, The Netherlands, 1989

Source: Alessie et al. (forthcoming, Fig. 4b).

Notes

1. Attanasio and Jappelli (1996) generalize the Deaton and Paxson analysis. Their empirical results, like those of Deaton and Paxson, do not reject the life cycle model.

2. Shorrocks (1975a, p. 639) refers to detailed unpublished data corresponding to the composition-corrected cohort estimates of wealth distribution reported in Shorrocks (1975b). He indicates that "data on the relative dispersion within age groups suggests that dispersion initially decreases with age but increases again in the retirement period". Atkinson and Harrison, 1978, p. 255 show a similar result (without the composition adjustment) for the male cohort born within 5 years of 1890 in the U.K. In using the U.K. estate multiplier data it is important to perform the adjustments set out in Shorrocks (1975b), and to follow a cohort, since the cross-section pattern for male individuals (although not for female) in the post-war period shows increasing inequality over the lifetime. (Atkinson and Harrison, 1978, Table 9.5, pp.. 256-257.)

3. Vaughan (1988) studies the overall distribution of wealth in an economy composed of life cycle savers where there is uncertainty both of labor income and interest rates. While his analysis is very rich, he does not study the implications for the age profile of wealth inequality.

4. It can be argued that unless account is taken of the consumption needs impact of the hump-shaped age profile of family size, introducing a hump-shaped earnings profile makes the exercise *less* rather than more realistic. See Davies (1988).

5. With the usual formulation in which shocks are multiplicative and lognormal, the variance of earnings rises at an increasing rate, in contrast to the linear behavior with (1). The convexity of the age profile of the variance in the lognormal case is, however, not particularly pronounced for realistic parameter values. Convexity of this profile should strengthen the relative tendency coming from permanent earnings shocks towards rising wealth inequality at later ages.

6. The prediction that consumption inequality rises with age is very robust, and is obtained in all the models considered in this paper. In order to economize on algebra, then, while numerical results are presented for consumption inequality the corresponding equations are omitted. They are provided in an appendix which may be obtained from the author.

7. $CV(E_t)$ is a little higher for those aged 22, but the value for those aged 25-30 is used instead. A decline in earnings inequality up until the late 20's is a typical finding, and could be accommodated in the earnings process used in this paper. However, that would require us to allow the distribution of ε_t to vary with t, which is not convenient for the derivation of the analytical results we are attempting to illustrate.

8. Note that, unlike the shape of the age profile. the *values* obtained for $CV(W_t)$ differ considerably from what is observed in the Dutch data (see Figure 1). The observed values are much higher. This suggests the importance of taking into account aspects of heterogeneity among families aside from earnings differences. See the discussion in Section VI.

9. The charts in Deaton and Paxson (1994) indicate that the variance of log consumption rises from about 0.2 to 0.6 over the working lifetime in Taiwan and the U.K., and from about 0.25 to 0.55 in the U.S. Taking the Taiwan and U.K. numbers, and assuming a lognormal distribution, $CV(C_t)$ should rise from about 0.47 to 0.91 over the working lifetime.

10. At the most advanced ages, differential mortality would tend to reduce wealth inequality as the remaining population would consist of a increasingly homogenous group of high income, and high wealth families. However, earlier in the lifetime (including the initial retirement years), before much mortality actually occurs, the impact would be to raise wealth inequality. This is because, while they are both still alive, for two individuals with the same earnings over the lifetime but differing life expectancy, the individual expecting greater longevity needs to have accumulated higher wealth. Real-world differences in longevity by social class, education, or income are very large. (See, e.g., Hurd, 1995.) Thus, this source of wealth inequality may be quite important.

Table 1

Simple LCM: Inequality by Age

I. $CV(W_t)$

$\beta = \ldots$

Age	0.0 (Random Walk)	.25	.5	.75	1.0 (White Noise)
22	.4001	.4292	.4658	.5082	.5550
27	.4456	.4319	.4200	.4103	.4030
32	.4944	.4585	.4276	.4031	.3862
37	.5439	.4879	.4397	.4026	.3797
42	.5949	.5195	.4542	.4044	.3763
47	.6482	.5535	.4706	.4075	.3741
52	.7052	.5908	.4894	.4116	.3727
56	.7677	.6324	.5064	.4170	.3716
57-76	.7677	.6324	.5064	.4170	.3716

II. $CV(C_t)$

Age	0.0 (Random Walk)	.25	.5	.75	1.0 (White Noise)
22	.4035	.3909	.3815	.3756	.3733
27	.5612	.4901	.4311	.3904	.3738
32	.7492	.6174	.5006	.4126	.3744
37	.9833	.7829	.5970	.4462	.3752
42	1.2867	1.0028	.7315	.4971	.3763
47	1.6971	1.3048	.9225	.5754	.3776
52	2.2822	1.7394	1.2036	.6987	.3795
56	2.9633	2.2478	1.5368	.8521	.3823
57-76	2.9633	2.2478	1.5368	.8521	.3823

Labor force entry age = 22; Retirement age = 57; Age of death = 77.

Table 2

Simple LCM: Sensitivity of $CV(W_t)$ to R and T

Age	$R = 57$ $T = 77$	$R = 62$ $T = 77$	$R = 62$ $T = 72$
22	.4658	.5108	.5842
27	.4200	.4329	.4534
32	.4276	.4366	.4506
37	.4397	.4471	.4583
42	.4542	.4604	.4703
47	.4706	.4760	.4854
52	.4894	.4939	.5037
56	.5064	.5104	.5212
61 to T	.5064	.5343	.5481
Age of Min. W_t	26	28	30
W_R/E_0	12.7	10.9	8.0

Table 3

Keynesian Saving Model: Inequality by Age

I. CV(W)

β = ...

Age	0.0 (Random Walk)	.25	.5	.75	1.0 (White Noise)
22	.4000	.4000	.4000	.4000	.4000
27	.4153	.4012	.3895	.3806	.3747
32	.4575	.4263	.4009	.3825	.3723
37	.4956	.4497	.4120	.3851	.3714
42	.5302	.4715	.4226	.3878	.3709
47	.5619	.4918	.4328	.3905	.3706
52	.5912	.5107	.4424	.3931	.3705
56	.6131	.5251	.4498	.3951	.3703
57-76	.6131	.5251	.4498	.3951	.3703

II. CV(C)

Age	0.0 (Random Walk)	.25	.5	.75	1.0 (White Noise)
22	.4000	.4000	.4000	.4000	.4000
27	.5185	.4699	.4315	.4064	.3972
32	.6058	.5250	.4580	.4121	.3947
37	.6752	.5703	.4807	.4171	.3924
42	.7323	.6084	.5004	.4216	.3903
47	.7804	.6409	.5176	.4257	.3884
52	.8216	.6690	.5327	.4293	.3866
56	.8571	.6889	.5459	.4320	.3853
57-76	.6131	.5251	.4498	.3951	.3703

Labor force entry age = 22; Retirement age = 57; Age of death = 77.

Interest Rate = 5%.

Table 4

Keynesian Saving Model: Sensitivity to Interest Rates

I. CV(W)

$r = ...$

Age	0.0	.05	.1
22	.4000	.4000	.4000
27	.3897	.3895	.3893
32	.4015	.4009	.4002
37	.4134	.4120	.4106
42	.4251	.4226	.4203
47	.4364	.4328	.4293
52	.4475	.4424	.4376
56	.4562	.4498	.4438
57-76	.4562	.4498	.4438

II. CV(C)

Age	0.0	.05	.1
22	.4000	.4000	.4000
27	.4352	.4315	.4280
32	.4678	.4580	.4495
37	.4982	.4807	.4664
42	.5269	.5004	.4798
47	.5541	.5176	.4905
52	.5801	.5327	.4991
56	.6000	.5459	.5048
57-76	.4562	.4498	.4438

Labor force entry age = 22; Retirement age = 57; Age of death = 77.