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GROWTH EFFECTS OF SUBSIDIES IN A SEARCH THEORETIC R&D MODEL: A QUANTITATIVE EVALUATION

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ABSTRACT

When growth depends on technological improvements resulting from R&D investment of the business sector, does it pay to subsidize the business sector? Does it matter whether subsidies are earmarked to R&D, or given as general capital subsidies? Based on a model of perpetual growth through optimal search for better technologies, calibrated on time series data from the Israeli economy, it is shown that capital subsidies produce a definite gain in expected growth rates, but those gain are invariant to the particular restrictions associated with the subsidies. Those restrictions, however, are not completely innocuous, and change the growth rates of total factor productivity.

1. INTRODUCTION

This paper quantitatively explores the impact of various schemes subsidizing investment in R&D and capital on economic growth. This exploration is based on a theoretical model of endogenous technological changes and growth which is calibrated to fit some key features of the Israeli economy. In particular we compare the performance of the model economy under subsidies aimed directly to promote R&D to its performance under more broadly based subsidy schemes.

There is some discussion in the literature concerning the impact of R&D subsidies (e.g. Helpman and Grossman (1991)). However, we are not aware of any analysis which quantitatively compares the growth effects of various subsidy schemes. In particular, the comparison of the effect of direct R&D subsidies and subsidies to capital cannot be undertaken within the framework of most endogenous growth models as these models do not include the accumulation of capital as a possible source of economic growth. Consequently, these models are not suitable to a theoretical comparison of different growth enhancing strategies actually undertaken by governments, such as those described in Young's (1993) discussion of Hong-Kong and Singapore. Our model, which is explicit about the role of technical change and the accumulation of capital, provides an appropriate framework for such an analysis. In fact, our experiment reveals that for the parameter values which fit the Israeli economy, all subsidy schemes are growth enhancing. Their impact on output growth is very similar, but they differ in their effect on total factor productivity growth.

Governmental role in promoting R&D is usually justified by the apparent externalities involved. It is assumed that the private inventor cannot appropriate all the benefits associated with his invention, and therefore the private returns to R&D fall short of the social returns. Indeed, some recent attempts at estimating the social returns come up with exceedingly high numbers.[1] Under such circumstances it seems obvious that government

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[1] For instance, Griliches (1992) found total rates of return on R&D of magnitudes up to 110%, Coe and Helpman (1993) estimated these returns for the

intervention in the form of subsidies is called for. These results are especially striking since economic theory identifies some counter-balancing effects as well. In particular, excessive R&D may be undertaken by the private sector due to the "creative destruction" effect (Jones and Williams (1995)). According to this effect, the monopoly position attained by innovators who replace existing products encourages too many resources to flow into the R&D sector. Another effect which may cause over-investment in R&D is the "congestion effect", which is caused, among other things, by duplication of R&D efforts.

The model used here, which is based on a model of endogenous invention cycles and growth developed earlier (Bental and Peled (1996)), generates only positive externalities of R&D. Accordingly, the growth effects of subsidies we obtain are an upper bound on their actual impact. The R&D externality exists because the knowledge created by the R&D process becomes public after one period. Firms, when determining the amount of R&D they undertake, fail to take this externality into account. As a result, R&D investment may be too low. There are no forces working in the opposite direction. In particular, firms in our model behave competitively in the product market. Therefore, incentives to conduct R&D are not further increased by the monopoly position obtained when a successful innovator replaces an existing producer and no negative externalities are generated by creative destruction.

We model the process of research and development as a search over a distribution of potential "untried" technologies for an improved technology of production.[2] The search for better technologies is conducted by profit maximizing firms in a sequential manner. Observing the outcome of the search process at each stage, the firms choose when to stop searching for further improvements, and adopt the best technology uncovered thus far. This search requires the investment of resources. If not used for search activities, these resources can be added to the existing capital stock, and, combined with

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G7 countries at up to 121.9%.

[2] The idea of modeling R&D as sequential search for better technologies has been suggested, in various forms, by Evanston and Kislev (1975), Nelson and Winter (1982).

labor, produce output using the currently available technology.

At the aggregate level, the discovery of improved technologies generates more output and additional income. Savings, which depend positively on income, increase. As a result, the amount of resources available to the business sectors increases as well. This enhances future search activities, and increases the probability of finding yet additional technological improvements thus perpetuating growth. However, the growth process is hampered by diminishing returns of two kinds. First, the production process of goods is characterized by the usual decreasing returns to capital, so that absent technological improvements, the economy would eventually stop growing. Second, the R&D technology is subject to decreasing returns.[3] Specifically, larger R&D investment is needed to improve upon better technologies. Consequently, as production technologies improve over time, an ever increasing amount of resources must be allocated to the search process in order to find further technological improvements.[4] Thus, to sustain growth, wealth must grow sufficiently fast to facilitate the required increased R&D investments. Under certain assumptions on the distribution of potential technological improvements, the declining R&D productivity is matched by a sufficiently rapid growth of the capital stock, and as a result growth is sustained.

A model of perpetual growth with capital accumulation allows us to examine the growth implications of government capital subsidies to the business sector. We investigate three subsidy schemes which involve a transfer of resources from consumers to the business sector.[5] The first two directly increase the amount

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[3] R&D is usually modeled as a constant returns to scale technology (see, for example, Jovanovic and Rob (1990), Romer (1990), Grossman and Helpman (1991a), and Aghion and Howitt (1992)). This assumption implies that R&D productivity is constant over time (Kortum (1994)). There is substantial evidence, both at the macro and micro levels, suggesting that this implication is counter factual (Griliches (1988), Coe and Helpman (1993), and Kortum (1994))

[4] This feature of the model fits well the observation that while long-run growth seems to be trendless, investment in R&D increases over time (see Jones (1995)).

[5] In a standard neo-classical growth model without technological change, transfers of this kind have only transient effects. Here, growth is bounded from below at a positive rate, thus allowing for possible long run growth implications of government subsidies.

of resources available to search, while the last increases only the amount of production capital. All subsidies are financed by otherwise non-distorting lump-sum taxes which reduce disposable income and private saving. National saving increases as the marginal propensity to save is smaller than unity. Therefore, the tax-subsidy scheme amounts to a forced saving program.

Under the first scheme, firms are allowed to dispose of the transferred resources as they see fit. In particular, they may decide to increase the amount spent on search or use the resources to increase production capital. The second scheme restricts the use of the transferred resources to search only. This scheme tends to increase R&D activities, thereby increasing the probability of finding an improved technology during the search process. However, on average less resources will be devoted to production relative to the previous scheme, and therefore, a numerical assessment is needed to determine which of the two policies is more conducive to growth. Finally, we restrict the firms to use the transferred resources for production purposes only. Superficially this scheme seems to have no impact on R&D. But since the allocation of resources to search activities depends also on the amount of resources to which any discovery will be applied, this subsidy has an indirect effect on search and growth.

The growth effects of these schemes are compared by simulating a calibrated version of the model. The parameters used in the simulations are chosen so as to fit some key features of the business sector of the Israeli economy. In particular, output, labor and capital levels are chosen to fit their observed counterparts in 1990, and the average growth rate of the economy is calibrated to fit the average output growth of the business sector in Israel between 1975 and 1990, assuming that the prevailing regime over that period was that of the unrestricted subsidy.

Using these parameters, we first simulate the economy without any intervention. We then compute the subsidy *levels* which are used for all three policy experiments. Following the typical subsidy rates in use in Israel, we set these at 30% of the average R&D expenditures in the intervention-free path. We find that subsidies impact annual growth, which increases by about 0.4 percentage points under all policies considered, including that of the subsidy restricted to production capital.

The similarity in the growth effects of the three subsidy schemes is not replicated in their impact on total factor productivity growth. The unrestricted subsidy and the subsidy which is restricted to search activities both substantially increase TFP growth over that of the intervention-free path. However, the TFP growth under the production capital subsidy is very similar to that which is obtained when no subsidy is involved. Accordingly, in our simulations, there is no obvious welfare advantage to policies which generate high TFP growth rates. These results may be compared with Young's (1993) findings in his study of Singapore and Hong Kong. Both city-states experienced almost identical growth rates between 1960 and 1985 (about 6% annually), but whereas Hong Kong's growth was due mainly to investment in education and knowledge, Singapore's growth process relied essentially on the accumulation of physical capital via forced national saving. According to our model, both policies are effectively identical in their welfare implications, and no judgment can be passed as to which dominates.

2. AN OVERVIEW OF THE MODEL

2.1 Consumers

We consider a simple variant of the Solow growth model. Time is discrete, and the population (which is also the labor force) grows at an exogenous constant rate. Every person is endowed with one unit of labor which is inelastically supplied. Saving is a fixed proportion out of disposable income.

2.2 Production and R&D

In line with the aggregate nature of the model we represent the business sector by a single firm. Each period this firm generates profits by engaging in two distinct but related activities: (i) at the beginning of the period, the firm may conduct a costly *sequential search* for a new "technology", with search costs financed by capital raised from time t-1 savers; (ii) The remaining resources (net of search costs) are added to the existing capital stock, and are combined with optimally hired labor input to produce output in a constant returns to scale production technology. This technology implements the firm's best discovery in its search activity, or the existing "technological fallback option". The fallback technology is the technology that has been used during the previous period. The firm's profits are returned to the economy, and constitute part of the population's income.

2.3 Optimal R&D Policy

The firm conducts its search by taking random draws from an infinitely large population of "untried" technologies. The firm examines random draws from that population sequentially, incurring a fixed sampling cost per draw paid out of the beginning of period resources. There is no time-cost involved in R&D efforts within the period.

A technology draw completely reveals its productivity level, and the sampling firm can then decide whether to adopt or reject it. Adopting a technology means stopping the search, and investing all remaining resources in that technology. Rejecting means taking at least one more draw from the distribution of technologies. In addition to having at hand the most recently sampled technology, the firm can adopt at any point during the current period the available technological fallback option, and avoid any further search costs. We assume below that draws from the technologies distribution are identically and independently distributed, so that sampling is done "with replacement".[6]

A *search strategy* of the firm at any period specifies the rule by which the firm decides when to stop the process of sequential sampling, given its remaining stock of the resource and the best technology known to the firm at that point in time. The optimal search strategy is characterized by threshold acceptance levels, such that a technology sampled at any stage of the search process is accepted if it exceeds the relevant acceptance threshold, and

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[6] The assumption that a technology draw which is rejected during the current search period cannot be returned to is made for convenience only. The characterization of optimal search "with recall" is more involved, and has little impact on the behavior of the model and its asymptotic properties.

rejected otherwise. As the amount of resources left declines, the acceptance thresholds decrease, and the firm becomes less fastidious (see Appendix A).

2.4 *Government Policies*

We consider three intervention schemes implemented by the government, all of which involve a tax-financed transfer of resources from the population to the firm. Clearly, all schemes increase the capital/labor ratio of the firm. With a fixed and unchanging technology, such increases would reduce the growth rate. The main issue to be examined is to understand the mechanism by which these subsidies affect growth when the technology improves over time endogenously.

The first scheme lets the firm decide how to use the resources transferred to it by the government. In particular, the firm adds this transfer to the resources it obtains directly from the population (implicitly through a capital market) and conducts search and production activities as described above.

In the second and third schemes the government "ear-marks" the funds it transfers to the firm. In the second scheme the firm is obliged to use the transferred amount to conduct R&D (search). In the third scheme the firms can use the government subsidy only as production capital. Thus the effect on R&D activities is indirect - since the capital base on which any new technology would operate is larger, an indirect incentive is created to increase the R&D effort.

2.5 *Equilibrium*

The amount of private saving which is available to the firm every period is predetermined. The amount of the government subsidy (and tax) is also pre-determined (in a way to be described below). Pre-search production capital is the amount of production capital of the previous period, minus depreciation. All of the income generated by the firm is channeled back to the economy, either as wages or as profits. The tax which finances the transfer is paid out of that income. In addition, there are some exogenous leakages (representing the remaining government activities as well as the foreign trade sector), which together determine the disposable income. The population saves a fixed proportion out of this disposable income. Part of the savings is exogenously allocated to non-industrial investments, (such as housing). The remainder becomes available to the business sector.

3. A COMPLETELY SPECIFIED ECONOMY

3.1 *The Economy without Subsidies*

Population grows exogenously at a rate denoted by X_N. Each agent supplies one unit of labor inelastically each period, so that labor supply, L_t, also grows at the rate X_N,

(3.1) $$L_t = (1+X_N)L_{t-1}.$$

Total output at time t is given by:

(3.2) $$Y_t = A\theta_t K_t^{\gamma} L_t^{1-\gamma}, \quad A > 0, \quad 0 < \gamma < 1, \ \theta > 1,$$

where θ_t is the index of the technology actually employed at t, K_t and L_t are, respectively, the capital and labor employed in production at period t.

In order to relate the model to the Israeli economy we have to identify the real life variables which those in the model approximate. We assume that the government runs a balanced budget policy, where tax revenue is used only for government purchases and subsidies to the business sector.[7] The alternative subsidy policies to be considered will differ only in the restrictions associated with them, holding government spending unchanged. Accordingly, disposable income is given by:

(3.3) $$Y^D{}_t = Y_t + (M_t - X_t) - G_t,$$

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[7] In fact the Israeli government ran substantial deficits during the period considered. To focus on the pure subsidy effects we ignore deficit financing.

where Y_t is total income, $M_t - X_t$ is net imports, and government purchases are G_t. Total income consists of the output of the business sector, Y^B_t, and the output of the government sector, Y^G_t, which is included for the purpose of calibrating the model to the Israeli economy.

Letting S_t denote total savings at time t-1, we have:

$$(3.4) \qquad S_t = \beta Y^D_{t-1}, \quad 0 < \beta < 1.$$

In a standard capital accumulation model, S_t is added to the undepreciated amount of capital left from t-1, to form the capital stock at time t. Here S_t is the amount of resources to be allocated to non-industrial investments, production of goods, and search for better technologies, (R&D). We denote by IX_t the non-industrial investment at time t, and regard it as exogenously determined. The amount of resources available to the business sector for R&D and production purposes, is denoted by Q_t, and is given by:

$$(3.5) \qquad Q_t \equiv S_t - IX_t.$$

Out of Q_t, the business sector invests R_t in R&D, a random amount determined during the search process. The rest is added to the stock of production capital. Thus, the law of motion of the stock of *production* capital, K_t, is:

$$(3.6) \qquad K_t = (1-\delta)K_{t-1} + (Q_t - R_t).$$

We regard the undepreciated portion of production capital as *installed* capital, which cannot be used for any purpose except production. We now turn to the description of the process of allocating Q_t between production of goods and technological improvements.

The allocation of new investment between its alternative uses is performed by the business sector during each period. This is done in two sequential but timeless stages: the search stage, and the production stage. First, a production technology - indexed by θ - is found by either adopting a default technology that was used in the previous period, or by investing in search for a better technology. When further search seems unwarranted, the remaining capital is combined with labor to produce the output.

We model the whole business sector as consisting of a single firm, which nevertheless behaves competitively in the labor market. This is a simplifying assumption which serves also to obtain an upper bound on the effect of the subsidies on the resulting growth.[8] The firm ends the search stage and enters the production stage with known levels of its production capital and technology, (k,θ), omitting the period subscript. The only decision left at that stage is the choice of labor input. Taking the wage rate, w, parametrically, the profit maximizing employment level for the firm is:

$$\text{(3.7a)} \qquad \ell^*(k,\theta,w) = k\cdot\left(A\theta\cdot\frac{1-\gamma}{w}\right)^{1/\gamma} .$$

The resulting output and profits are given, respectively, by:

$$\text{(3.7b)} \qquad y(k,\theta,w) = (k\cdot\theta^{1/\gamma})\cdot A^{1/\gamma}\left(\frac{1-\gamma}{w}\right)^{(1-\gamma)/\gamma}$$

$$\text{(3.7c)} \qquad \pi(k,\theta,w) = \gamma(k\cdot\theta^{1/\gamma})\cdot A^{1/\gamma}\left(\frac{1-\gamma}{w}\right)^{(1-\gamma)/\gamma} .$$

The optimal search for better technologies takes the form of a sequential and costly sampling from a pool of technologies. These technologies are described by the cumulative distribution function of their productivity index, $H(\theta)$. We specify $H(\cdot)$ to be the Pareto distribution,

$$\text{(3.8)} \qquad H(\theta) = 1 - \theta^{-\lambda}, \quad \theta \geq 1, \ \lambda > 1.$$ [9]

The firm pays α units out of the new resources made available to it at the beginning of the period for each successive draw. It can stop the process at

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[8] We have examined the behavior of the model with multiple firms in Bental and Peled (1996). The first two moments of the growth process decrease when the number of firms comprising the business sector increases.

[9] When the search is conducted from a Pareto distribution the equilibrium can support sustained growth rate bigger than the population growth rate even when the latter is zero, unlike other endogenous growth models, (Kortum (1994), Jones (1995)).

any point and exploit the most recently found technology, or the default technology available to it from the previous period.

The search strategy is chosen in order to maximize the expected profits in (3.7c). These profits are random as of the beginning the period, being a function of the random results of the firm's search activity, (k,θ), and the wage rate.

Under the above assumptions, the search problem of the firm has the simple "reservation technology" character. The firm continues the drawing process until it finds a technology index which exceeds a threshold function, which depends on its remaining search capital as well as on its already installed production capital. We denote by $\theta^*(q,(1-\delta)K)$ the threshold technology, when q units of investment are still available for R&D and production, and the installed production capital is $(1-\delta)K$. We refer the reader to Appendix A, where it is shown that the threshold function which determines the sequential R&D investment process is the solution to the following recursive relation, for $q \geq \alpha$:

$$(3.9) \qquad \theta^*(q,(1-\delta)K) =$$

$$\text{Max}\left\{\left(1 + 1/(\lambda\gamma-1)\cdot\theta^*(q-\alpha,(1-\delta)K)^{-\lambda}\right)^{\gamma}\cdot\theta^*(q-\alpha,(1-\delta)K)\cdot(1-\alpha/(q+(1-\delta)K))^{\gamma}\ ,\ \theta^0\right\},$$

where θ^0 is the default technology available at the beginning of the current period search. At the beginning of period t, $q = Q_t$ from (3.5), and with each additional draw at that period q is reduced by α. Absent government subsidies to R&D, and given initial conditions consisting of: production capital, K, a default technology, θ^0, labor supply, L, and new resources available to the business sector, Q, the stochastic equilibrium path is completely determined by equations (3.1) - (3.7), the distribution of untried technologies specified in (3.8), and the optimal sequential R&D investment strategy in (3.9).

Growth is driven in this model by R&D which increases θ. However, sustaining the growth requires an ever increasing R&D investment, which is both feasible as the economy becomes richer, and more profitable as the results of R&D are applied to a larger capital base. Accordingly, government policies that subsidize R&D directly, or increase the capital base of the business sector are likely to have growth effects.

3.2 Government Subsidies

We assume that the subsidies are fully financed by taxes. Accordingly, the disposable income is amended to:

$$(3.3') \qquad Y^D_t = Y_t + (M_t - X_t) - (G_t + SUB_t),$$

and a fraction s of that sum is channeled to the business sector in the form of invested savings.

Government subsidies to the business sector affect the optimal search behavior. All three subsidy schemes provide additional incentives to R&D, but they differ from each other in the precise way these incentives are created. We now explain, in terms of the threshold function characterized by (3.9), how each policy works.

Unrestricted Subsidy

With this subsidy (3.5) is amended to:

$$(3.5') \qquad Q_t = S_t - IX_t + SUB_t.$$

Thus this subsidy increases the beginning of period resources that can be allocated at will to R&D or production. From (3.9), this will raise the threshold function used by the searching firm, (holding K fixed). Consequently, we expect this policy to result in more R&D investment, and higher output growth than would be the case absent subsidies. The increased growth stems from higher technology levels and possibly higher levels of production capital.

Search Capital Subsidy

This policy effectively forces the firm to first spend the subsidy amount on search at the beginning of the period, thus possibly improving upon the default technology θ^0 in (3.9). Specifically, the firm will conduct $[SUB_t/\alpha]$

draws up front, and set θ^0 to the best technology among those sampled and the original default technology for that period. It is likely, therefore, that θ^0 in this case will be larger than it is under the previous subsidy. However, after completing this government funded phase of its search, the firm has a smaller amount of resources compared to the amount, (including the subsidy), available under the previous scheme. Therefore, the comparative effects of these subsidy schemes need to be numerically evaluated.

Production Capital Subsidy

The subsidy is added to the installed production capital, $(1-\delta)K$, which appears in (3.9). It can be shown, from (3.9), that increasing the amount of installed capital raises the acceptance threshold given q, thereby creating indirect incentives to R&D investment.

4. CALIBRATION

4.1 *Basic National Accounting*

The basic time series we use as a guideline for the purpose of calibrating the model is that of the output of the business sector in Israel, 1975 - 1990. Growth was quite low during this period, and no major real changes occurred.[10] Business output grew over this period by an average of 3.8% annually. Output was 21.5 billion shekels in 1975, and 37.9 billion shekels in 1990 (all in 1986 prices, see Appendix B).[11] These values are assumed to have been generated by a government policy approximated by our unrestricted capital subsidy, at the rate of 30% of R&D costs, and are used in order to determine some of the key parameters of the model, as described below.

Disposable income is divided into three components: the output of the business sector, the taxes needed to finance the subsidies, and all the rest, according

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[10] The period was characterized by major nominal disturbances, with inflation peaking at 30% per month in June of 1985 and a stabilization program which followed. A major immigration wave has started in 1991, which to this date has increase Israel's population by over 15%.

[11] The average exchange rate in 1986 was 1.5 shekels per $US. From this point we measure all relevant magnitudes in billions of 1986 shekels.

to:

$$(4.1) \qquad Y^D = Y^B + Y^G + (M - X) - (G + SUB),$$

where Y^B is the output of the business sector, Y^G is the output of the public sector, and $Y \equiv Y^B + Y^G$.

Of the ingredients of disposable income, Y^B, the output of the business sector, is endogenous to our model, SUB is determined by us as modelers. The remaining elements are treated by us as fixed in the sense that they do not directly respond to subsidies to the business sector.[12]

The series of $Y^G + (M - X) - G$ during the sample period does not display any clear pattern over the sample period (see Appendix B), so we set its value in the simulation to its average over the sample period, 5.069.

In order to find the amount of new resources made available annually to the business sector, Q, we use the average private sector saving rate of 0.3 out of disposable income to compute private savings, and subtract the non-business investment, IX_t, (mainly government investment in infrastructure). We treat the latter form of investment as exogenous. Using a simple AR(1) model, we find that the non-business investment follows:

$$(4.2) \qquad IX_t = \underset{(2.27)}{1.425} + \underset{(3.99)}{.622} \cdot IX_{t-1}$$

with t-statistics in the parentheses, and $R^2 = .51$. For the purpose of our calibration, we took IX to equal the steady state of this process, rounded off to 4.

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[12] A significant portion of economic activity was generated in Israel during the period by the government, which directly owned some public utilities, some industrial conglomerates, the railroad company and the national airline, as well as several industrial and commercial banks. Although the public sector was responsible for some 28-45% of total output during the period 1975-90, (with that share declining monotonically), we choose to focus on the output of the business sector, which seems to be more responsive to profit maximization than the government owned sector.

4.2 *Parameters Choice and Simulation Method*

As stated above, the production function is given by a constant returns to scale Cobb-Douglas function,

$$(4.3) \qquad F(k,\ell;\theta) = \theta A k^{\gamma} \ell^{(1-\gamma)},$$

where k denotes the production capital, and ℓ denotes labor input.

As we intend to study different scenarios which would apply for hypothetical policy changes, we start the analysis with initial values taken from the last period in our sample, of 1990. Accordingly, the initial value of k is set to equal 70, while ℓ is set to equal 1 (million workers), which are the observed capital stock and employment in the business sector in 1990. The depreciation rate of capital, δ, is set to equal 8% annually, and the growth rate of the labor force, X_N, is set to its average over the period 1973-90 of 1.7% per year. The distribution of untried technologies over which search is conducted has one parameter, λ, as in (3.8).

Remaining to be specified are four parameters, A, α, λ, γ, and some initial conditions for the simulation. Specifically, we have to choose values for the first simulated period for the stock of production capital, $(1-\delta)K_0$, the new resources made available to the business sector, Q_0, and the default technology, θ_0.

The parameter values of γ and λ are jointly determined by the restriction necessary to sustain growth:

$$(4.4) \qquad 1/\lambda + \gamma = 1.$$

Given restriction (4.4), the remaining parameters were chosen as follows: for any (λ,γ) satisfying (4.4), we choose (A,α) so as to match the output and capital stock of the Israeli business sector in 1990, (see below). Then we run simulations of the model for 25 periods to produce an average annual growth rate. This procedure resulted in $\lambda = 2.04$, (and $\gamma = 0.5098$) as the only parameter combination which matches under the unrestricted subsidy regime the observed growth rate of 3.8%.[13]

The scaling parameter of the production technology, A, and the search cost, α are inter-related in a particular form. With initial output of the business sector set to its value of 1990 of 38, and $Y^G + (M - X) - G$ set to about 5, we obtain a disposable income of 43, assuming no subsidies (and no additional taxes). The amount of resources available to the business sector (given the saving rate of 0.3) and the non-business investment of about 4 is approximately 9, which is the value we choose for the initial new capital made available to the business sector, Q_0. To simplify the computations, we momentarily assume that no search takes place, so that the total amount is available for production purposes. To complement the capital stock to 70, we set the value of the installed capital $((1-\delta)K_0)$ to 61. To justify the assumption that no search takes place, we set the default technology, θ_0, to the computed acceptance threshold level $\theta^*(9,61)$, which corresponds to 9 units of "new" capital and 61 units of installed capital. This value was computed for several alternative values of the unit cost of R&D, α, (see equation (3.9)). Finally, given the value of θ_0, the value of A is determined by requiring that output at the initial period be 38.

We simulate 20,000 such trajectories for 25 consecutive periods. The number of simulations was chosen to be large enough to render differences between sample means of the growth rate statistically insignificant given the sample standard deviation. Part of the simulation procedure involved randomizing the initial stock of installed capital so that its mean across all 20,000 simulations is 61.[14] We use the same set of 20,000 values for the initial stocks of installed capital for each of the simulated policies, (including the intervention free regime).

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[13] Notice that the value of γ exceeds the capital share in income of about 0.3. This kind of deviation has been observed and commented upon in previous papers which used endogenous growth models to account for the data (see Romer (1987), Barro and Sala-i-Martin (1992), Young (1993), and Mankiw (1995)).

[14] We draw the initial installed capital stock from a uniform distribution [31,91]. With these values the mean and variance of search activities in the initial period are similar to subsequent periods.

Lacking any data on the cost of a single technology draw, (the unit cost of R&D), α, we tried four different values for that parameter, 0.1, 0.3, 0.6, and 0.9, corresponding to 100 to 900 million shekels of 1986, which constitute 0.26% to 2.37% of the annual output of the business sector in 1990. These seemingly high values of α need a clarification. One should remember that we consider a single "aggregate" searcher, whereas there were about 200 firms officially registered as being involved in R&D in Israel in 1990. Their search efforts were conducted independently, while our single searcher aggregates all their search activities. Accordingly, when dividing the range of 100 to 900 for α by 200 firms we obtain a search cost per firm of 0.5 to 4.5. To put this range in perspective, we translate it to engineer-years, using the average employer's cost per engineer of 90,000 shekels. Thus, the above range for α corresponds to 5 to 45 engineer-years per draw for a single firm.

4.3 Subsidies

Our goal is to compare different restrictions associated with the same amount of subsidies to the business sector, holding everything else constant. Consequently, we use the same subsidy amounts for the different policies that we simulated. The actual subsidy amounts were set using the official subsidy rate of the Israeli government of 30%, according to the Law of Promoting Capital Investment, which covers a broad range of industrial activities, including R&D. We first ran 20,000 simulations of the model economy with no government subsidy, and computed for each of the 25 simulation periods the average R&D investment. We then defined the subsidy for each period of the simulation as 30% of the average R&D expenditure for that period under the intervention-free regime.

This completes the description of the calibration of the model. To summarize, we use $\beta = 0.3$, subsidy rate = 30%, $\lambda=2.04$, $\gamma = 0.5098$, $Q_0 = 9$ (on average), $(1-\delta)K_0 = 61$ (on average), $\theta_0 = \theta^*(9,61)$ given α, $\delta = 0.08$, $Y^G+(M-X)-G = 5.069$, and $IX = 4$. We tried four different values for the sampling cost, α, and for each of those we found a value of A to fix the output at the initial simulation period to 38. The values of A and α under which each simulation was run are reported in Table 1 below.

5. COMPARISON OF ALTERNATIVE POLICIES

As noted before, we have based the calibration on 20,000 simulations, 25-periods long each, of the intervention-free model. We report sample averages of the entire sample period for the policies considered. For each of those we report in Table 1 below the following statistics:

(i) Annual output growth rate, (GR). For each 25-period long simulation we average the 24 annual growth rates, and GR is the sample mean of that statistic over our sample of 20,000 such simulations.

(ii) Annual total factor productivity growth, (TFP). For each simulation, in each period, we compute the improved productivity in the usual way. Letting $Y(t) = \theta(t)K(t)^{\gamma}L(t)^{(1-\gamma)}$, we have $\Delta Y/Y \cong \Delta\theta/\theta + \gamma\Delta K/K + (1-\gamma)\Delta L/L$, so that the measure of improvement in technology over time is approximated by:

$$\Delta\theta/\theta \cong (Y_{t+1}-Y_t)/Y_t - \gamma(K_{t+1}-K_t)/K_t - (1-\gamma)(L_{t+1}-L_t)/L_t.$$

We average this measure over the simulation horizon, and report the sample mean of this statistic across all 20,000 simulations.

(iii) Sample standard deviations. Under the sample mean of each statistic we report (in brackets) its standard deviation in the sample of 20,000 simulations. While these standard deviations of both GR and TFP appear to be large relative to their means, the sample size is large enough to give us confidence in those statistics. In particular, given n i.i.d. observations on g_i, with sample average $\bar{g}$ and sample standard deviation $\hat{\sigma}$, an unbiased estimator of the standard deviation of the sample mean is given by $\hat{\sigma}/(n)^{1/2}$. As can be seen below, the sample standard deviations of the statistics rarely exceed 8 percentage points, so that with 20,000 observations the standard deviation of those statistics is 0.057 of one percentage point. This suggests that a two standard deviations interval of such statistics would be about 0.1 of one percentage point. We ran our simulations with a different seed for the random number generator, and got sample means which differed from those reported by less than 0.1 percentage points for almost all the statistic reported above.

The important implication of this discussion is that while the growth rate over 25 periods generated by our model displays considerable variability, we can compensate for this by large enough samples to get reliable estimates of *mean* growth rates under alternative policies.

Table 1: Annual Output Growth Rates and TFP Under alternative Policies:
(in percentages, standard deviation in brackets)

Policy		Intervention Free		Unrestricted Subsidy		Search Subsidy		Production Subsidy	
Parameters		GR	TFP	GR	TFP	GR	TFP	GR	TFP
α	A								
0.1	0.1	3.38 (6.4)	1.23 (6.0)	3.76 (7.8)	1.51 (7.4)	3.72 (7.6)	1.46 (7.2)	3.74 (6.7)	1.30 (6.4)
0.3	0.17	3.45 (6.6)	1.27 (6.2)	3.76 (7.7)	1.49 (7.3)	3.78 (7.8)	1.51 (7.5)	3.70 (7.8)	1.26 (7.5)
0.6	0.24	3.54 (7.0)	1.34 (6.6)	3.82 (7.9)	1.52 (7.4)	3.83 (8.0)	1.54 (7.6)	3.81 (7.0)	1.33 (6.5)
0.9	0.295	3.40 (7.0)	1.37 (6.7)	3.84 (8.4)	1.65 (8.1)	3.83 (8.4)	1.66 (8.2)	3.76 (7.1)	1.40 (6.8)

Given the above discussion about the standard deviations of the reported sample means, we conclude that differences of 0.2 percentage points or more are significant at the 5% level. Accordingly, the main finding in Table 1 is that the subsidies, regardless of the associated restriction, increase average growth rates over the simulated period by 0.3-0.4 percentage points. This is a non negligible effect, given that the subsidies involved with these policies amounted to about 2% of total annual output, on average. The second conclusion from Table 1 is that there are very small and statistically insignificant differences between average growth rates obtained under the different active policies.

The increase in growth highlights the importance of the incentive to invest in search for better technologies. It is also instructive to note that all three policies have positive R&D incentives. In particular, even the "production capital subsidy", intensifies the R&D effort, by inducing higher threshold for accepted technologies, motivated by the larger capital base on which better technologies, if found, can be applied.

The difference between the policies seems mostly evident in the total factor productivity results. The two policies which allow firms to use the subsidy for search, produce the highest TFP means, and with hardly noticeable differences between them. The third policy which provides only indirect support to R&D shows TFP scores that are hardly distinguishable from the "intervention free" regime, but significantly lower than those of the other two subsidy schemes that directly increase the R&D incentive of the business sector. The differential R&D impact of the different policies is similarly reflected in the shares of R&D expenditures from output, with annual averages of 5.6 to 6.0% for the "intervention free" and the "production capital subsidy" regimes, and 6.9 to 7.4% for the "search capital" and the "unrestricted subsidy" regimes.

Finally, notice that significant differences, up to a factor of 9, in the R&D cost parameter, α, along with the implied changes in the scale parameter A, (needed to generate the initial output level), have no noticeable impact. This robustness is important because there are no direct observations on the value of α.

6. CONCLUSION

This paper demonstrates how a rather abstract model with explicit micro underpinning can be used to obtain meaningful quantitative growth implications of alternative subsidy policies. We have pinned down most parameters by either directly observing their value, (saving rate, subsidy rate and initial conditions), or by matching simulated moments to time series observations, (growth rate). Two parameters, the scale parameter of the technology and the search cost are not separately identified by the data, although the data imposes a joint restriction on their values. Our simulation demonstrates that the results are robust to alternative choices of these two parameters which are consistent with that restriction. All parameters were chosen under the assumption that the observed data was generated under the "unrestricted subsidy" regime. It is therefore remarkable that the simulated growth rates under all other policies considered, including the intervention free regimes, were also insensitive to the choice of the scale and the search cost parameters.

Our analysis clearly indicates that growth promoting subsidies have a quantitatively significant long-run impact. Moreover, this impact is evident and similar under all forms of subsidies considered. The differences between the alternative policies manifest themselves in the total factor productivity growth. Whereas policies aimed at promoting R&D generate growth through TFP, the capital subsidy works through the more traditional channel of factor accumulation.

The impact of the policies could also be evaluated by comparing them period by period. We found this to be a much less reliable way of evaluating the policies due to the large variance in the performance of the simulated economy in any given period, even under a fixed policy: the period by period growth rate averages show much larger variability than the 25-period growth rate averages reported in Table 1.[15] This is hardly surprising given that the Pareto distribution with $\lambda = 2.04$, which was used to represent the pool of untried technologies, has a variance of 23.11. We believe that the 25-period averages reflect better the differential long-run impacts of the alternative policies.

The overall desirability of the intervention schemes depends on the usual

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[15] For instance, the 10th period growth rate average in our 20,000 simulations under the intervention free regime was 3.1% with standard deviation of 17.9%. With such high standard deviations, differences of 0.4 of one percentage point would be insignificant at the 5% level.

trade off between short-term sacrifice and long-term gains. In our case, an annual tax of about 2% (on average) of total output was necessary to finance the subsidies, which in turn increased annual growth rates from about 3.4% to 3.8%. Since the tax reduces disposable income and consumption, we may use the length of the period during which consumption (and output) return to the intervention-free level as some crude measure of cost-benefit. With an increase in annual growth rate of 0.4% this pay back period amounts to 5 years.[16] Any increase in the subsidy levels tends to increase the gain in the growth rates, but also extends the pay back period. Thus, the choice of an "optimal" subsidy rate can be obtained only with an explicit intertemporal welfare measure. Furthermore, given the similar growth effects of the alternative policies considered here, the choice of a subsidy method must depend on additional considerations, such as simplicity in administration.

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[16] The five years pay back period is based on, $0.98 \cdot (1.038)^t = (1.034)^t$, which implies $t=5.23$.

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APPENDIX A: Optimal Search Strategy

Let $\varphi(q,K,\theta)$ be the profit to the searcher's when the technology θ is operated with the installed capital, $(1-\delta)K$, plus the remaining q units of new capital. Given the assumed Cobb Douglas production function, $A\theta k^{\gamma}\ell^{1-\gamma}$, when labor is hired optimally at the wage rate w, we have:

$$(A.1) \qquad \varphi(q,K,\theta) = A^{1/\gamma}\gamma\cdot\left(\frac{1-\gamma}{w}\right)^{(1-\gamma)/\gamma}\cdot\left[q+(1-\delta)K\right]\theta^{1/\gamma}.$$

The searcher seeks to maximize the expected value of $\varphi(\cdot)$ by choosing a strategy that maps sampled technologies and remaining new capital into the binary decision "accept" or "reject". Accepting means stopping the search and operating the technology at hand, θ, with all available capital, $q+(1-\delta)K$. Rejecting means sampling at least once again, at the cost of α units of new capital. The search is conducted over draws from the distribution $H(\theta)$, $\theta \in [\underline{\theta},\bar{\theta}]$. As noted in the text, we choose the Pareto distribution, where $H(\theta) = 1 - \theta^{-\lambda}$, $\theta \in [1,\infty]$, $\lambda \geq 1$.

The Bellman equation that summarizes the optimal decision is:

$$(A.2) \qquad V(q,K,\theta) = \text{Max}\left\{\varphi(q,K,\theta)\ ,\ E\,V(q-\alpha,K,\tilde{\theta})\right\},$$

where the expectations are taken with respect to the random result of the new draw, $\tilde{\theta}$. Solving (A.2) yields the optimal search strategy, to be denoted $\theta^{*}(q,K)$, such that the search process is stopped, and the technology θ is utilized with $q+(1-\delta)K$ units of capital as soon as $\theta \geq \theta^{*}(q,K)$.

Since $\varphi(q,K,\theta)$ increases in θ, (see (A.1)), the threshold technology level can be found by equating the two terms in the maximand in (A.2), utilizing the fact that:

$$(A.3) \qquad V(q-\alpha,K,\theta) = \begin{cases} \varphi(q-\alpha,K,\theta), \text{ if } \theta \geq \theta^{*}(q-\alpha,K) \\ \\ \varphi(q-\alpha,K,\theta^{*}(q-\alpha,K)), \text{ if } \theta < \theta^{*}(q-\alpha,K). \end{cases}$$

In particular, (A.3) implies:

$$(\text{A.4}) \qquad E\, V(q-\alpha,K,\tilde{\theta}) = H\Big(\theta^*(q-\alpha,K)\Big)\cdot\varphi\Big(q-\alpha,K,\theta^*(q-\alpha,K)\Big) + \int_{\theta^*(q-\alpha,K)}^{\infty}\varphi(q-\alpha,K,\theta)dH(\theta).$$

Equating the two terms in the maximand of (A.2) using (A.4), together with the particular specification of $\varphi(\cdot)$ and $H(\cdot)$, we get $\theta^*(q,K)$ as the solution to the recursive relation:

$$(\text{A.5}) \qquad \Big(q+(1-\delta)K\Big)\cdot\theta^*(q,K)^{1/\gamma} = \Big(1-\theta^*(q-\alpha,K)^{-\lambda}\Big)\Big(q-\alpha+(1-\delta)K\Big)\cdot\theta^*(q-\alpha,K)^{1/\gamma} + \int_{\theta^*(q-\alpha,K)}^{\infty}\Big(q-\alpha+(1-\delta)K\Big)\theta^{1/\gamma}\lambda\theta^{-\lambda-1}d\theta.$$

Equation (A.5) allows us to solve for $\theta^*(q,K)$ recursively. Specifically, for any initial quantity of new capital, Q, we start from $q_0 \equiv Q - \alpha\cdot[Q/\alpha]$, where $\theta^*(q_0,K) = \underline{\theta}$, and use (A.5) to find $\theta^*(q_0+\alpha,K)$, $\theta^*(q_0+2\alpha,K)$, ..., $\theta^*(Q,K)$. This procedure yields equation (3.9) in the text, where the remaining new capital is simply denoted by q.

APPENDIX B.

Israeli Data: 1975 - 1990, in 1986 millions of shekels, except N_b which is in thousands of workers.

YEAR	Y^b	Y^g	G	M-X	I^g	I^d
1975	21533	10448	13299	8924	1446	3278
1976	22308	10185	12086	7120	1191	2993
1977	22509	10601	10507	5455	1070	2358
1978	23364	11070	11651	6702	1160	2304
1979	24462	11522	10621	6867	504	2569
1980	25318	11782	14590	4947	889	3386
1981	26786	11984	15538	6064	912	3457
1982	26813	12383	14575	7303	947	3347
1983	27725	12568	13992	8366	1100	3193
1984	28446	12774	14817	6293	964	2991
1985	29982	12804	15410	4863	933	2632
1986	31691	12678	13946	5953	978	2115
1987	34166	12811	16327	8451	1239	2290
1988	35026	13200	16027	8176	1173	2340
1989	35711	13287	14601	5620	1219	2503
1990	37931	13547	15225	7372	938	2939

YEAR	YX	XI	K_b	N_b
1975	6072.06	4724.31	45173.16	877
1976	5218.85	4184.18	48500.25	882
1977	5548.52	3427.76	47270.75	903.4
1978	6120.1	3464.22	49161.58	934.1
1979	7767.47	3073.06	51472.17	956.5
1980	2139	4275.83	54097.25	961.6
1981	2510	4369.22	55828.36	976.5
1982	5111	4293.61	57447.39	992
1983	6942	4292.96	59228.26	1033.3
1984	4250	3955.33	61952.76	1048.6
1985	2257	3565.05	63873.29	1050
1986	4685	3092.52	65342.38	1054.3
1987	4935	3528.78	66910.59	1102.2
1988	5349	3513.26	68917.91	1135.7
1989	4306	3721.97	70571.94	1130.8
1990	5694	3876.65	71559.95	1154.5

Legend:

Y^b - Output of the business sector

Y^g - Output of the public sector

G - Government consumption

M-X - Net import

I^g - Public investment

I^d - Housing investment

$YX = Y^g + (M-X) - G$

$XI = I^g + I^d$

K_b - Capital stock of the business sector

N_b - Employment in the business sector

Data Sources:

Bank of Israel: Annual Report, various years.

Central Bureau of Statistics: Statistical Abstracts of Israel, various years.