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How to Escape Dutch Books in Dynamic Choice

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Abstract

This paper discusses two ways in which decision makers facing sequential choices can make themselves invulnerable to Dutch books. The first is based on the observation that if preferences are quasi concave, then even if Y is preferred to X , there may be an $\alpha \in (0, 1)$ such that $\alpha X + (1 - \alpha)Y$ is even better. Therefore, a decision maker with such preferences will prefer to randomize when he is offered to replace X with Y . The second situation in which Dutch books are ineffective is when the decision maker knows what zero probability event happened in the first period.

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1 Introduction

One of the strongest normative justifications for the independence axiom is based on the assertion that violations of this axiom expose decision makers to Dutch book manipulations. That is, these subjects can be manoeuvred into accepting a sequence of transactions that leave them in a position which is stochastically dominated by their initial position.

Suppose that a decision maker prefers X to Y but $pY + (1 - p)Z$ is preferred to $pX + (1 - p)Z$. He begins holding the compound lottery $(X, p; Z, 1 - p)$.¹ For a small $\varepsilon > 0$ he is willing to exchange it for the lottery $(Y, p; Z - \varepsilon, 1 - p)$.² If the Z event occurs, he has lost ε . If the Y event occurs, the decision maker (who prefers X to Y) is willing to trade Y for $X - \varepsilon$. Ex post the decision maker ends up trading $(X, p; Z, 1 - p)$ for $(X - \varepsilon, p; Z - \varepsilon, 1 - p)$, thus losing ε for sure (see Green [7] and Machina [10]).

It turns out that the successful execution of this procedure strongly depends on the way decision makers update preferences. Green [7] shows that this kind of Dutch book fails when preferences are quasi convex (see a discussion in Section 5 below). Machina [10] and McClennen [11, 12] show that Dutch books can be avoided if preferences at each node are updated to agree with the predicted induced preferences for this node. Therefore, preferences depend on events and outcomes that could occur but did not. This requirement is weakened in Segal [13] to prevent updated preferences from converging to expected utility. Wakker [14] on the other hand claims that updated preferences should also depend on decisions that could have been made, but were never made because the decision maker did not reach the relevant decision nodes.

These models assume that if a decision maker is offered an option that is superior to the one he holds, he will take it. This is not necessarily true, because the decision maker may prefer to flip a coin over what he holds and what is offered to him. This is the case when preferences are quasi concave. In Section 3 I show that a decision maker with such preferences cannot be manipulated.

¹Assume that preferences satisfy the reduction of compound lotteries axiom. By this axiom, the decision maker is interested only in the probability of reaching a certain outcome and not in the probabilities of the sequence of stages leading to it.

²For a lottery Z and a number ε , the lottery $Z + \varepsilon$ is obtained from Z by adding ε to all possible outcomes in Z .

The major aim of this paper is to show that the vulnerability of a decision maker to Dutch books depends not only on his initial preferences and on the way these preferences are updated, but also on the information that is revealed to him. It turns out that unless they know exactly what state of the world happened, some decision makers will be vulnerable to manipulations. This is not due to an asymmetry of information between the decision maker and the possible manipulators. Everyone has precisely the same information in the present model. But having full information about what happened in the past enables the decision maker to update his preferences in a way that will immunize him against manipulation. This point is discussed in Section 4.

The next section surveys some notions of dynamic consistency without the expected utility hypothesis. Some references to the literature and few remarks on the present approach are delayed to section 5. Proofs appear in an appendix.

2 Dynamic Consistency

The Dutch book in the introduction depends on the assumption that preferences over lotteries do not change from one period to another. Therefore, if X is preferred to Y at period 0 ($X \succeq_0 Y$), then these preferences will also hold at period 1, after the decision maker knows that he won a ticket for X in the lottery $(X, p; Z, 1 - p)$. (That is, $X \succeq_1 Y$). To solve the problem posed by this Dutch book, Machina [10] and McClennen [11, 12] suggest that preferences change after some uncertainty is resolved. Suppose that in the previous period the decision maker played the lottery $(X_1, p; X_2, 1 - p)$, and the p -probability event happened. If he is now asked for his preferences between X and Y , the decision maker should not ask himself whether he preferred X or Y last period, but whether he preferred $(X, p; X_2, 1 - p)$ or $(Y, p; X_2, 1 - p)$ then. Formally,

Axiom 1 *Suppose that at period 0 the decision maker's preferences were \succeq_0 . If at period 0 he played the lottery $(X_1, p_1; \dots; X_n, p_n)$ and won the lottery X_i , then his preference relation \succeq_1 at period 1 is given by*

$$\begin{aligned} \forall X \forall Y, X \succeq_1 Y &\iff \\ (\dots; X_{i-1}, p_{i-1}; X, p_i; X_{i+1}, p_{i+1}; \dots) &\succeq_0 \\ (\dots; X_{i-1}, p_{i-1}; Y, p_i; X_{i+1}, p_{i+1}; \dots) & \end{aligned}$$

The updated preferences in period 1 depend on what could have happened, but did not happen, and on the probability p_i of the event that did happen. Obviously, updating preferences according to Axiom 1 immunizes the decision maker against the Dutch book of the introduction. However, it turns out that as the probability p_i becomes small, the induced order \succeq_1 becomes arbitrarily close to expected utility (see Border and Segal [1]). This has two obvious implications. First, if the initial uncertainty is continuous, rather than discrete then, whatever the outcome of the lottery at period 0, the probability of receiving this outcome is zero. Therefore, the updated preferences in this case must be expected utility. Secondly, over time, as more and more uncertainty is resolved, the probability of reaching any particular node of the multi stage decision tree goes to zero. Therefore, even if at each period's uncertainty is discrete, over time, preferences will converge to expected utility.

To solve this problem, I suggested in [13] a weaker concept of dynamic consistency. To prevent the above Dutch book, it is sufficient to restrict only the updated preferences between the actual outcome of period 0 and other possible lotteries. Formally,

Axiom 2 *Suppose that at period 0 the decision maker's preferences were \succeq_0 . If at period 0 he played the lottery $(X_1, p_1; \dots; X_n, p_n)$ and won the lottery X_i , then his preference relation \succeq_1 at period 1 must satisfy the following restriction.*

$$\begin{aligned} \forall Y, X_i \succeq_1 Y &\iff \\ (\dots; X_{i-1}, p_{i-1}; X_i, p_i; X_{i+1}, p_{i+1}; \dots) &\succeq_0 \\ (\dots; X_{i-1}, p_{i-1}; Y, p_i; X_{i+1}, p_{i+1}; \dots) & \end{aligned}$$

Preferences that are updated according to this axiom do not have to converge to expected utility, as this axiom limits only one indifference curve of the updated preferences, the one through the actual outcome of the lottery of period 0. (Note that the updated preferences defined by Axiom 1 also generate this indifference curve).

Suppose that the original distribution at period 0 is continuous over a set of lotteries A (see Figure 1). Denote the reduced form lottery of this distribution by Z_0 . So Z_0 is the average lottery the decision maker anticipates receiving once the uncertainty of the first period is resolved. In [13] I proved

that if the outcome of this lottery (over lotteries) is the lottery X , then at period 1 the indifference curve through X of the new preferences \succeq_1 will be affine.³ Moreover, this indifference curve is derived from the local utility (Machina [9]) function $U_0(\cdot; Z_0)$ of the preference relation \succeq_0 at Z_0 . This claim has a very simple geometric interpretation. The indifference curve of \succeq_1 through X (denoted $I_1(X)$) is affine, and is parallel to the tangent at Z_0 to the indifference curve of \succeq_0 through Z_0 (denoted $I_0(Z)$). Other indifference curves of \succeq_1 are not restricted, beyond the obvious fact that they cannot intersect each other.

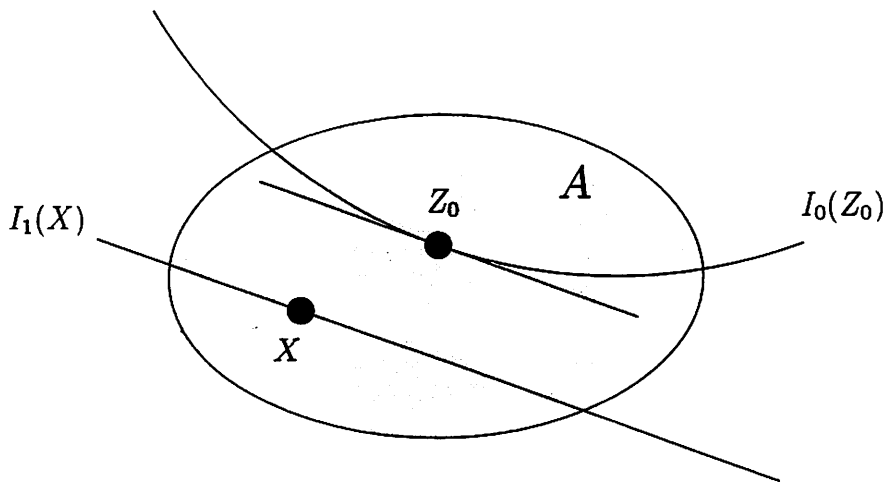


Figure 1: The indifference curve of \succeq_1 through X .

In the next section I show under what conditions the updated preferences of Axioms 1 and 2 make the decision maker invulnerable to Dutch books. For this I need a formal definition of such manipulations. To conform with the literature, I present the definition in terms of random variables. They are denoted $\{X_\beta, s_\beta\}_{\beta \in \mathcal{B}}$, where $\{s_\beta\}_{\beta \in \mathcal{B}}$ is a partition of the sure event.

Definition 1 *The decision maker's preferences admit a Dutch book if there are two distributions over lotteries $F = \{X_\beta, s_\beta\}_{\beta \in \mathcal{B}}$ and $G = \{Y_\beta, s_\beta\}_{\beta \in \mathcal{B}}$ such that*

1. *If at period 0 the decision maker holds G , and is offered to replace it with F , he will agree, but*

³An indifference curve I is affine if $Y, Y' \in I$ implies for all $\alpha \in [0, 1]$, $\alpha Y + (1-\alpha)Y' \in I$.

2. For all $\beta \in \mathcal{B}$, if s_β happens the decision maker will agree to replace X_β with Y_β .⁴

3 Dutch Books and Quasi Concavity

I will use the following definitions below.

Definition 2 Let \succeq be a preference relation over lotteries.

- \succeq is quasi concave if $X \sim Y$ implies $\forall \alpha \in (0, 1)$, $\alpha X + (1 - \alpha)Y \succ X$.
- \succeq is quasi convex if $X \sim Y$ implies $\forall \alpha \in (0, 1)$, $X \succ \alpha X + (1 - \alpha)Y$.
- \succeq satisfies betweenness if $X \sim Y$ implies $\forall \alpha \in [0, 1]$, $\alpha X + (1 - \alpha)Y \sim X$ (see Chew [3], Dekel [4], and Fishburn [6]).

It is sometimes argued (see footnote 5 below) that the Machina-McClenen's notion of dynamic consistency implies betweenness. These arguments apply for Axiom 2 as well. In this section I show that if anything, these arguments imply that preferences should be quasi concave. In the next section I show that these arguments fail if preferences are updated with respect to zero probability events, regardless of the nature of the original preferences.

Consider the following situation. At period 0 a fair coin is tossed and the decision maker has the following preferences (which violate betweenness):

$$(X, H; Y, T) \sim_0 (Y, H; X, T) \succ_0 (X, H; X, T) \succ_0 (Y, H; Y, T) \quad (1)$$

Suppose that the decision maker holds the lottery $(Y, H; Y, T)$. We assume that all preferences are continuous, therefore, for a sufficiently small ε , each of the following transactions will be accepted.

Procedure 1

1. Offer the decision maker to exchange $(Y, H; Y, T)$ for $(X, H; X, T)$.
2. (a) If H happens, offer to replace his outcome with $Y - \varepsilon$. According to Axiom 2, and certainly according to Axiom 1, the decision maker should make this exchange for a sufficiently small ε .

⁴By continuity, it is sufficient that only one of the preferences, either at period 0 or at period 1, will be strict.

- (b) If T happens. offer to replace his outcome with $Y - \varepsilon$. Similarly, he should agree. ■

The decision maker is thus willing to replace the lottery $(X, H; X, T)$ with the lottery $(Y - \varepsilon, H; Y - \varepsilon, T)$, which is inferior not only to $(X, H; X, T)$, but is dominated by his initial holding $(Y, H; Y, T)$.⁵

But this analysis is misleading. Consider again the preferences given by eq. (1). The decision maker initially holds $(Y, H; Y, T)$. The first step in the manipulation assumes that when offered a chance to replace it with $(X, H; X, T)$ he will take the latter lottery. This is not true, as the decision maker may randomize. Denote the utility functional that the decision maker employs at period 0 by V_0 , and let $\alpha^* \in [0, 1]$ maximize $V_0(\alpha X + (1 - \alpha)Y)$. We know that $\alpha^* \in (0, 1)$. Let H^* be an event that is statistically independent of H and T , such that $\Pr(H^*) = \alpha^*$ and let T^* be the event “not H^* .” When he is offered a choice between X and Y , the decision maker can choose any convex combination $\alpha X + (1 - \alpha)Y$. Therefore, when we offer to replace Y by X , he will replace Y by the lottery $Z = \alpha^* X + (1 - \alpha^*)Y$ and play the random variable $(X, H \cap H^*; X, T \cap H^*; Y, H \cap T^*; Y, T \cap T^*)$. Suppose that he played this lottery and H happened. According to step 2(a) in Procedure 1, he is now offered a chance to replace the outcome he holds with $Y - \varepsilon$. If T^* also happened, he will certainly refuse, as he already holds Y . But Axioms 1 and 2 imply that he will decline this offer even if H^* happened and he holds X . To see why, let $\alpha' = \Pr(T \cap H^*)$. Then by the definition of α^* ,

$$\begin{aligned} (X, H \cap H^*; X, T \cap H^*; Y, H \cap T^*; Y, T \cap T^*) &\sim (X, \alpha^*; Y, 1 - \alpha^*) \succeq_0 \\ (X, \alpha'; Y, 1 - \alpha') &\sim (Y, H \cap H^*; X, T \cap H^*; Y, H \cap T^*; Y, T \cap T^*) \succ_0 \\ (Y - \varepsilon, H \cap H^*; X, T \cap H^*; Y, H \cap T^*; Y, T \cap T^*) & \end{aligned}$$

Similarly, the decision maker will refuse to replace his holding with $Y - \varepsilon$ in case T happens. So although he is willing to utilize new options that are offered to him, the decision maker will not fall for the Dutch book of Procedure 1. The following lemma generalizes the above analysis.

⁵This problem was pointed out to Machina by Dekel and Segal and discussed by him in Section 6.5 of [10]. Machina’s analysis is completely different from the one offered here. I discuss his analysis in Section 5 below.

Lemma 1 *Suppose that the decision maker updates preferences according to Axiom 1 or to Axiom 2, and that his preferences at period 0 are quasi concave. Then the decision maker is not vulnerable to the Dutch book of Definition 1.*

4 Dutch Books and Quasi Convexity

Consider now the following preferences (which are compatible with quasi convexity).

$$(X, H; X, T) \succeq_0 (Y, H; Y, T) \succ_0 (X, H; Y, T) \sim_0 (Y, H; X, T) \quad (2)$$

Let the decision maker hold the lottery $(Y - \varepsilon, H; X - \varepsilon, T)$. Assume for simplicity that for every ε , $(X - \varepsilon, H; Y - \varepsilon, T) \sim_0 (Y - \varepsilon, H; X - \varepsilon, T)$, and expose the decision maker to the following manipulation. Here too, for a sufficiently small ε , all transactions will be accepted.

Procedure 2

1. Offer the decision maker to exchange $(Y - \varepsilon, H; X - \varepsilon, T)$ for $(X, H; Y, T)$.
2. (a) If H happens, offer to replace his outcome with $Y - 2\varepsilon$. As in Procedure 1, it will be accepted.
- (b) If T happens, offer to replace his outcome with $X - 2\varepsilon$. He should similarly agree. ■

The decision maker is thus playing $(Y - 2\varepsilon, H; X - 2\varepsilon, T)$, which is clearly inferior to $(Y - \varepsilon, H; X - \varepsilon, T)$.

This problem seems to be similar to the one posed by Procedure 1. Indeed, Machina's [10] analysis handles this problem similarly to the way it handles the former problem (see next section). The analysis of section 3, however, cannot be extended in the same way. This analysis shows that if the decision maker's preferences are quasi concave, then he may not replace Y by X , but by a mixture of the two. However, the preferences depicted by eq. (2) are quasi convex and mixing will not be the decision maker's best choice.

The success of Procedure 2 depends strongly on the information the decision maker and the manipulator have at the end of period 0. Suppose that the uncertainty in period 0 is about the outcome of a uniform distribution

over $[0, 1]$, where $H = [0, 0.5]$ and $T = (0.5, 1]$. In this case, each resolution of the uncertainly has zero probability. Therefore, the indifference curve of the updated preferences through the outcome of period 0 (X or Y) will be affine and will be parallel to the tangent to the indifference curve of \succeq_0 at $Z_0 = (X, \frac{1}{2}; Y, \frac{1}{2})$ (see Figure 1).

Consider the case depicted by Figure 2. The decision maker went through the first stage of Procedure 2, and is now holding $(X, H; Y, T)$. Suppose the outcome of the uniform distribution of period 0 is in H , and the decision maker holds X . Since Y lies below the new indifference curve $I_1^H(X)$ through X , the decision maker will not accept the offer to replace X with $Y - 2\varepsilon$. ($I_1^S(\cdot)$ is the indifference curve through \cdot when a number in S is observed, $S = H, T$). On the other hand, if the observed value is in T , then the decision maker receives Y , and X is above the updated indifference curve $I_1^T(Y)$ through Y . In this case he will agree to replace Y with $X - 2\varepsilon$, thus playing $(X, H; X - 2\varepsilon, T)$. For a sufficiently small ε , this lottery is indeed superior to $(Y - \varepsilon, H; X - \varepsilon, T)$. Similar analysis implies that the decision maker will end up with $(Y - 2\varepsilon, H; Y, T)$ if Y is above $I_1^H(X)$ (and therefore X is below $I_1^T(Y)$). Once again, for a small ε , this lottery is superior to $(Y - \varepsilon, H; X - \varepsilon, T)$. Finally, if Y is on $I_1^H(X)$ (and X is on $I_1^T(Y)$), the decision maker will keep $(X, H; Y, T)$, which he prefers to $(Y - \varepsilon, H; X - \varepsilon, T)$. Note that the above analysis strongly depends on the fact that after a zero probability event happens, the updated indifference curve through the new holding is affine, and parallel to the tangent at Z_0 to the indifference curve of \succeq_0 through Z_0 . This is why if Y is below (on, above) $I_1^H(X)$, then X is above (on, below) $I_1^T(Y)$.

The following lemma generalizes the above analysis. I still assume that when the decision maker is offered a choice between two lotteries, he will pick the best randomization over the two. Without this assumption, it is possible to create examples where the decision maker updates preferences with respect to zero probability events, but still falls for a Dutch book as in Definition 1.

Lemma 2 *Suppose that the decision maker updates preferences according to Axiom 1 or to Axiom 2. If all events in the partition \mathcal{B} have zero probability, then the decision maker is not vulnerable to the Dutch book of Definition 1.*

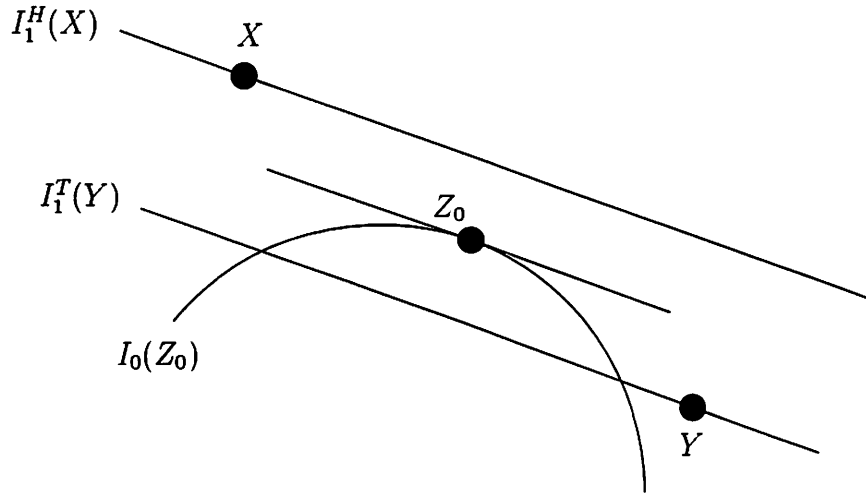


Figure 2: The case where at period 1 X is preferred to Y .

5 Discussion

In this section I discuss three different approaches to dynamic consistency, offered by Green [7], Machina [10] and McClennen [11, 12], and Wakker [14]. Green shows that a decision maker is *not* vulnerable to Dutch book manipulations only if his preferences are quasi convex. This seems to be contradicted by Lemma 1 above. But Green's model is completely different from the one discussed above (and indeed by all the rest of these papers) as he assumes that preferences do not change from one period to the next.

Machina's [10, Section 6.5] response to the Dutch books described by Procedures 1 and 2 is that they represent hidden nodes. If the decision maker is aware of these procedures, then he will plan to accept an exchange if H happens, but not if T , or if T happens, but not if H . However, since such a decision is unobservable by any other agent, it is not clear how such a theory can be verified. Whatever the outcome of the first period, the decision maker's choices may be dynamically consistent regardless of whether or not he is willing to accept an exchange.

Wakker's model is aimed to show that, unlike Border and Segal's [1] claim, updated preferences after the realization of small probability events do not

have to be close to expected utility. The solution he offers is the following. Assume that after the uncertainty of the first period is resolved the decision maker finds himself at decision node N . For each possible option Z he now has, he will find what would have been the best plan (at period 0) for all other nodes, given that at node N he is to play Z . Denote this plan $A(Z)$. The decision maker will choose the option Z that maximizes the ex ante utility from the combined lottery $(Z, A(Z))$.

Although this program implies that if Z was optimal for N at period 0, it is still optimal after the decision maker reaches node N , it is not clear how this theory handles the Dutch books presented above. For this, the decision maker has to know not only what is offered to him now, but also what would have been offered to him at other nodes. Since beliefs are not observable, any kind of behavior can be supported by this notion of dynamic consistency, as there can always be a set of beliefs about other possible offers that would support any transaction that does not violate first order stochastic dominance. The advantage of the analysis presented in this paper is that it does not require any knowledge about what could have happened in other nodes, and it does not use any unobservable data.

The analysis presented in this paper assumes that both the decision maker and the manipulator have the same information. Another assumption is that both follow probability theory in updating their beliefs. For Dutch books involving violations of the reduction of compound lotteries axiom and Bayes' rule, see Epstein and Le Breton [5], Border and Segal [2], or Kelsey [8].

A Proofs

Proof of Lemma 1: Let F and G be as in Definition 1, where $F \succ_0 G$. To simplify notations, assume that \mathcal{B} is a finite partition, and write $F = (X_1, s_1; \dots; X_n, s_n)$ and $G = (Y_1, s_1; \dots; Y_n, s_n)$. Suppose that the decision maker holds G , and is offered to replace it with F . Let $\alpha^* = \arg \max_{\alpha \in [0,1]} V_0(\alpha F + (1 - \alpha)G)$, and let H^* be an event, independent of the events s_1, \dots, s_n , such that $\Pr(H^*) = \alpha^*$. As before, let T^* be the event "not H^* ." Obviously, $\alpha^* > 0$. The decision maker will now choose to play $(F, H^*; G, T^*)$, for which he will be willing to pay a positive amount of money. So he now plays

$$F^* = (X_1, s_1 \cap H^*; Y_1, s_1 \cap T^*; \dots; X_n, s_n \cap H^*; Y_n, s_n \cap T^*)$$

Suppose that for every $s_i \in T^*$, the decision maker is willing to pay to replace the outcome of F^* with Y_i . For $j = 1, \dots, n$, let

$$G_j = (X_1, s_1 \cap H^*; Y_1, s_1 \cap T^*; \dots; Y_j, s_j; \dots; X_n, s_n \cap H^*; Y_n, s_n \cap T^*)$$

According to Axioms 1 and 2, such preferences are possible only if $G_j \succ_0 F^*$ for all $j = 1, \dots, n$. I now show that these preferences, together with F^* being the best mixture over F and G , violate the assumption of quasi concavity. Let t_1, \dots, t_n be independent of s_1, \dots, s_n and of H^* such that for every i , $\Pr(t_i) = \frac{1}{n}$, and let $G^* = (G_1, t_1; \dots; G_n, t_n)$. Since for every i , $G_i \succ_0 F^*$, it follows by quasi concavity that $G^* \succ_0 F^*$. But

$$G^* = \frac{n-1}{n} \alpha^* F + \left(1 - \frac{n-1}{n} \alpha^*\right) G$$

In contradiction with the definition of α^* . ■

Proof of Lemma 2: Let F , G , α^* , H^* , and F^* be as in the proof of Lemma 1. Denote the local utility at F^* by $U(\cdot; F^*)$. By [13, Theorem 1] (see Figure 1 above), when the decision maker learns that event s_β happened, he will be willing to replace X_β (with distribution function F_β) with Y_β (with distribution function G_β) if and only if

$$\int U(x; F^*) dG_\beta(x) \geq \int U(x; F^*) dF_\beta(x) \quad (3)$$

Suppose that there exists \mathcal{B}^* with positive probability such that for $\beta \in \mathcal{B}^*$ the above inequality is strict. Since G is a distribution over the lotteries Y_β , it follows from eq. (3) that

$$\int U(x; F^*) dG(x) \geq \int U(x; F^*) dF^*(x)$$

But then, for α sufficiently close to 1, $\alpha F^* + (1-\alpha)G \succ_0 F^*$, in contradiction with the definition of α^* . ■

References

- [1] Border, K.C. and U. Segal, 1994(a): "Dynamic consistency implies approximately expected utility," *Journal of Economic Theory*, 63, 170–188.

- [2] Border, K.C. and U. Segal, 1994(b): "Dutch book arguments and subjective probability," *Economic Journal*, 104, 71-75.
- [3] Chew, S.H., 1983: "A generalization of the quasilinear mean with applications to the measurement of income inequality and decision theory resolving the Allais paradox." *Econometrica*, 51, 1065-1092.
- [4] Dekel, E., 1986: "An axiomatic characterization of preferences under uncertainty: Weakening the independence axiom," *Journal of Economic Theory*, 40, 304-318.
- [5] L.G. Epstein M. LeBreton. Dynamically consistent beliefs must be Bayesian, *J. Econ. Theory*, 61 (1993), 1-22.
- [6] Fishburn, P.C., 1983: "Transitive measurable utility," *Journal of Economic Theory*, 31, 293-317.
- [7] Green, J.R., 1987: " 'Making book against oneself,' the independence axiom and non-linear utility theory," *Quarterly Journal of Economics*, 102, 785-796.
- [8] Kelsey, D., 1995: "Dutch books arguments and learning in a nonexpected utility framework," *International Economic Review*, 36, 187-206.
- [9] Machina, M.J., 1982: " 'Expected utility' analysis without the independence axiom." *Econometrica*. 50. 277-323.
- [10] Machina, M.J.. 1989: "Dynamic consistency and non-expected utility models of choice under uncertainty," *Journal of Economic Literature*. 27, 1622-1668.
- [11] McClennen, E.F, 1988: "Dynamic choice and rationality," in B. Munier, ed., *Risk, Decision. and Rationality*, Reider, Dordrecht, Holland.
- [12] McClennen, E.F., 1990: *Rationality and Dynamic Choice*. Cambridge: Cambridge University Press.
- [13] Segal, U., 1996: "Dynamic consistency and reference points," *Journal of Economic Theory*, forthcoming.

[14] Wakker, P., 1995: "Not only foregone outcomes are relevant for updating in resolute choice, but also forgone decisions." mimeo.